Introduction to Atmospheric Science

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Lecture - 26
Static Stability-Brunt-Vaisala Frequency

Okay, so Good Morning. So since yesterday's class we started looking at static stability. The stability of an air parcel if it is pushed from its original position as you all know this is got to do something with the buoyancy force and the buoyancy or the difference in I mean the buoyancy force which is related to the difference in densities. The difference in densities are related to the difference in temperatures and so on.

And so the difference in temperature is brought about brought out by the difference in the lapse rate. Are you getting the point? The difference in temperature is brought out by the difference in the lapse rate. Why is there a difference in the lapse rate? The air parcel will follow the gamma d. The air will follow gamma. The air parcel will follow gamma d till it is saturated. After that it may follow gamma s.

That is why when you consider static stability you have to first subdivide into static stability for unsaturated air and static stability for saturated air. In yesterday's class we looked at static stability for unsaturated air right. So please recall this diagram. For the sake of completeness I will go through this again.

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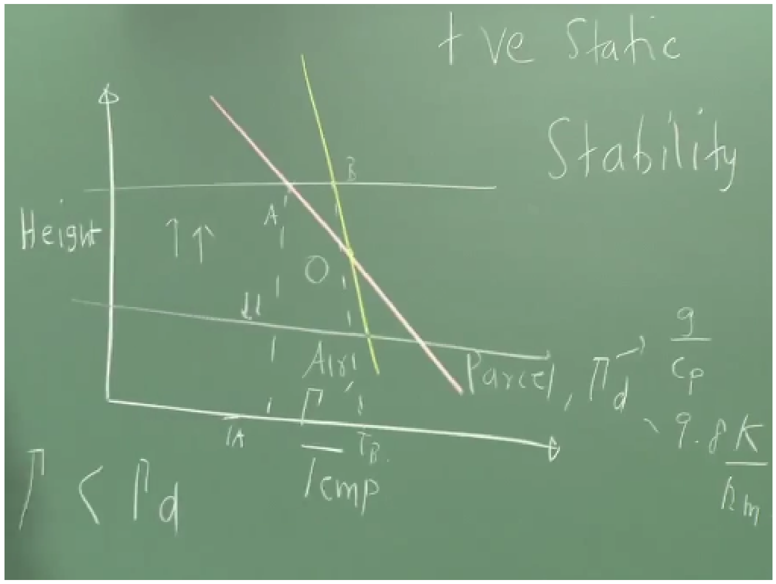

This is the height. There is the point O. The air parcel is at its equilibrium initially at point O and this was the parcel correct, this was air. So the parcel has got a lapse rate of gamma d. The air has got a what is generally gamma d value? Gamma d is 6? 9 point c g by cp correct. So approx 9.8 K/km. This may be 4, 5, 6. This cannot be obtained from theory. This has to be obtained by measurements. A good way to get it is from radiosonde.

We are now trying whether from satellites also we can get it. In fact one of the projects which I am doing for INSAT-3D is to get this gamma using a infrared sounder and then solve the radiative transfer equation and then using the radiative transfer equation find out what will be the radiance to be captured by the top of the satellite. Keep on correcting the temperature profile till such time you get a temperature profile.

Which when substituted into the radiative transfer equation will give a radiation intensity at the top of the atmosphere as picked up by the satellite which will be closer to the measurements. This will not be done in one channel. This will be done in several channels. This is called passive remote sensing. We will look at it in little more detail when we actually go to the chapter on radiative heat transfer of the atmosphere okay. So this is O and then we thought we saw took a case where this parcel is lifted up and this was A, this was B correct.

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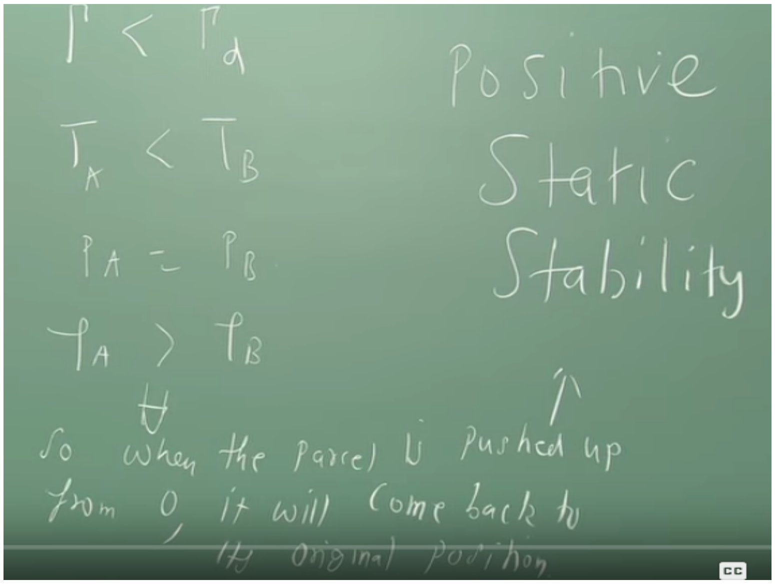

So gamma is less than gamma d. Though the yellow line looks steeper it is a height versus temperature okay and then T. So if the parcel is lifted up from O, left to itself will come back to its original position okay. So it is so when the parcel is pushed up so this is called positive static stability okay. So the restoring force I will clean this up. So gamma < gamma d?

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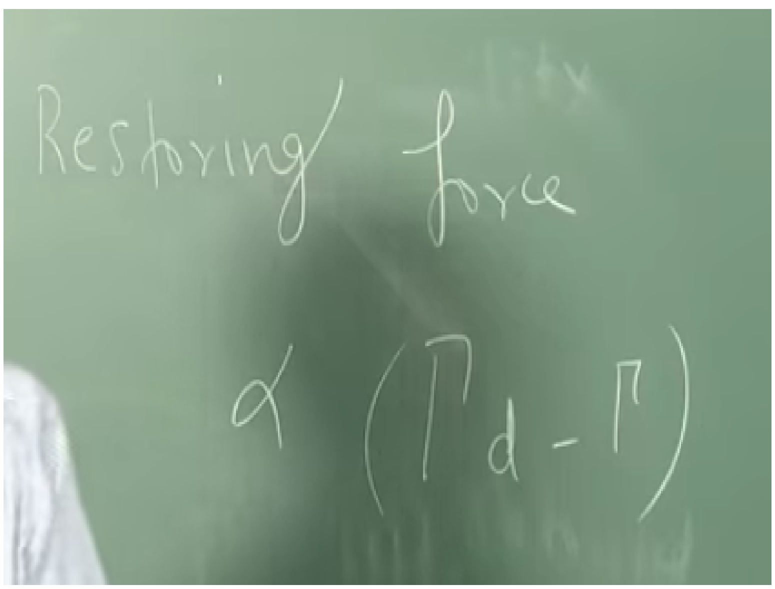

Greater the gamma d minus gamma, greater will be the restoring force which will bring it back to O. Are you getting the point. So this is very important. Okay, now for the sake of completeness we will have to draw the second figure where it is negative stability. Instead of negative stability we can call it instability okay. Please draw that figure. Please draw the, so it is negative static stability or simply instability.

So this is okay by the same token people who missed yesterday's class by the same token instead of pushing it up we push it down. Then what will happen is this T C will be less than T B, T D and then you can have the same arguments and then actually it will become denser or something or lighter. It will become lighter. It will go back to its original alright. So whether it is displaced pushed up or pushed down it will always come back to O.

So there is a possibility that it can oscillate around O and we will have to find out the natural frequency. That natural frequency as I told you yesterday is named Brunt-Vaisala frequency in honor of the 2 scientists, Brunt and Vaisala. So actually the radiosonde balloons are called Vaisalasonde because this family started the company and then they are now that Vaisala company it is in the Scandinavia so Vaisala Sonders are popular radiosondes are popular all over the world okay. Now I will use this part of the board.

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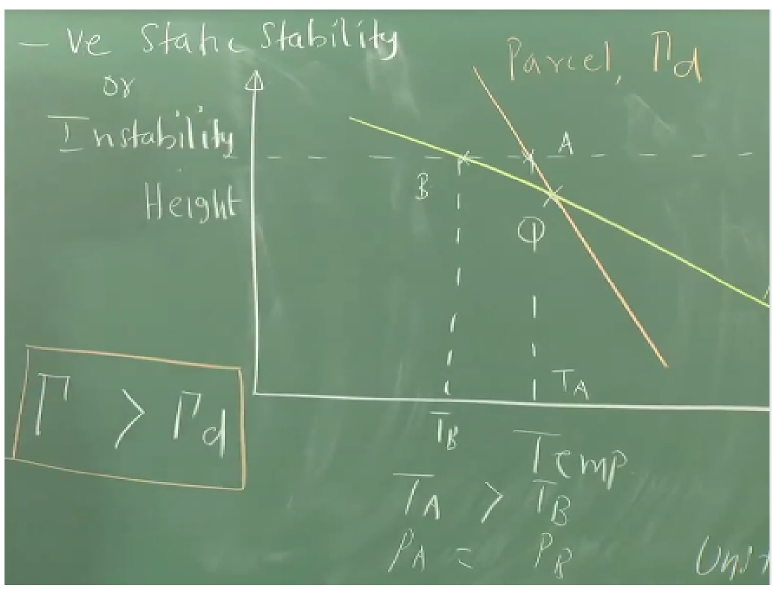

Height okay. So initially both the parcel and the air are at equilibrium at O. We are pushing the parcel little up. A was always the temperature of the parcel correct. This is okay. Rho A. So the parcel will go out of control. We push it up it will try to go up right okay. So unstable parcel. So negative static stability or instability alright. Any doubts so far okay. Problem number next one 39. Problem 39. Please take it down.

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An unsaturated air parcel has a density of rho plus, temperature T plus and the pressure and the density and temperature of the surrounding air are sorry an unsaturated air parcel has density rho plus and temperature T plus and the density and temperature of the ambient air are rho and T respectively. Derive an expression rho plus, T plus rho and T; rho plus, T plus for parcel, rho and T for surrounding air.

Derive an expression for the upward acceleration of the air parcel in terms of so you need to get the, figure out the expression for acceleration. The variables are z is the height. So d z by d t will give you velocity; d by d t of d z by d t will give you acceleration which can be taken as d square z by d t square. So do the force balance, get an expression for the use the ideal gas equation whatever you want hydrostatic equation whatever you feel is relevant.

Use all this and get an expression. Then we will proceed to problem 40 where we will solve the ODE and get the frequency. That frequency will be the Brunt-Vaisala frequency. So we are considering this situation, no some situation does not matter.

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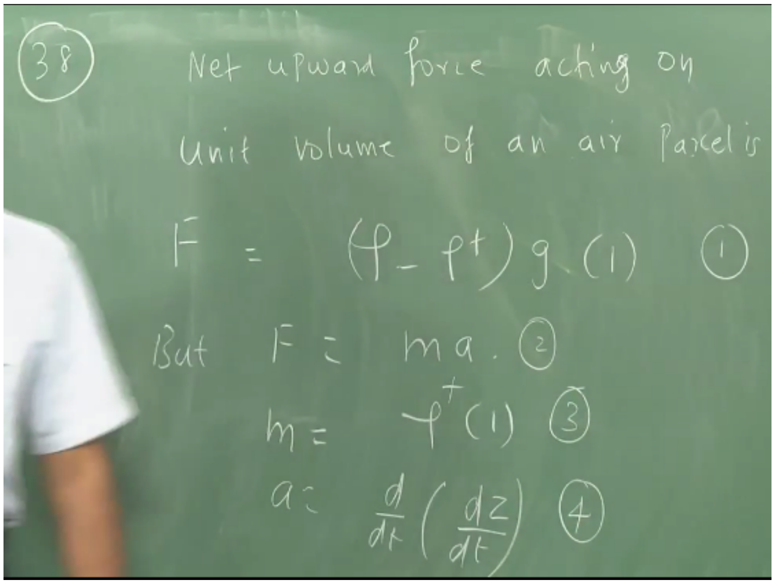

Let us look at net upward force acting on unit volume of an air parcel. F equal to rho v is mass into g is weight which is the force alright. So 1 is the volume. Did I make a mistake? Rho or rho plus? Watch out because we are looking at the parcel okay. Is it okay?

“Professor - student conversation starts” Sir if the first one is so the next F equal to ma is, it is supposed to be m by rho right. Why m by rho. The first one is force acting on per unit volume. Ya I multiplied no into 1. It is fine. Dimensionally it is consistent. **“Professor - student conversation ends”.**

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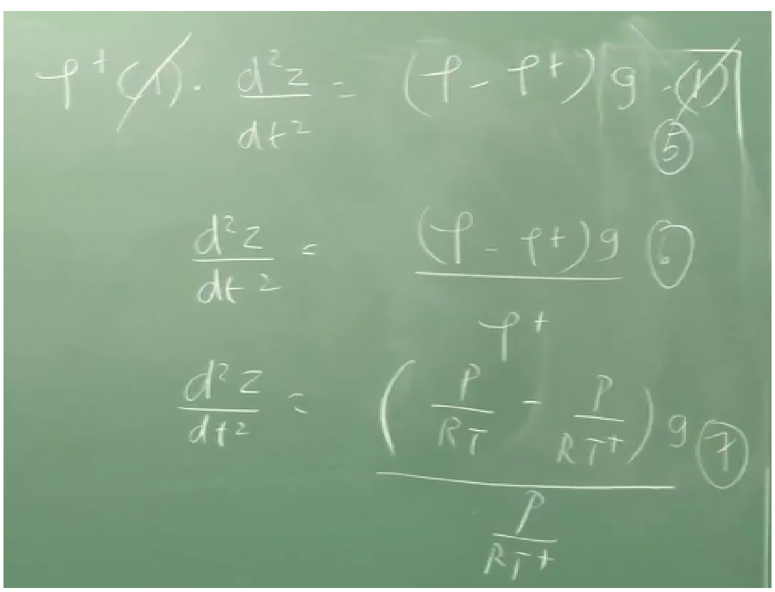

Hey that 1 and 1 is getting cancelled right, so what is your problem? We can make it 2, 3, 4, 0.1 whatever you want. Now it should be small that is important right because otherwise you cannot lump the properties right. Please remember from the definition of an air parcel the pressure in an air parcel adjusts to the pressure of the surroundings. Temperature will not adjust okay. Is it okay.

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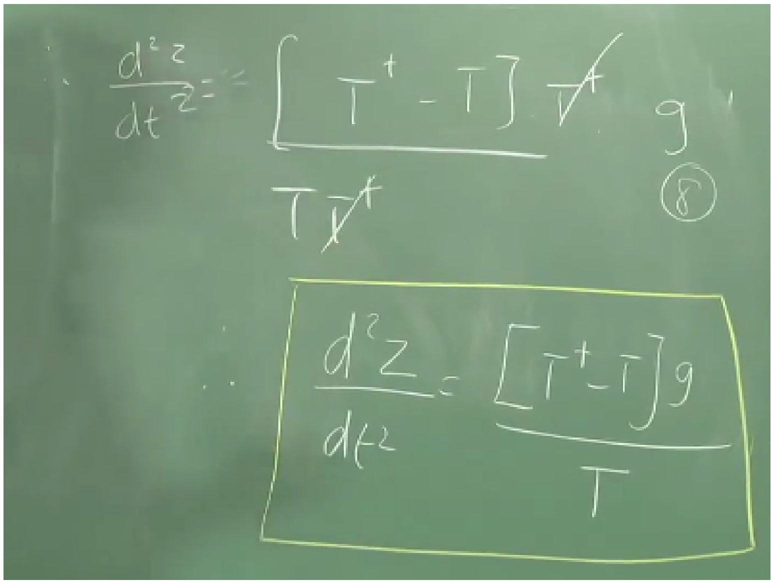

So this simple expression which gives the acceleration of an air parcel. Greater T plus - T which is a consequence of gamma d - gamma that will be more will be the force alright. So strictly speaking instead of T what should be used? Instead of T T v should be used, the virtual temperature should be used but we already saw that virtual T v - T is only 0.1, 1, 2, 3 K. So for all practical purposes the T v can be approximately equal to T and then we will use the because it is more difficult to work with the virtual temperature.

So we will work with the actual temperature alright. So problem number problem number 40. The air parcel discussed in problem 39 is now displaced, the air parcel discussed in problem 39 is now displaced upward from its equilibrium position by a distance by a distance z plus from its equilibrium position z not okay is displaced upwards from its equilibrium position by a distance z plus from its equilibrium position z not.

That is z plus = z - z not okay to a new level where the ambient temperature is capital T to new level where the temperature is T. The air parcel is then released to a new level where the ambient temperature is T. The air parcel is then released. Derive an expression to new level where the ambient temperature is T, the air parcel is then released.

Determine sorry derive an expression that describes the subsequent derive an expression that describes the subsequent vertical displacement of the air parcel as a function of time. Derive an expression that describes the subsequent vertical displacement of the air parcel as a function of time in terms of in terms of T, gamma, and gamma d. Try this. Okay so in the interest of time I will just wait just for a minute.

You just start and let there be a phase lag I will start working. When you do not follow me you will just crosscheck that is all. Right you can start now.

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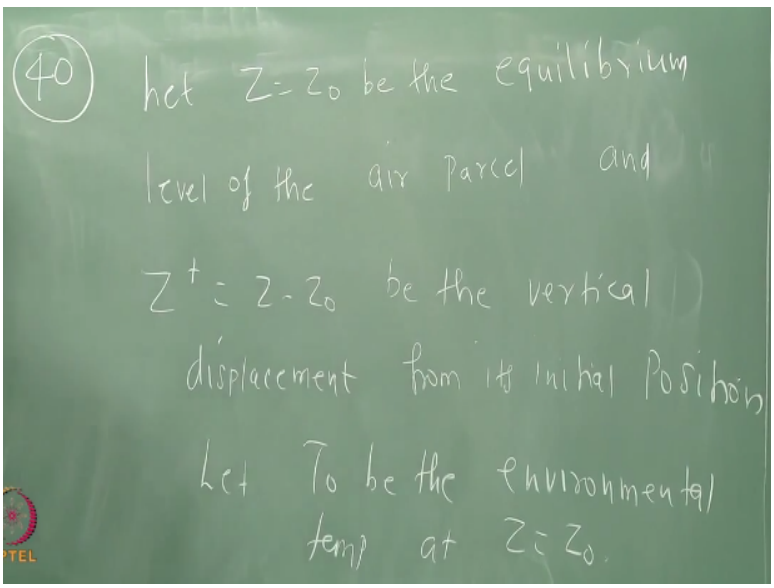

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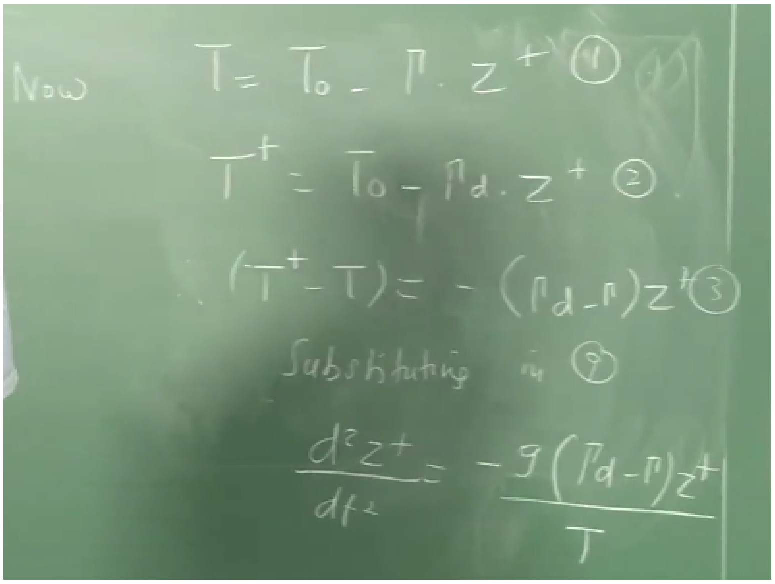

Now, T not - T is the environmental temperature. It will follow, what is its dharma. It will follow gamma or gamma d? gamma, gamma into let us not complicate d z. We will keep it as z, z plus I said z plus is okay. In my notes I have z dash, does not matter. Z plus fine z plus. Okay this will be you want to keep it as T plus or T, T plus will be for the parcel minus. Therefore t plus - T equal to I want to use it here. Are you able to see the logic.

I want to get there. Therefore, what is this now? Very good. D square z by d t square is the same as d square z plus by d t square correct? If I change it to because z not is constant okay. No problem right. I can change the variable. Therefore, I substitute it, I will substitute in 9. Minus g by what do you get, T? By T. I keep the T no problem. Okay so I have taken it as plus okay we will bring it to the other side.

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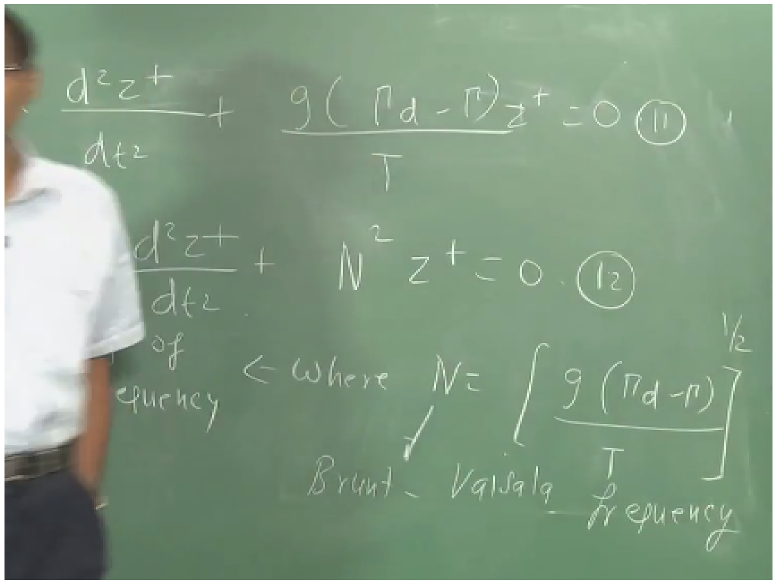

I call it as N square z plus where N square is this okay. Check the units of N. Please check the units of N. It will be radian per second. Is it 1 by second or. Got the units of frequency. So that N is called the Brunt-Vaisala frequency okay. So N. I will give you some tidbits on Sir David Brunt.

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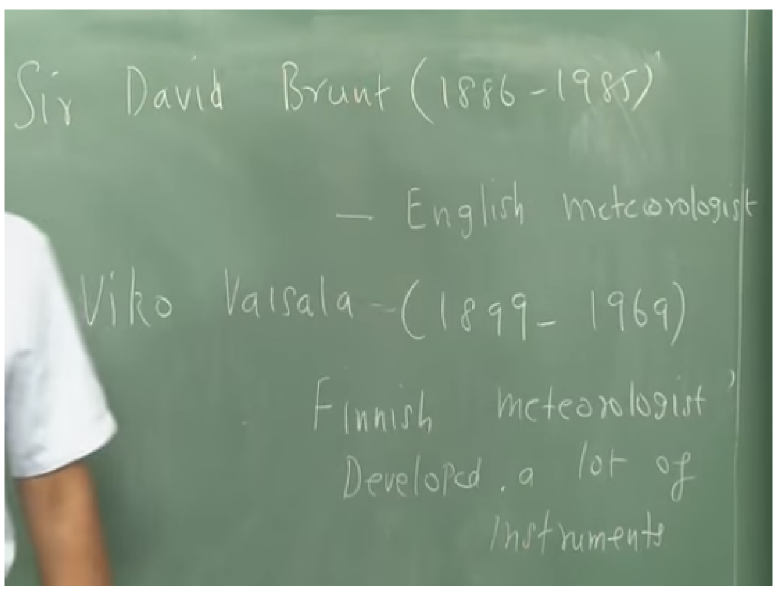

Is English meteorologist, Sir David Brunt. So he was a Finnish meteorologist who developed lots of instruments. So it is called the Brunt-Vaisala frequency. He lived too much? What is your problem? Almost there no. He might have registered for the course or he might have offered the course. This will be edited. Okay. Equation 12 is what mathematically. Is a second order ordinary differential equation. Please note it down. Equation 12 is a second order ODE.

Equation 12 is a second order ODE and equation 12 is a second order ODE and if gamma d is greater than gamma N will be real okay. If gamma d is greater than gamma that is gamma is less than gamma d which means positive static stability. If positive static stability condition is satisfied, gamma is less than gamma d. Therefore gamma d - gamma is positive. N is real. Please find the roots of the equation.

Please find sorry not roots please find the solution to equation 12. For the case of real N please find the solution. You require 2 initial conditions because it is second order. So I at T equal to 0 z plus is z plus of 0 and initial velocity 0. At T equal to 0 z plus is Z plus of 0 right. I will give you the conditions.

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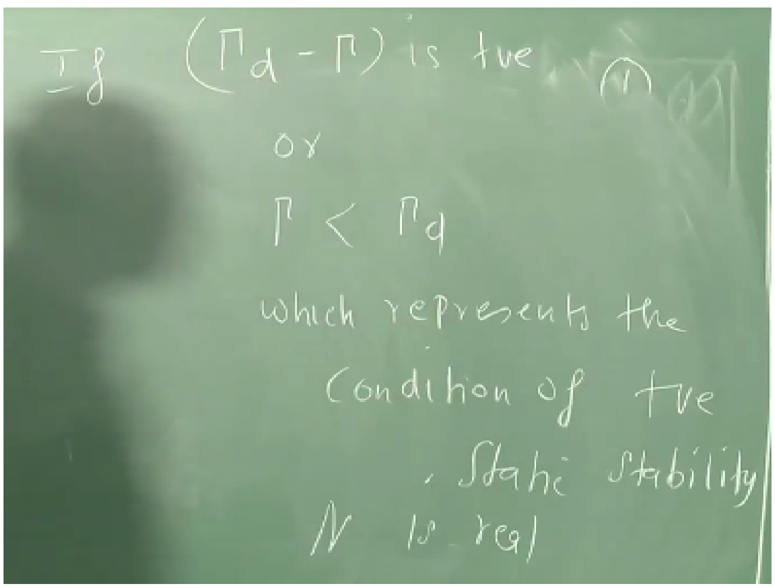

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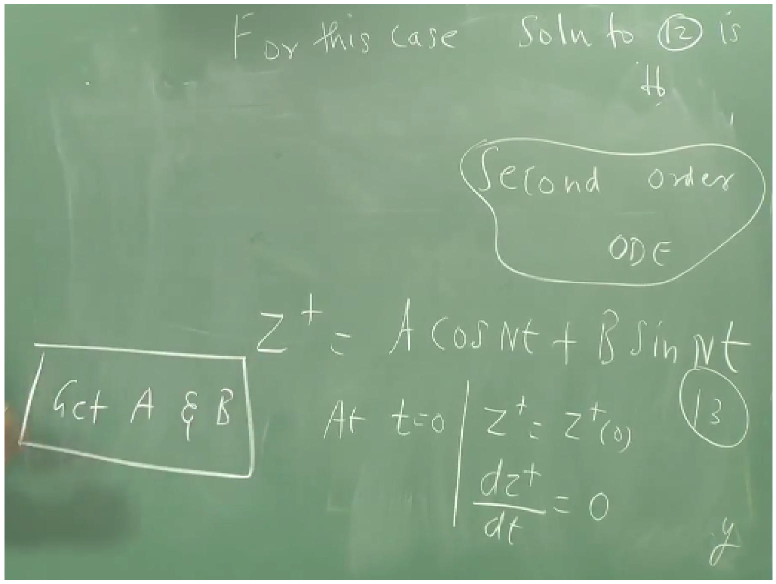

Board is moving okay. What is the solution? Agree, correct? Now you have to get the 2 constants. Initially you are giving only displacement but velocity is 0. Please find out A and B. I will give you 2 minutes. Please get A and B. So I will write. You can show this. Please get A and B. People who have got it just hang on for 2 minutes. First you should recognize it is a periodic solution okay and N is the frequency, characteristic frequency.

So this air parcel will oscillate with a characteristic frequency. That is decided by the value of N and N of the many things, N is critically N is dependent on g, rho d minus rho, and T; g is not under our control. Therefore, gamma d - gamma and T. T also will be 280, 290. So that gamma d - gamma will be the singularly most important factor which will decide okay the response of the system to an initial disturbance.

And greater the value of N greater will be the stability. It will get restored. So the Brunt-Vaisala frequency is a characteristic measure of the stability of the air parcel okay. So the Brunt-Vaisala frequency is a good measure of the stability. Greater the Brunt-Vaisala frequency more stable it is. Is it okay. Now let us complete this.

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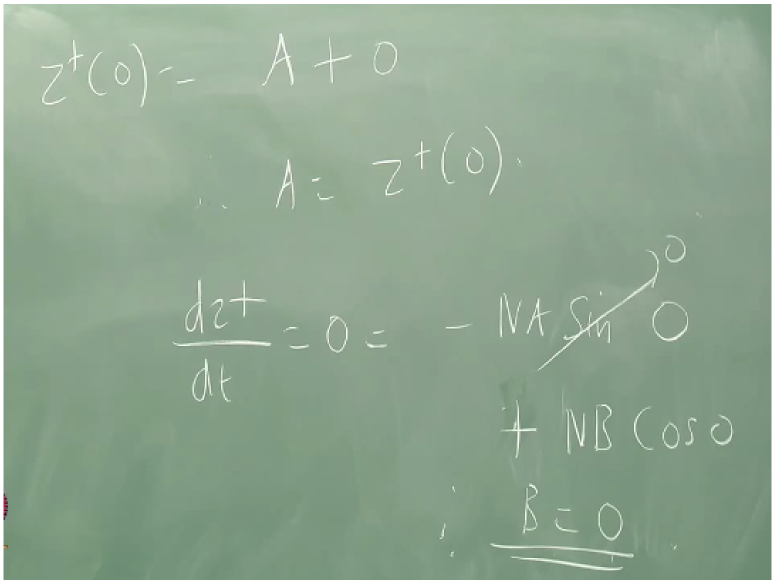

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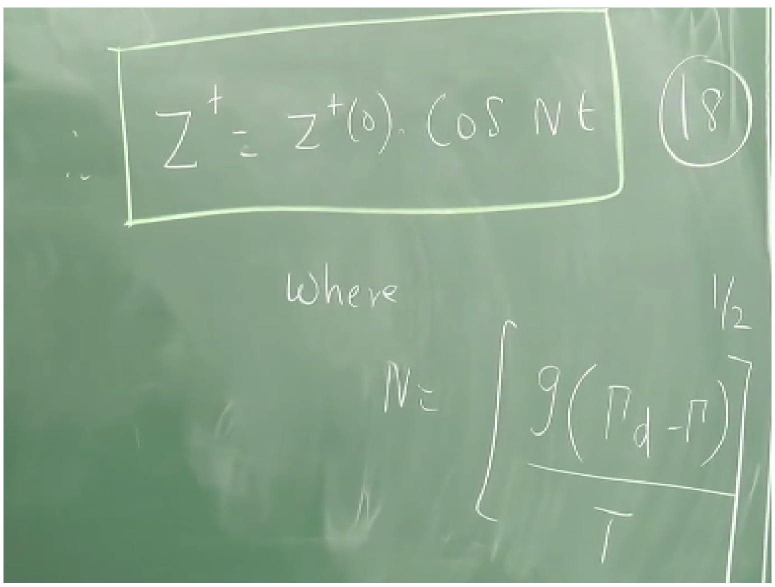

Therefore, if an air if an unsaturated air parcel is originally at its equilibrium level O, it is pushed and is given a displacement, initial displacement of z plus of 0. Then it will execute oscillations, periodic oscillations with a frequency N given by g, gamma d - gamma by T whole to the power of half which is called the Brunt-Vaisala frequency and an amplitude which is given by the initial displacement.

Higher the Brunt-Vaisala frequency greater will be the stability of the air parcel. So this gives an gives us an idea about the stability of the air parcel okay. So we can say. So in the next class what we will do is we will take a layer of air which is flowing through some mountains and I will give you some gaps the characteristic length between the mountains then you can find out what will be the characteristic time period and the frequency.

Then if there is some disturbance in the air I will give you also the lapse rate and all that. You find the Vaisala frequency. If both are matching then it will lead to resonance okay. We will see what will be the velocity at which it will create some trouble. Then we will move on to saturated air parcel what will be the condition for stability and then we cannot stop with this.

Once it has crossed the lifting condensation level whether it will be convectively stable, unstable, partially stable, neutrally stable there are other conditions, after it crosses LCL it becomes more and more difficult. So more and more intricate. We will see that in the next 1 or 2 classes. Then we will have to go to the second law or thermodynamics. Quickly go through the Kelvin-Planck and Clausius statement.

After that we have to get to the Clausius-Clapeyron equation which will give you the relationship between latent heat and all that okay. After all that some considerations of entropy then we will put an end to this thermodynamics chapter which has been going on for such a long time and then you feel that forever it is going. I also feel the same way. So because this is the most interesting part but we will have to stop somewhere.

Then we will go to radiative heat transfer 2 weeks. Then climate change and climate science 2 weeks and this will be just a week or so the, we have only 5, 6 week. The dynamics will be so we will already we are in 26 or 27 class. We will go to 40 or 42 and then your presentations will be there.