Introduction to Atmospheric Science

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Lecture - 23
Equivalent Potential Temperature and Wet Bulb Potential Temperature

Good morning. So last couple of weeks we have been looking at moist air thermodynamics. We have been defining quantities one by one. We are and we are solving problems of graded difficulty and graded complexity. So now you are in a position where you can get the lifting condensation level. You can find out the potential temperature, you can find out the dew point, you know what the wet bulb temperature is and so on.

With all these definitions we should be able to solve problems where some atmospheric processes are involved. There is an air parcel which is going up. Then it is condensing. Then it is coming down. It is coming from the left side of the mountain. There is a wind. It goes up to the mountain and then it reaches some height. It sheds its moisture. It comes from the other side. It comes back through the other side of the mountain.

It is descending and what happens to the temperature; whether the temperature on the windward side and the leeward side of the mountains are one and the same. This will lead to various phenomena and then meeting of the cold front and the warm front which leads to these are called frontal (()) (01:13) to several interesting atmospheric phenomena and so on. So we are not yet done with our definitions of thermodynamic quantities.

So we will have to look at 2 very important quantities in today's class. We will look at the formal derivation of this. Then we will see how to get these quantities on the Skew-T ln P chart. Then we will turn around and then solve one or two problems where we will start from first principles and calculate all the quantities up to what we discussed today and in the next class in Tuesday's class we will actually solve.

We will just take up an atmospheric phenomena and from our knowledge of moist air thermodynamics and the expertise you have gained in using this Skew-T ln P chart with minimum request to formulae and maximum use of this Skew-T ln P chart can we solve problems involving atmospheric processes through the use of charts alone okay and then after you use the charts and all that you will get some values.

For example you can get so many g/kg moist this dry bulb wet bulb temperature and dew point and so on okay. So the quantities we are going to discuss today are the equivalent potential temperature and wet bulb potential temperature okay.

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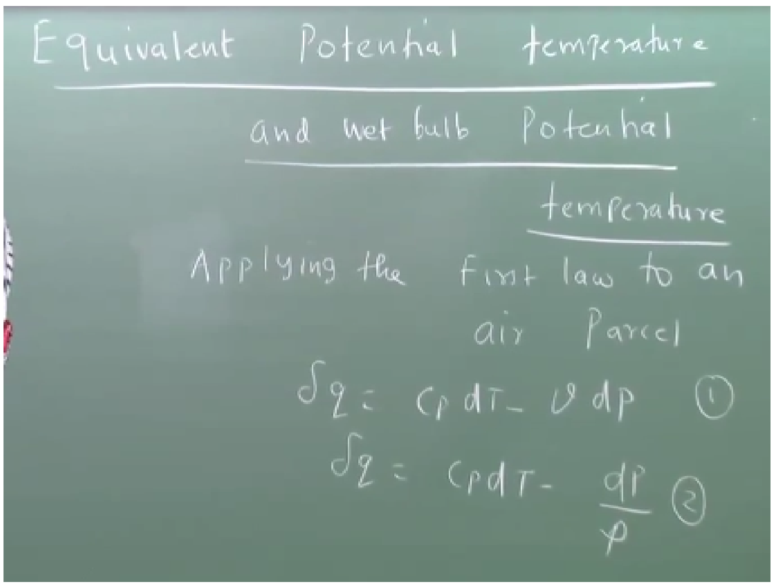

The next 2 important quantities are sir we already saw potential temperature why are you saying equivalent potential temperature. Good question but the answer is the potential temperature looked at dry air but now we have graduated. We have started looking at moist air. So if there is an adiabatic process involving moist air then theta equal to constant will not do. There is some other line okay. That line you already know.

Those curves already we saw yesterday but now we will go through a formal derivation. So watch this carefully. Applying first law correct the net heat added is equal to work done and V dp correct, is this correct? How about this, is this alright? I used the ideal gas equation and replaced P in the denominator sorry replaced rho in the denominator okay.

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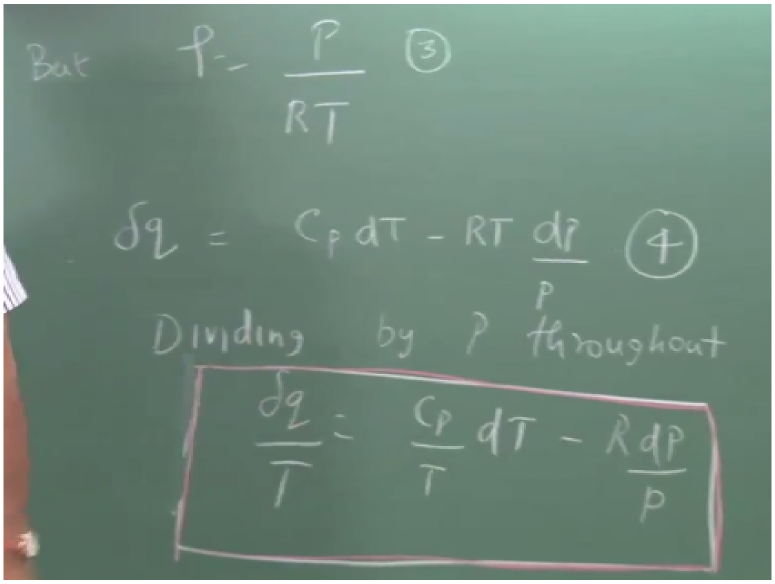

It is quite alright is it not? I want to divide by P throughout. So keep this result.

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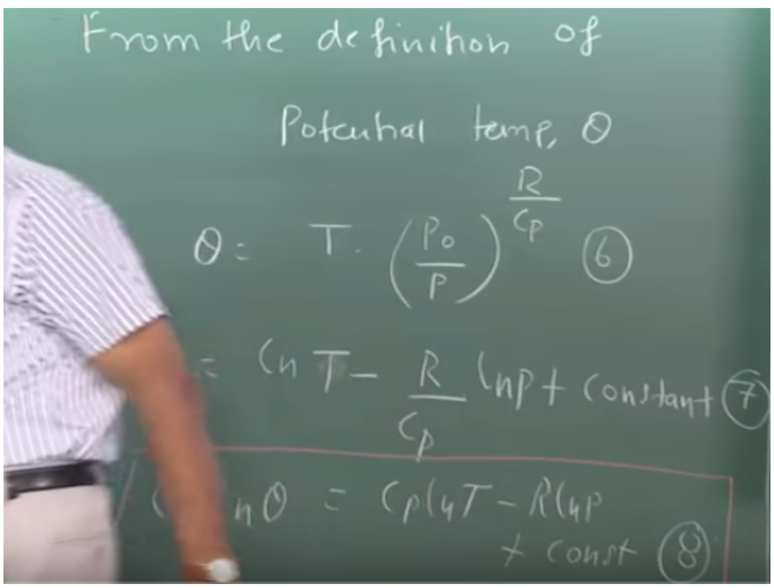

Then to the power of. Take ln both sides. Is this correct? It is quiet alright. That constant is some P not into R by something right. - R is correct right. - R by C p ln P is correct because P is in the denominator. I want to multiply by C p throughout. For the sake of completeness let me write taking ln both sides, taking ln on both sides okay so we are nowhere near equivalent potential temperature but whatever we know we first write.

Knowledge always proceeds from the unknown to the known right. So whatever we know about this air parcel we keep on writing. We apply the first law, we apply this potential temperature all this and keep this milestones and then we will look at the and the last but one step we will invoke the definition of equivalent potential temperature and pull out whatever we want okay. Now I will multiply throughout by C p. This can be some other constant.

Do not bother about this. So far so good. There is nothing new we have discussed so far which we have not seen in earlier part of the course. But now, I want to introduce an arbitrary quantity and we want to see its variation and then looking at the variation of the arbitrary quantity which involves the latent heat of vaporization and the saturation mixing ratio it lead to some intermediate steps.

Then we will try to connect up to either equation 3, 4, 5, 6, 7, 8, 9. Keep all these results and then we will mix and match and we will do a kichadi and finally we will pull out the equivalent potential temperature. Is it okay?

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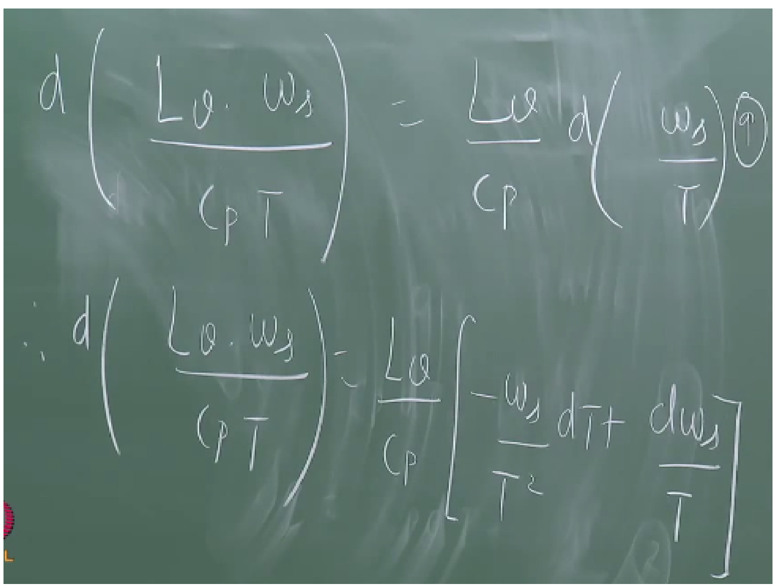

I want to look at the quantity d of the latent heat of vapourization into the saturation mixing ratio. What does it give actually? Leave the d off. What is the numerator? Latent heat right. kJ/kg into kg per kg. So for 1 kg of air, what is the latent heat? Denominator is specific heat, some energy content. It may not be you will argue with me C p delta T sir. But C p T is the amount of heat stored or something like in the form of sorry sensible heat.

So this ratio we want to look at. Now since we are considering small ranges of temperature and the processes are not taking place over wide ranges of temperature, I want to pull out L v and C p out of this. Can I do that? Now please pull out the partial derivatives from this. Please write the, please expand this. I will start with though it is not an equation. First term, - omega s by T square d T correct. Second term, d omega s by T.

Now, I will have to do an order of magnitude analysis to find out whether both the terms are significant or one of the two term is of leading order. That is one term is 9, other term is 1 or one term is 9.5 the other term is 0.5 so that we can remove one of the two terms so that we can proceed further with the analysis because it may lead to some simplification. Therefore, I want to look at this quantity.

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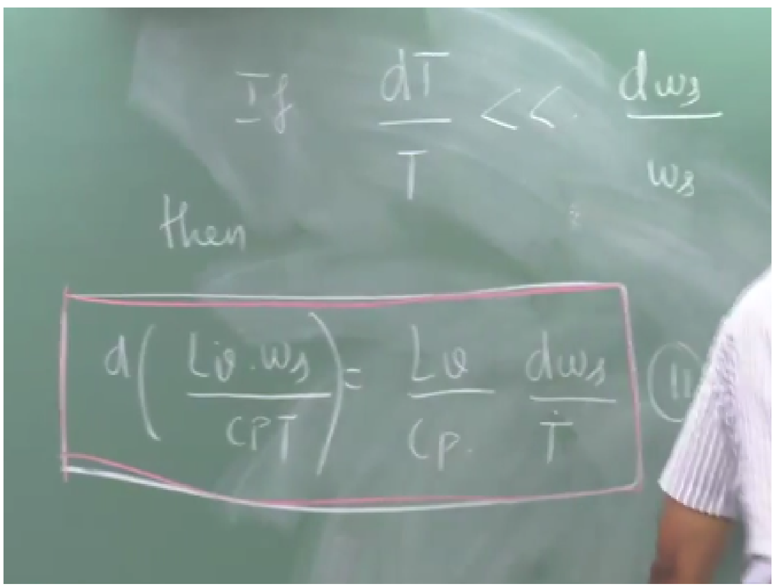

So if omega s is about 15 g/kg and d omega s is 3 or 4 kg per 4 or 5 g/kg. This d omega s by omega s might be of the order of half, one-third and so on. But if it is only 5 K change and T is 300 K please look at the magnitudes of d T by T and d omega s and omega s and tell me whether d T by T is approximately equal to d omega s by omega s; d T by T is much greater than d omega s by omega s or d T by d T is much smaller compared to d omega s by omega s.

d T by d T much small very good. I will also say if then I will say generally it is applicable in the atmosphere okay. If d T by d generally d T by d s but here we will make a conditional statement, if d T by d s is less than this then. So you get L v by what you get. Which is more important, what you get L v by C p by T alright. It is quite alright.

Now, this is fine. Equation number 11. Let us put some colour. Where are we heading? Are we going in an aimless direction? Where is the connection, please let me know. What is sir doing? That question may be uppermost in your mind now. What am I trying to do? What is the result from the first law? D q by d equal to minus R something is there. So we have worked out L v, C p all that. Now we have to link up the two. So tell me a way out.

What will be d q if only latent heat change is involved? Then you are hitting the bulls eye. Is the question clear? D q will be equal to - L v d omega s. When it is just about to change phase, d q will not be equal to C p delta T because there is no sensible heat okay. So we are talking about saturation where some water is changing into water vapour or water vapour is changing into water. Therefore d q will be equal to L v d omega s.

Then the connection is automatically established and this series of these steps which you have done can be put to good use okay. Now, is the connection clear now?

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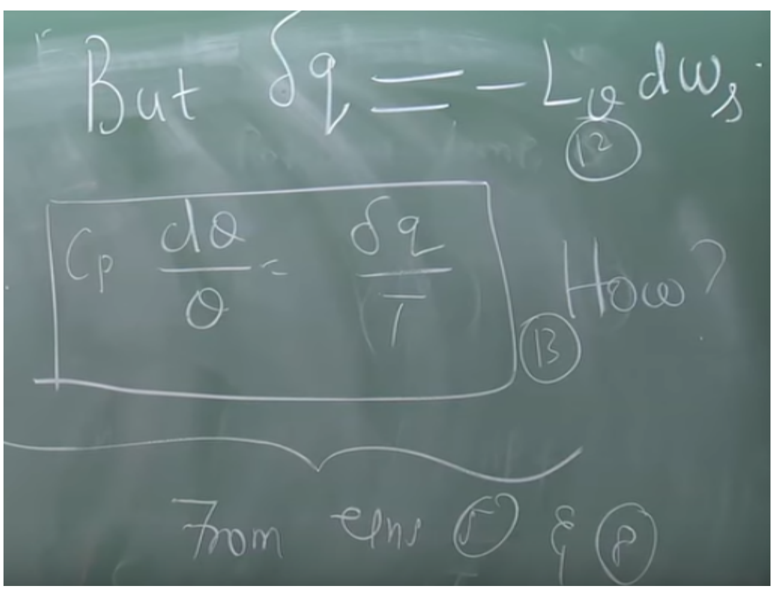

Therefore, that minus is heat removed from the system all that. I want to be consistent. Is it okay. Therefore, the other equation is also there no? d theta by theta okay ya so you had the equation, I forgot the numbers. You had an equation d q/T is equal to C p d T - R d p/d p then d theta by, compare these 2 equations and are you getting this. Are you getting this? I made a mistake. Where is the C p? Very good, correct very good. How?

Please help me from equations, which are the 2 equations. Tell me the number 5 and 8. Is it correct? It is going to be shown to others. Is it correct? Sai Sneha 5 and 8 okay. So I am going to write here. So by comparing equations 5 and 8 we got that C p d theta/theta = d q/ t. Watch, what I am going to do now. D q I know is L v d w s. Therefore this will be - L v d omega s/T.

But L v d omega s/d and d of that is nothing but d of L v omega s/d is this. Then it becomes a perfect differential. D theta by d theta and d of this thing I integrate then I will look at what happens when omega s by T goes to 0. When omega s goes to 0 when all the water vapour, all the moisture is removed it reaches a temperature that is theta e, equivalent potential temperature. Is it okay? Let us complete the derivation. So everybody is wearing a puzzled look. Ya, is it too much or no you have to learn this.

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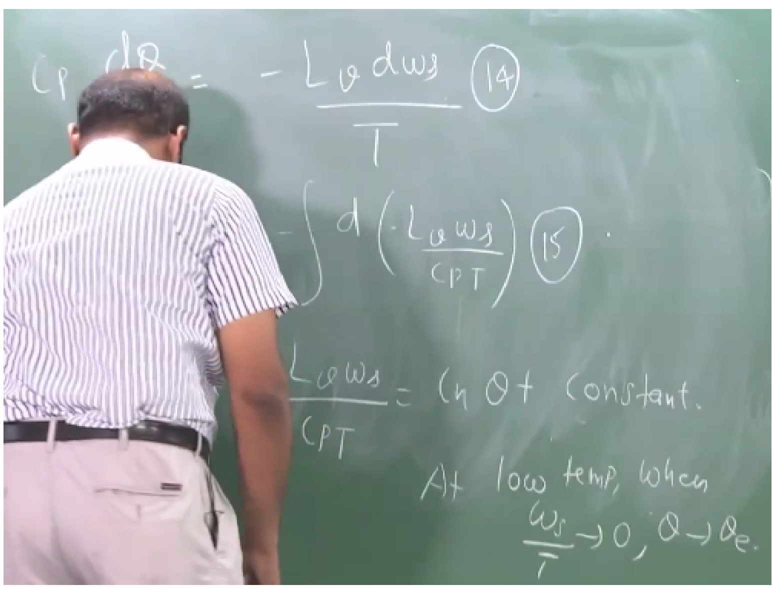

What is it, by T? This is nothing but minus d of, will you agree with me okay. 14. Please integrate. So, what do you get? I think we get minus. At low temperatures when omega s by T goes to 0 the air parcel is completely removed of all its moisture. This theta will approach an equivalent will approach a temperature called theta e which is a equivalent potential temperature okay. So let us complete the derivation. We can make it 16.

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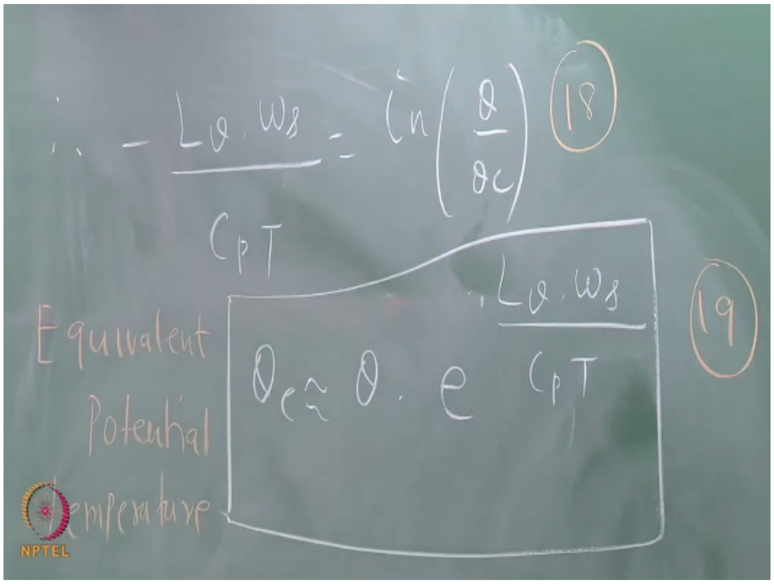

Plus, fine? So theta e is called the equivalent potential temperature. Is it okay? Let us take the formal derivation for this. So hang on, so theta e is equivalent potential temperature is the potential temperature of an air parcel when all the vapour, all the vapour all the water vapour has condensed so that its saturation mixing ratio is 0. There is no water vapour available. Everything is condensed. I come again. Theta e is the potential, I will formally state it.

You first understand. Theta is the potential temperature of the air parcel is what happens to it what happens to the temperature when all the water vapour is condensed so that its saturation mixing ratio is 0 okay. Theta e equivalent potential temperature is defined as the temperature reached by the air parcel. Theta e the equivalent potential temperature is the temperature reached by a parcel of air when all the water vapour has condensed so that its omega s equal to 0 okay.

How to get the theta e? How to get the theta e, tell me from this Skew-T ln P chart. I will tell you. Just listen to this carefully. The equivalent potential temperature theta e of an air parcel may be found by lifting the air parcel adiabatically until all the water vapour has condensed. It has released its latent heat and the condensed water vapour has fallen out. It is having no moisture now. But it is at some height. It is not at its original height.

From that height it is compressed in a dry adiabatic fashion all the way up to its initial temperature, all the way up to its optimal pressure of let us say usually 1000 hPa okay and the point where it intersects the 1000 hPa isobar the dry adiabat is called the equivalent potential temperature okay. So if an air parcel it lifted, first it will go dry adiabatically. It will go dry adiabatically up to what level, lifting condensation level.

Then suppose we say it is lifted further then it will go to a it will go through a saturated adiabat. At that point I declare that this fellow has left out all its water. Then we have to rework the point for that pressure for a new omega s which is close to 0. Then from there all the way dry adiabatically compress it back to 1000 hpa. It is quite different from the theta or the potential temperature where the question of omega s does not arise. Are you getting the point?

So what are the definitions of the potential temperature theta? The potential temperature theta was conserved during dry adiabatic transformations. Please note down, please take down. The equivalent potential temperature theta e is conserved during both dry and moist process or it is conserved both during dry and saturated adiabatic processes. So I come again, the potential temperature theta is conserved during dry adiabatic processes.

The equivalent potential temperature theta e is conserved during both dry and saturated adiabatic processes. The last one is a wet bulb potential temperature.

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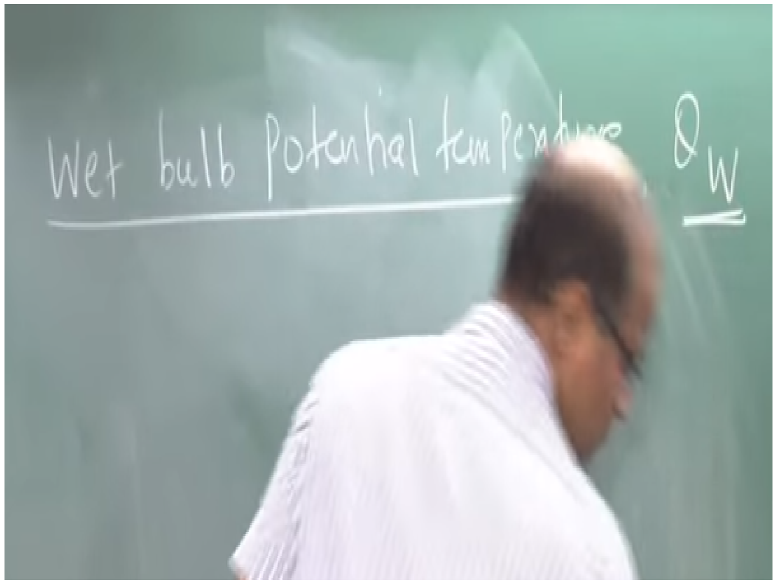

Please take down. If a line of constant theta, if the line of constant theta e that passes through T w of an air parcel is traced back if line of constant theta e that passes through the T w that is the wet bulb temperature is traced back on this Skew-T ln P chart, it intersects the 1000 millibar isobar. The temperature at this point of intersection is the wet bulb potential temperature theta w.

Please note equivalent potential temperature and wet bulb potential temperature both correspond to 1000 bar 1000 millibar, but the processes are different. For the equivalent potential temperature you go through a dry adiabat. Here it will go through a saturated adiabat okay, now everything is very confusing. Once you solve, once we solve a problem things will become clear. Now it is high time we stop the discussion and solve a problem involving all these quantities.

Let it take half an hour, it does not matter. I will dictate the problem, then work on your chart okay. Now, we are going to solve a problem which involves condensation and release of water and all that. Problem number 37. Please take down. A parcel of air with an initial temperature of 25 degree Celsius. A parcel of air with an initial temperature of 25 degree Celsius and a dew point of 10 degree C.

A parcel of air with an initial temperature of 25 degree Celsius and a dew point of 10 degrees C is lifted adiabatically from is lifted adiabatically from the 1000 hPa level. Determine the LCL. Determine the LCL and the temperature at that level. Determine the temperature, determine the

LCL and the temperature at that level. If the air parcel is lifted, if the air parcel is lifted by a further 250 hPa, this is new, you have not done problems like this so far.

If the air parcel is lifted by a further 250 hPa above its LCL what is its final temperature and how much water is condensed and how much water or I mean how much liquid water and how much liquid water is condensed during this rise? How much liquid water is condensed during this rise? Somebody needs a Skew-T ln P chart, anybody needs this? Anubhav do you have this. Give one to Amritha. So let me write down the data.

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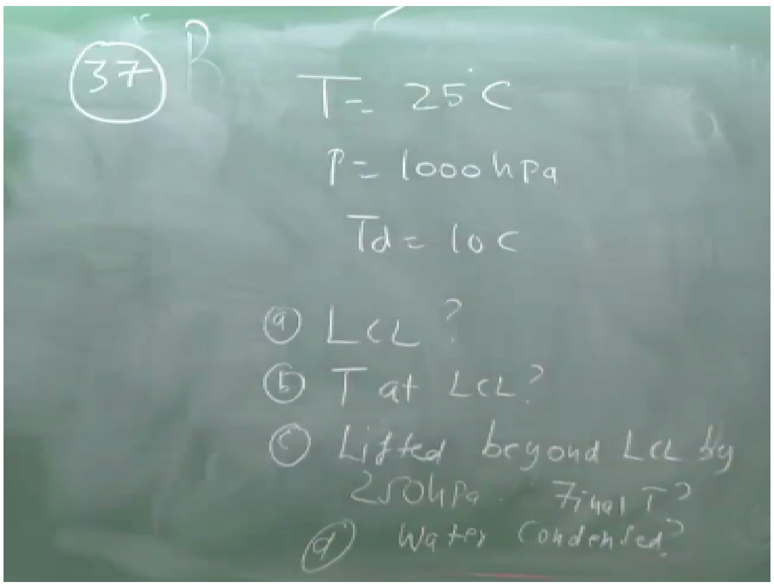

So which is the moisture parameter which is specified in the problem? What? T d. Dew point is given, temperature, pressure are given, LCL. The temperature at LCL is lifted beyond LCL by 250 hPa. What is the final temperature? How much is the water content. It is a standard problem. Take a good 15 minutes. Finished? It will take about 15 minutes. A standard quiz question. I may complicate it little further by asking you theta w, theta e whatever.

I may ask you 8 or 9 quantities okay. I may also ask you to tie up the Skew-T ln P chart along with the answer sheet. Is it okay. So problem 37.

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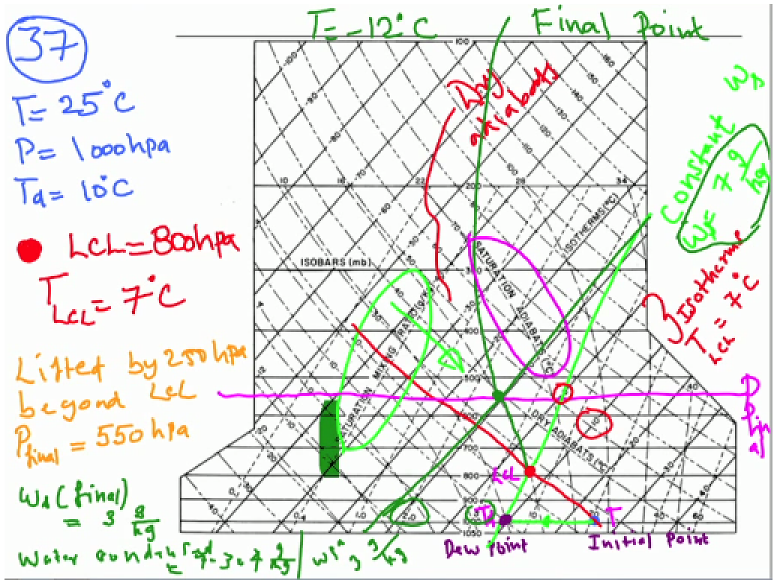

What is the T, 25? So is this okay? Is this okay, initial point. Dew point is 10. What do you mean by that? Here? Is that the dew point. What colour do you want? Initial point. Okay very good. Dew point is over. Next. Oh this green we will use for the process, green colour. So from dew point what are you taking? You are taking the so you have to use the not constant T constant omega s okay. So saturation, so constant saturation mixing ratio is coming like this.

That is the dashed line right? Correct? What is the value, 8? Class. 6, 7 okay. We will take arithmetic mean. Alright. Next. From here dry adiabat. I will change the colour now. Blue, red. So these are the dry adiabat okay. Bad? It is okay? Alright. 800? Okay. So let us make a big circular on 800 and so very good. The LCL lifting condensation level is 800. What is the temperature? How? Okay, yes. So this is 0. Sai Rahul is it okay? So let us write that. Nice.

Now comes the story. Saturation adiabat. From here where is he going okay. So P final 5. What is this? Quantity. What is that I am drawing, isobar? P final. Is everybody with me? Now, this is LCL. This is T d. How do you escape this? Good. This is T c. So this is T. Now from LCL I have to take a saturation adiabat. This one? I do not want magenta now. Okay. So final point. What is the question? Temperature you please tell me, what is the temperature? Minus?

So it is becoming cooled. Obviously, it is going up. T equal to minus. Ya small variations I will allow. So problem number 37 LCL 800 hPa. Temperature at LCL 7 degree C. Lifted beyond LCL by 250 hPa. Final T - 12 degree C. Only quantity we have to calculate item number d, question number d, water content. So you told me that oh you already told me that omega s for this fellow is 7. Now this fellow is already saturated.

So is omega is equal to omega s. Is omega is equal to omega s. Who, that fellow who already went up to 550. What is his omega s? No I cannot see that right okay. This is 2. Is it correct? This is 5. Correct? What are you saying, 3? So water removed, what is the question? Water condensation. 4 grams per. So this air parcel has condensed. Then the other story is if it is condensed and is thrown out and then it is coming back to 1000 hPa what will happen.

If it is coming back with the same this thing what will happen. That will lead to equivalent potential temperature equivalent temperature. It is already 8:15. So we will solve some problems involving theta w, theta e and all that in the next class. So now we know how to use all the lines on this. So in fact for each of this you can put P e to the power of gamma is equal to constant. Omega s, L v s and you can solve the whole thing using equations.

But we started at 8:38 or something, in 13 minutes we solved the whole thing without using any equation. Why, because all the equations are inbuilt into the chart. It is good right and we also used all the colours okay. Thank you very much.