

Introduction to Atmospheric Science
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Lecture - 39
Climate Sensitivity and Feedback

Okay. So good morning so we will continue with our study of climate sensitivity and climate dynamics okay.

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Climate Sensitivity

$$F_s = \sigma T_E^4 \quad (1)$$
$$T_E = \left[\frac{F_s}{\sigma} \right]^{1/4} \quad (2)$$
$$\ln T_E = \frac{1}{4} \ln F_s - \frac{1}{4} \ln \sigma \quad (3)$$

Taking "ln" both sides

$$\frac{dT_E}{T_E} = \frac{1}{4} \frac{dF_s}{F_s} \quad (4)$$

Correct, what is that? What is the problem we considered in the last class? Problem number 50 was if the solar flux reaching the earth $F_s = \sigma T_E^4$ where T is equilibrium temperature considering the earth to be a black body that turns out to be 255 kelvin therefore what do you get? $1/T_E$ or dT_E/T_E ?

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$$\therefore \frac{dT_e}{dF_s} = \frac{1}{4} \frac{T_e}{F_s} = \frac{\partial T_s}{\partial F}$$

↓
Climate sensitivity in the
absence of any
feedback

Is this correct? But we know that this is also equal to $\frac{dT_s}{dF}$. So if you look at this, there is a very fine distinction, T_e is the equilibrium temperature of the earth, T_s is the global mean surface air temperature okay but the rate of change of the global surface air temperature with respect to F that the partial d of T_s with respect to F will be the same as dT_e/dF_s in the case I mean there is no feedback from other things.

So basically the temperature as such is a consequence of the solar radiation coming in and the emission that is the basic assumption. Then, like pizza you add on, toppings are there, those toppings are water vapour feedback, carbon dioxide feedback, base is something is coming otherwise earth will not get heated at all. Something is coming and something is emitted back that equilibrium causes some basic temperature.

Now we are trying to see what is the sensitivity with respect to change in some of these forcing. If that F_s itself changes, what happens that is are you getting the point? Instead of the 239 watts per meter square it becomes 240 watts per meter square what will happen okay. So everything is benchmark with respect to the change in that F_s so that is how we define the radiative forcing okay.

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$$\frac{\partial T_s}{\partial F} = \frac{1}{4} \times \frac{255}{239.4}$$

$$\frac{\partial T_s}{\partial F} = 0.266 \frac{K}{\left(\frac{W}{m^2}\right)}$$

So how much was this? 255, so the sensitivity has got the units of kelvin/watts per meter square.

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Taking the reciprocal of this,
We have

$$\frac{1}{0.266} = 3.76 \frac{\left(\frac{W}{m^2}\right)}{K}$$

It looks like units of heat transfer coefficient watts per meter square per kelvin correct. Mechanical engineers will it will ring a bell with units of h okay. Do not get confused, it is far away from the h, it is nowhere related okay. Let us switch at some point, I made some CRC complex kind of definition in the last class, so the earth's equivalent black body temperature rises by 1 kelvin for every 3.76 watts per meter square per kelvin of downward radiative forcing incident on the earth from the top of the atmosphere okay.

Now all these climate change experts are trying to calculate what is the watts per meter square okay or I will just begin okay.

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The image shows a chalkboard with handwritten mathematical work. The top part shows a fraction $\frac{1}{0.266} = 3.76$ followed by a unit $\left(\frac{W}{m^2}\right)$ over a unit K . Below this, it shows $1K \Rightarrow 3.76 \frac{W}{m^2}$.

So we can leave it like this also 1 kelvin requires 3.76 watts per meter square. So all these climate change and climate science experts are trying to figure out if the albedo reflectivity changes how many watts per meter square? If carbon dioxide changes how many watts per meter square? Water vapour changes how many watts per meter square? So you add all these and these forcings are additive.

You calculate the forcing separately, for calculating the carbon dioxide forcing you have to solve the radiative transfer equation. Are you getting the point? If the carbon dioxide changes, how will the forcing change? That is not so easy to obtain. So what do you do is you will solve the equation of radiative transfer in the atmosphere with 390 ppm and get one value.

Then with 395 ppm or 400 ppm will get a value and then find out how this changes with respect to so I will tell you later on that there is a relationship called \ln of C/C_0 *something that is how it change with respective carbon dioxide. So you will have to do or you have to take control there, you have to do ice core measurements, you have to go deep into Vladivostok where somewhere north high up right.

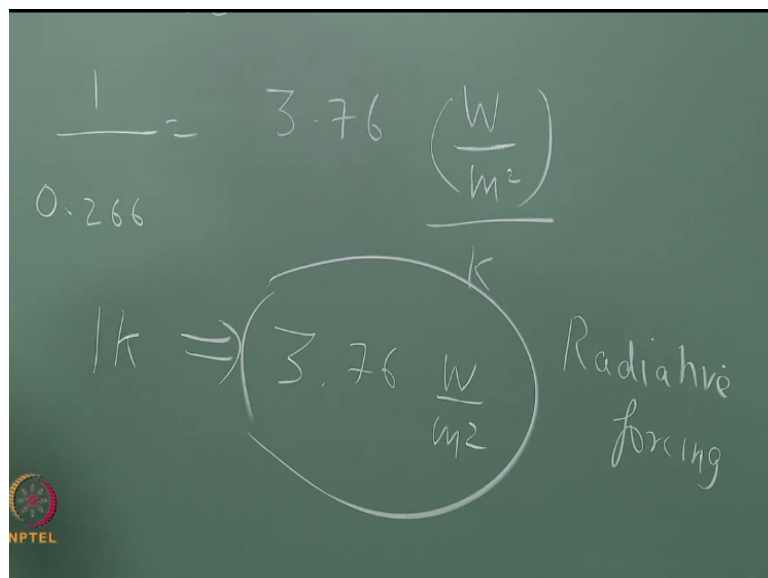
You go to Vladivostok and then drill whole kilometers into the snow then get the ice core, in that ice core air will be trapped. Then, there is a dating formula, will find out for that depth what will be the time period? That time period that air will contain carbon dioxide, find out what is the carbon dioxide concentration in the atmosphere at that point in time. Then, the

funda that carbon dioxide concentration is universal, then you apply that whatever is in Vladivostok or whatever is the same in Velachery okay.

Then you reconstruct the carbon dioxide concentration though you do not have direct measurements before 1945 when some Keeling set up that biggest laboratory to measure carbon dioxide in Hawaii. Is it okay? That is how, these are all (()) (09:09) scientifically established, it is not like alchemy or somebody's astrology or something, it is not like that. So this carbon dioxide concentration people are because normally when you say how are you confident? How are you confident?

People who do not understand know they will say how are you confident? How did people measure carbon dioxide? 100 years back where was the instrument? 200 years back where was the instrument? All that air is trapped know. Now I have instrument then Archeology or Paleoclimatology can never be studied. Man has found out ways of studying the past alright. So this is the very important this thing. So something like this is called radiative forcing.

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So 2 things, for 1 kelvin it is 3.76 watts per meter square and you are talking about radiative forcing not forcing by wind, forcing by convection. Radiation is very important for climate. Convection, winds and all these pressure distributions and all these orography, topography all these things are land breezy. Breeze is important for weather. What will happen in the evening? What will happen in the afternoon all that is weather?

But compared to 20th century what happens, 21st century, 22nd century is a bigger picture, is a bigger problem. So here radiation has to be taken into account okay now this is fine. Let us get back. Though we solved the problem in the last class I think now you understood it better right okay. Are you done? I think I went all over the place, you were not able to be seen, Bharath Dharshan, you figure out, you ask him.

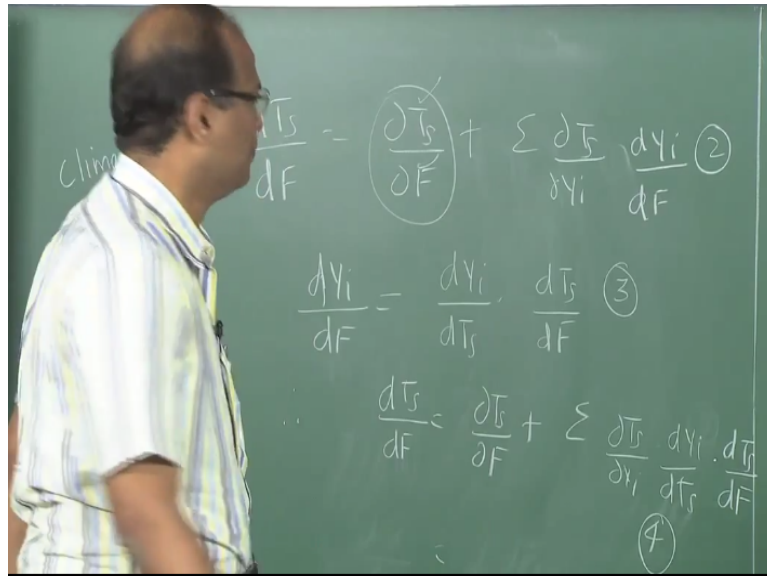
You do Delhi and Bombay dharshan if you are on a flight because normally it is very difficult to get ATC clearance, you will keep on circling Bombay right, free, time is lost but alright okay. I promised to you I will teach at least 2 hours of atmospheric dynamics but I forgot to realize that there is a very important lecture on climate change, which is a fast paced PowerPoint presentation which I will have to deliver tomorrow.

So we are just left with only one lecture for the atmospheric dynamics. I will quickly give the governing equations and some simplification and tell you 2 approximations, the cyclone approximation and the geostrophic approximation. We will solve a simple problem and then close the course. That is the plan on Thursday. Tomorrow's plan is to bombard you with the PPT okay.

I will convert into PDF and send it to you. By tomorrow I will also send you the PDF which gives the schedule whatever from 9 o'clock to 1 o'clock. At 1 o'clock Sai Rahul will come I think 12:45 you are. This cloud seeding 2 people are coming, 2 groups so 10:15 and 10:30. So everybody has to be present, okay you just cannot afford to come only for your presentation unless you have some this exam that exam and so you are at your creative best.

So if it is a genuine reason it is okay. Why you are late to class? Brilliant expression of creativity 1, 2, 3, 4, 5, 6, it will just come like missiles okay (()) (12:58) how creative students can be alright.

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Now we are looking at so dT_s/dF , this is the lambda right, yeah what was the equation number for this? Was it 1? We wrote it in one of the earlier classes 2 okay. Now see why T_s is the global mean of this air temperature okay, F is the forcing flux. So this is dT_s/dF is how much the temperature will change for a given change in the forcing? That is the lambda, that is the climate sensitivity right.

So we are qualifying the whole of climate with just one quantity what is it? T_s and climate sensitivity by one more quantity which is lambda okay. Climate means global mean surface air temperature; sensitivity means with respect to forcing. This has to be understood, so this is the partial d is partial to a T_s to partial F is basically when there are no feedbacks plus you add the feedback but the feedbacks were including auxiliary variable.

These auxiliary variables are carbon dioxide, water vapour, clouds all those things okay. Now this dY_i/dF can be taken to be this okay. This is just some playing with calculus partial differentiation okay. Therefore, $dT_s/dF = \partial T_s/\partial F + \sum (\partial T_s/\partial Y_i) \cdot (dY_i/dT_s) \cdot (dT_s/dF)$ is too much okay alright okay.

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$$f_i = \frac{\partial T_s}{\partial Y_i} \frac{dY_i}{dT_s} \Rightarrow \text{Feedback factor} \quad (5)$$

$$f = \sum f_i \quad (6)$$

$$\frac{dT_s}{dF} = \frac{\partial T_s}{\partial F} + \frac{dT_s}{dF} \sum f_i \quad (7)$$

but $f = \sum f_i$

$$\frac{dT_s}{dF} [1-f] = \frac{\partial T_s}{\partial F} \quad (8)$$

This is called the feedback factor. It is qualified by subscript i where i can be 1, 2, 3, 4, 5; 1 could be water vapour, 2 could be snow cover, 3 could be cloud cover and so on okay. Now $f = \sum f_i$ therefore okay.

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$$\frac{dT_s}{dF} = \frac{\left(\frac{dT_s}{dF}\right)_{\lambda_0}}{[1-f]} \quad (9)$$

$$\lambda = g \rightarrow \text{Gain} \quad (10)$$

Provided $f < 1$

$$g = \frac{1}{(1-f)} \quad (11)$$

$f > 1 \Rightarrow$ Unstable equilibrium or infinite sensitivity.

So is that okay? So we are able to relate. We are able to calculate all the feedback factors, get the sum of the feedback factors and call that as F and find out 1-f and then do the simple $dT_s/d(F/1-f)$ you can find out the climate sensitivity. Of course this equation will cup if $f=1$. That is alright that is okay but now some equation number for this. So this is λ_0 , this is λ correct.

λ is the sensitivity of the climate with feedback, λ_0 is the sensitivity of the climate without feedback. Therefore, λ/λ_0 should be what conceptually? It is a

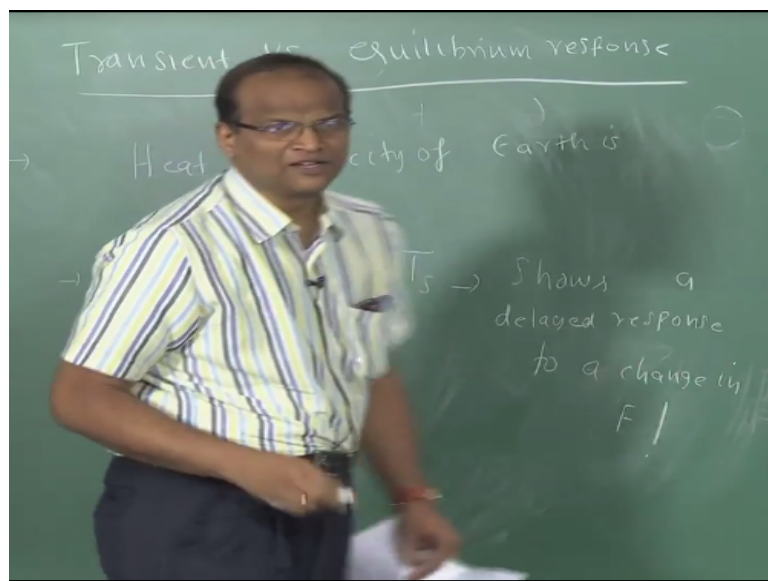
gain, it is g right. What is the climate sensitivity with feedback/what is the climate sensitivity without feedback. So now from equation 10 it is clear that g is $1/1-f$ okay provided f is <1 .

So f is ≥ 1 infinite sensitivity, unstable alright. So challenge is to be able to calculate this f , so you can use your zero order model, first order model, second order model and whatever. You can use your full sealed radiative transfer model and all these. If you do all these, calculate these feedbacks and give projections and you are a climate science or a climate change expert okay alright, so much about climate sensitivity okay.

But we have not studied one thing so far whether it is the atmosphere or the ocean, it has got the finer inertia or you just change the forcing by another 2 watts per meter square next second will the temperature change? So there is something called the transient response and the equilibrium response, even if you change that instantaneously solar load is changed instantaneously the next second the temperature will not become 255 because there is a mcp of the atmosphere.

Are you getting the point? There is an mcp of the ocean correct, there is a thermal inertia so we will have to look at this response. So then we have to solve initial value problem now. So the most simplified approach will be considering it to be a first order system okay. Let us consider a simplified first order system and then proceed okay. This will be transient versus equilibrium response.

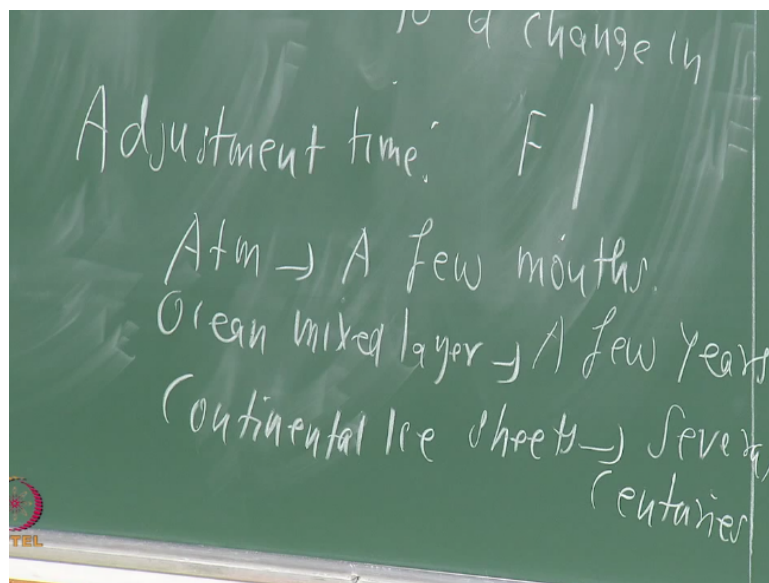
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What do you mean by this $f \gg 1$? Even small changes in the forcing can cause large changes in the temperature, which are the highly unstable equilibrium okay. I do not think we are in that situation now. As of now yes as of now we are not in that situation okay it does not happen tomorrow, Wednesday or before the exam okay. So transient heat capacity of earth is very large, hence T_s shows a delayed response to what? To a change in f .

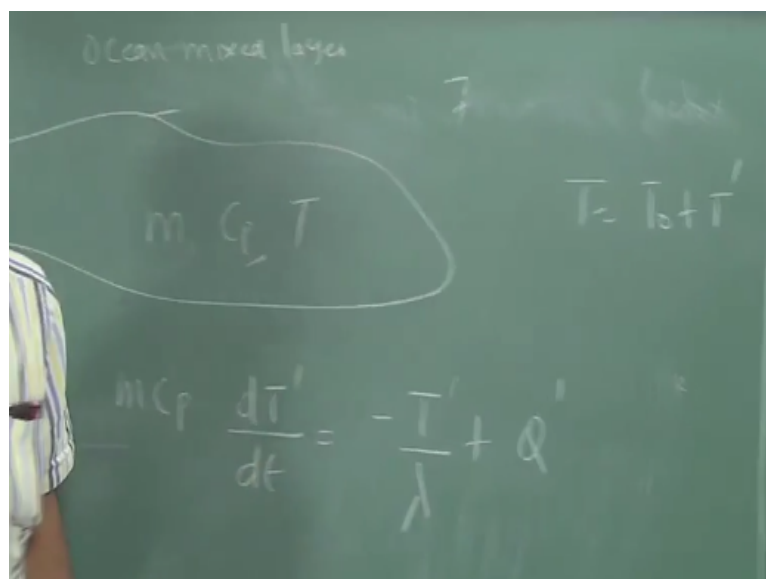
This is the first point you have to understand. So this is called the adjustment time okay.

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For the atmosphere it is a few months; for the ocean mixed layer take a guess? 71% it takes a few years, it does not mean that we can be irresponsible what we are saying is it would not happen overnight; continental ice sheet, it will take several centuries okay.

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Now we are going to consider an example. Will consider ocean mixed layer? okay. So I am not going to consider layer wise and then make it a big complicated problem. There is a m , there is a c_p , there is a temperature T okay. Your baseline temperature of T_0 and the temperature increase or fluctuation is the T' the change. Now we are going to write an equation for this so okay. Is this okay?

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Handwritten equation on a chalkboard: $T = T_0 + T'$. Below T' , there is a downward-pointing arrow leading to the text "Transient response to forcing".

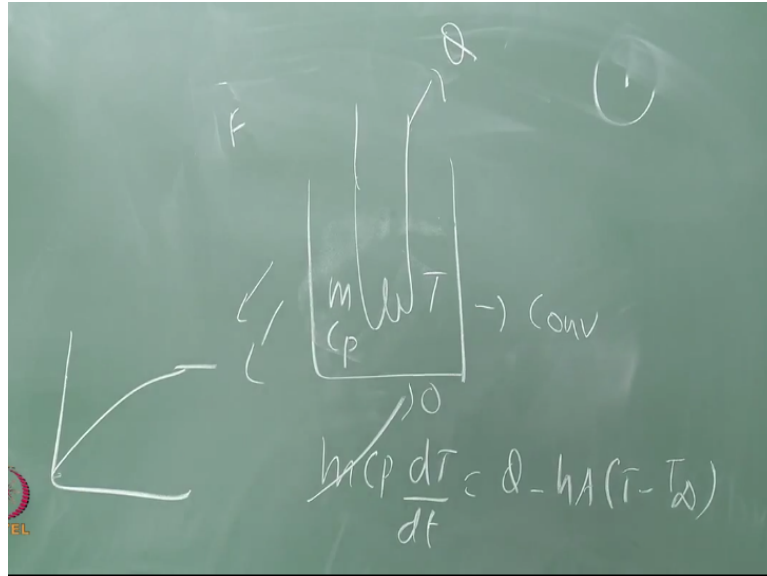
So T' is the transient response to forcing. Which is the forcing? Which is the force term? Q correct okay.

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Handwritten differential equation on a chalkboard: $m c_p \frac{dT'}{dt} = -\frac{T'}{\lambda} + Q'$. Below Q' , there is a downward-pointing arrow leading to the text "Forcing".

So if you are at a loss to understand what is going on okay. So you are not able to conceptually what is going on.

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You take a bucket of water; you are heating it in the hostel for example you want to take hot water bath. There is a dipper heater okay; it gives you a heating input of Q okay. So this has got the mass of m and c_p , this mass can be heater+wall+water and all that. Let everything will be at one temperature. Now it is also losing heat by convection because radiation also. Let us neglect that.

So the equilibrium response of this will be $m c_p \frac{dT}{dt} = Q - hA(T - T_0)$. As it starts getting heated, this temperature will increase therefore it will lose heat by convection but heat is continuously input. So the curve it will go like this and it reaches a steady state. This term will vanish, that will be the equilibrium temperature of the bucket. If it is more than 100 degrees, you are finished.

When will it become more than 100 degrees? You switch on without the water okay. Otherwise you can do your math and it will come to 80 or 85 degrees if you are using 1 kilowatt. I gave this problem in one of the courses right which one was that? You forgot which? Heat transfer okay, one of these courses. Now you are able to instead of the heater we have some change in carbon dioxide or something.

Instead of this water in the bucket we have water in the ocean okay alright. So you will be able to solve this? λ is whatever your gain okay.

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$$m_{cp} \frac{dT'}{dt} + \frac{T'}{\lambda} = Q'$$

lambda -> climate sensitivity

So we can bring the equation we can. Lambda is climate sensitivity okay. Yeah, please solve this. **“Professor - student conversation starts.”** What is that? Yeah because the forcing term will be positive this will be negative is not it? No, you solve it, we will see. Now please solve this equation. Let us do I will answer your question okay.

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lambda -> climate sensitivity

let $\tau = m_{cp} \lambda$.

$$\frac{dT'}{dt} + \frac{T'}{\tau} = \frac{Q'}{m_{cp}} \quad (3)$$

Now let tau be $m_{cp} \lambda$ therefore plus Q dash/what is the right hand side? Q dash/ m_{cp} . So please solve equation 3, it will consist of 2 parts that complimentary function and the particular integral. So the solution will answer your question. **“Professor - student conversation ends.”** So you can also have it as Q dash*lambda/tau, you can keep it that way also.

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Soln.

CF + PI

CF: $T - Ae^{-t/\tau}$

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PI: $Q \cdot \lambda$

$T = Ae^{-t/\tau} + Q \cdot \lambda$

Is this correct? Particular integral is? What is the particular integral for this? $Q \cdot \lambda$. Is it? Go figure out what is this particular integral? That particular integral if you take the derivative the first term will vanish and we substitute then left hand side will be equal to right hand side then it is a particular integral alright. Is it correct? Sneha I am believing what you are saying okay.

Can we proceed? Okay so T dash. How do you get A now? A is the constant and time $t=0$ what is T dash? If time $t=0$, T dash is 0 because it is equal to the initial temperature T_0 . What is T dash? That is why that $-T$ dash/this thing is coming. Now you got it. If there is no forcing left to itself this will decay, understood okay fine.

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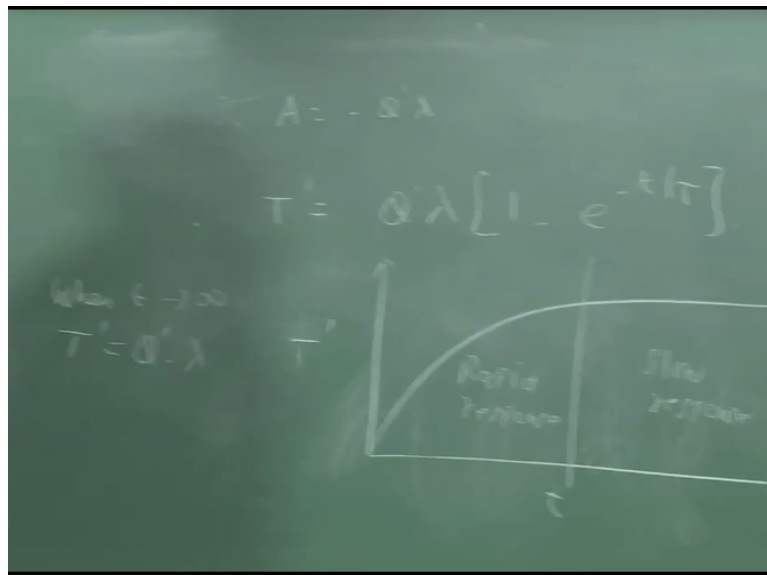
$$T' = A e^{-t/\tau} + Q'$$

$$\text{At } t=0, T'=0$$

$$\therefore 0 = A + Q'$$

So at $t=0$, $T' = 0$, $0 = A + Q' \lambda$.

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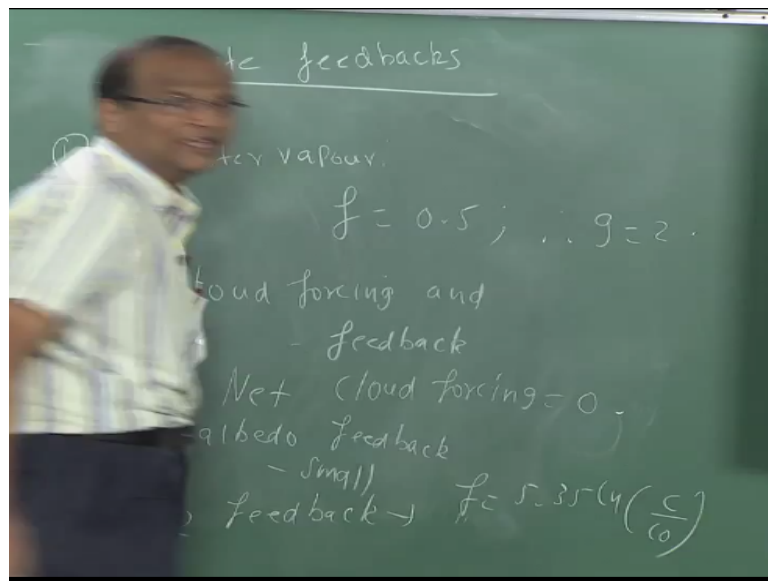
Therefore, $Q' \lambda$ okay. So there will be a rapid response initially, after some time depending on the time constant there will be a slow response. When t tends to infinity, we thought that if we apply a forcing immediately it will reach $Q' \lambda$. That $Q' \lambda$ will take a long time, that $t = \infty$ is few months for atmosphere, few years for the ocean, few centuries for the continental ice sheet okay.

This is the characteristic time. Are you getting the point? So any first order system will behave like this. So if you put the thermometer in the mouth, doctor will not get the temperature immediately, for him he has to wait his time constant is 2 minutes. If you put a

thermocouple you will wait for no change in reading for 10 minutes, you will say it should not change by more than 0.1 degrees 10 minutes you will wait.

Some make sure will wait for hours, so there is a time constant for everything. Doctor will not get the exact temperature, at the end of 2 minutes you will get the reasonable temperature, anyway it is Dolo 650 1-1-1. Is not it? Whether it is 101 or 102? Okay alright so this nice right. So you got, how it would response also we have seen. Here we saw the sensitivity and all this. Now what about the feedbacks?

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I will give you some stats, climate feedback and then we will solve a problem and close. Water vapour $f=0.5$ people have calculated. If $f=0.5$ $g=2$ very good that you understood okay. Next cloud, clouds contribute to greenhouse effect with the cloud cover is more. Watch what I am saying very carefully. If the cloud cover is more whatever radiation is coming back from the earth, this cloud will intercept it and it will not allow it to go out okay.

So the cloud will continuously build up and correct that is what you are thinking but the cloud will also reflect the sun's radiation from the top. If there is no cloud, everything will come is not it? So the net effect is 0. Clouds are not causing climate change otherwise big anti-cloud movement would have taken place. So is that okay? Cloud also created by nature know why unnecessarily you have to trouble.

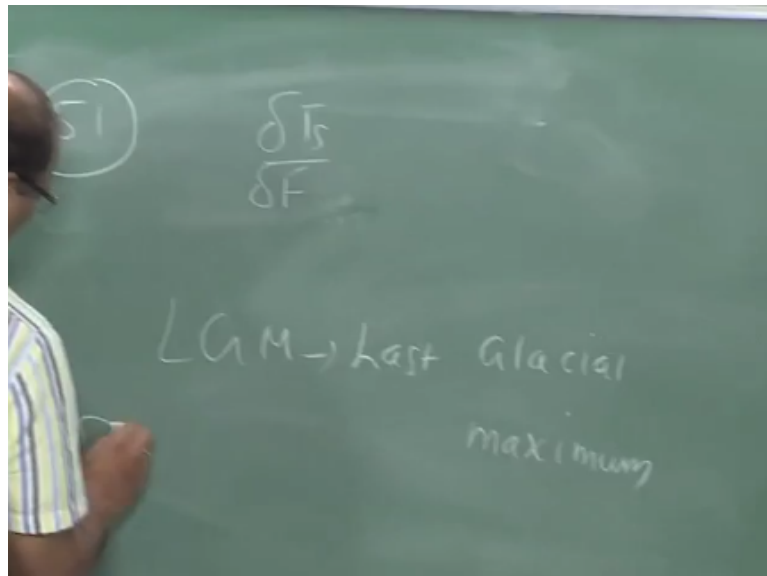
So net cloud forcing is I am not going to write it down, there is some cloud computing also right so $g=0$ okay. So clouds are harmless. Ice albedo feedback, it is very small, it is not a

cause for concern. There is something called snowball earth where the earth when it is at tipping point this ice albedo feedback can make the whole earth freeze but we are not at that tipping point now.

Who is taking this snowball earth? If you reach that tipping point they will present that everything will work in such a way that the feedback will infinite do loop, it will repeat and this thing and then earth will keep on it will like crazy, it will build up ice and snow and all that but now we are not there. So we are fine okay. So no worry, so the national logo of Australia you know is you know T-shirt what they wear no worries Australia right.

So no worries ice albedo feedback. Now carbon dioxide feedback is dangerous. It positively contributes to GHG greenhouse gases and people will figure out. If C_Y is your initial concentration and C is your new concentration, 5.35 that is arrived by radiative transfer calculation. If you do that then you have to add that f and then calculate the λ and then you will calculate the g and so on okay.

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Now problem number 51, estimate the apparent climate sensitivity, the apparent climate sensitivity is given by what do you mean by this? The temperature T_s at one point in time-another point in time/ F at one point. This is like they see way of otherwise you have to get the curve and then calculate dy/dx and then so it is an apparent climate okay.

Estimate the apparent climate sensitivity based on differences between T_s and F between the current climate and the climate 20,000 years ago called LGM last glacial maximum okay

with the following data, T_s current- T_s LGM is 5 degree centigrade okay. CO2 LGM 180 ppm, CO2 now 360 ppm okay, change in albedo 0.01 that is all only 2 effects. So can we proceed?
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LGM → Last Glacial

$$\frac{\Delta T_s}{\Delta F} = \frac{T_s(\text{current}) - \overset{\text{maximum}}{T_s(\text{LGM})}}{F(\text{current}) - F(\text{LGM})}$$

NPTEL

You can calculate this ΔF okay so numerator is how much? 5 so the challenge is to calculate the denominator okay.

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$$\frac{\Delta T_s}{\Delta F} = \frac{5}{\left[\text{CO}_2 \text{ feedback} + \text{Albedo feedback} \right]}$$

$\text{CO}_2 \text{ feedback} = 5.35 \ln(2)$
 $= 3.71 \frac{\text{W}}{\text{m}^2}$

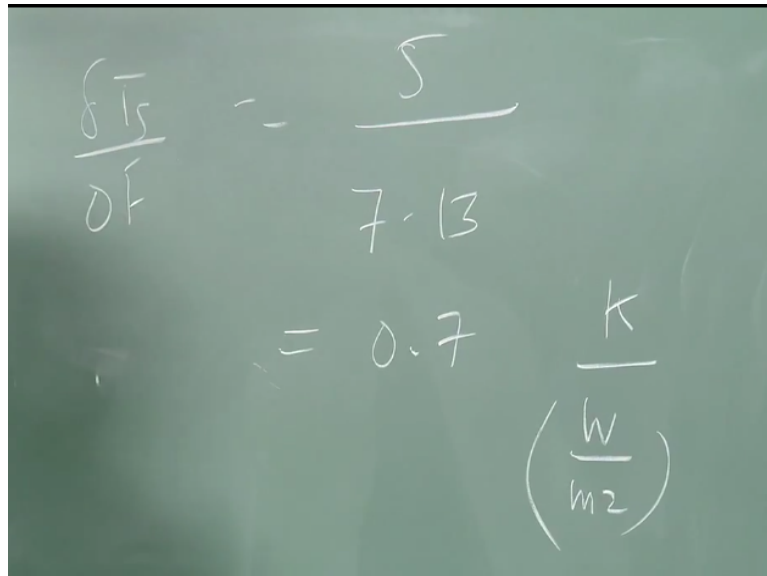
$\text{Albedo: changed by } 0.01$
 $\therefore F = 342 \times 0.01$
 $= 3.42 \frac{\text{W}}{\text{m}^2}$

$\downarrow F_s$
 $342 - 239.4$
 $\frac{\quad}{0.7}$

There are only 2 feedbacks carbon dioxide. How will you calculate the carbon dioxide feedback? $5.35 \ln 2$ how much it is? Okay 3.71 watts per meter square. Correct? 342 is not it? Correct, so I was able to get this 3.42 okay, 239 what is that F_s ? 239.4. What is the albedo of the earth 0.7, so incoming is 342 but what is contributing to σT to the power of 4 is 342×0.7 .

We have done this again and again in the problem that results in 239 okay. So out of this 342 if again changes by 0.01 that is 3.42, so please add this. So you are all becoming climate change experts now. Given the data you know how to calculate but this is coming from some model and all that, you can improve your models, you can consider interaction effects okay and then you can look at all these changes over 30 years, 40 years, 50 years you can put your current consumption of oils and this thing and CO₂, emission all that.

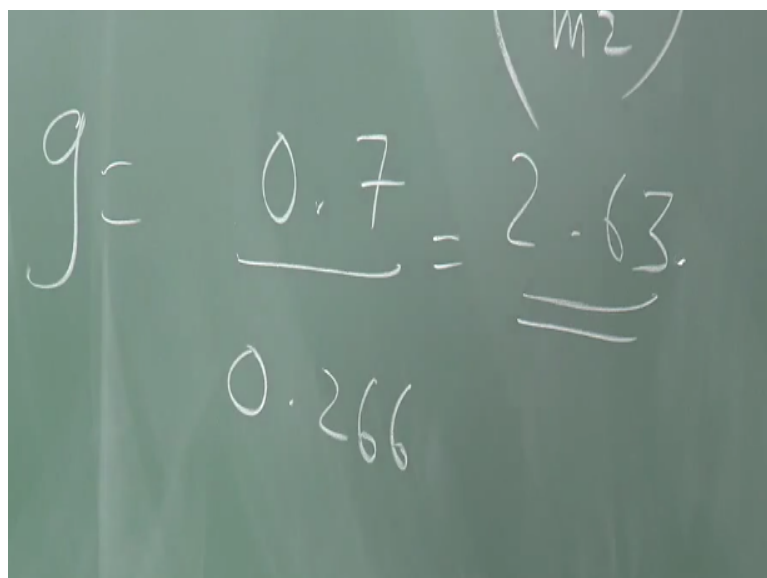
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$$\frac{\delta T}{\delta F} = \frac{5}{7.13} = 0.7 \frac{\text{k}}{\left(\frac{\text{W}}{\text{m}^2}\right)}$$

Then, it will become a stud model. Now we are using every simple model okay, 5/7.13 this is 0.7 kilo watts per meter square. What do you infer from this?

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$$g = \frac{0.7}{0.266} = 2.63 \text{ (m}^2\text{)}$$

G is 0.7/0.266 we solved the problem number 50 know, it is 0.266 without feedback right. That is 1/0.266 gives 3.76 for 1 kelvin. So this is the gain for the amplification of the

sensitivity because of feedback. How much is this? How much is it? 2.63 okay. So in simple English due to the 2 feedbacks the apparent sensitivity increased 2.6 times what was an original value of 0.266 which is coming because of solar radiation.

Now 2 things are changed, the reflectivity of albedo has changed, carbon dioxide has changed. So if they are going at this level, then we are in trouble but this is 20,000 years. The time we are talking about is 20,000 years 5 degree centigrade but now next 5 degree centigrade many things will melt will lose surfaces, will lose continents, will lose countries and so on.

But it may happen much more before 20,000 years because if it has taken 20,000 years for carbon dioxide double from 180 to 360, now it is doubling at a mad rate, 360 now it is 390 already in calculations you are doing 390, 400 so tomorrow I will show you a plot where carbon dioxide last 100 years just it goes like this. So this 5 degree is not a great thing but the time is coming down okay that is the problem.

I will stop here. Tomorrow will run through a presentation. I will give you a PowerPoint on climates change and then will wind up the course on Thursday with just a very brief one lecture course on dynamics.