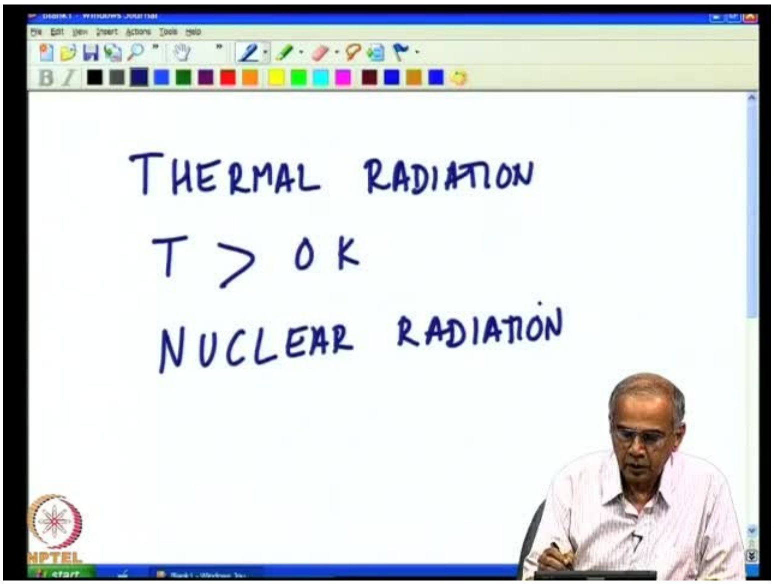
Radiation Heat Transfer

Prof. J. Srinivasan
Centre for Atmospheric and Oceanic Sciences
Indian Institute of Science, Bangalore

Lecture -01
Introduction

(Refer Slide Time: 00:21)



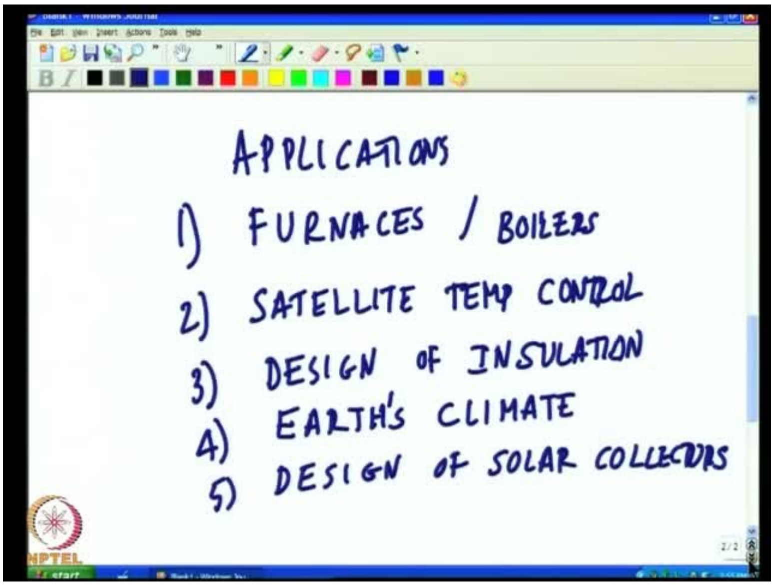
THERMAL RADIATION

$T > 0\,K$

NUCLEAR RADIATION

This course is on Thermal Radiation Heat Transfer. Thermal Radiation means radiation emitted by surface or gas, because its temperature is greater than ^{0}K. Thermal Radiation is different from nuclear radiation, the latter occurring due to reactivity on account of change in mass of the substance. Thermal radiation has many applications.

(Refer Slide Time: 01:15)



APPLICATIONS

1) FURNACES / BOILERS
2) SATELLITE TEMP CONTROL
3) DESIGN OF INSULATION
4) EARTH'S CLIMATE
5) DESIGN OF SOLAR COLLECTORS

The first application is in the area of furnaces, and boilers. These have very high temperature, hence radiation transfer plays an important role in the design of these furnaces.

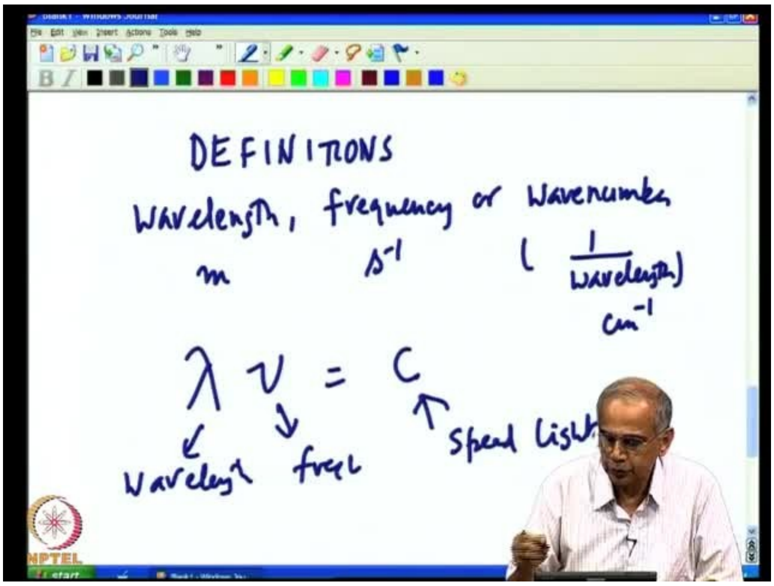
The second important application is in satellite's thermal control systems. On a satellite all the heat transfer must be done through radiation. The need to understand what kind of materials must be used in order to reduce or increase absorption or to reduce increase reflection in satellites, are of importance.

The third application of thermal radiation is in the design of insulation. It should be designed in such a way that all conduction, convection and radiation must be reduced.

The fourth important area of application is in predicting Earth’s climate. Earth’s climate is controlled by radiation primarily because Earth receives radiation from the sun, which it is partially absorbed and partially reflected. The Earth emits radiation from the surface of the atmosphere into space. The balance between the absorbed solar radiation, and the emitted solar radiation controls the Earth’s main temperature. The Earth’s temperature has gone up by around 0.7 ^{0}C due to Co_2.

The fifth application is in the design of solar thermal collector such solar water heaters. The radiated heat transfer process that occurs in these devices needs to be understood before designing it.

(Refer Slide Time: 05:36)



DEFINITIONS

wavelength, frequency or wavenumber

m s^{-1} ($\frac{1}{\text{wavelength}}$)

cm^{-1}

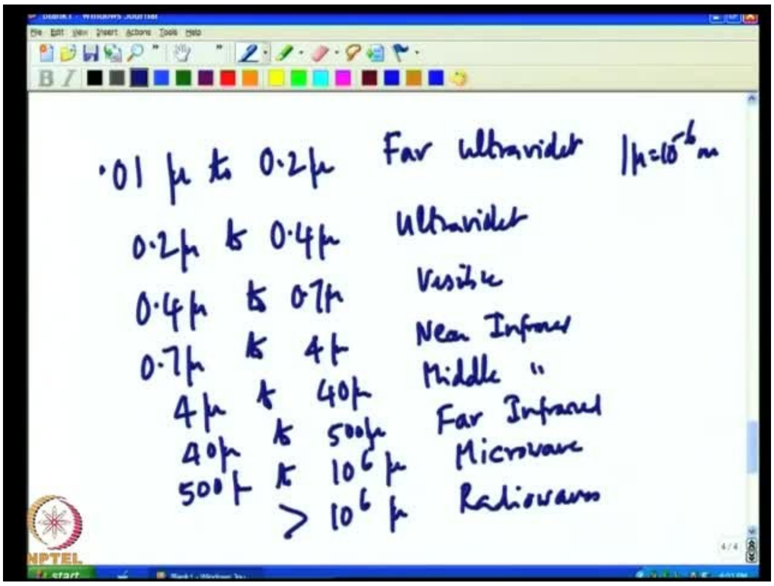
$\lambda \nu = c$

wavelength freq speed light

The first thing we need to know is the wavelength (λ) range required for radiation heat transfer to occur. Radiation is defined in terms of its wavelength which is measured in meters, or in micrometers (microns). Frequency is measured in per second or Hertz (Hz). Wave number which is the reciprocal of a wavelength is measured in centimeters (cm^{-1}). All the above terms are used interchangeably to measure radiation.

As light travels from one medium to another its speed changes. The wavelength of the radiation will also change, but not the frequency. From the point of analysis and theory, frequency is preferred to describe radiation because it is invariant. On the other hand, to make observations of radiation, wavelength is easier to measure. In certain areas wave number is used to measure radiation. Hence wavelength, frequency and wave number are all interchangeable and used to measure the radiation. We can also convert one measurement to another very easily. In radiation different wavelengths are characterized by different names for convenience.

(Refer Slide Time: 08:37)



$\cdot 01\ \mu$ to $0\cdot 2\mu$ Far ultraviolet $1\mu = 10^{-6}$ m

$0\cdot 2\mu$ to $0\cdot 4\mu$ Ultraviolet

$0\cdot 4\mu$ to $0\cdot 7\mu$ Visible

$0\cdot 7\mu$ to 4μ Near Infrared

4μ to 40μ Middle "

40μ to 500μ Far Infrared

500μ to $10^6\ \mu$ Microwave

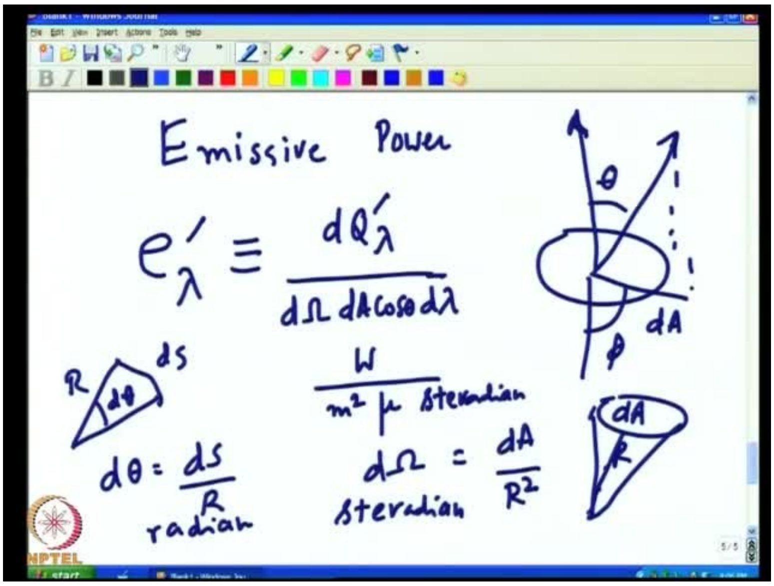
$> 10^6\ \mu$ Radiowaves

For example: The wavelength range from 0.01 micron to 0.2 micron is called the far Ultra Violet (UV), whereas 1 micron is 10^{-6} meters. The wavelength range from 0.2 micron to 0.4 micron is called the Ultra Violet (UV), and 0.4 to 0.7 micron is the visible radiation. The wavelength 0.7 micron to 4 micron it is called the near Infrared (IR). These are radiations emitted by the Sun which are visible. The wavelength from 4 micron to 40 micron is called the middle Infrared (IR). These radiations are emissions by the surfaces around us at room temperature.

As we go from 40 micron to 500 micron, it is called the far Infrared (IR) and usually objects at very low temperature can emit lot of radiation, in this wave length range. The wavelengths from 500 micron to 10^6 micron are called the microwave. The radiation is emitted by objects at very low temperatures. Our mobile phones use microwave frequencies. Wavelengths above 10^6 micron come in the domain of radio waves.

The above mentioned are the wide range of possible wave lengths, covered by thermal radiation. Our primary focus will be on the visible and the infrared radiation, deliberating on emission by temperature, emission by objects at room temperature and radiation emitted by the sun.

(Refer Slide Time: 11:39)



Emissive Power

$$e'_\lambda \equiv \frac{dQ'_\lambda}{d\Omega\, dA\cos\theta\, d\lambda}$$

$$\frac{W}{m^2\,\mu\ \text{Steradian}}$$

$$d\theta = \frac{ds}{R} \quad \text{radian}$$

$$d\Omega = \frac{dA}{R^2} \quad \text{Steradian}$$

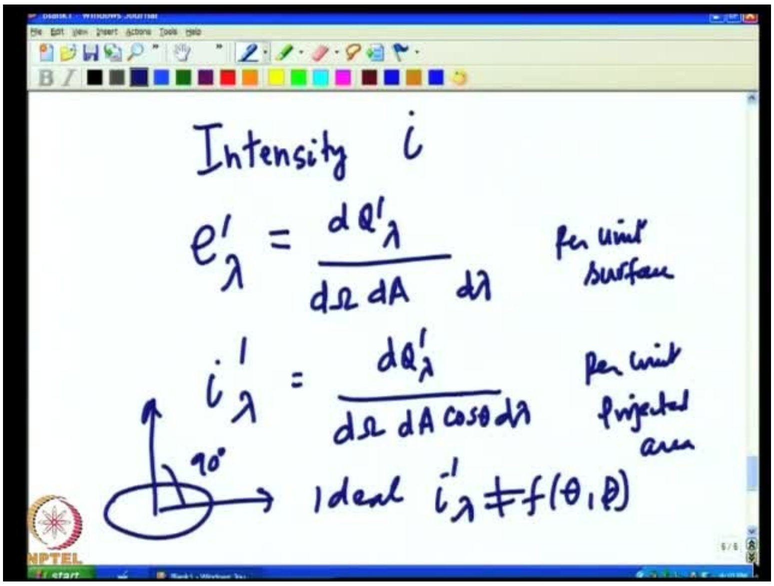
Emissive power is term use to characterize the energy density of the radiation. For, any quantity, radiation has to be defined initially, in terms of a given direction and given wave length. This is because radiation is emitted in all directions by any object. For example when we calculate radiation from the surface, the factor dA in the above shown equation (Refer Slide Time: 11:39), variation can deviate in any direction (angle θ) from the normal. We also characterize the ray by another angle called phi (φ). The ray is characterized by angle θ and φ, hence we define every quantity in radiation in terms of these angles and every wavelength (λ).

Let us use the symbol 'ẻ$_\lambda$' for emissive power. It is defined as the radiation emitted by object in a given direction and a given wave length, divided by unit solid angle, which is the area projected normal to that ray per 'd' unit wave length. The unit of this quantity, is Watts per m^2 per micron (μ) per steradian. This is radiation emitted per unit area per unit wave length and per unit solid angle.

Solid angle is a new concept. We now extend 2 dimensional solid angle to a 3 dimensional solid angle. The 3 dimensional solid angle dΩ (omega), is the area (dA) by R^2. This is similar to 1 dimensional equation. It can be easily shown from early geometry that dS by R (Radiation) is dθ, but in the case of solid angle the 3 dimensional presentation would be dA by R^2 as dΩ also called as Steradian.

In 3 dimension phenomena, the radiation occurs in all directions. Another quantity that comes into picture is intensity.

(Refer Slide Time: 16:31)



Intensity i

$$e'_\lambda = \frac{dQ'_\lambda}{d\Omega\, dA\, d\lambda}$$ per unit surface

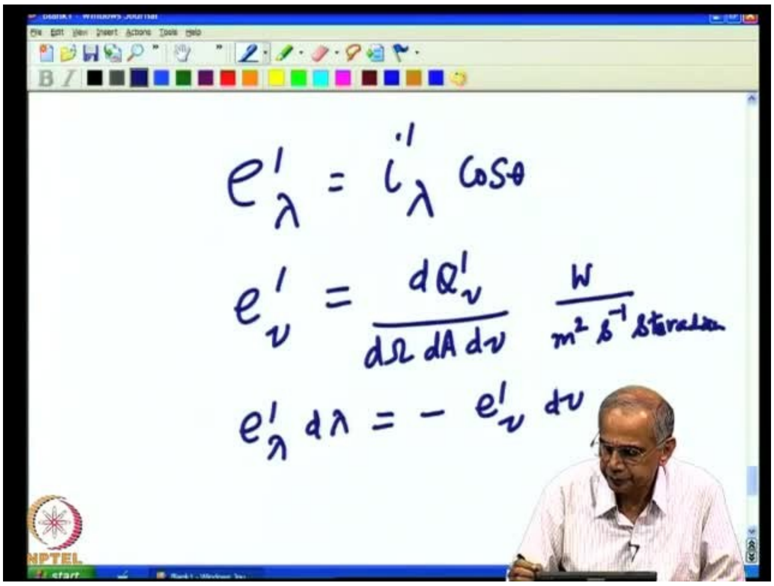
$$i'_\lambda = \frac{dQ'_\lambda}{d\Omega\, dA \cos\theta\, d\lambda}$$ per unit projected area

Ideal $i'_\lambda \neq f(\theta, \phi)$

Intensity is a concept different from emissive power. We use the symbol 'i' for intensity. If you recall, we define emissive power ($\dot{e}_\lambda$) as dQ'_λ divided by $d\Omega$, dA and $d\lambda$. i'_λ is defined similar to emission power equation, but per unit surface area ($\cos\theta$ term is also included).

Intensity needs to be defined per unit surface area and per unit projected area. On many surfaces, the emissive power in any direction (that is when θ is close to 90), will be very small. Emissive power shows a strong angular dependence. The highest value is attained when θ is 0. As the angle increases, the emissive power will decrease and approach 0 as it reaches 90. As intensity was dependent on angle another quantity per unit projected area was defined which was independent of angle. So, intensity was defined in a such that for an ideal surface, i'_λ would not be a function of angle θ and φ. Real surfaces will show some dependence, but ideal surfaces will have some dependence on emissive power with angle.

(Refer Slide Time: 20:40)



$$e'_\lambda = i'_\lambda \cos\theta$$

$$e'_\nu = \frac{dQ'_\nu}{d\Omega \, dA \, d\nu} \qquad \frac{W}{m^2 \, s^{-1} \, \text{Steradian}}$$

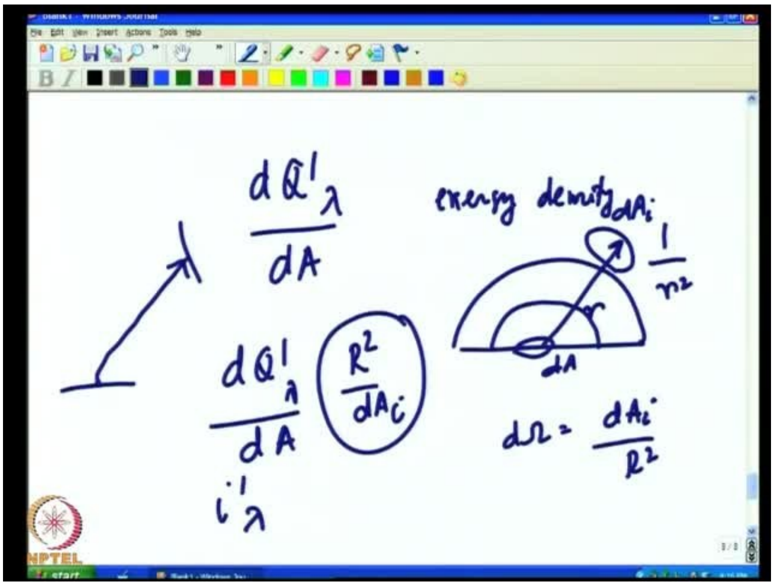
$$e'_\lambda \, d\lambda = - e'_\nu \, d\nu$$

Now, let us see the connection between i'_λ and ẻ$_\lambda$. We see that even if i'_λ was not a function of angle θ and φ, ẻ$_\lambda$ is a function of angle, and it will depend on $\cos\theta$.

All though we have define these quantities in terms of per unit wave length (λ), we can also define the above quantities in terms of per unit frequency (Hz). The frequency ẻ$_\mu$ is defined as dQ'_μ by $d\Omega$, dA and $d\mu$. This will be in Watts per m^2 per unit frequency per unit solid angle.

Hence ẻ$_\mu$ and ẻ$_\lambda$ both have different units. Also they will have different numerical values, but the product of both equations must be equal, the latter having a minus sign This is because when wave length increases the frequency has to go down.

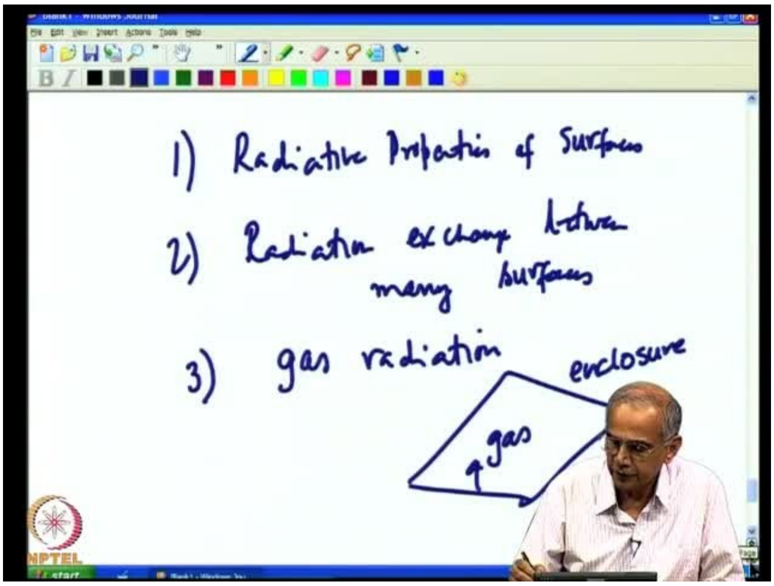
(Refer Slide Time: 23:04)



When we look at the quantity dQ' $_\lambda$, dA which is the energy density of power per unit area, we realize that as we move away from the surface, this will go down as 1 by r^2 where r is the distance from the surface. We define a new quantity that does not vary as we move away from the surface for a longer distance. We call this quantity as dΩ.

The concept of solid angle defined using a quantity like intensity, will be invariant in space if there is no absorption or scattering of the photon remitted by the surface. In gas radiation, when gases are there in the medium, which observe or scatter radiation, we find that this quantity will start to decline. but if there is only vacuum between one surface and another, then radiation intensively will remain invariant until it reaches the other surface. In other words the intensity will remain invariant in space if photons are not absorbed or scattered in the intervening medium. This is facilitated by the introduction of the concept 'solid angle'.

(Refer Slide Time: 26:42)



1) Radiative Properties of Surfaces

2) Radiation exchange between many surfaces

3) gas radiation

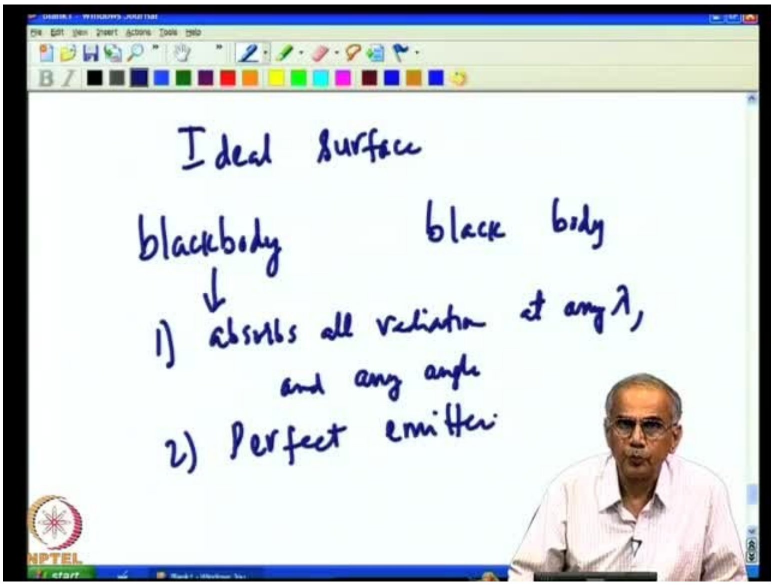
Radiative properties of surfaces.

In order to characterize radiation emitted by surface, we need to know the properties of the surface, by various definitions. Then we look at radiation exchange between many surfaces. We would like to know how radiation leading a surface, what fraction reaches another surface and how to calculate that fraction. This is purely a problem in 3 D geometry solid state forward, but there are various ways of doing this calculation.

Once we have understood how to calculate the radiation emitted by real surface and then we have to understand how radiation exchange occurs in surfaces. We now take a look at a situation, where there are surfaces and there is a gas in between the surfaces. We need to estimate how much radiation is leaving a surface. Then we try to calculate how the gas which is now absorbing or scattering. will modify this radiation and finally, understand how much radiation reach the other surface.

This can be calculated in an enclosure containing surfaces, and gases. It is very relevant in the design of furnaces and boilers because furnaces and boiler have an enclosed surface subjected to various temperatures, and gas inside it absorbs and scatters radiation.

(Refer Slide Time: 29:26)



Ideal surface

blackbody black body

↓

1) absorbs all radiation at any λ,
and any angle

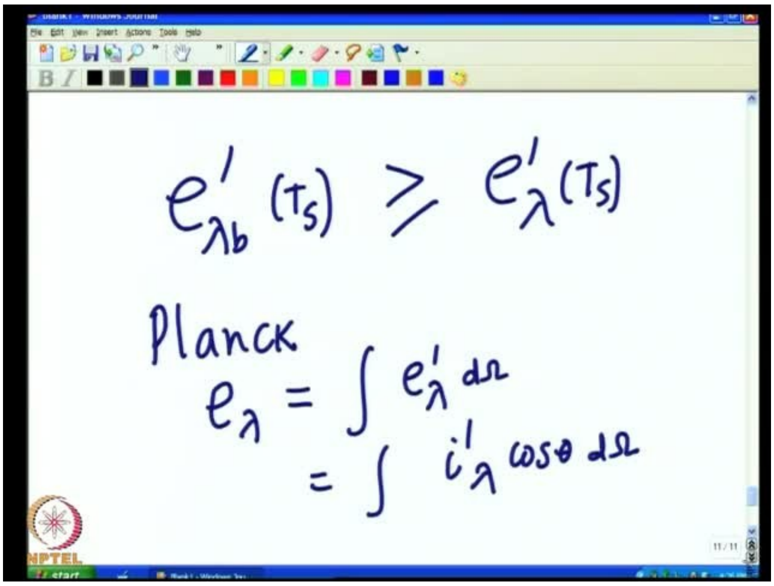
2) Perfect emitter

We first take the concept to define an ideal surface. The concept of the Carnot cycle in thermo dynamics, is an ideal cycle, consisting of reverse root processes.

We can now build Carnot cycle in the real word. The concept of an ideal cycle is very useful in thermo dynamics because we can compare the performance of all real cycles with an ideal cycle. Similarly, in radiation heat transfer we want to compare the properties of real surfaces with that of an ideal surface. This ideal surface is called the blackbody. This ideal surface is one in which can be used to send radiation at any wave length at any angle.

For example you take a typical black color object. It absorbs most of the radiation that is visible, but reflects a small amount of radiation. This body absorbs all radiation that falls on it, at all angles and wavelength. It is also a perfect emitter. By perfect emitter we mean, that when we take emissive power of this blackbody at a given wave length, given angle and a given temperature, it will be always be greater than or equal to emissions by any real surface at the same given temperature, angle and wavelength. It is able to achieve that because it absorbs perfectly and also emits radiation better than any other object.

(Refer Slide Time: 33:09)



$$e'_{\lambda b}(T_s) \geq e'_{\lambda}(T_s)$$

Planck

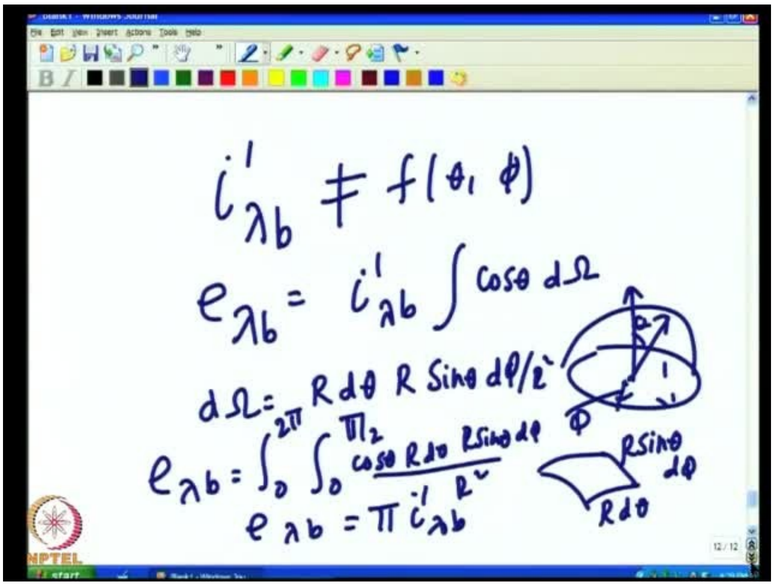
$$e_{\lambda} = \int e'_{\lambda} \, d\Omega$$
$$= \int i'_{\lambda} \cos\theta \, d\Omega$$

People have tried to measure, the variation of emissive power with wave length of a blackbody and have developed an analytic expression for the radiation emitted by a blackbody.

Planck derived theoretical expression for the emissive power of a blackbody. This was the first concept in quantum mechanics, that there is quantum nature of matter. Planck was interested more in the hemispherical quantity. The hemispherical quantity is nothing but averaging of the spectral directional quantity overall solid angle, to integrate the directions spectral emissive power of a surface over all angles, then you get the emissive spectral quantity.

From the definition of this quantity we can also derive i'_{λ} cosθ and dΩ. Since a blackbody absorbs and emits radiation equally well in all direction, the directional spectral intensity of a blackbody is not a function of angle θ and φ. Since intensity is independent of angle θ and φ one can easily calculate ẻ$_{\lambda}$ as i'_{λ}.

(Refer Slide Time: 36:55)



$$i'_{\lambda b} \neq f(\theta, \phi)$$

$$e_{\lambda b} = i'_{\lambda b} \int \cos\theta \, d\Omega$$

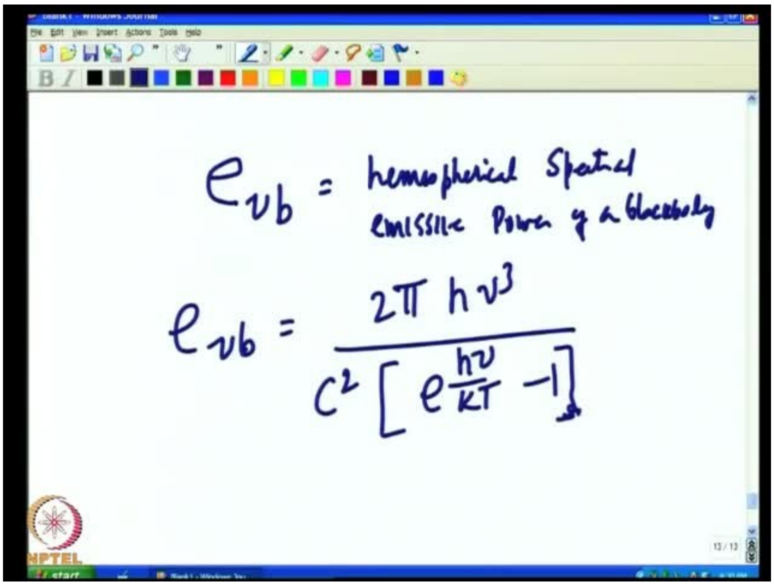
$$d\Omega = R\,d\theta \; R\sin\theta \, d\phi / R^2$$

$$e_{\lambda b} = \int_0^{2\pi} \int_0^{\pi/2} \frac{\cos\theta \, R\,d\theta \; R\sin\theta \, d\phi}{R^2}$$

$$e_{\lambda b} = \pi \, i'_{\lambda b}$$

We have to integrate costθ and dΩ. Now, dΩ is the solid angle and you can work on the Rθ coordinate on a hemisphere. Thus dΩ can be rewritten as the product of R dθ R sinθ dφ by R^2. When we integrate θ from 0 to ɲ/2, and integrate φ azimuth angle from 0 to 2ɲ, ultimately we will get an expression which says $e_{\lambda b}$ is equal to ɲ times $i'_{\lambda b}$. This derivation is hemispherical spectral emissive power.

(Refer Slide Time: 39:26)

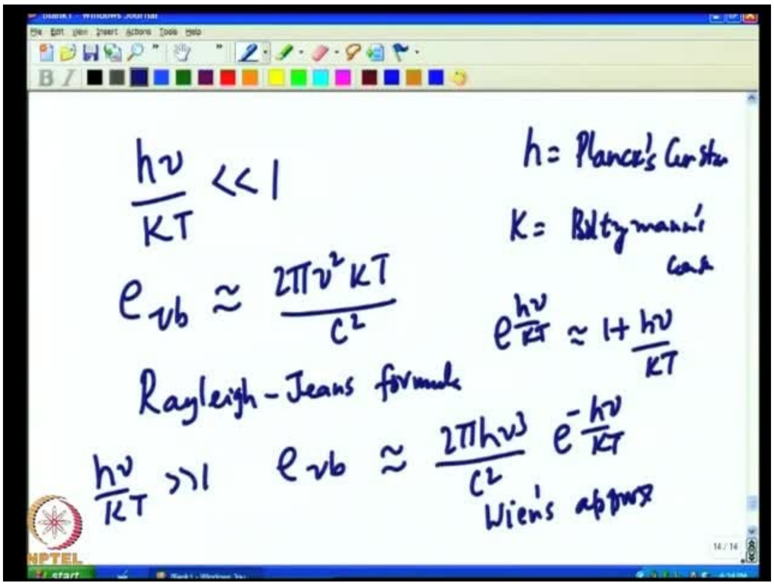


$e_{\nu b}$ = hemispherical Spectral emissive Power of a blackbody

$$e_{\nu b} = \frac{2\pi h \nu^3}{c^2 \left[e^{\frac{h\nu}{kT}} - 1 \right]}$$

Next Plank derived an expression for the frequency $e_{\mu b}$, which is the hemispherical spectral, emissive power of a blackbody per unit frequency. Planck showed that this quantity is the product of 2ɲ h u^3 divided by c^2 e into (hμ by kT) minus 1. Planck's variation method demands some basics in statistical mechanics, and quantum mechanics. We will use Einstein's methodology instead.

(Refer Slide Time: 40:55)



$$\frac{h\nu}{KT} << 1$$

h = Planck's Const

K = Boltzmann's Const

$$e_{\nu b} \approx \frac{2\pi\nu^2 KT}{c^2}$$

$$e^{\frac{h\nu}{KT}} \approx 1 + \frac{h\nu}{KT}$$

Rayleigh-Jeans formula

$$\frac{h\nu}{KT} >> 1 \quad e_{\nu b} \approx \frac{2\pi h\nu^3}{c^2} e^{-\frac{h\nu}{KT}}$$

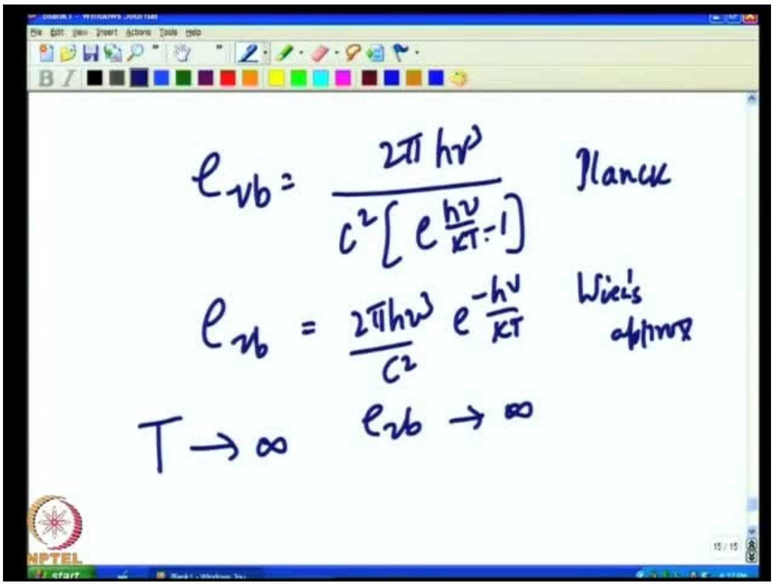
Wien's approx

In the equation quantity hµ/kT, 'h' is a Planck constant and 'k' is a Boltzmann’s constant which was known earlier. If this quantity is much less than 1, the expression for blackbody can be approximated as 1 + hµ/kT.

We can see that in this result, 'h' is playing no role, because in this limit 'h' approaches a classical limit, where quantum nature of mater is not manifested. This is called Rayleigh-Jeans limit. The Rayleigh-Jeans limit which was derived earlier, based on classical electromagnetic theory, is not accurate at high frequencies. This is good only for low frequencies, as high frequencies does not agree with observations.

When we look at high frequencies where hµ/kT is much greater than 1, we can derive another approximation by neglecting the denominator (that is denominator =1). This is called the Wien’s approximation. This expression was derived by Wien before Planck obtained his result. Interestingly this Wien’s approximation agrees with the approximation, given by Planck.

(Refer Slide Time: 43:57)



$$e_{\nu b} = \frac{2\pi h\nu^3}{c^2\left[e^{\frac{h\nu}{KT}}-1\right]}$$ Planck

$$e_{\nu b} = \frac{2\pi h\nu^3}{c^2} e^{-\frac{h\nu}{KT}}$$ Wien's approx

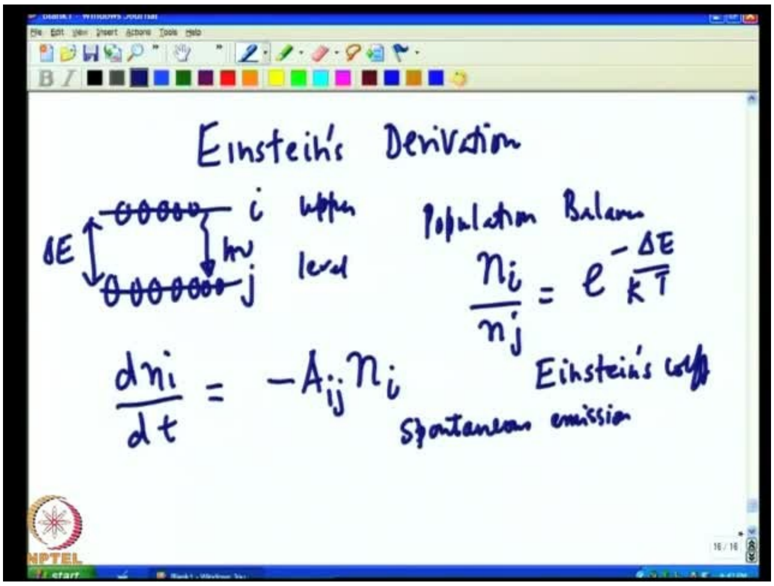
$T \to \infty$ $e_{\nu b} \to \infty$

Let us compare the Planck’s formula with Wien’s formula. When the denominator goes to 0 in Wien's formula, then the quantity $e_{\mu b}$ goes to infinity. Planck insisted that when temperature (T) goes infinity, $e_{\mu b}$ should go to infinity. This is logical because as temperature increasingly absorbs radiation, emissive power must also go on increasing.

Only Planck’s formula could meet the above requirement, which is the major contribution that Planck made in his proposal for quantum nature of matter. He recognized that this system 1 has to be there in order to ensure that temperature goes to infinity. The hemispherical spectral emissive blackbody also goes to infinity. Although Beams expression is quite accurate in most conditions, it did not consider the requirement in the condition of T going to infinity.

Planck’s derivation played a major contribution to recognize the importance of the limit T tending to infinity, but there was some uncertainty. A more meaningful derivation that provided insight about the role of that minus 1 was made by Einstein later.

(Refer Slide Time: 47:08)



Einstein's Derivation

i upper level

j

δE

hν

Population Balance

$$\frac{n_i}{n_j} = e^{-\frac{\Delta E}{kT}}$$

$$\frac{dn_i}{dt} = -A_{ij} n_i$$

Einstein's coeff

Spontaneous emission

Einstein's derivation of the blackbody formula, provides more insight into the nature of emission. The simple idea of Einstein was, that he looked at a two layer system with an upper level and a lower level, we can think of these as two energy levels of a molecule. He did essentially, what is known as population balance. As we know that many molecules will remain at the lower level, and a few molecules will remain the upper level.

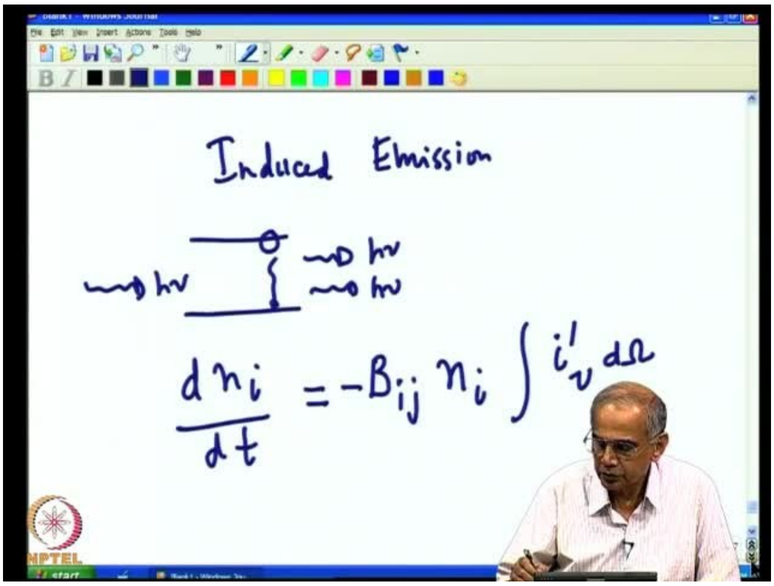
The energy gap between these two levels is delta E (δE). Under equilibrium, a number of molecules at the upper level, n_i to the number of molecules at the lower level n_j has to be of certain form. The radiation must go down as the number of molecules go from lower, to upper level of any. This kind of distribution of molecules at variation occurs at equilibrium.

If there is sufficient number of collations between molecules to transfer energy between molecules then this kind of state will be attained. But there can be special situations where, if there are insufficient collations or insufficient time the distribution made of the energy may depart from the surface. In such cases we cannot derive simple expressions for radiation. Hence we assume that equilibrium is already present, and that the number of molecules in the upper level are always lesser than that the lower level.

Now, let us ask ourselves a question as to what controls the number of molecules at the upper level. The first thing is when molecules are left by itself, the molecule will come down and spontaneously emit radiation, as it comes down. When this happens the number of molecules at the upper level will go down, it will be minus depending on how many molecules are there in the upper level. Due to the emission, the number of molecule at upper level will go down proportionately to the number of molecules in the upper level.

This proportionality constant is called A_{ij} which is the two level system. The molecules going from upper level 'i' lower level 'j', will reduce the number of molecules in the upper level to a certain rate. This rate is determined by the constant A_{ij}, also called as Einstein's co-efficient because Einstein proposed it first. This process is called spontaneous emission. Spontaneous emission, is the emission of photon by a molecule because one other molecule in the upper exited state, comes down to lower state and realizes a photon. This occurs spontaneously without any trigger or any other stimulation.

(Refer Slide Time: 52:11)



Induced Emission

$$\frac{dn_i}{dt} = -B_{ij}\, n_i \int i'_\nu \, d\Omega$$

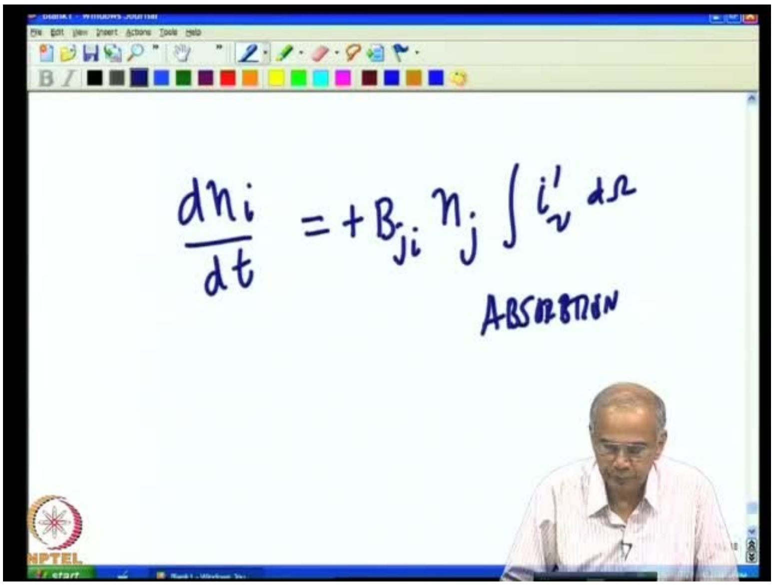
There can also be another form of emission which is called induced emission. This concept was not known earlier when Einstein proposed it. Einstein's concept was that a photon comes along and stimulates the system to drop one of the molecules from upper level to lower level, and allows the photons to go out. Here one photon strikes the substance and triggers another molecule to come down, which emits another photon.

Einstein's hypothesis was that, there will be decrease in the number of molecules in the upper level due to individual emission. The proportion of the number of molecules at the upper level is proportional to the photons coming in, which is recommended by the directions spectral intensity, integrating all solid angles. All the photons come in all directions and strike the substance. They induce photons to be emitted which is proportional to both the number of photons coming in and the number of photons present at the upper level.

There will be the minus sign because the number of molecules is going down, and this term is called constant of proportionality which is another Einstein's constant called B_{ij}. We now have defined two kinds of emissions, the spontaneous emission which occurs on its own with no trigger from any photon, and the induced emission which occurs because of photons striking objects and triggering another photon which is emitted by the object.

The main difference between the induced emission and spontaneous emission are, in the case of induced emission, the number of molecules coming down, depends on the number of photons striking the object and the number of molecules in the upper level. In the case of spontaneous emission, the second term here is not present, that is the i'_μ dΩ, which is a number of photons that are there, is not present in the formula.

(Refer Slide Time: 55:30)



$$\frac{dn_i}{dt} = +B_{ji}\, n_j \int i'_\nu \, d\Omega$$

ABSORBTION

Finally there is absorption, because due to absorption, the number of molecules is upper level goes up. This is because of photon is absorbed by the object. This absorbed proportion to the number of molecules at the lower level n_j is equal to the number of photons striking the object. The number of molecules in the upper level is going up, and the constant of proportionality is called B_{ji}.

The expression dn_i by dt is related to spontaneous emission. B_{ji} is related to induced emission, and n_j i'_μ dΩ is related to absorption. In the next class, we will look at a balance between these three phenomena, and arrive at an equilibrium situation where all the three are in balance. In this lecture, we have covered the basics of radiation transfer, defined quantities like intensity, emissive power, and how they are related. We looked at the Planck's blackbody radiation law, which is the basic law. We are in the middle of derivation of Planck's blackbody radiation formula through a method propose by Einstein. We will complete this in a next course of lecture.