

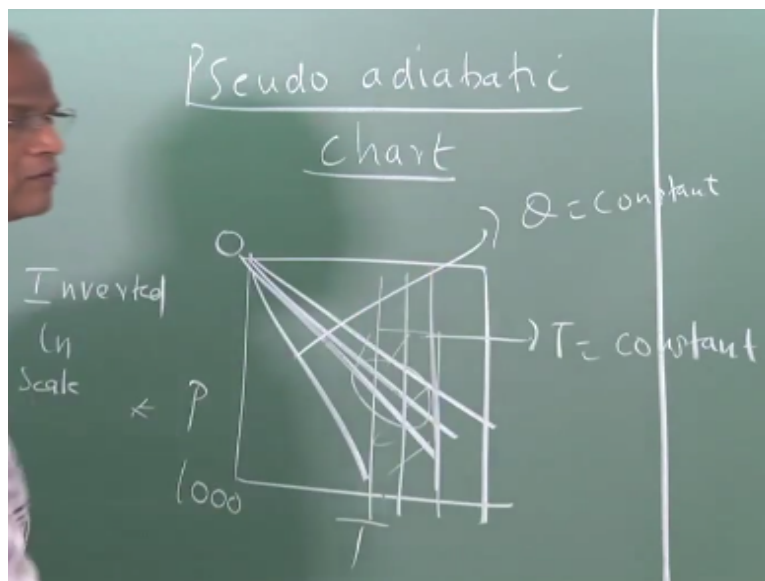
**Introduction to Atmospheric Science**  
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**Department of Mechanical Engineering**  
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**Lecture - 17**  
**Problems using Skew-T In-P Chart**

Okay, good morning. So we looked at the pseudo adiabatic chart in the last 2 classes and we tried to solve some problems using the pseudo adiabatic chart as well through formulae. Then the problem was, the constant theta lines and the constant temperature lines were very close to each other in the pseudo adiabatic chart.

The, as you recall the constant temperature lines that is isotherms are verticals and this in the pseudo adiabatic chart the constant theta lines that is the constant potential temperature lines called as the dry adiabat or the pseudo adiabat, we say pseudo because the moisture has not been considered. So there one was sloping down if you recall it was like this.

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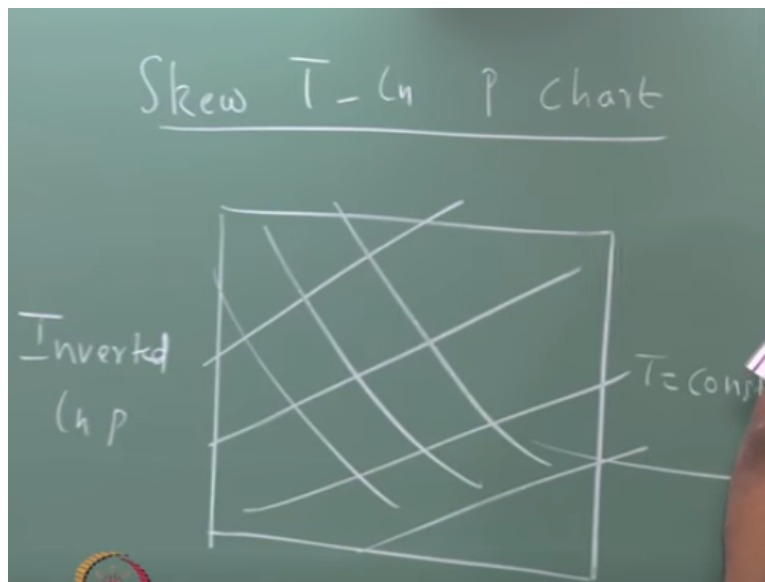
P was in a and this is T. So the funda is an inverted ln scale correct, is an inverted ln scale. What is a inverted? This top is 0, bottom is 1000. We never draw like that. From childhood you have said that it is 0 to this thing. So it is inverted but it is for convenience because bottom represents ground, 0 represents top. So since we are always dealing with variation of so many quantities

with respect to height in the atmosphere it is convenient for us to imagine and comprehend the situation okay.

So the point was these are  $T$  is equal to constant. So the  $T$  is equal to constant and this  $\theta =$  constant are very crowded. So to overcome this we people figured out this Skew- $T \ln P$  chart. Then we also looked at the region of interest because for the kind of pressures and temperatures encountered in the atmosphere there is one particular region, I think from 220 Kelvin to something okay.

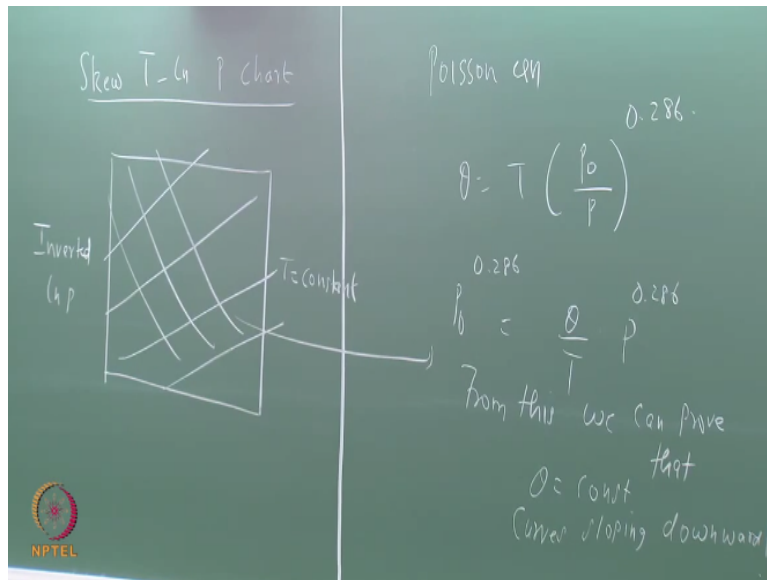
And some pressure to some pressure this is the region of interest for us when we want to work out problems okay or we have to study phenomena in the atmosphere okay. Later on we saw that an advantage is this is the pseudo adiabatic chart.

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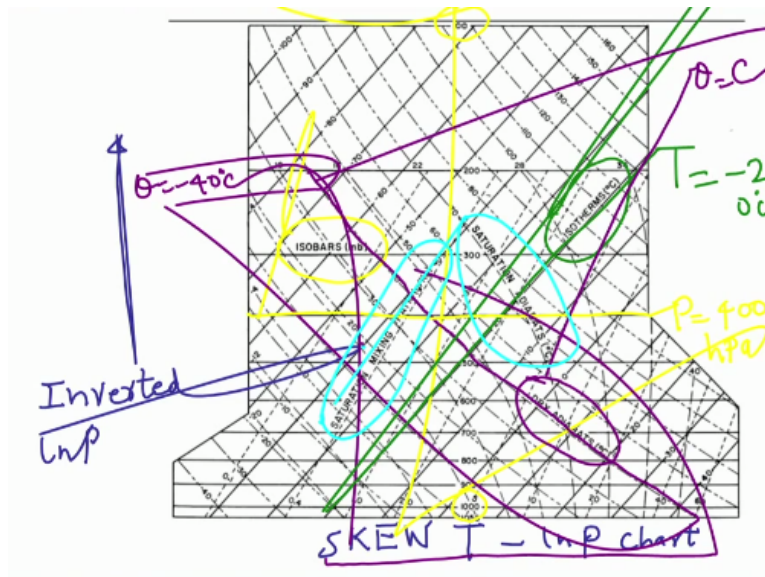
This Skew- $T \ln P$  chart overcomes this in the sense that this inverted  $\ln P$  okay but so the x axis is kind of fussy is it not because the it is kind of fussy. So you have to when you mark points on this you have to be careful okay. So we saw that we wrote the equation of this pseudo adiabatic or potential temperature which is called the Poisson equation.

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Theta = P to the power of 0.286 or minus okay. Then we took ln P both sides and this thing and all that and we found out that on this chart the curve sloping downwards okay. I will give you the pseudo adiabatic sorry the Skew-T ln P chart. Take a look okay.

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Alright. So all of you have this okay. So we are looking at this chart. So this so this is inverted ln P correct? I do not know why it happened. So inverted ln P. So please look at the isotherms. Please look at the isotherms. One isotherm for example okay. So the isotherms are like this. One isotherm is, what is the problem. Okay, so this is one isotherm. What is that isotherm? T equal to -20 degree C, is it okay? So isobar okay.

So one isobar is what is this isobar, correct okay. Where is the 1000 hPa? Yes, 1000 hPa is here okay? Where is the 100 hPa? Topmost. So you are having between 1050 and 100. This straight line only I do not know how to, anyway it is still making sense. So 100 hPa okay. Now, can you spot the dry adiabats theta equal to constant lines? Let me show you. I am using the magenta. So these are the dry adiabats okay and those are the theta = constant okay.

How are they going? So this is theta equal to okay. Alright, so you are going up it is a decreasing pressure from bottom to top. The isotherms are skewed by 45 degrees. That is why this Skew-T. It is inverted ln P, that is why it is a Skew-T ln P chart okay. Most of the actions which are taking place in the atmosphere for the pressures and temperature which you normally encounter are, latecomers can you can, one of you come and do not cross the camera.

You can keep something. Rohan keep a few on your desk okay alright. But there are some more things which we have to see which I will just I will allude to this now but after we flush out the definitions only we will understand what these are. We will choose another colour. There is something called the saturation adiabat. They are also sloping downwards but the nature of the curve is different from that of the dry adiabat correct?

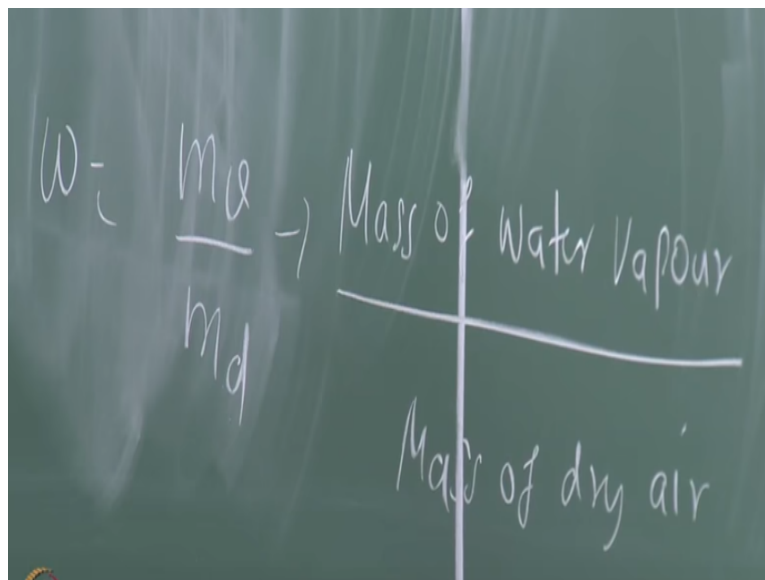
Okay and then we have one more set of lines which are the saturation mixing ratio okay. It is a really involved chart where, how many quantities are there or how many kinds of curves or lines are there, 5. Isobar, isotherm, dry adiabat, saturation adiabat, saturation mixing ratio. I come again. There are 5 quantities in this chart isobars, isotherms, dry adiabat, saturation adiabat, saturation mixing ratio. Out of these 5 isobar, isotherm, dry adiabat you already know.

So what you do not know right now is saturation mixing ratio and saturation adiabat. Even for this we can write the formulae. We will start with the formulae and these are incorporated as lines and it is very comprehensive. You can choose to solve the problems either by using formulae or quickly you can solve any of these problems using the charts just like that aircraft cabin air problem we solved by using both the chart and the formula.

Using the chart will be little bit faster but it will be less accurate. **“Professor - student conversation starts”** Sir, ya. Dry adiabats are drawn over here. Ya. There is a confusion as to which one, which side is positive and which side is negative. You can see. Which side is positive? As it go to the right it is positive correct? As you go to the right it is positive okay. **“Professor - student conversation ends”**.

Okay for the benefit of, the question was that minus is not clear so when you go to the right it is positive, alright okay. Shall we proceed? So you have to now consider water vapour in air.

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The image shows a chalkboard with a handwritten equation defining the mixing ratio  $\omega$ . The equation is 
$$\omega = \frac{m_w}{m_d}$$
 where  $m_w$  is labeled as "Mass of water vapour" and  $m_d$  is labeled as "Mass of dry air".

So we have to now flush out some terminologies or definitions concerning moisture parameters okay. So mixing ratio. The mixing ratio omega is given by the ratio of the mass of the water vapour to the mass of dry air okay. So what is the maximum content of moisture in the atmosphere when we saw in the earlier table? 5% no? or the order of 5 % or 10%, 5%. So kg by kg is it a good unit? Kg of vapour/kg of dry air. Is it a good unit?

What is the better unit, grams. Grams of water vapour by kg of air. That is a good unit. In 1 kg of air how many grams that is decent. You will say 5 grams, 4 grams, 10 grams. Kg means 0.000 it will you will miss one point and 0 and then go wrong okay it is not an appropriate unit. Therefore, mass of water vapour grams mass of dry air in kg. It is called mixing ratio. How much

water vapour is mixed along with the dry air? So the total mass of air is  $m_d + m_v$  please remember correct, specific humidity. That is the next quantity. So I will skip this.

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The image shows a chalkboard with the following handwritten text and equations:

⑥ Specific humidity  
 $\phi$   
$$\phi = \frac{m_w}{(m_w + m_d)}$$
$$= \frac{1}{1 + \frac{m_d}{m_w}}$$
$$\phi = \frac{\omega}{(1 + \omega)}$$

An NPTEL logo is visible in the bottom left corner of the chalkboard image.

Denoted by the symbol  $\phi$ , some people call it as  $\phi_i$ , let us call it as  $\phi$ . I would prefer the bracket but if you are comfortable without the bracket in the denominator that is also okay so long as you do not get confused okay. So the specific humidity is the mass of the water vapour divided by the total mass of air which is mass of dry air plus mass of water vapour. So it is slightly different from mixing ratio. Now I will give you one minute. Please convert the specific humidity formula in terms of the mixing ratio.

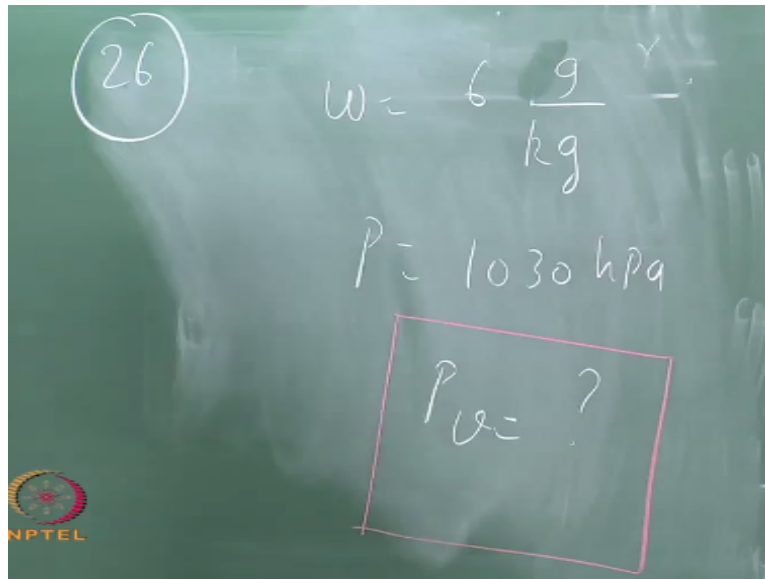
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The image shows a chalkboard with the following handwritten text and equations:

Invariably  
 $\omega \ll 1$   
$$\therefore \phi \approx \omega$$

Or invariably therefore phi is approximately equal to omega which means the numerical value of the specific humidity, the numerical value of the mixing ratio are nearly close to each other. They are indistinguishable. Please solve this problem, problem number problem 26.

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If air contains water vapour with a mixing ratio of 6 grams per kilogram and the total pressure is 1030 hPa calculate the vapour pressure  $p_v$ . I come again for those people who missed. If air contains water vapour with a mixing ratio of 6 grams per kilogram and the total pressure is 1030 hPa, calculate the vapour pressure. I will just write the, mixing ratio is what? Omega. Question is what is the vapour pressure? Just take 5 minutes. I will take attendance so that we can do parallel processing.

Ranjit kumar, K. Vinay, Vivek Kumar, Akil, balakumar, V.K. Depak, Ashwin, Srivastava, Srivatsava, Amritha, gautham, Lakshmikanth, Digvijay, SaiSneha, Rajesh, Rishita, Venu, Piyush, Saichand, Praturaj, Vivek Kumar Gupta, Chaitanya Ravindra, Saipawan, Daniel Akaash, ChithaDeepak, Rohan, Vivek Siddharth, Nimith, Noel Jose, Ranjith, Rohit Omar, Udit, Vishwajith Bhat, Arun Kumar, Anubhav, Amit Soni, Arpan Das, Divyashree, Soujanya, Aditya, Anusha, Maurya, Mohit Reddy, Bhargavi, SaiRahul, Sanjay, Marius, Ajayendra, Vivek Harb.

Done? What is the vapour pressure? 9 point, let me check. That is quite good. How many of you are still doing? Just raise your hands. Any problem? Where are you getting stuck? Is somebody clueless? Shall I solve? Shall I solve on the board, okay.

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The image shows a chalkboard with handwritten equations. At the top, the partial pressure of water vapor is given as  $p_w = \frac{n_w}{n_d + n_w} \cdot P$ . Below this, the equation is rewritten in terms of mass and molecular weight:  $p_w = \frac{m_w}{M_w} \cdot P$ . A boxed equation shows the ratio  $\frac{M_w}{M_d} = \epsilon$ . The final equation is  $p_w = \frac{m_d}{M_d} \cdot \frac{m_w}{M_w} \cdot P$ .

So the partial pressure is the number of moles of vapour plus the number of moles of dry air plus number of moles of the vapour into the total pressure okay. The total pressure is given. So now this number of moles is nothing but mass of this thing by  $M_w$  molecular weight of water vapour divided by mass of dry air by molecular weight of dry air plus mass of water vapour divided by molecular weight of water vapour into  $P$  okay. Everything is given. You can straightaway solve it. I have actually simplified it further.

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$$P_w = \frac{6}{18} \times 1030$$

$$\left[ \frac{1000}{28.9} + \frac{6}{18} \right] \text{ hPa}$$

$$P_w = 9.84 \text{ hPa}$$

$$= 0.98 \text{ kPa}$$

Can we write it as  $P_v$ ? What for this simplification is possible or you are just doing  $6/18$  you are doing that is it okay do that 6 by plus actually it comes 9.84 hPa correct in kilo Pascal. We call it as it is nearly 1 kPa.

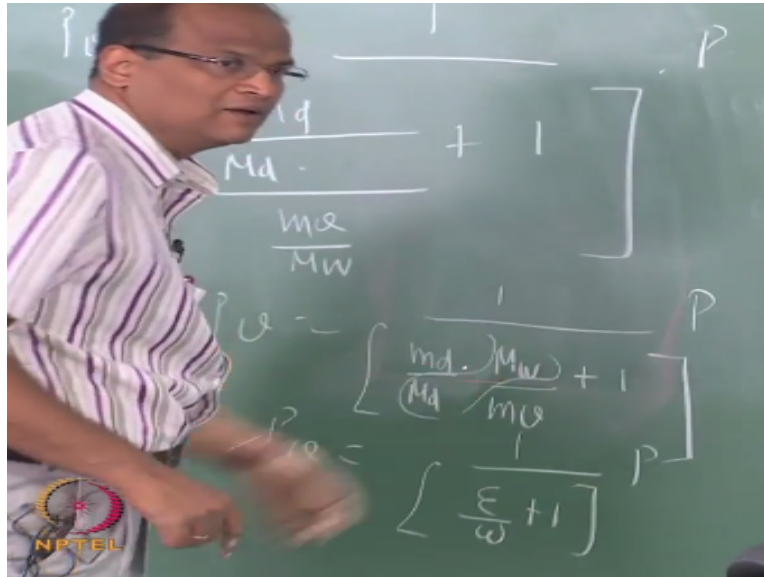
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$$\frac{P_w}{P} \sim \frac{10}{1000} \sim \frac{1}{100}$$

Very Small

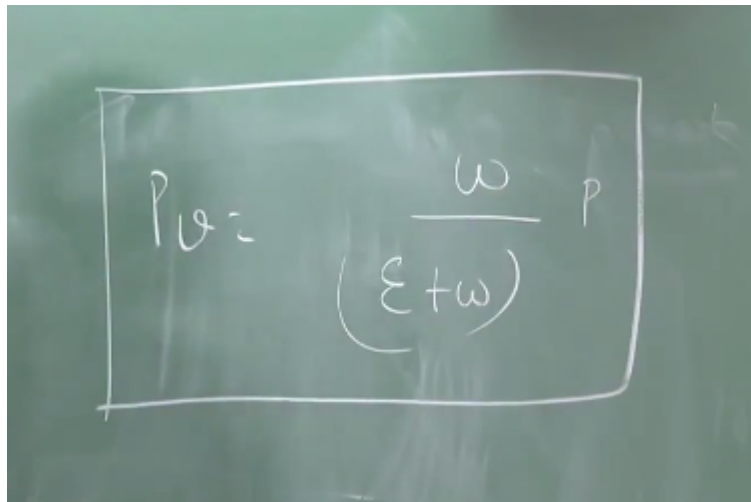
What is  $P_v/P$ ? Correct? It is very small right? Inference. But some funda is there. We will make it little bit. I want to generalize this.  $M_w$  by  $M_d$  is epsilon right. Just do a quick 2-minute exercise. Put this in terms of omega and epsilon. A quick exercise. People who have completed problem number 26 just write the formula in general term in terms of omega and epsilon. Question is clear? Just to look for generalization okay. I will clean up this side. What you want to do. Ya, now what are the steps. You are dividing throughout by  $M_v/M_w$  is it?

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Okay, so  $M_v$  will come here? We are dividing both the numerator and denominator by 1. Is quite alright okay. 1 by,  $\epsilon/\omega + 1$ ,  $\epsilon + \omega$ ,  $\omega$ .

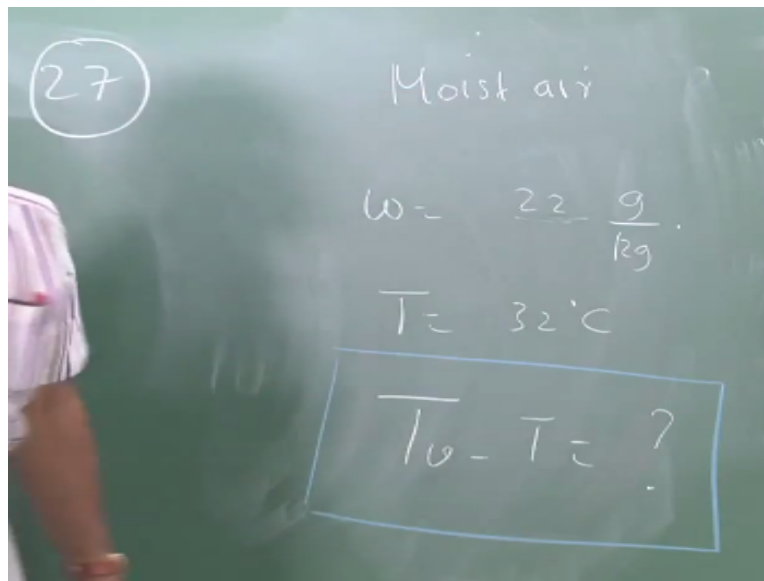
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Another smart way of doing it. Fine? So mixing ratio 6 is given that is point 6 means you have to take it as 0.006 do not take it as 6. So point 0.006 + 0.622 that will be in the denominator;  $1/0.628$  is it correct? No,  $0.006/0.628$  into 1030. We will get this alright. Problem 27. Calculate the virtual temperature correction. Calculate the virtual temperature correction for moist air.

Calculate the virtual temperature correction for moist air at 32 degree C that has a mixing ratio of 22 grams per kilogram okay that has a mixing ratio of 22 grams per kilogram.

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Given moist air, for moist air omega is 22 grams per kilogram. T is 32 degree C. Question is  $T_v - T$ . We have seen that for all kinds of situations dry air, hot air,  $T_v - T$  very less, does not exceed a few degree centigrade. If we get some 40 or 50 degree centigrade please revisit your answer or revisit your calculation. So you will get an answer which is a few degree centigrade. That few could be 1, 2, 3, 4, 5 that you solve.

Now after defining this mixing ratio and so on that saturation adiabat and saturation mixing ratio we have to define and then we have to again go back to this chart. It is already messed up. So we will take a new chart and consider these lines; 4 correct it is 4. I got it as 4.1. Ya that is okay around 4. Rohan, Amritha solving? Rohit Kumar. You have the calci, cellphone okay. 4? People who have already completed do the next exercise.

Put it in general put it as a general formula which can be used later on without the numericals without the value, epsilon by omega and all that. **“Professor - student conversation starts”** What happened you got it? I just forgot the formula. What about you SaiRahul? The formula is there or  $T_v/T$  whatever. How many of you are stuck? Just try. Give it a shot. **“Professor - student conversation ends”**.

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$$\left[ 1 - \frac{P_w(1-\varepsilon)}{P} \right]$$

From the previous problem

$$P_w = \frac{\omega}{\omega + \varepsilon} P$$

Write the formula for  $T_v$ , correct? Rahul, you have to start it from here. Now, funda is from the previous problem  $P_v = \omega$  by. Substituting for  $P_v/p$ .

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$$T_v = \frac{T}{\left[ 1 - \frac{\omega}{\omega + \varepsilon} (1 - \varepsilon) \right]}$$

$$T_v = \frac{T (\varepsilon + \omega)}{[\omega + \varepsilon - \omega + \omega \varepsilon]}$$

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$$T_v = \frac{T(\omega + \epsilon)}{\epsilon(1 + \omega)}$$

$$\omega \ll 1$$

$$\therefore \frac{1}{1 + \omega} \approx (1 - \omega)$$

$$T_v = \frac{T(\omega + \epsilon)(1 - \omega)}{\epsilon}$$

What 1 plus. I will not stop here. I will agonize it further. Agreed? Therefore, oh again you can cross multiply and simplify. Let us do that.

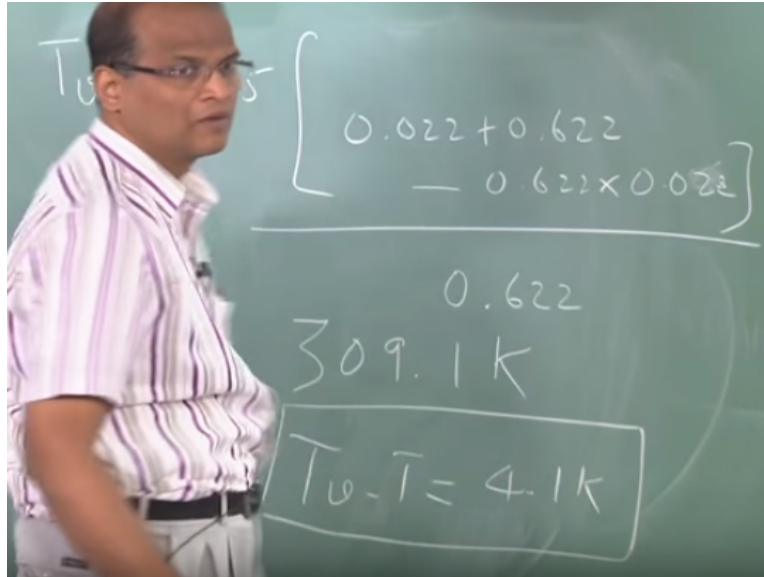
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$$T_v = T \left[ \frac{\omega + \epsilon - \omega^2 + \epsilon\omega}{\epsilon} \right]$$

$$T_v = T \left[ \frac{\omega + \epsilon - \epsilon\omega}{\epsilon} \right]$$

Ya please help me. Please tell me  $T_v$  after all that clean up all that what you get?  $\omega + \epsilon$  plus minus, nothing gets cancelled? Okay so okay we will leave him like this. Ya, Chaitanya what you can do is you can use this approximation, work out you need not use this approximation, stop here, find out what is  $T_v - T$  do approximation level a, level b, level c and find out what is the difference. Now, let us shove in all the values.

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$T_v$ , 32, you are converting to Kelvin right? 3 not. Not 4.1 point, no point 4.1. So what is the learning which has taken place? What is the further learning which has taken place in the last few classes as compared to chapter 1 is though we got the formula for virtual temperature correction from moisture parameters we were able to calculate the same thing because we are able to link up the ratio of  $P_v/P$  to the moisture parameter. So you do not require the pressure at all.

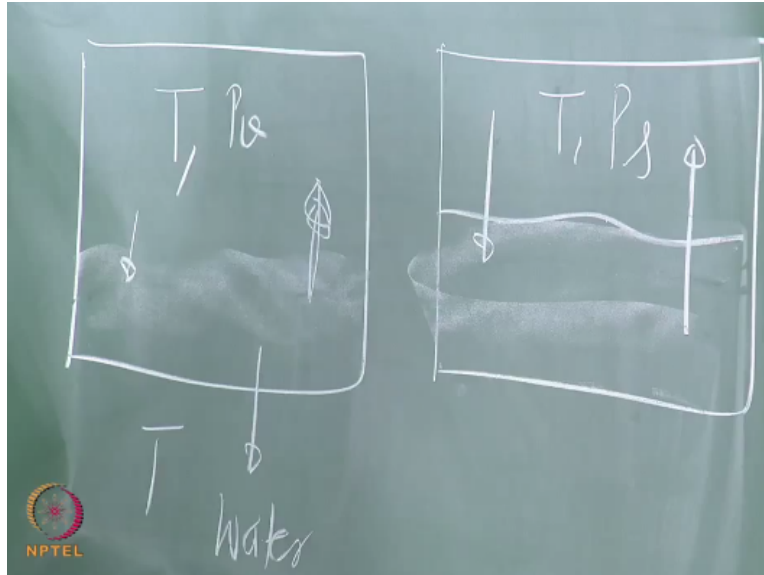
If you know the moisture parameter you can play with them and get virtual temperature. Once you get virtual temperature you can put  $z_2 - z_1$   $rtv$  by  $g$  into  $\ln$  of  $p_2$  all that. So you can start working out okay. So this specific humidity and mixing ratio are big time. They are a big deal because they let you work out problems by ignoring the pressure, number of moles all those things are all gone we can work you can work you work out problems based on this right.

So the next would be to figure out this saturation adiabat and saturation mixing ratio. What you think will be the, conceptually what could be saturation mixing ratio. We will define formally in the next class. We have got only couple of minutes. **“Professor - student conversation starts”** The mass of water vapour when the air parcel is saturated. Ha, mass of water vapour when the air parcel is saturated compared to the dry air okay **“Professor - student conversation ends”**.

So the saturation mixing ratio must be more than generally the mixing ratio correct? So this saturation mixing ratio we should be in a position to calculate the saturation mixing ratio. If you

have to calculate the saturation mixing ratio you have to define a new quantity called the dew point temperature. Then first we have to find out define what is saturation okay. So let us just start. I will just give a conceptual framework and then we will get into this in the next class.

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Consider a beaker of water, consider a container of water. Let it be a particular temperature  $T$  okay. Now, there will be there will be some water vapour here because there will be some evaporation even at that temperature. It need not boil. There will be some so what will happen is there will be some water which is evaporating and becoming water vapour and the reverse condensation will also take place okay. These 2 fluxes need not be balanced okay.

As more and more as slowly the evaporation increases the vapour pressure here also increases so the condensation the reverse condensation also increases. So at one particular stage whatever is going up will be exactly equal to whatever is recondensing. So now what was  $T$  and  $P_v$  has become  $T$  and  $P_s$ . That  $P_s$  is called the saturation. Is it okay? Consider a small box the floor of which is occupied by water, a plain sheet of water at temperature  $t$ .

Initially, we assume that it is completely dry. There is no water vapour here. Then there is a pressure difference. The water vapour normally it will start boil it will start evaporating. Once it starts evaporating the partial pressure of the water vapour begins to rise in the top half in the top

of this okay at the same temperature itself okay bubbling will take place. Then what happens is the rate of condensation is less than the rate of evaporation.

So this rate of so continuously the pressure will build up. The vapour pressure will build up on the top half. One stage will come when the reverse condensation is the same as the rate of evaporation. Then the water is said to be completely saturated okay. So the water vapour, not water, the water vapour is completely saturated. So we say the humidity, relative humidity has reached, actually I have not defined I have to define relative humidity.

Relative humidity will reach 100%. That is why we say it is very sweaty , it is humid. That means it is about to rain in the atmosphere. So it is fully saturated, fully laden with water. So if it is fully laden with water, it is an air parcel. All these things are conglomerating. It is forming a cloud. It just requires little bit it rises you know there is a lapse rate in the atmosphere. When it rises a little bit, it is full of moisture, it will start crying. So it will rain okay but everything may happen.

There may be big wind. So the clouds may form in one place rain may go to some other place. Or suddenly something will happen, they may just breakup. So on many days you see it is dark and cloudy, rain is not happening. There, some trigger is there correct. But if regular convective activity see if it is very hot during the day particularly in Kerala, people from Kerala will know evening there will be heavy rain in many of the place.

Even in for example in Chennai if you know it is very hot there is a good chance that there will be a thundershower in the evening. So the breeze brings the rain clouds into the land okay. So now we will define saturation mixing ratio, relative humidity. What is the connection between these two? Go to this Skew-T ln P chart, identify all this and start solving problems using this Skew-T ln P chart.