Introduction to Atmospheric Science
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Lecture - 31
Atmospheric radiation - Radiation laws

Okay, so good morning, in the last class we started radiative transfer. So we will continue with the basic definitions terminologies. We will also look at the 2 important radiation laws namely the Stefan-Boltzmann law and the Wien's displacement law, which can be derived from the Planck's blackbody distribution function, the Planck's distribution function got him the Nobel Prize, Max Planck got the Nobel Prize in 1980 okay.

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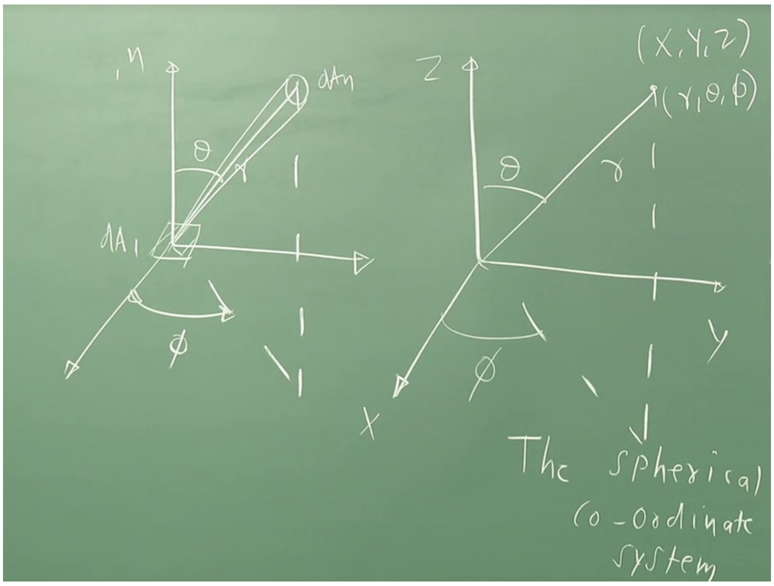

So just have a quick recap, if you see the last class we did this, there is a small area dA1, n is a unit vector, the point in space okay, so there is another elemental area dAn okay, so this is the radius r, this is the theta, theta is the zenith angle and phi is the azimuthal angle. So basically this is fine no xyz is that okay, so r this phi is measured from x okay bottom that x, y plane. So a point can be specified as a x, y, z, you can also be specified as r theta phi okay.

So if you specify the radius, the zenith angle as well as azimuthal angle then you are able to specify that point, so this is basically the spherical co-ordinate system alright. So we must now look at the difference between plane angle and solid angle.

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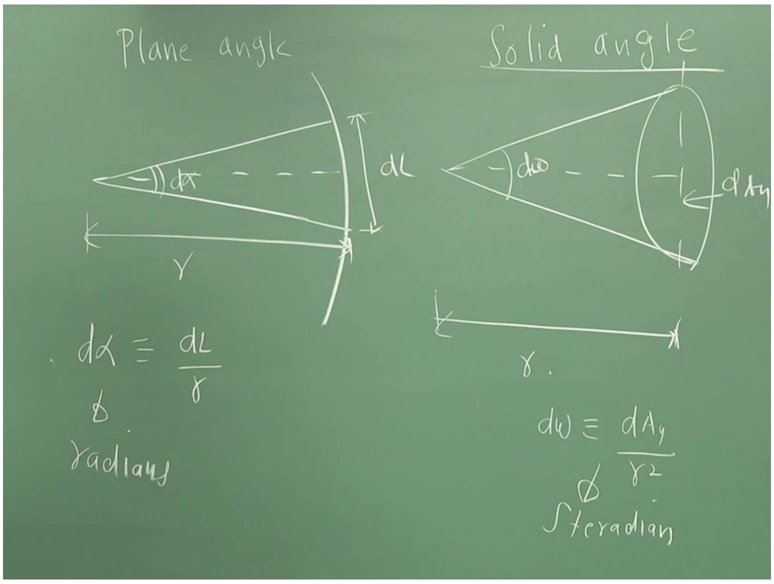

If d alpha, d alpha =dL/r radiance the unit for that is radiance, this is the plane angle 2D. 3D, this is solid angle dAn, n denotes area normal okay the normal area, so this is got the units of steradian okay.

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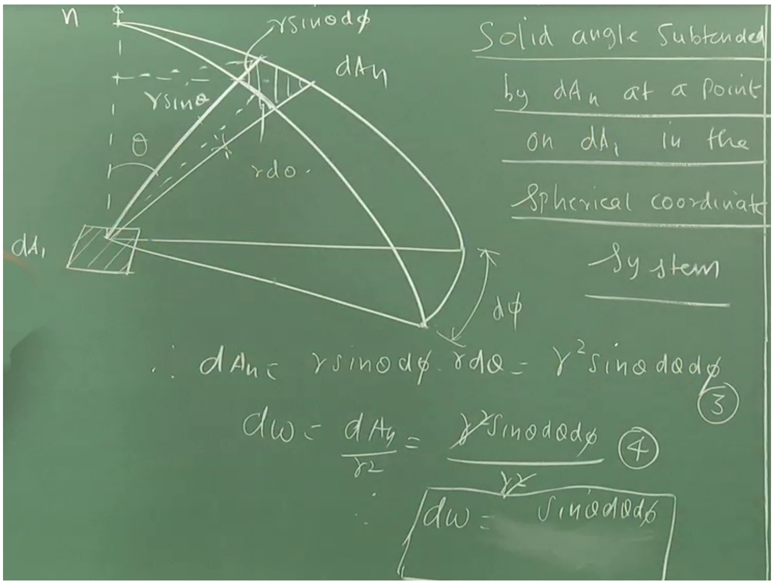

Now let us go little deeper, this is small this radius is r, this theta, this d phi, this is dA1, this is your unit vector and all that, that is dAn okay, is it clear. Now the next step is I am obstructing you. The next step is to write an expression listen to me carefully, next step is to write an expression for dAn in terms of the basic coordinates r theta and phi, and find an expression for the d omega solid angle in terms of r theta and phi.

And then we will integrate between the limits and find out what is the maximum solid angle for this hemisphere, if you multiplying by 2 you will get the maximum solid angle for a sphere, that is the exercise we are going to do for over the next 5 to 6 minutes. So first write dAn in terms of r theta and phi sin theta cos theta tan theta whatever you want to do, then get an expression for omega as a function of r theta and phi.

So therefore, we will be in a position to express the solid angle in terms of r theta and phi, so this spherical co-ordinate system is very useful for us okay. And then we will find out the maximum solid angle over a hemisphere okay. Now I need to give a proper caption for this figure, so this is got, so the right caption for this sphere would be solid angle subtended by the elemental area dAn at a point on another elemental area dA1 in the spherical co-ordinate system okay.

All these are r right, what is this? r sin theta. What will be this? So this is same as this, this is r sin theta, what about this? r d theta okay, no r square.

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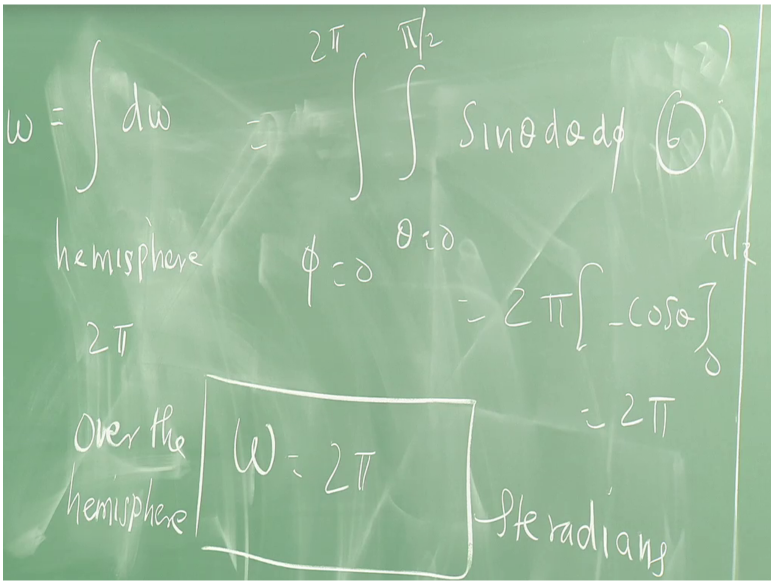

Okay, so hemisphere is 2 pi, complete this please complete this integration. So omega over the hemisphere is 2 pi steradians. Why we are worried about hemisphere is we are looking at radiation onto a surface and radiation from the surface. If you consider radiation from a volume and I mean a particle is burning or something or you are considering a gas molecule and all that then we will consider the 4 pi okay, so omega=2 pi.

Now we are able to see the d omega is proportional to okay. So what about the flux density? We wrote an expression for the flux density in the earlier class, I do not think I gave this okay.

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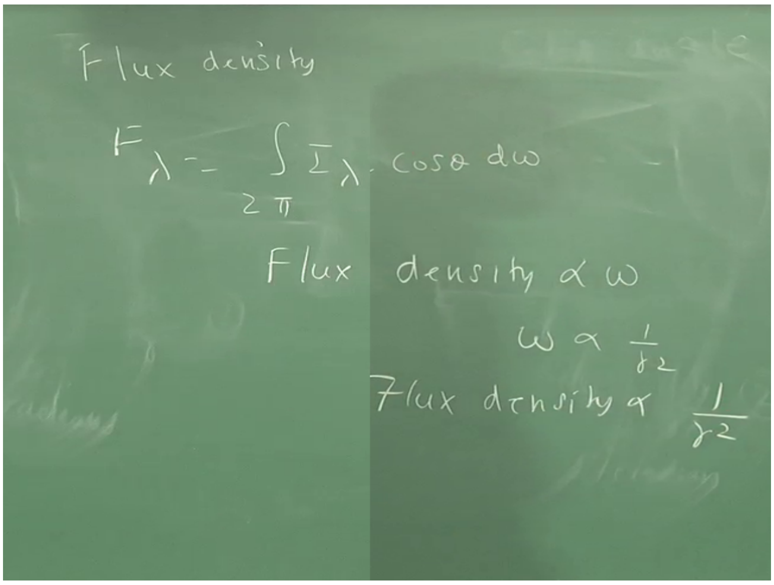

Flux density

$$F_\lambda = \int_{2\pi} I_\lambda \cdot \cos\theta \, d\omega$$

Flux density $\propto \omega$

$$\omega \propto \frac{1}{r^2}$$

Flux density $\propto \frac{1}{r^2}$

So this is an important formula which converts I to F, I is basically a scientific quantity, F is an engineering quality alright. I is got watts per meter square per steradian per micrometer okay, so this will become the steradian will go, it will become watts per meter square per micrometer. Then if you integrate with lambda 1 to lambda 2, it will become watts per meter square, if you multiply by the dA1 it will finally become watts alright.

So this is the formula for flux alright, so flux density is proportional to solid angle right, yes or no? Okay, suppose you do not believe me, consider this dA1 is this A4 sheet of paper this dA1, this duster is dAn okay, this is dAn this is dA1, assuming the both are very small, connect them centroid of this to centroid of this, then I keep it at this side I can find the solid angle okay, both the areas are the same.

Now this is a 1000 degree centigrade let us say, this is at 30 degree centigrade, you are finding out how much, this fellow duster is receiving heat from this fellow. So it will temperatures have the same, other conditions are the same, it will receive some heat right. I kept the duster here, what happens to the solid angle subtended by the duster at a point on dA1 is it increasing or decreasing? Decreasing, because what is actually the solid angle?

The solid angle intuitively can feel that the solid angle is nothing but if this duster is here, I draw an imaginary hemisphere with the radius =this distance okay, I am trying to find out what is the ratio of the area of the duster to the 2 pi r square. And even in the duster area where to remain the same as these are keeps on increasing, the duster occupies an increasingly smaller area of the hemisphere 2 pi r square.

Therefore, the flux density is inversely proportional to r square which is called the Inverse square law which your physics teacher must have taught somewhere okay. So flux density is omega towards the d omega is proportional to 1/r square. So it is not only the size of earth and the sun, and the temperatures of the sun which decides, the temperature the earth, the earth sun distance. we will solve the problems, are you getting the point? So the distance is also very important in radiation all right, now this is over.

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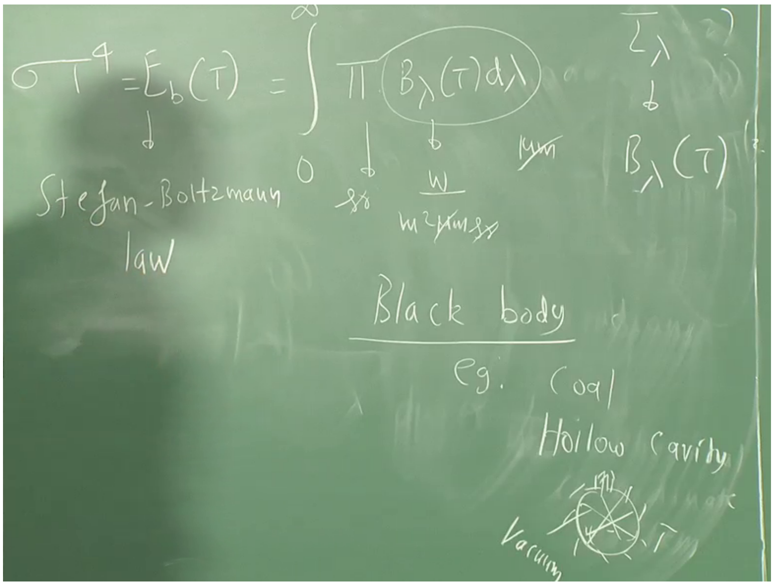

See for a blackbody instead of I lambda you can call it as B lambda, this would be a function of temperature right, so the spectral radiance or the monochromatic radiance or radiance intensity will be a function of lambda okay, and for each temperature it will have a distribution correct. So why that I lambda is replaced by B lambda is I lambda can be for reflection, I lambda can be for transmission, I lambda can be for absorption, and I lambda can be everything anything.

But B is basically for emission, now I have to multiply by the pi, pi is a solid angle it takes care of that proof we will see later, now I do not have time. So you multiply that pi by the lambda integrate from 0 to lambda alright, what do you get here? watts per meter square will be the emissive power of a blackbody, this you know is sigma T to the power of 4 for blackbody, we are still discussing blackbody right.

This result that it is =sigma T to the power of 4 was already known to all the people before Planck also, because it was verified by experiment, you have to just find out the emissive power of a body, and then no I am not defined blackbody yet right okay. I will do that but anyway this relationship for blackbody was known to people previously, but what is that value of B lambda which are multiplied by pi and integrated from 0 to lambda will give the sigma T to the power 4.

And what is that B lambda which will for all temperatures tend to 0 when temperature is 0, which will tend to infinity when temperature is infinity, which will tend to 0 when lambda =0, which will tend to 0 when lambda tends to infinity, this is a big puzzle. See many people many physicists worked on this and they were unsuccessful, because they were trying to explain everything through classical physics.

Then Max Planck found out an expression first in 1901, he found out on expression which agreed exactly with the experiments and proposed the Planck's law. And he worked on the proof for 15 to 17 years, then he figured out that for this to be constant for this to be true, e must be =h nu okay, that is how he got the Nobel Prize. Now before this we have to get, we have to define what blackbody is.

Please take down, a black body is one which absorbs all incident radiation regardless of its direction and wavelength. Coal and then hollow cavity, these are something which are close to blackbody, hollow cavity is like this, it is maintained at temperature t, there is vacuum inside okay, small opening is there. So there will be multiple internal reflection right, then the small the pencil of radiation which is coming out can be deemed to be a blackbody radiation at the temperature t.

This is how you stimulate blackbody in the lab okay. Coal will be having so this is defined by a quantity called emissivity, if emissivity is =1 we will say it is blackbody okay, rather a real body is the emission of a real body divided by the emission of a blackbody at the same temperature in wavelength. So these are all some of the examples, and then for this blackbody what is the I lambda for emission is the B lambda.

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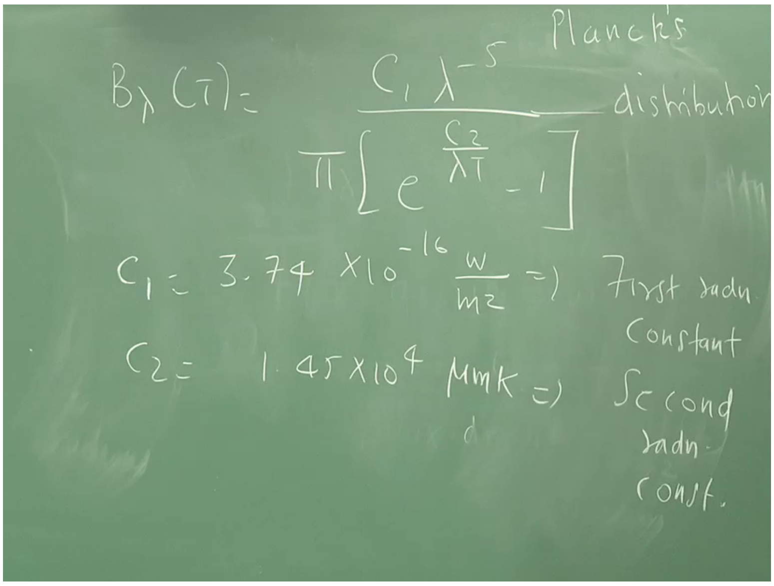

So of the various distributions only Planck arrived at the correct distribution, and Planck got this as, I think it is -16, this is called the first radiation constant, this is called a second radiation constant okay. Rayleigh–Jeans got a distribution for B lambda, which is C1 lambda to the power of -5/pi*e to the power of C2/lambda T, Planck added -1 and he got the Nobel Prize. He just added -1, when he added -1 his experimental curves and this model agreed very well.

So first it was curve fitting, he just added-1 and it worked. Then he worried why it work, then he tried all those, he took some of those atomic oscillators, if you take my radiative heat transfer course or you watch my YouTube lecture in NPTEL, radiative heat transfers into 2 full classes I will explain the derivation of this Planck's distribution from first principle, you have to take an atomic oscillator you have to take kinetic energy and potential, and you keep on doing.

And finally you will come to a stage where e=h nu, you find the total number of oscillators in a particular volume, and the total energy of all the oscillators in a volume, then total energy divided by total number gives you the average energy, then the average energy then you have 3 degrees of freedom and this thing from, so you borrow something from statistical mechanics, and then proceed, this is how you figure out okay.

So but this is asymptotically correct okay that and it satisfies all fundamental, and it satisfies all mandatory requirements what are the mandatory requirements? Lambda tending to 0, it has to tends to 0, if it tends to infinity it will become a singularity, then all physicist will concentrate on lambda tend to 0, and produce lot of energy right okay, same thing with lambda tending to infinity.

So when lambda tending to 0, and lambda tending to infinity, B lambda must tend to 0 regardless of the temperature. Then when T=0, T tends to 0 lambda, B lambda must tend to 0, that is intuitive. And when T tends to infinity B lambda tends to infinity, and finally if it is integrated with over pi and then B lambda 0 to infinity it was resulted in the Stefan-Boltzmann law. So it is an inverse problem, what is that B lambda which will satisfy all this? and which will agree with experiments.

So answer to this was finally given by my explain. Now if you look at this if you plot this it is very interesting.

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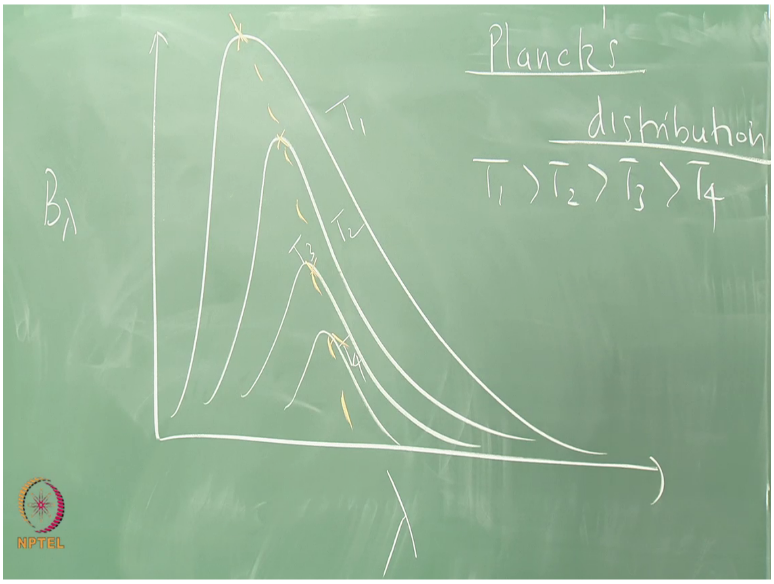

So B lambda versus lambda, this is like this, this is experimentally obtained or you can take it as a Planck's distribution. Please take down, features of the Planck's distribution, number 1: B lambda of T continuously varies with lambda for every d okay, or is a B lambda varies continuously lambda for every d whatever. B lambda varies continuously lambda, so it is that means I am saying it is not a discontinuous function.

Number 2: for every temperature there appears to be a peak in the distribution, for every temperature there is a peak value of B lambda, or there is a value of lambda at which B lambda is a maximum. 3: this maximum decreases with temperature, that is what I have indicated right, the locus of the points will be a straight line, we are going to do that proof a little while. Now 4th point: at a given lambda B lambda increases with temperature correct.

If it is not so it violates which law of thermodynamics? It will violate the second law of thermodynamics, what does it mean? As a consequence of 4 you can write within brackets, no 2 curves can cross can intersect each other, I mean no 2 curves for 2 different temperatures can intersect each other, higher temperature curve will be above the lower temperature correct, no 2 curves will intersect each other okay.

Now quick question, please multiply this by pi and all that, what will be the area under the curve area under each curve? Sigma T to the power of 4 by pi correct, if you multiply by pi it will be sigma T to the power of 4. So the area under the curve gives us Stefan-Boltzmann, so this is called the Stefan-Boltzmann law, we will see that little later, Joseph Stefan was an Austrian professor, Boltzmann is a German PhD student, they figure this out using thermodynamics.

And then they use experimental data to get the value of sigma, but without quantum statistics. Max Planck came later, and then figured out, so you can see that in radiation the contribution of Germans is very, very high, the development of radiative transfer okay. Now we will have to do something about this orange line, do not wait for me find out the peak of every curve. So differentiate your B lambda, and find out where it becomes stationary.

We lost track of the equation number, what is this equation number let us used no not, one from the previous 1, 7 okay 8 okay.

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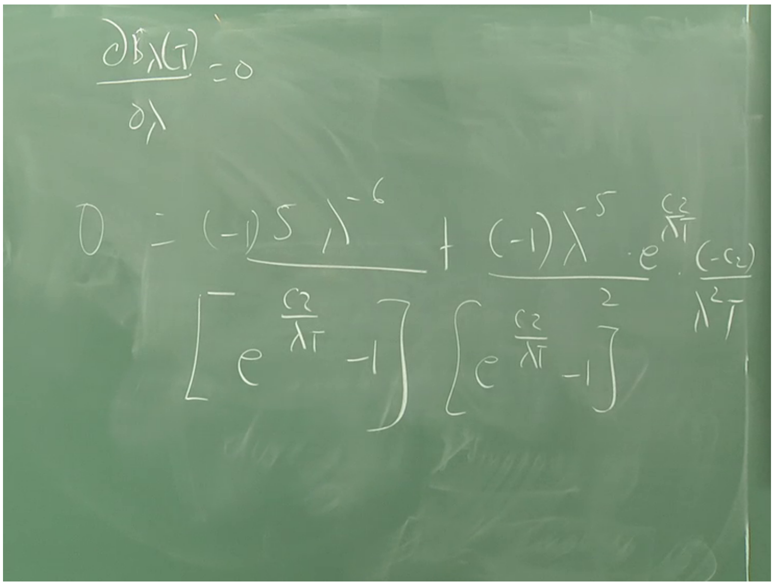

So first term is -5 lambda- blah blah, I leave the pi is -1 lambda is that okay.

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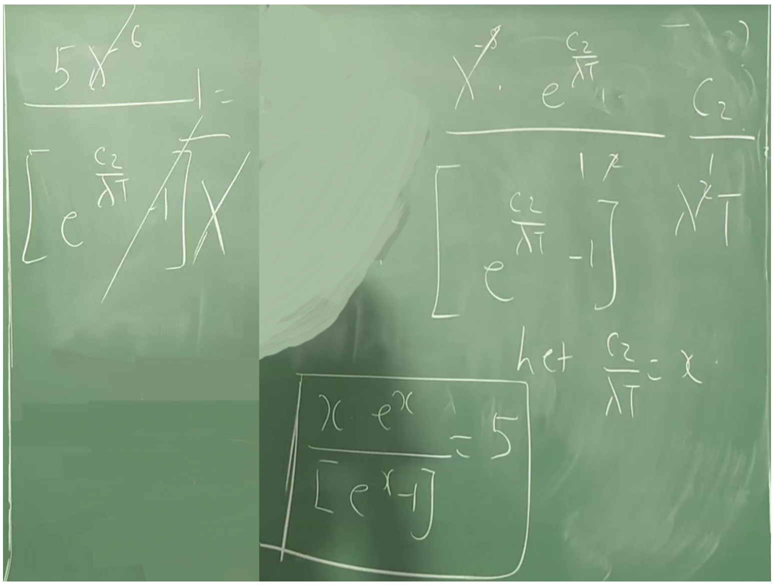

How many of you are still doing? We will just give you a minute, is just a partial differentiation okay.

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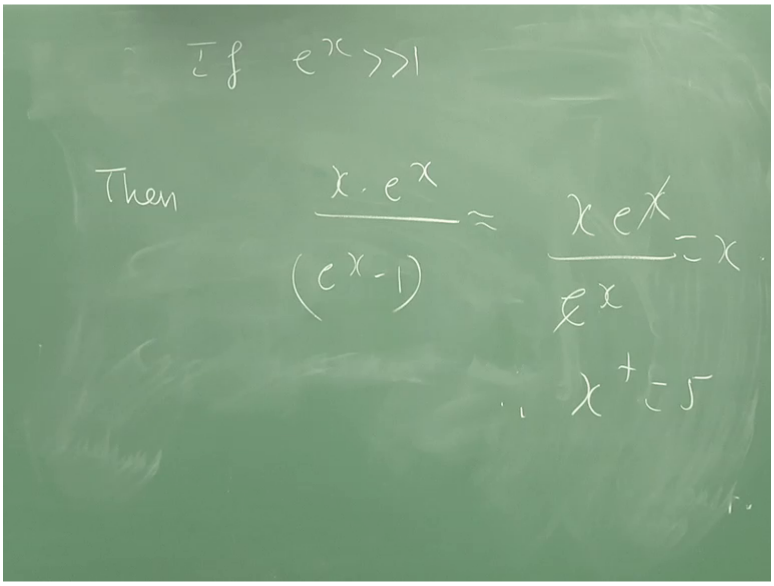

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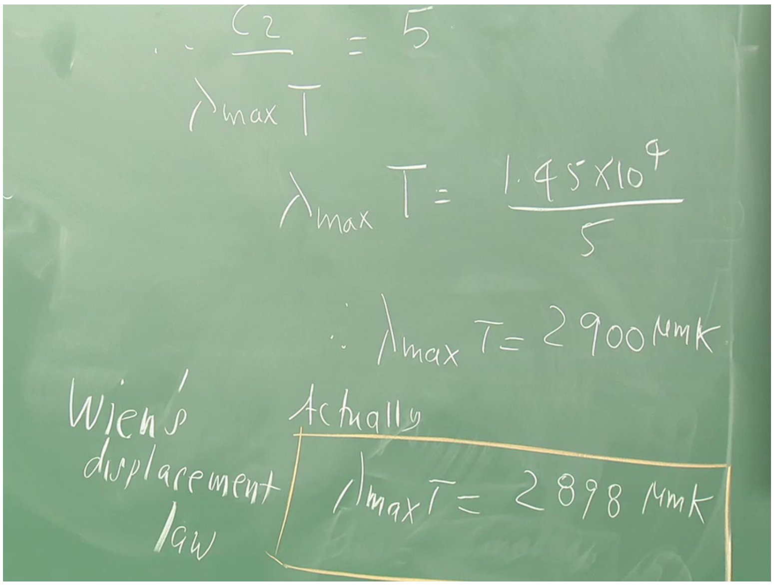

Now, but if you solve it numerically, what was C2? 1.45 10 to the power of 4. How much is this? 2900 okay. So this is called the Wien’s displacement law, so Wien is another German professor he also gave a blackbody distribution, which was correct only in one part of the spectrum. Rayleigh–Jeans gave a distribution which is correct in the other part of the spectrum, but Planck gave a distribution which is correct in the all the parts of the spectrum okay.

Now as T increases what happens to lambda max? Decreases, alright, that is why with increasing temperature the lambda max tends to degrees, which was evident from the distributions we plotted okay, this is one thing. The sun's temperature is about 5800 Kelvin, what will be the lambda max for the sun's radiation? 1/2 which is the visible part of the spectrum okay, that is why the sun's radiation is very important, it gives that cool daylight.

Now the earth is at a temperature of let us say 280 Kelvin, 289 let us say, it is 255 depending on which one you will be doing all that, let us take 289. Therefore, lambda max will be 10 micrometers okay, this will be in the infrared part of the spectrum. And is greenhouse gases absorbed the radiation in the infrared part of the spectrum, whereas they allow the radiation in the shortwave or in the visible part of the spectrum.

So these greenhouse gases are this water vapour, carbon dioxide and methane and all those gases, so when you can continuously burn this fossil fuels, incoming they allow and outgoing they do not allow. So this leads to a positive feedback which leads to what is called a radiative forcing, and which upsets the equilibrium temperature of the earth. This also leads us to the concept of colour okay, so today I am wearing some hopefully it is a blue trouser.

What is the wavelength of the blue? 0.5 okay, 0.4 is violet right, let us say 0.5 if you put 0.5 what is the temperature 6000, I mean it is like what will happen to me with 6000 Kelvin. What is this colour then? What is colour, what is the colour which you see? The colour you see is not emission, the colour you see is the reflection, whatever colour you see is the refraction it absorbs all the colours but this gives out blue colour okay.

But if you want to see real emission colour in your workshop you heated that iron piece and then it becomes orange in colour in smithy shop in forging shop that is an emission temperature. Otherwise, whatever you see as colour, the Chaitanya’s t-shirt is bright yellow, because it reflects yellow colour. If nothing is coming out, then it will be black that is why it is the blackbody. so black is not a colour black is the absence of colour.

So that is the deep philosophical concept, black is not actually black is not a colour okay. But our eyes are capable of detecting only from detecting only from 0.4 to 0.7 okay, so we cannot judge that body to be radiatively black based on 0.4 to 0.7 for all you know <0.4, and >0.7 it may not absorb okay. So it has to be radiatively black for 0 to infinity lambda, you should absorb all there, but when it absorbs in 0 to infinity guaranteed that 0.4 to 0.7 also will absorb.

Therefore, radiatively black object will be a visually black object, but a visually black object need not be a radiatively black object. For example, snow with some white colour is seen, it is a high absorber, for most part of the spectrum its absorption is so high, and by virtue of its emissivity it is 0.9597 and so on. So blackbody means you immediately think Asian Paints or this thing blackboard or that you conjure a visions of something being very very dark right okay.

So that is basically absence of colour was denoted by black. Next question comes why is umbrella black, all these questions will come to you, whether it is good to have white umbrella or black umbrella, popular science question you answer, let us not answer in an atmospheric science course alright, I do not know you figure out why should the umbrella be black. We will solve a problem and then close.

Problem number 44: Use Wien's displacement law, Marius I am pronouncing rightly. Use Wien's displacement law to compute the colour temperature of the sun, for which the wavelength of the maximum solar emission is observed to be 0.475 micrometer, no degree Celsius. let us get out kelvin.

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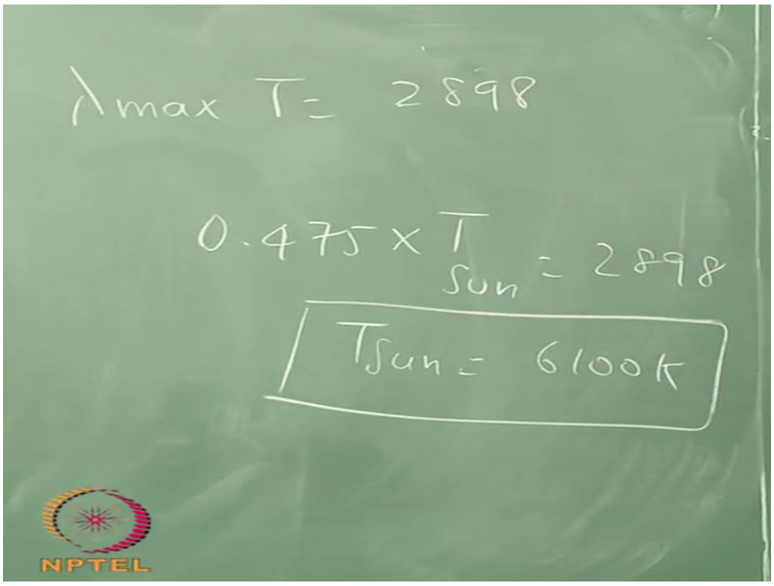

$$\lambda_{max} T = 2898$$

$$0.475 \times T_{sun} = 2898$$

$$\boxed{T_{sun} = 6100 K}$$

44 actually the sun's blackbody distribution does not correspond perfectly to black body, it is likely jagged, so it is approximation. That is why the actual sun temperature is 5800, it is not the core temperature of the sun where it is the photosphere that is outer portion of the sun, this is about 5800 Kelvin. So it is approximately is 6000 Kelvin, but the tube light is working at tube light is low temperature.

The incandescent bulb is working at 3000 Kelvin, the tungsten filament bulb is working at 3000 Kelvin, so the peak radiation is occurring in the infrared, so it will produce heat and light. The LED, they say cool LED because it produces more light than heat okay, for technology is completely different. As you know the Nobel Prize has gone to the people to figure out the LED bulb okay, so this is called the colour temperature.

So the colour temperature is based on the emission, so my pants colour temperature will be something okay, but now it is reflection, so it is the same temperature as me, assuming that it reached equilibrium okay alright. So in the next class we will see what happens when you integrate this Planck's distribution from 0 to infinity, Stefan-Boltzmann law, we calculate the sun's temperature, we calculate the earth’s temperature.

I will give you all this geometry solid angle and all that. And then we will proceed the emissivity, absorption, transmissivity. Eventually, we will lead to, we have to go to a stage where you have to consider absorption emission and scattering in the atmosphere, we should be able to find out the variation of the intensity in the atmosphere alright.