Introduction to Atmospheric Science
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Lecture – 32
Planck's distribution and Inverse square law

Yesterday we were looking at radiation loss. So the correct black body distribution was proposed by Max Planck, it s a theoretical distribution. I agreed exactly with experiment. So in order to make the theory agree with the experiment it was necessary to move away from classical physics and then propose a d = h* nu. So if you go away from the classical physics.

And propose that energy transfer takes place in multiples of h * nu where h is not infinitesimally small tending to 0. h is of finite constant so fundamental constant of nature, which is the Planck's constant which is 6.627 * 10 to the power of - 34 joules second. Then we looked at the distribution. This is the Planck's distribution.

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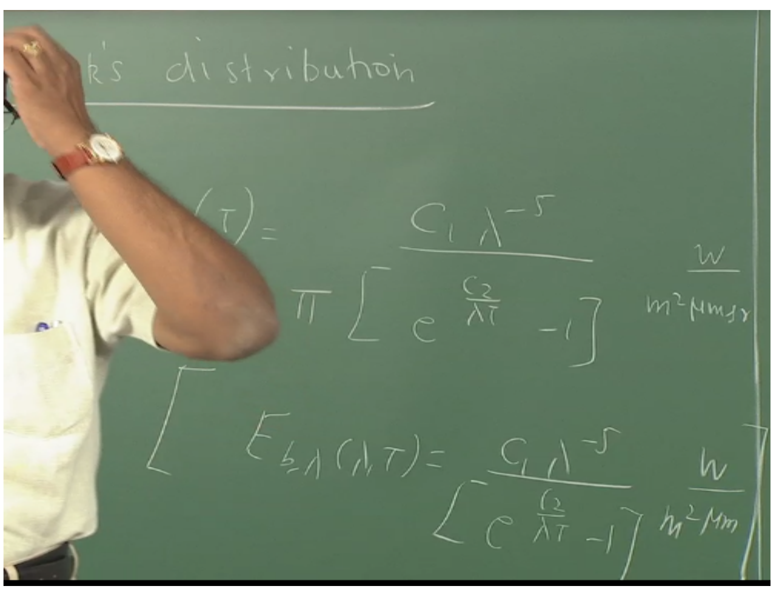

Monochromatic spectral radiation intensity of black body. In engineering radiation, we will call it as so that pi is taken care of the steradian will not come. So that is e takes care of the solid angle.

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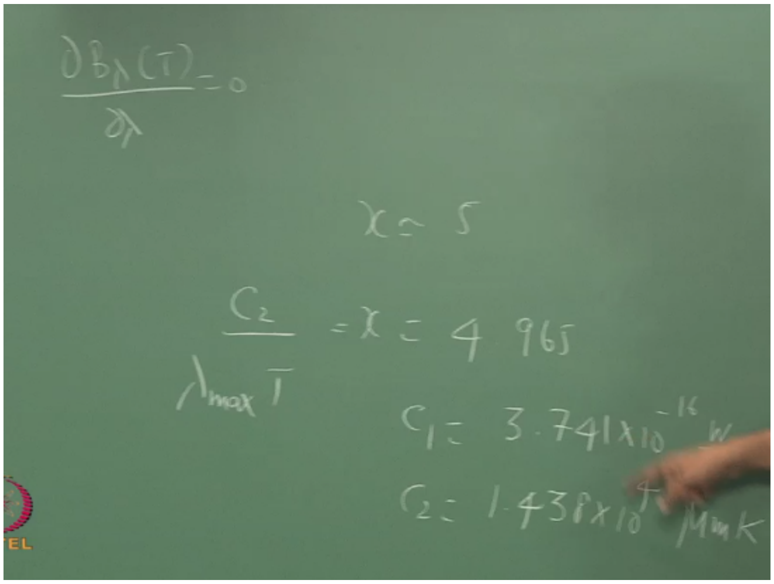

Now we did this in the last class. We got this. x = So let us use better values for C1 and C2. C1 will be actually. So I gave it as 1.45 * 10 to the power of 4 that is approximate is actually 1.0. So this is the called the first radiation constant. This is called the second radiation constant.

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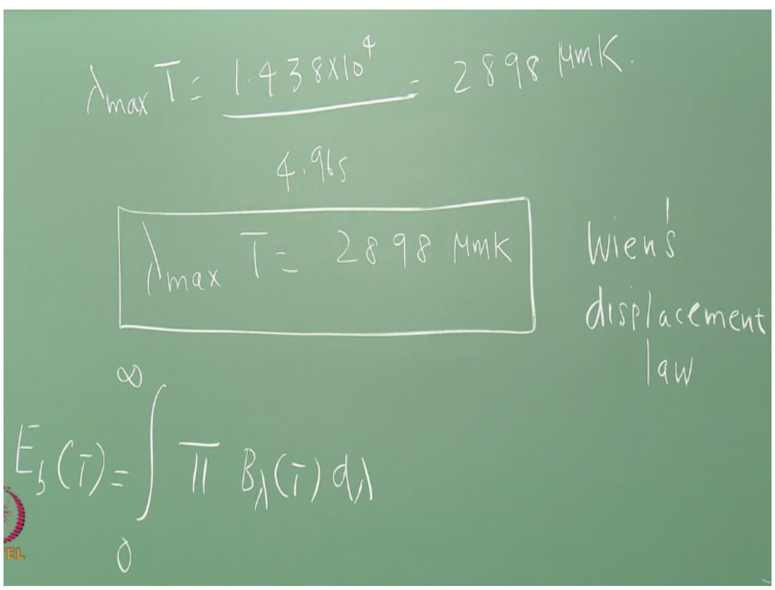

So lambda max T is that explains why as the temperature increases, lambda max decreases. So is the Weiss displacement law. So you can also look at the colour temperature emission spectra and try to figure out the temperature from that and so on and that comes from the Weiss's displacement law. Yesterday, we worked out a problem where I gave you the maximum what is that lambda max for solar radiation I gave it as 0.475.

We found out the colour temperature to 6100 Kelvin. The distribution of radiation from the sun does not exactly correspond to a black body therefore there is a minor difference. So actually the equivalent black body temperature the photosphere of the sun is about 5800 Kelvin. So what will happen if this should give you the black body emissive power at a particular temperature. You will already know that it is = sigma T to the power of 4 which you are learning from high school. So let us see whether we can do something to derive this from the Planck's distribution.

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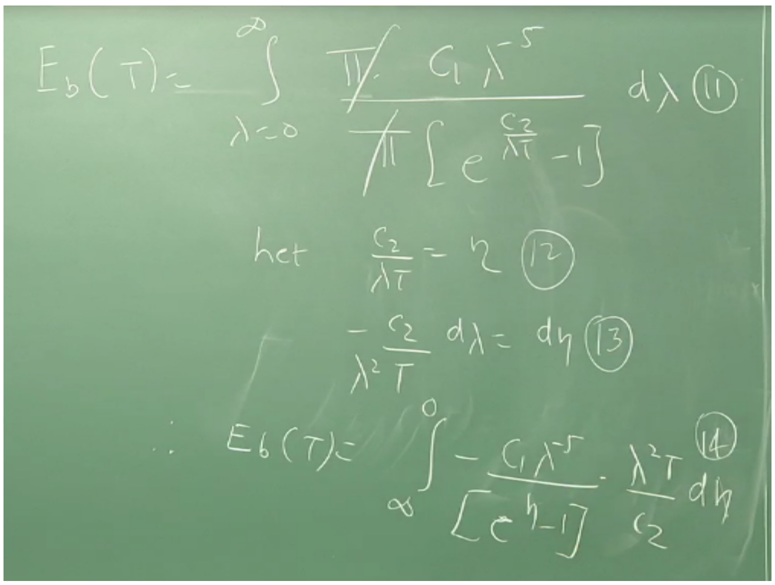

We gave some equation number yesterday? Weiss's displacement law what is the number. No numbers? We will forget it. So we will be there was a number this was 9. Let C2/lambda T be eta. Is it correct? Is that okay?

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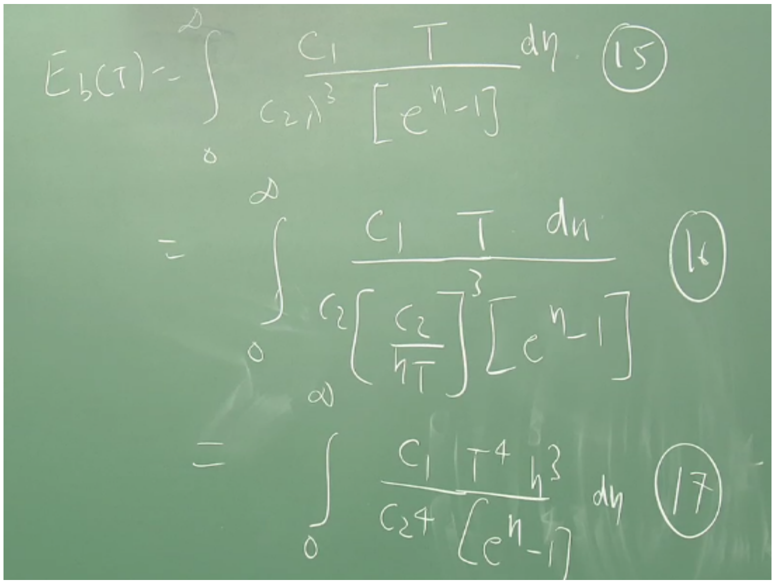

I changed the limits from 0 to infinity to infinity to 0 because the eta and this are eta is 1/lambda then there was a - I again changed it 0 to infinity. I think we are doing fine. Now T is the numerator. Is it fine? Now what is here? We are doing it only for a particular temperature. C1 is a constant can be pulled out. C2 to the power of 4 is a constant can be pulled out. T is a constant can be pulled out.

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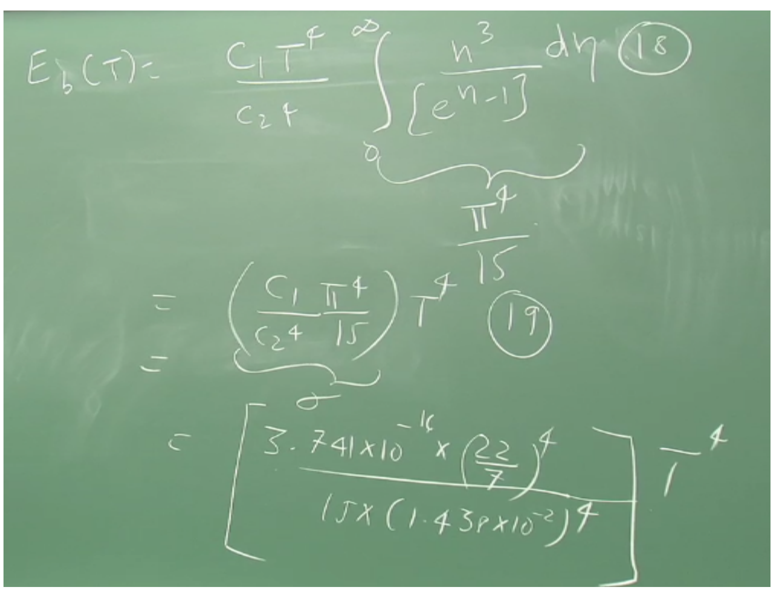

So this eta cube e to the power of eta - 1 d eta cannot be easily integrated. It is very tough. so you introduce elliptical integrals and all that so it is cos theta, sine theta all that you have to do, but you can numerical integrate using Simpson's rule, Trapezoidal rule, Gauss quads, Gauss quadrature whatever, it is known that it gives you result of pi to the power of 4/15. If that is the case, so this is nothing but pull out your calculators and check for yourself whether, you get a value of 5.67 * 10 the power of – 8.

Because this is your sigma the Stefan's Boltzmann constant. Please pull out your calculators and check you know the value of C1, you know the value of C2, you know the value of pi, 22/7 just check it out, but C2 may be micrometer Kelvin so convert into 10 to the power of - 6, C2 to the power of 4 means 10 to the power of - 24. Just check it. 10 to the power of - 2, it was 1.43 * 10 to the power of 4 micrometer 10 to the power of - 6 so it becomes - 2. Ravindra, Chaitanya done, 5.67.

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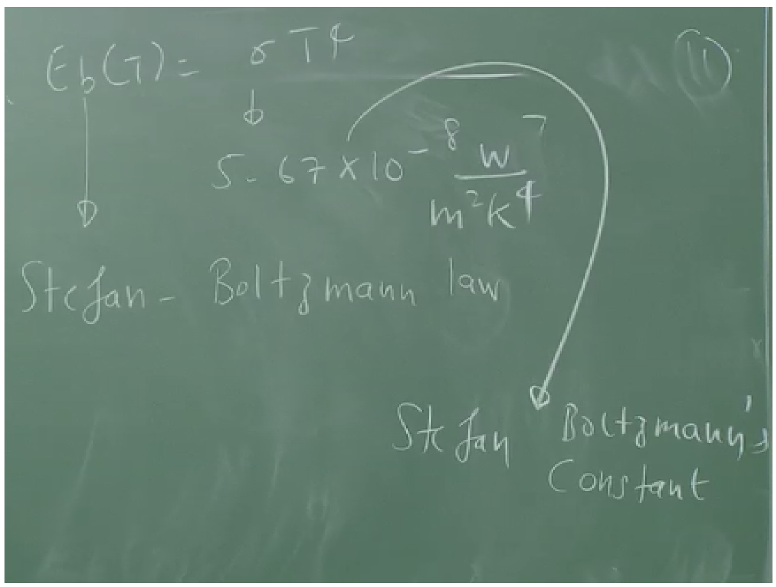

Watts per meter square per Kelvin to the power of 4. So, this is called the Stefan-Boltzmann law. This law was known much before Planck figured out the Planck's distribution, because this has come from thermodynamics. From thermodynamics, we can prove that the radiation, the emissive power of a black body is proportional to T to the power of 4, however for getting the sigma we cannot get it purely from thermodynamics you require experiments.

So if a T to the power of 4 if it is matched to the experiments that a will turn out to be 5.67 * 10 to the power of - 8, however if the Planck's distribution is also integrated from lambda = 0 to infinity it will result in the same value of 5.61 * 10 to the power of - 8 therefore it further validates Planck's black body distribution function. This is called the Stefan-Boltzmann's law and this is called the Stefan-Boltzmann's constant.

So if you have 1 meter by 1-meter square which is 1000 Kelvin it will have emissive power of 56700 watts 56.7 kilowatts. 5.6 * 10 to the power of - 8 * 1000 is 10 to the power of 3, 10 to the power of 12 so that will be 10 to the power of 4, so 5.6 * 10 to the power of 4, 56.7 * 10 to the power of 3, 56.7 * a 1 meter square 56.7 kilowatts. So 1 meter by 1-meter plate which is at 1000 Kelvin has an emissive power of 56700 kilowatts.

The solar constant, the solar radiation which is entering the earth that has got the power of 1353 watts per meter square on the equator, on a clear day at 12 'o clock in the noon that is the value 1353 watts per meter square. Therefore, if you have 1-meter square area the maximum you can get is only 1.35 kilowatt if it is cloudy and this thing you correct for altitude, latitude, all that so it will be much lower than that then take care of these rainy days, and you take care of the evening and this thing and all that.

Finally, it will be a nonzero value, but still, but you will have to take into account all this, fine. So we have derived both the laws from first principles. Now it is time to solve some problems involving these. Problem number: 45. So we will hit half century very soon. Problem: 45. Calculate the equivalent black body temperature any doubt. Everything is clear. Anyway those people who are mathematically inclined.

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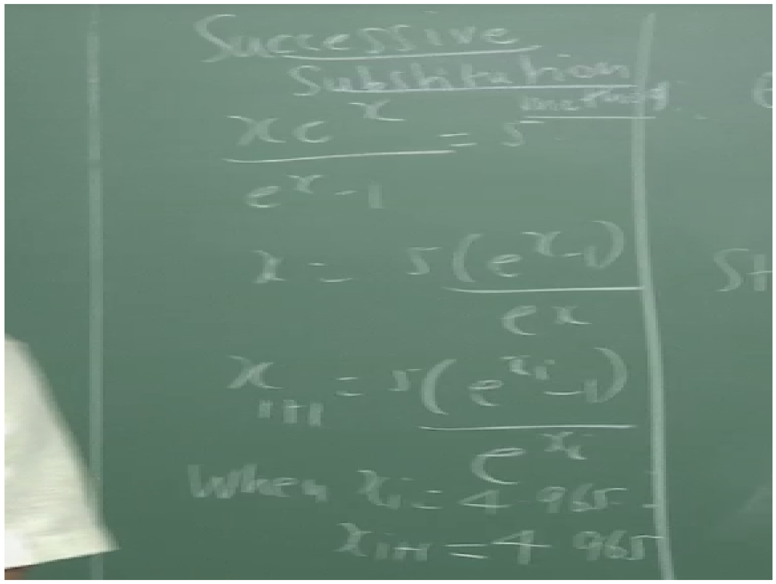

So this is the algorithm for what is called the successive substitution method. You will start with the value of x, let us say x = 3 using e to the power of 3 -1/ e to the power of 3 * 5 will be xi + 1 that will not be 3. Then a new value again substitutes, find the new value of xi + 1 and keep on doing it till the left side and right side are very close to each other. So this algorithm, so this is called the trial and error. This is the called the successive substitution method.

If you apply the method, you will get a value = 4.965. You already told it by saying that e to the power of x is much > 1 and therefore e to the power of x -1 and this can get cancel x is almost = 5. So those people who are very mathematically inclined you asked this question yesterday I am trying to answer this. We can solve it using this. It will turn out that the answer is 4.965 which will lead to 2899 micrometer Kelvin rather than 2999-meter Kelvin. So this is called the successive substitution method.

For the sake of completeness somebody take the value of 4.965 and let us get re-assured that right side and left side are 4.95. When xi is 4.965, so doubting Thomas, put your doubts to rest. 4.965 is indeed the correct answer. If you want to be more threateningly formal please go ahead and use the Newton Raphson method, put it as f of x then x of I + 1 is x of I - f of x/f dash of x keeps on integrating. We will get the same answer 4.965. I got distracted. We are supposed to do problem 45. Problem: 45, calculate the equivalent black body temperature Te.

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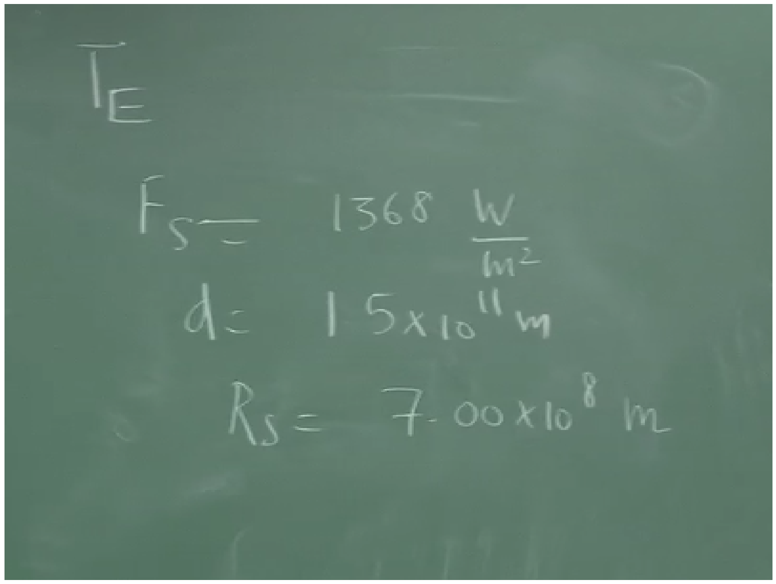

Calculate the equivalent black body temperature TE of the solar photosphere (outermost visible layer of the sun) solar photosphere is the outermost visible layer of the sun based on the following information. The fluxed density of solar radiation reaching the earthF5, just now I told you. I told you 1353 I think it is 1368 not a big difference. The flux density of solar radiation reaching the earth Fs is 1368 watts per meter square, the earth's sun distance d and the radius of the solar photosphere, the radius of the sun 7 * 10 to the power of 8 meter.

If you have the flux density on the outer layer of the photosphere you have done right. If you have the flux density on the outer layer of the photosphere, then that flux density = sigma t to the power of 4. How do you have a flux density in the outer layer of the photosphere you know the flux density at a far off place and you know there is something called the inverse square law which is at work, now use it. You know the flux density at a far off distance so find the flux density at a lesser radiance. **"Professor – student conversation starts"** Venu, you understood?

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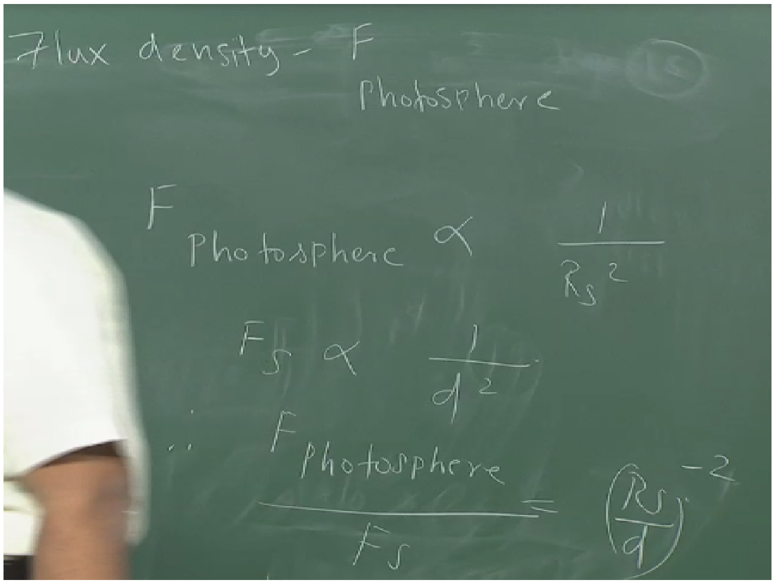

What was the distance? What did I call it d. what did I say earth? Fs what do they call that? fs, Rs. Is it correct? I messed it up. It is correct right. Divyashree, what is the problem? T/R, is it correct. What is d that I called it as d. then this is correct? Is it okay? What is troubling you Anusha? What is going like this?

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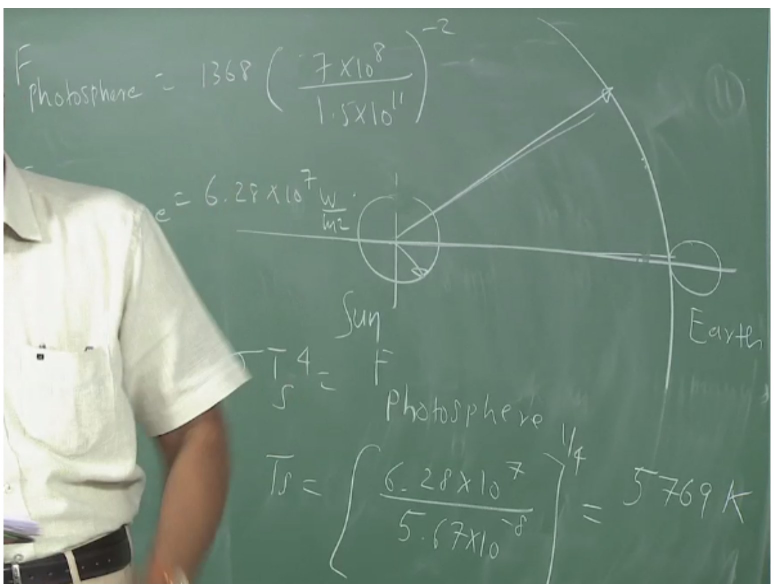

So E is like this. Now we know the flux density here. We want to find out the flux density here. That is, it. Sneha, is it okay? **"Professor – student conversation ends"** So now tell me what is here photosphere? Sun. See at this radius if it is having 1368 at this radius if it is having 1368 at this radius it will be much more. What is that much more it is = sigma T to the power of 4 that is it. So what is this? 1368 * Rs is what: 7 * 10 to the power of 8/1.5 * 10 to the power of 11.

So F photosphere how many is it 6.28 * 10 to the power of 7 watts per meter square. No Ts. **"Professor - student conversation starts"** What is the doubt? Too low what made you think so? You may think about many things, but according to the value I gave it is it correct or not. Just hang on. We can check it. If I get 5800 Kelvin, then it is correct. **"Professor - student conversation ends"** So let us see what the girls are saying is correct. 6.28 * 10 to the power of 7/sigma whole to the power of how much is it. 37.6.

What is the problem? I am also getting watts per meter square. What is the problem? Just check relax it should be fine. So this is 5769 K we approximate it into 5800. So 1368 watts per meter square it is possible for you to measure. So, if you are able to measure the radiation falling on the earth and from astronomy you get the sun earth distance, but somebody has to give you the measurement of the radius of the photosphere.

That somebody is on physics people from cosmology somebody is giving you based on this data it is possible for you to extrapolate or I can directly get from emission, from the emission spectra I can take sigma d to the power of 4 and figure out that it is 5770. Inversely we can work out if 5770 is correct then temperature is correct and then you can actually work out astronomy from radiation. You can actually work out the distance if you know the radius and all that.

Any way let us not get into all that. Actually they are moving, the stars are all moving they will find out how it is moving, they will measure the radiation spectra, and then it can be related to the solid angle. Problem number: 46. To complete the discussion let us do the ultta, let us calculate the earth temperature.

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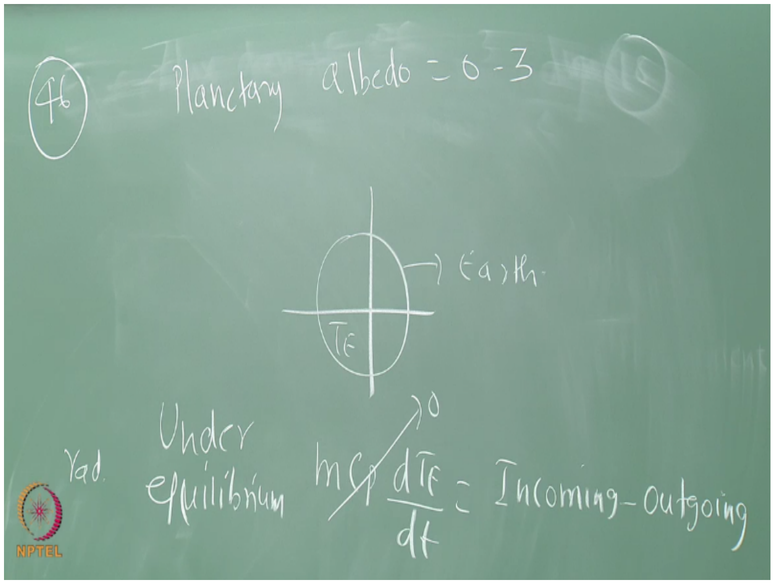

So that your understanding is complete and one more thing is required for calculating the earth's temperature that is the reflectivity of the earth that is called the albedo. The reflectivity is called the planetary albedo. Albedo means how much it reflects back. I will give you the data. Calculate the equivalent black body temperature of the earth. Problem number: 46. Calculate the equivalent black body temperature of the earth assuming a planetary albedo of 0.3.

Open the brackets (fraction of the incident solar radiation) I am defining albedo (fraction of the incident solar radiation that is reflected back into space without absorption), assume that the earth is in radiative equilibrium. So planetary albedo. People are working n this presentation. The planetary albedo can change if the ice cubes are melting the snow cover changes and that lead to a feedback effect and all that. Snow ball earth will be the opposite of that.

Albedo is a very critical factor in earth, how much it reflects. Now what is radiative equilibrium? radiative equilibrium means you are considering the whole let us say that the whole earth is at 1 temperature TE now under equilibrium the rate of change of enthalpy of the earth is incoming - outgoing. The incoming is from the solar and the outgoing is basically emission.

Under equilibrium, temperature does not change with time. Now, this 1368 watts per meter square that is based on the projected area of the earth. Lakshmikanth, that 1368 is based on the projected area of the earth. What is that you have any doubt? So you will have to be careful. So if you want to solve problem 46.

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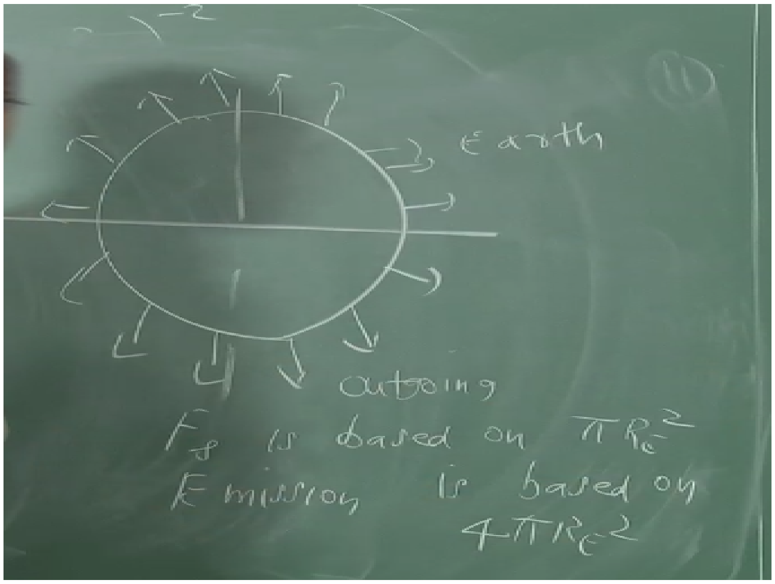

So the radiation is coming from the sun. This 1368 this is the earth is a sphere, that 1368 is based if you cut like this what happens if you cut like this what do you get a circle pi r square. So the 1368 is based on pi r square, but the emission back is based on 4 pi r square, please use this. So this is incoming this is outgoing. Emission is based on 4 pi R E square and please take factor in the albedo. What would be the absorption, absorptivity 0.7 that you have learnt somewhere? I think T step first you have already learnt is somewhere. So that funda is required now.

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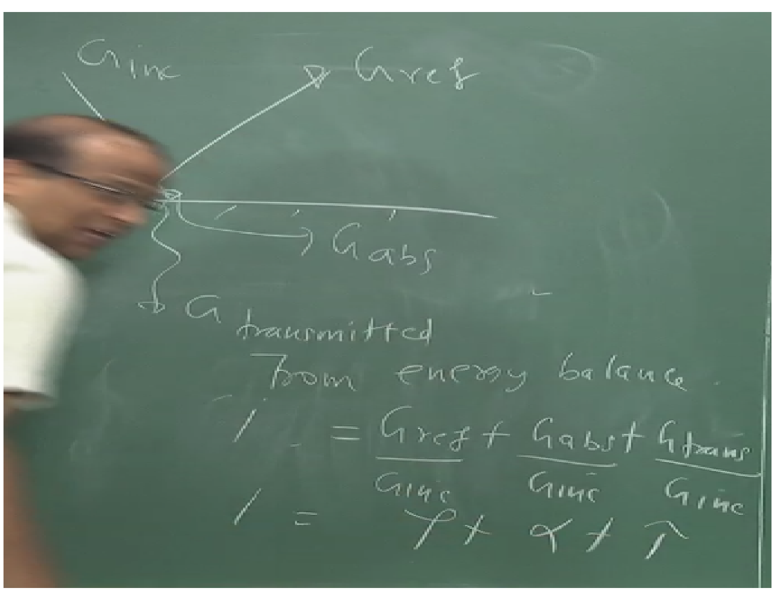

So if radiation is falling on the surface water is also falling, it is not only radiation is just falling. So G incident what can happen to this radiation if it falls on a surface, this can be reflected. This can be observed or this can be transmitted. There is no chance of transmission from the earth. If it is glass it is possible.

So using energy balance G incidence, G is the commonly used nomenclature for incident radiation G is irradiation then we qualified the subscripts, reflection, transmission, and absorption. I will divide by G throughout and his 1. So 1 = reflectivity + absorptivity + transmissivity.

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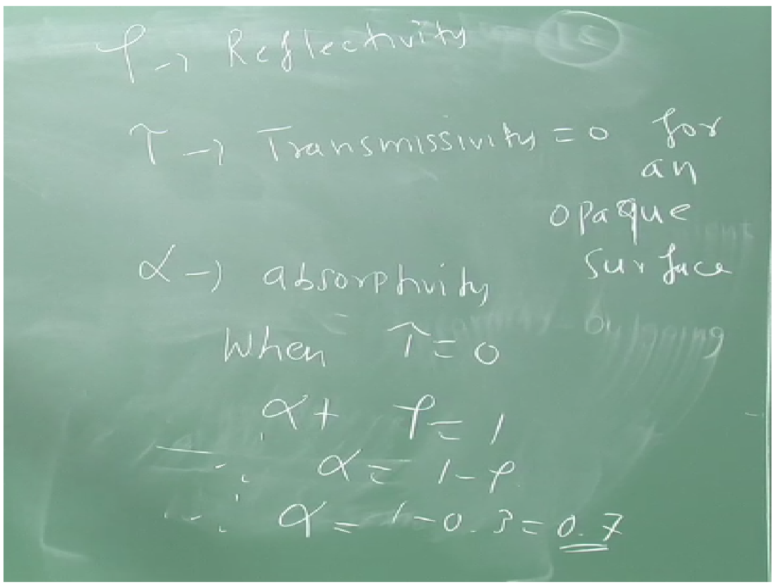

Transmissivity is 0 for the earth, but for the earth's atmosphere it is not 0 for an opaque surface. When tau = 0, therefore the absorptivity of the earth planetary absorptivity will be 1 - 0.3. In simple English 1 - 0.3 is 0.7 * 1368 watts per meter square will be absorbed for every meter square of the area.

And what is the total area which is absorbing this pi RE square, but this must be equal to the emission. The emission is 4 pi Re square * sigma Te to the power of 4. That is the energy balance please do it we will complete it. We will get the temperature that is 255 is the temperature of the earth. You are at equilibrium now. You figure it out. Let us finish this.

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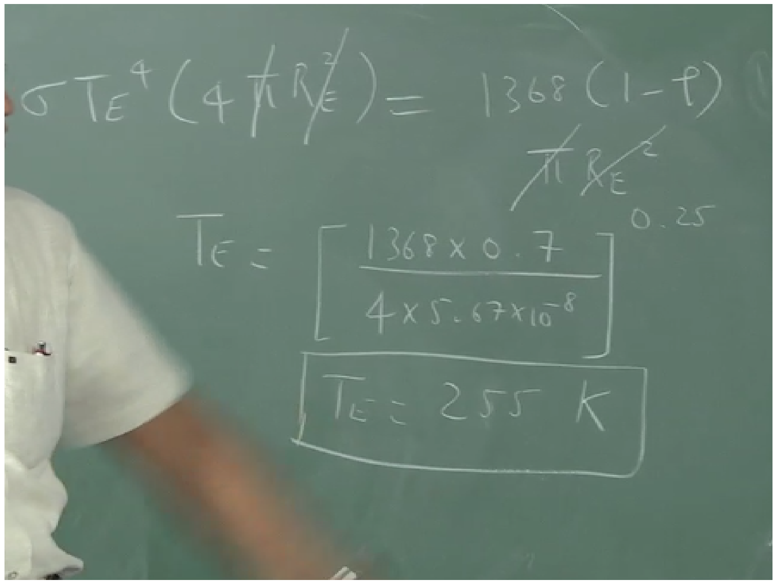

Chaitanya for this also the radiation of the earth is not required. 4 pi Re square = 1368. So the temperature is about - 18 Kelvin. Do not worry about the temperature in Chennai that is something. Overall it is about you also have the poles, you have got Siberia, you have got all these places the average temperature is assuming the earth's temperature to be 1, the average is 255, assuming the earth's temperature is 1 is also a bad idea.

But you had to do that it is 255 and their other assumption. When you do climate predictions they assume something called aqua planet. The aqua planet means the earth contains the earth contains 100% water. So there are simulations with aqua planet stimulations it is easy to do, but otherwise you have to take 3/4th, 71% water 29% land.

Then you have to have emissivity for land, emissivity of ocean, the problem gets more complicated. So first level modeling they will say aqua planet that means everything is water 100% water and they will do the simulations. Now this power point I want to show. Just 3 minute. It is already there.

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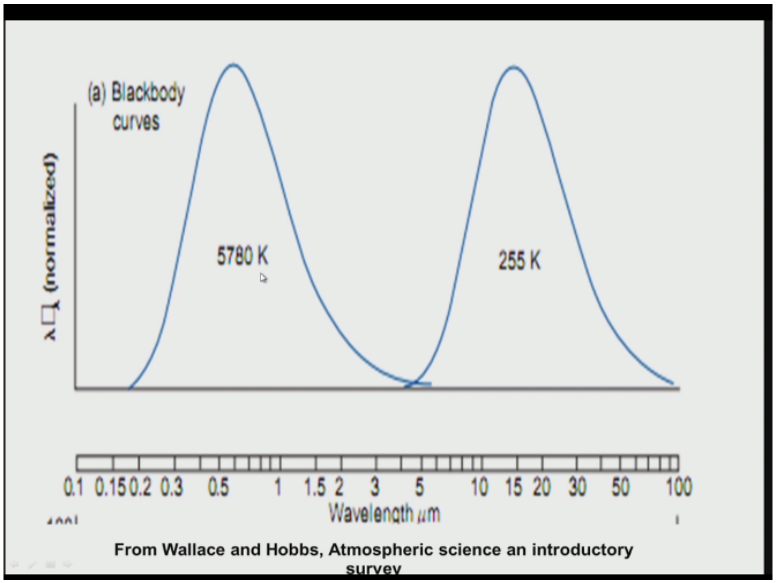

So these are the black body radiation curve, so if you see this is from the great book Wallace and Hobbs much of my material is taken from that book, atmospheric science and introductory survey. So you see the Planck's black body distribution drawn side by side. So the left curve basically the units have been normalized because 255 Kelvin it cannot be so much. That should be a big mountain, this should be hillock, but we have adjusted that.

Now if you see just look at the nature of the curves. So the peak is around 0.5 that is what we saw in yesterday's class 0.475 for the solar it is about for the solar incoming radiation I mean radiation in the sun it is about 0.5 Kelvin so this is the spectrum black body spectrum for the sun, this is the black body spectrum for the earth.

Therefore, the incoming radiation is largely in the visible part of the mere infrared part which is < 4 micrometer. So actually for the radiations on the earth it is always in the infrared part of the spectrum. So this is the first major difference and all these green house gasses do not absorb much in this part of the spectrum and they absorb a lot in the other part of the spectrum till if you do not believe me.

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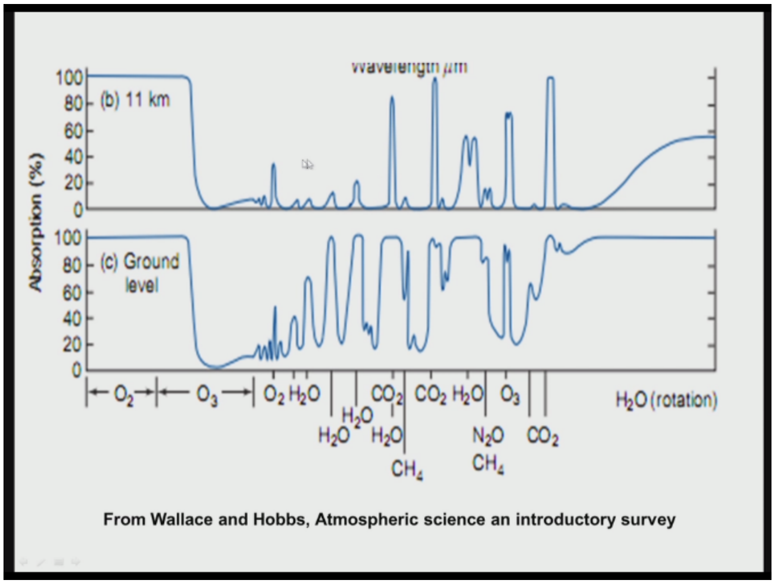

So this is the absorption spectrum. So if you see this is the absorption spectrum as measured so at 11 kilometers, at the top of the atmosphere and at ground level. At the top of the atmosphere is what is of interest to us so same wavelength if you see 0.1, this is 1, 5 and this is 100. So remember this. So in the incoming radiation you see the absorption is very less, but whereas in the outgoing part the absorption is very high.

So because of these the greenhouse gases so the top if basically carbon dioxide, the top is at 11 kilometers, the bottom is ground level and the various gases absorption are given here oxygen, ozone and so on. So you can see that there is very little absorption in this part of the spectrum where there is lot of absorption therefore these gases mostly diatomic gases because of the electric dipole.

Because of the particular property the nature of the properties they allow that incoming radiation, incident radiation that come through whereas they do not allow the radiation which is going back from the earth. This is responsible for a continuous build up of this radiation in balance, the budget is affected. The radiation budget is affects these leads to global warming and so on. Always water vapour is there.

Water will evaporate and water vapour is not under our control. It is a byproduct of the natural hydrological cycle, but this carbon diode and methane are under control so the whole idea is to reduce this carbon dioxide so that all this stay within limits. So in the next class we will go little deeper and then look at the green house effect and then we will have to look at the equation of transfer, how radiation gets transferred in an absorbing and emitting atmosphere.