Introduction to Atmospheric Science Prof. C. Balaji Department of Mechanical Engineering Indian Institute of Technology-Madras

# Lecture - 18 Problems using Skew-T ln-P Chart

Okay, so in today's class the first 10 minutes we look at saturation vapour pressure and complete that concept and then we will take a break. Then I will distribute the quiz papers. Then I will work out the solution right and then we will continue with relative humidity and so on in the next class, alright.

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So saturation vapour pressure okay. Consider a container containing water at the bottom and then it is at a particular temperature T. The top is empty now. There is a vapour pressure difference between the water and the top. Automatically evaporation will take place okay. So what will happen is slowly water vapour molecule will start going up okay. There will be recondensation of some molecules. So that is given by the downward arrow.

Evaporation is given by the upward arrow. But I have deliberately made the upward arrow longer than this because as the vapour the vapour pressure keeps building up and the vapour pressure keeps building up because of a consequence of difference between the rates of evaporation and the rates of condensation okay. When it is fully this is a unsaturated, b when it is saturated what happens is there is no change in the temperature. Now it becomes T and P s okay. So now the arrows are equal. So this is vapour pressure, so this is saturation vapour pressure okay.

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Completely any with time, water evaporates, Pull As Pull, rate of condensation of Mater vapour back to water also Increases. 7) When Yak of Evap & Vake of Condensation Pulls Unsaturated

So initially, upper part of container is completely dry correct? With time, water evaporates and P v increases correct? With time, water evaporates and P v increases. As P v increases, rate of condensation also increases. When rate of evaporation is less than rate of condensation the box is said to be or the container box is said to be unsaturated correct?

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Now when P v is such that when P v is such that, rate of evaporation yes sorry correct thanks correct. So this P s is called the saturation vapour pressure. How do the saturation vapour

pressure change with temperature. That is what, any guess? Decreases? It increases with temperature. So let us see that.

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This should be what some millibar, appropriate units. So this is T sat. When T sat increases so in Ooty for example, 2000 meters above sea level the temperature is less. So the water will boil at less than 1.013 atmosphere. So the cooking will not be proper if it is open. So you have to do, but pressure cooking is different okay. Pressure cooking the whole advantage is the whole thing is done at higher pressure so higher temperature, latent heat is more because you are working at a higher pressure. Is it okay, compared to normal cooking at atmospheric pressure okay.

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So this if you look at for those people who are doing mechanical engineering you know that so this will be like this. The temperature entropy diagram for steam is like this. So this is the critical temperature. This is 374 degree C and the critical pressure is 221 bar. So this is the okay. So this is liquid, liquid plus vapour, s is entropy okay alright. So but this is too high for atmosphere.

This is engineered right you heat it and you have to burn coal and this thing and transfer the heat to water, water will become steam, you pressurize it and all that okay. After saturation vapour pressure you have to look at we looked at mixing ratio. We will have to now look at saturation mixing ratio. Then the concept of relative humidity and then the concept of dew point which is a big indicator of human comfort.

So we will do all this in the next class, saturation, missing ratio, relative humidity, dew point. I will explain all the concepts and all the formulae. Then with the Skew-T ln P diagram given to you we will solve a problem where you will plot the state of moist air on the chart, I will use this the tablet and then I will ask you to determine the dew point and all that and then if time permits that is in tomorrow's class if time permits we will solve one more problem so that you become familiar with the chart.

Then we will look at some advanced concept like lifting condensation level of an air parcel and when air parcel is going beyond that level and then it condenses and it releases some moisture as rain or water what happens and all that and then we will look at some phenomena okay like Chinook wind and all that and try to understand all these phenomena using the Skew-T ln P chart which will be extremely useful to us okay.

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Okay so now we look at the solution to the quiz paper okay. So this is the problem set involving 5 problems okay. So the first problem is the total mass of the atmosphere is known to you okay to all the people who have attended this course in one of the earlier classes you have done this 1.004 into 10 to the power of 4 is it kg/m square and then 6373 km is the radius of the earth, so 4 pi re square if you multiply by this you will get the mass as 5.11 into 10 to the power of 18 kg. The kinetic energy in the atmosphere is of the order of 10 to the power of 21 J. Hence, give an order of magnitude estimate of the average wind speed okay.

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$$\begin{array}{l}
0 \\
\text{m-} & 5.11 \times 10^{18} \text{ kg}. \\
\frac{1}{2} & \text{mv}^2 = & 10^{21} \\
\frac{1}{2} & 5.11 \times 10^{16} \times (& y^2) = 10^2 \\
& & y^2 - & 391 \\
& & & y^2 - & 391 \\
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\end{array}$$

So the total mass in the atmosphere is known to you that is mass is 5.11 into 10 to the power of 18 kg okay. So half mv square kinetic energy is known to be 10 to the power of 21 J. So half into

so you will get an order estimate of the velocity v square is 391. So the velocities, the average velocities in the atmosphere air velocities are of the order of tens of meters. You can say 10 even if you have put tens of meters of seconds it is alright.

It comes to 19.76 or something right 20. Is it okay? So this is a ballpark estimate 20 m/s. That is how much km/h, 72 km/h. That is a lot no okay.

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Problem 2				
[2] The composition of the atmosphere of Venus is given below:				
	S.No	Constituent	Fractional concentration by volume	
	1	$CO_2$	95.2 %	
	2	$N_2$	3.4 %	
	3	$SO_2$	150 ppm	
	4	Ar	70 ppm	
	5	$H_2O$ (vapour)	20 ppm	
Deterrit				

The second question is the composition of the atmosphere of Venus is given to you. So carbon dioxide is 95.2%, nitrogen is 3.4%, sulphur dioxide is 150 ppm, argon is 70 ppm, and H 2 O is 20 ppm. The carbon dioxide is very high okay. So it can lead to it is a greenhouse gas so it can lead to what is called like runaway inflation, runaway increase in temperature right. So the Venus cannot support life.

Convert the volumetric analysis to a gravimetric analysis [6]

First oxygen is not there and the temperature is expected to be very high okay. Carbon dioxide is too much alright. Now, what is the apparent molecular mass. Somebody who went very deep and found out what is, you, normal atom mass and apparent molecule, I just wanted simple molecular mass, that is okay. That is the first part. Then convert the volumetric analysis to gravimetric analysis.

Is a standard question so that you are familiar in converting from volumetric or molar analysis to mass based or gravimetric analysis and once the composition is given of any atmosphere you should be in a position to calculate the molecular weight. From the molecular weight you will get the gas constant and then you will use the ideal gas and you will start working out all problems. The first step is to be able to calculate the molecular weight okay so which you studied long back in high school.

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Molecular Weigh

So molecular weight Veducian or Venusian what is the correct spelling for that. Is this okay? Okay, so this is nothing but m of Venus 95.2 into 44 carbon dioxide + 28.3 sorry 28 into nitrogen 3.4 + 64 sulphur dioxide is 64 right; 32 + 16 into 2, argon is 40. Different people have come out with different 36, 80. Some people have put argon into and left it. That is a very honorable exit that is called argon into something they have put okay; 64 into 0.01.

When you do like this do not forget to divide by 100 okay. If you do this. Let us see, so 42.85. Then the next part is to do the gravimetric analysis.

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#### Solution

Molecular weight of Venusian atmosphere,

$$\frac{[95.2 \times 44 + 28 \times 3.4 + 0.015 \times 64 + 0.007 \times 40]}{100} = 42.85$$

Gravimetric Analysis:

=

$$CO_{2}: \frac{44}{42.85} \times 95.2 = 97.75 \%$$

$$N_{2}: \frac{28}{42.85} \times 3.4 = 2.22 \%$$

$$SO_{2}: \frac{64}{42.85} \times 150 \ ppm = 224 \ ppm$$

$$Ar: \frac{40}{42.85} \times 70 = 65 \ ppm$$

$$H_{2}O: \frac{18}{42.85} \times 20 = 8.4 \ ppm$$

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So CO 2. I have cut some mark for some people who have just left it with N 2 right. You should continue with the, other things may not be very this significant but you can write it in ppm right. Some water is coming from somewhere. SO 2 is how much 224 ppm correct and then H 2O was 8.4 ppm. Now I have given you the breakup of marks okay so it is also given here 97.75 carbon dioxide, nitrogen is 2.22, SO 2 is 224, argon is 65, and H 2 O is 8.4 ppm. It is straightforward. Many of you have done it. There is no problem at all okay. Let us go to the third question.

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# Problem 3 [3] Using the hypsometric equation show that the pressure decreases with increasing height at about 1 hPa per 15m at the 500 hPa level. (6) Solution: $Z_2 - Z_1 = \frac{-R_d T}{g_o} \int_{p_1}^{p_2} \frac{dp}{p}$ $g \ dz = -R_d T_v \ \frac{dp}{p}$ $\frac{dp}{dz} = \frac{-gp}{R_d T}$

Third question is, using the hypsometric equation show that the pressure decreases with increasing height at about 1 hPa per 15m at the 500 hPa level. I come again. So the question is using the hypsometric equation show that the pressure decreases with increasing height at the rate of about 1 hPa per 15m at the 500 hPa level okay. There is several ways of solving this. You can find out what is the height at the 501 hPa level or 499 hPa level and be done with it or you can take the delta P to be 1 and then solve it. There are several ways of doing it. So you can use the actual hypsometric equation in its actual form or in its differential form also okay.

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So how I have done it is so gdz, so you can take T to be 255 K the mean temperature of the atmosphere, Rd okay.

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Substitute this. Ya please tell me what this value is. People who have not solved it can solve it now, 0.067 minus watch out the minus will come pascal per no hPa per. Some people have taken 500 and 499 that is also okay. P 2 = P 1 into e to the power of -z/h okay but they would have taken a h of 7.5 or 8 km okay so z 2 - z 1. So you can calculate the delta z and all that, that is also possible.

But if you use 7.5 or 8 kms a little bit difference will come. I have not penalized anybody for doing that. Is it okay? This is a better way of doing. The only assumption is the 255 but 255 is acceptable. That is the mean temperature okay, fine. Let go to, let us go to question number 4. It was little tricky. So the fourth question is where many people bungled okay. Some assumptions have to be made. See this paper I have told you that it is not like JE or whatever.

So I told you whatever data is not there you can make a reasonable assumption. One assumption you have to make was the lapse rate. If you make an assumption of the lapse rate things would have become very easy for you okay. So let us look at the question.

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# Solution contd:

But, T = 255 K,  $g_o = 9.81 \ m/s^2$  and  $R_d = 287 \ J/kgK$  $\frac{dp}{dz} = \frac{-9.81 \times 500}{287 \times 255}$  $\frac{dp}{dz} = -0.067 = \frac{1}{15} \ hPa/m$ 

Okay, the third question T is 255, g is 9 point, R d is 287. So dp/dz is 1/15 hPa/m okay. That is the solution given there okay. So this was some 6 marks depending on this if you have left in between I have given 3 marks, 4 marks, 6 marks whatever, alright.

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# Problem 4

[4] The gross weight (balloon, basket, fuel and passengers but not the gas in the balloon) of two balloons is the same. The two balloons are cruising together at the same altitude, where the temperature is 5°C and the ambient air is dry. One balloon is filled with helium and the other with hot air. The volume of the helium balloon is 850  $m^3$ . If the temperature of the hot air balloon is 87°C, what is the volume of the hot air balloon? (10)

Fourth question. The gross weight (balloon, basket, fuel, and passengers but not the gas in the balloon) okay so we are comparing 2 balloons. One is a hot air balloon, the other is a helium balloon. Both will go up in the atmosphere. Both will rise for different reasons. Helium is lighter than air, so it will rise. Even if its temperature is the same as the surrounding air but the hot air balloon will rise for the same reason that the density is less consequent upon the fact that temperature is less therefore density is less.

It will keep temperature sorry temperature is higher density is less it will keep on rising till the density inside and outside becomes equal plus whatever weight of the balloon blah blah all that. So you can do a force balance and but that is not the question here. Watch carefully. The gross weight (balloon, basket, fuel, and passengers but not the gas in the balloon) of two balloons is the same. So m into g what is that m into g is the same okay.

The two balloons are cruising the two balloons are cruising together at the same altitude, where the temperature is 5 degree C of the outside air and the ambient air is dry so that we do not want it to get into a spin and use the moist air thermodynamics and all that. One balloon is filled with helium. Other balloon is filled with air. The volume of the helium balloon is 850 meter cube okay.

Now what is given, the temperature of the other balloon the temperature of air in the other balloon is given, 87 degrees. Therefore the simple, the questions is what is the volume of the hot air, what is the volume of the hot air balloon okay. So this is the question. How do we go about solving? So it is basically you have to write the do a force balance. The weight will be equal to the buoyancy which is valid for both the balloons and we will equate it to mg, mg is the same.

Therefore you get a buoyancy relationship between the two balloons okay. Then there will be a delta rho for the two balloons so your smartness lies in calculating the delta rho appropriately. If you calculate the delta rho appropriately one volume is given the other volume can be found out. So the whole challenge in this problem boils down to getting the delta rho for the helium and delta rho for the hot air okay. Let us solve this.

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Helium and hot air

Okay, V hot air correct? V hot air into rho infinity - rho into g is equal to M into g. V helium, equating 1 and 2. Equating 1 and 2 and getting rid of the g okay what do you get? (Refer Slide Time: 27:34)



So I have to, temperature is 5 degree C okay. I will make two assumptions. Ground temperature is 30 degrees. You can make it 25 whatever you want. I will take a lapse rate of 6.5. So this is assumed. Therefore, I get an estimate of the height at which the balloon are cruising. It is about 3.8 km. This is important. Otherwise, you will make some assumption in the pressure, equal to 1 point because it is delta rho you may still get close.

Sir, I also got 3000 meter cube but you are getting the correct answer for wrong reason. The correct way okay. So this is you can question me sir why not 30, 35 whatever you would have taken I would have appreciated. Some lapse rate 6 to 7 we told that data is also given in the notes. Assume some reasonable. So get the height. What is the funda of getting the height? Once you know the height hypsometric equation we can get the hydrostatic equation.

What is that hypsometric equation, using the hypsometric equation you can get the pressure. Temperature 5 degrees is given. So rho is obtained. What is this rho? Rho infinity. So with this so there is a way to get rho infinity if you know only the temperature. Many people in the quiz they ask me sir pressure is not given. That is for normal students. Not for students taking atmospheric science course.

You can calculate the pressure okay fine. So even if we little bit here and there instead of 3.8 you got 4.4 or 4 point ya **"Professor - student conversation starts"** Sir why is the assumption that pressure of the helium inside the balloon and the temperature outside. I have not even come to helium, wait **"Professor - student conversation ends"**.

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Okay, so okay so P, P equal to 278. (Refer Slide Time: 31:38)

Now this is the same pressure for the helium also okay. P is the same as P helium. So the P helium is basically rho. So Rohan is asking me why are you using 278? Why cannot you use 330 degrees is it?

**"Professor - student conversation starts"** I was asking that outside temperature is 5 so why is the assumption that temperature of helium is also 5 invalid. Only now I am making that assumption. You can make 30 also no problem. Some, that means I am assuming some heat transfer equilibrium somewhere you have to make an assumption. If we can cancel out the whole pressure terms then. How. 3 by r t then I am not able to do that. We can sir, we can sir. **"Professor - student conversation ends".** 

Just hang on. So let me complete this. So I will take it as a so rho helium I got it as 0.105. We will see what is the difference it makes to the result okay. Now 850, what is that 0.59 where did I get the 0.59, from the 87 degree C okay. Okay, so I am not saying this is the solution. This is a reasonable solution. If you get 30 degree C you may if you take 30 degree C you may get little bit different because the helium anyway density is very low.

So the leading order term, the leading order term is 0.764 only. If you make 30 degrees also it will make little difference to the answer. Is that okay. Yes, now proceed. **"Professor - student conversation starts"** Sir, actually then if you are simply assuming the temperature of helium

then you do not even need to find pressures and all because directly it will just cancel out in the equation. That is okay but I am going through I am using a systematic procedure where what all we learnt in the course we have used **"Professor - student conversation ends"**.

If you have got approximately that 3000 something I would have given maximum marks but I wanted you to go through the steps okay. What is the other assumption? See if I have to know more, Rohan if I have to know more about the helium temperature I should know the heat transfer process whether it is the elastic membrane I would apply first law of thermodynamics and we can do that in ME course. This is also ME course okay now alright.

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Solution	
	$V_{\text{latter}} \left[ a_{\sigma \sigma} - a \right] \sigma = M \sigma$
	$V_{He} \left[ \rho_{\infty} - \rho_{He} \right] (g) = M (g)$
	$V_{hotair} \left[ \rho_{\infty} - \rho \right] (g) = M (g)$
	$V_{He} \left[  ho_{\infty}$ - $ ho_{He}  ight] = V_{hotair} \left[  ho_{\infty}$ - $ ho  ight]$
	Given Data: Temperature = 5 °C, Lapse rate = 6.5 K/km
	Ground temperature = 30 °C, Height = 3.8 km

Okay V is equal to whatever I have written on the board g g gets cancelled lapse rate okay. (Refer Slide Time: 34:32)

# Solution contd:

$$Z_{1} - Z_{0} = \frac{-R_{d}T}{g_{o}} \int_{p_{0}}^{p_{1}} \frac{dp}{p}$$

$$Z_{1} - Z_{0} = \frac{-287 \times 293}{9.81} ln \frac{p_{1}}{p_{0}} \dots [Z_{0} = 0]$$

$$\frac{p}{p_{0}} = e^{\frac{-Z}{H}}$$

$$\frac{p}{1013} = e^{\frac{-3.8}{7.5}}$$
Hence, p = 610 mbar = 0.61 bar

So the pressure is 0.61 bar.

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Solution contd...

• Now, p = \rho_{\infty} R T

• Substituting T = 5 °C, R = 287 J/kg K and p = 0.61 × 10<sup>5</sup>

bar,

0.61 × 10<sup>5</sup> = \rho_{\infty} × 287 × 278

\therefore \rho_{\infty} = 0.764 \ kg/m^3

• For helium, R = 2078.5 J/kg K,

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0.61 × 10<sup>5</sup> = \rho_{He} × 2078.5 × 278
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Now substituting this thing I am getting the rho infinity to be 0.764. For helium R is 2078.5 because 8314/4 that you should remember, molecular weight is 4. So you can get the rho helium. (Refer Slide Time: 34:53)

# Solution contd...

• Substituting values of  $\rho_{\infty}$  and  $\rho_{He}$ ,

 $850[0.764 - 0.105] = V_{hot \ air}[0.764 - 0.59]$  $\therefore V_{hot \ air} = \frac{561.85}{0.174} = 3229m^3$ 



So you can get about 3000, approximately it is around 3000. If we cancel out the pressure also you are getting like that right okay alright.

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So the last problem in the quiz was calculate for the earth's atmosphere the height of the surface where the pressure is 0.15 of the surface value. Calculate for the earth's atmosphere the height of the surface where the pressure is 15% of the surface value assuming (i) uniform temperature of 290 K, (ii) surface temperature of 300 K and a uniform lapse rate of 10 K/km. So the idea is to find out, given the pressure find out the height.

It is very easy using the hypsometric equation but that is for an isothermal atmosphere. If lapse rate is given some integration has to be done. No approximation. You have to use an integration. So the part b of the problem I have made it as 3 marks + 5 marks. The first part is pretty straightforward. So total was 8, so it is 3 + 5. Is it okay? Let us solve this problem now. So any cribs you have to meet me on Thursday, Thursday or Friday afternoon okay. Tomorrow I am busy with the comprehensive viva exam.

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So 5 (i) alright. Some people have got 16 kms, that is alright. You have taken a scale rate of 8 kms.

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So the interesting part is the part, second part okay where gdz. So please write an expression for the temperature in terms of the lapse rate okay. Now okay. So let y be okay. Make the substitution okay.

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So getting an error of 2 kms. You got 15.16 kms. Now you are getting 12.8. So you get an error of 2 kms as a consequence of an assumption of the isothermal atmosphere. Somebody got 2 km this thing and all that is all weird. I think somewhere the integration went haywire. You thought 300 and 1/300 is very easy to do. it is better what we do is equal to z, z = y or x something if you do like that then you will not make mistakes.