Introduction to Atmospheric Science
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Lecture-12
Hypsometric Equation and Pressure At Sea Level

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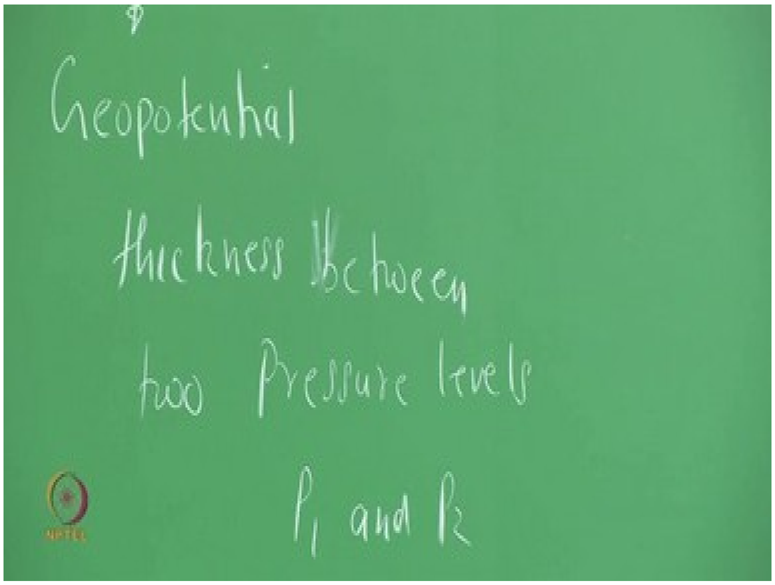

Okay, so I think we are trying to derive the equation for the difference in geo-potential thickness between 2 levels so we got some well we got an expression like this R by Rd by g naught integral no P1 to Pz okay what is the equation number okay so this was the equation we did in the earlier class. So, this is the geo-potential thickness level between 2 pressure levels P1 and P2.

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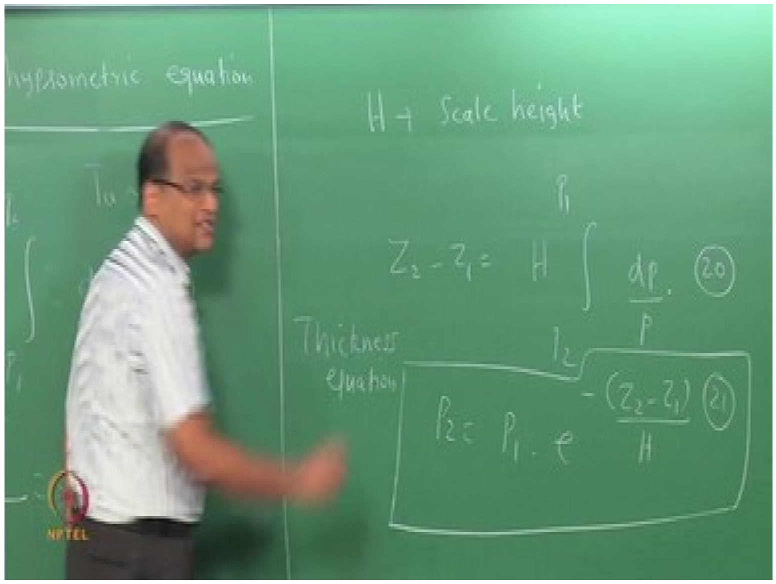

Now we will look at the look at the concept of scale height and the hypsometric equation okay. If Tv is so very close to T, so this will become okay, so this is or we change it to okay. We will stay with this or you can change it to RT by g naught integral P2 to P1 and take care of the - thing whichever way is comfortable okay. So, now this RT, we are trying to figure out the units of this earlier class now we will just, we will just clean this up.

So, this R1 Joules you use Kelvin okay, goes let us take this fellow, kilogram meters per second square, correct people are saying it is dimensionless and all that in the morning I also thought I was worrying way the fundas, suddenly getting wacky okay. So, how do I get a scary okay? So, this Rd T by g naught is called H, the H which is the scale height of the atmosphere. Once you have the scale height Z2 - Z1, please check whether the - sign is all right, I have done something here okay.

So, this form, this form, this form they are all various forms of the hypsometric equation the if hypsometric equation is also called the thickness equation that is it gives you an idea of the thickness but in 2 pressure levels P1 and P2 okay. So, if you know the scale a to the atmosphere a priori and you know the 2-level Z1 and Z2 and Z1 we start 1 1 if you start with the pressure P1 it is possible for you to calculate the pressure at the level corresponding to 0 if you know what the okay if we know the 2 levels Z2 and Z1.

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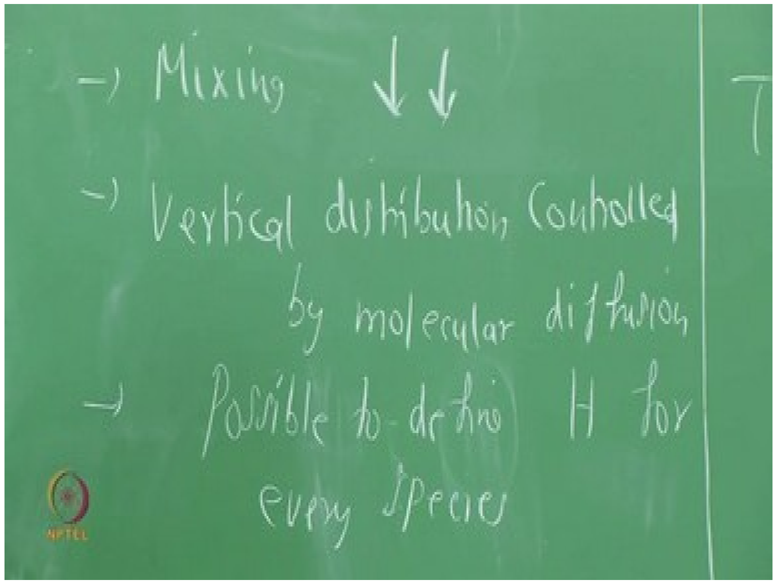

Now, let us work further on this what is that hey H what is this value 29.3 T, so, between the stratosphere and troposphere you can assume a mean value of the temperature of the atmosphere to be 255 Kelvin, somewhere between troposphere and stratosphere. What will the value H will be which becomes 7.5 kilometers. So, 7.5 kilometers is a good estimate of the scale height of the atmosphere okay that is something you have learnt in today's class?

Now what is the story this is a troposphere and stratosphere and all this be on turpopause what about mixing beyond turpopause, mixing is very, very less okay. So, this vertical distribution is mostly controlled by molecular diffusion okay. So what, what is the big deal, if mixing is very, very limited beyond the turpopause and the vertical distribution is controlled only by molecule diffusion. It is possible for you to define the H for every species okay.

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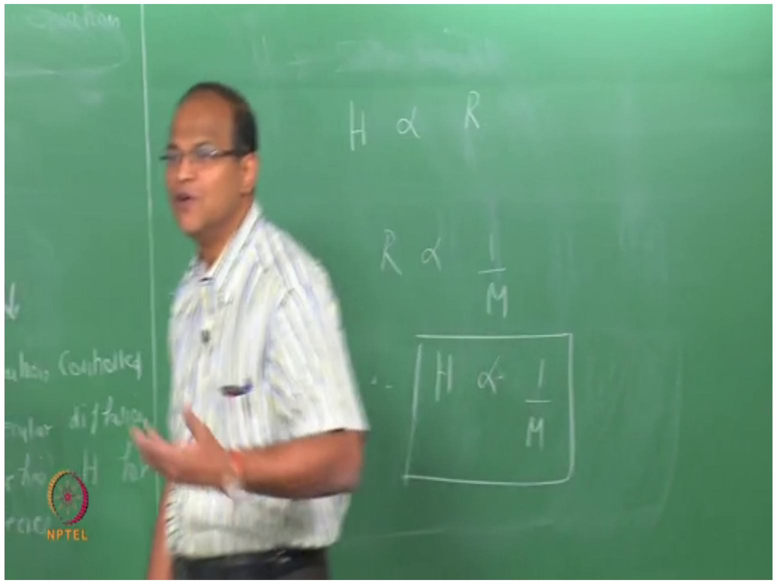

For each gas what is the relationship between this H for each gas and the molecular weight, H you will find heavier gases at the top or lighter gases at the top light I gases at the top so for heavier gases the concentration will exponentially decayed very quickly right. So, the lighter gases have a chance to escape like hybrid correct okay. So, this is so far as the hypsometric equation in scale light is concerned.

Now no no no that was on is somewhere it is coming only in between, so not all gases are present at all levels. We are talking about 100 kilometers whatever is present its distribution will be characterized made this. This equation is not applicable for troposphere okay. I already told you that it is only in the stratosphere some you look Ozone is maximum only in the stratosphere okay. So, we have lot of problems in today's class.

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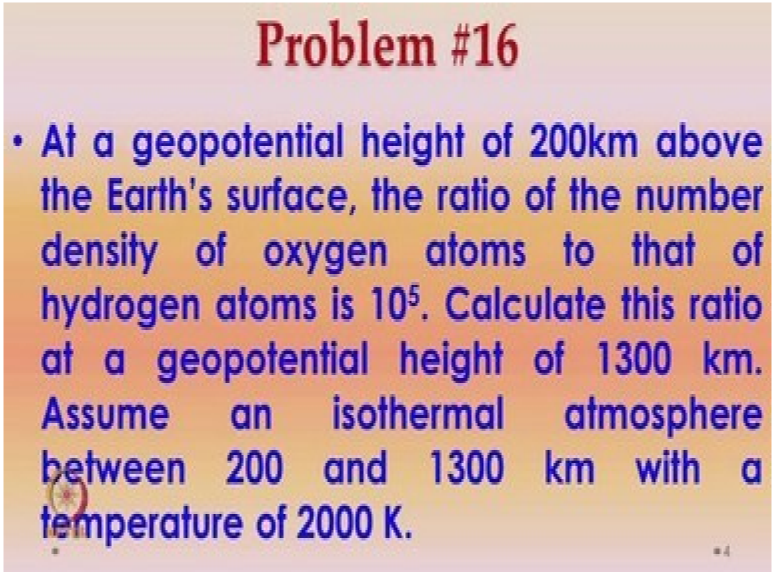

Problem number 16 at the geo-potential please take down problem number 16 at the geo-potential height of 200 kilometers above the Earth's surface a typical quiz question okay. At the geo-potential height of 200 kilometers above the Earth's surface the ratio of the number density of oxygen atoms to that of hydrogen atoms is 10 to the 5, calculate this ratio at a geo-potential height of 1300 kilometers at the geo-potential height of 200 kilometers above the Earth's surface the ratio of the number density of oxygen atoms to that of hydrogen atoms is 10 the 5 10 to the power of 5.

Calculate this ratio to geo-potential height of 1,300 kilometers; please assume an isothermal atmosphere between 200 and 1300 kilometers to make the problem tractable. Assume an isothermal atmosphere between 200 and 1300s with a uniform temperature of 2000 Kelvin okay temperatures will be very high right. It is much above all this turbo polynomial we are going to do the math and solve the problem before that do you expect that ratio to come down or what.

It will come down drastically correct, so we just now saw oxygen is much heavier compared to hydrogen. It has got a very high ratio that is for every hydrogen atom you have 10 to the power of 5 oxygen atom at the ground. At 200 kilometers you nobody can survive. In fact at 43, 44,000 feet, that is 43,000 feet is a service ceiling for most commercial jetliners what did 44,000 feet you just the cabin is exposed to the outside for a few seconds all people will die.

There would not be enough oxygen okay there is one theory about this MH70 know, MH370 he went even beyond 45,000 feet and all okay. The service ceiling for example the Boeing triple777 300 ERN so on, he is 43,100 feet okay. The pilot has to ensure that he stays below that level all the time okay even on a transatlantic or a trans polar flight okay. Please solve this problem, problem number 16.

So, when you fly in an airplane you it has to be the cabin has to be pressurized to get enough oxygen. So, it cannot be at the same pressure as the outside, oxygen will be very low okay. Of course temperature here the pilot has to heat the inside outside temperature will be - 52 degree centigrade at a height of 12 kilometers from mean sea level.

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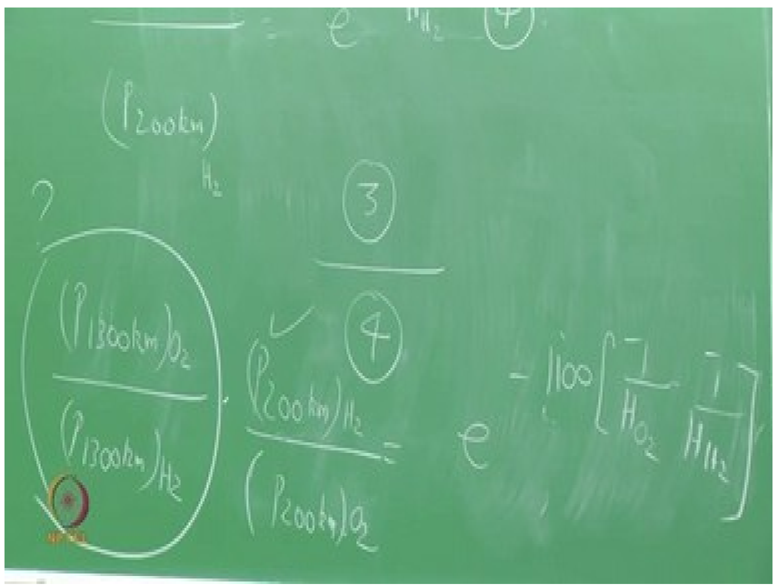

Problem number 16; so, let me start off the first step which is the most difficult step a lot of you have ignition trouble okay. So, P 1300 of oxygen divided by P 200 kilometers we do not; I told you that half 100 kilometers you can define a edge for each of these okay Z2 - Z1, what is Z2 - Z1is at 1000 - that is =, yes correct. So, please note it is in kilometers okay that is the similar equation for; write a similar equation for hydrogen divide one equation by the other P 1300 of a; P 200 of oxygen divided by P 200 of hydrogen is 10 to the power of 5, already given okay.

Now I am given right I have given it to you, So, work out a relationship with which you will be able to get P 1300 of oxygen divided by P 1300 off what is that hydrogen as a function of only the H of oxygen and the H of hydrogen build. The last 2 steps involved the calculation of scale eight for oxygen and hydrogen substitute all the values plug it into your formula then you will get the results.

Please do it, so, P now I do 3 divided 4 any problem he is getting strange results, no that is Kelsey problem. So, P divided by this would be P200 1 by H2 – or + 1by H is it okay. Now what we want is this Ray, this is what we want this I gave you okay this and this if you calculate you are done.

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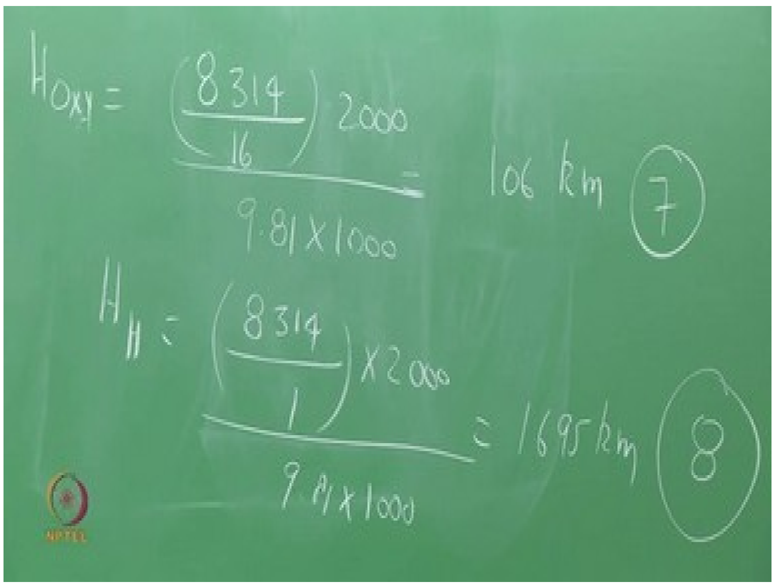

Let us do that what is H of O2 Rd, Rd by g naught 32 what is this take care of this equation is in kilometers H is in kilometers. How much is this, what is it wrong that hang over of the 250, 500 yes, yes how much is this I get something 8314 oxygen is 62 or 32 or 16 no it is capping. So, this has to be lower, then I will say hang on H no problem no sweat both are atom no but this one by H minute that you get but okay.

So, let us make it 16, I get some 103 kilometres something okay fine 106 kilometers, that is too bad the normal atmosphere yeah when you are looking troposphere and stratosphere II so what is the scale height 7.5 scale 8 is too much then H of or input X of oxy whatever okay. So, H of H is 8314 by 1, it is too much okay. Now this is 5, 6, 7, 8 okay.

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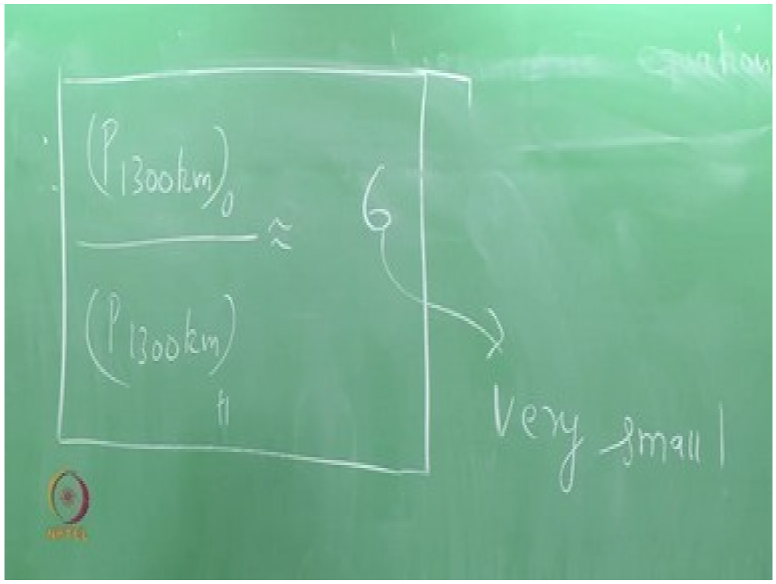

Substituting for what does this over head 10 to the power 0f +5 or - 5 at the geo-potentially the ratios okay P, I got it at O okay now please tell me into, that is very good e to the power of – 1100 1 by 1 by 106 you can pack that 1695 you do not contribute anything correct. Will be just a 10th or t20th of that okay, how much is this 10 power simple, okay very nice okay.

So, so the ratio is only 6 though this is 16 is to 1, so the ratio is oxygen is in really bad supply and those places O by very small with respect to the 10 to the power of 5 with which you started out okay. Bisrajith are you checking, check, check no problem. So, what is the bottom line what is the take home message from this problem oxygen concentration rapidly decreases with height okay. Now you cannot we have just to use the T the original scale height and hypsometric equations that is the Tv was inside the integral.

It is not possible for you to always neglect this variation and then just say that Tv = d and put it and pull it out of the integral it cannot always be neglected okay so I think now what I am going to write is a more general hypsometric equation.

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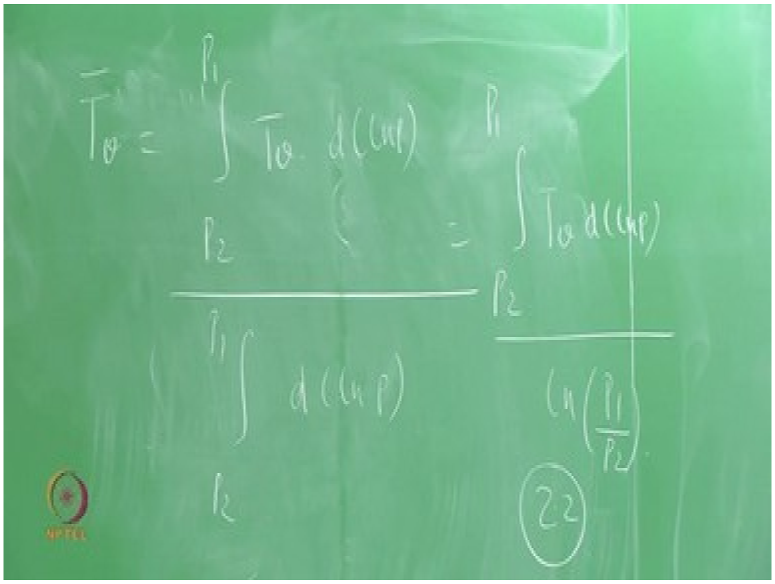

So, we have to define put Pv bar so you have to define a mean virtual temperature which is called Tv bar which is given by integral P2 to P1 Tv d of non P divided by p2 to p1 d of non P. What is this, non yes please non okay so keep this result now. We go back to the equation numbers when we started out today's class is it this would be 22 or 23, 22 okay. Let us take stay with 22 okay. So, what we are saying is generally that Tv = T will not work. So, this is the more general form.

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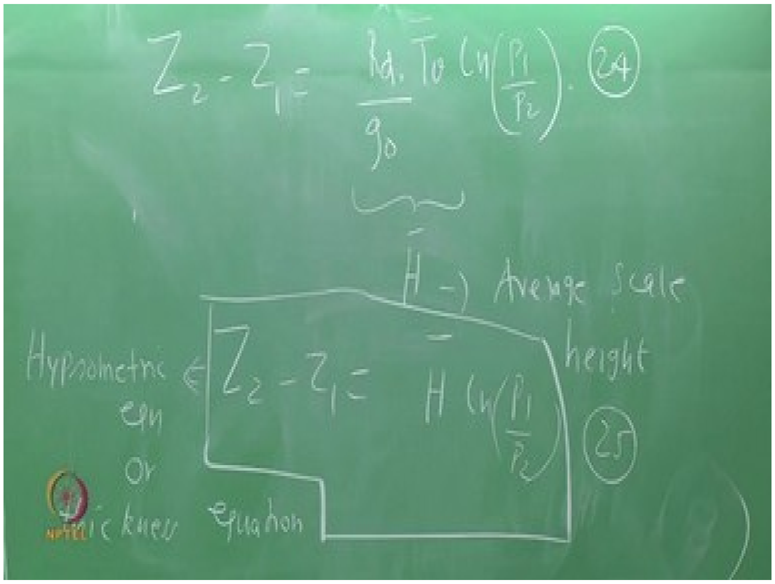

Now compare this with your original equation what does the original equation like let me see where is the original equation okay. So, we had this equation correct the d of non P can be taken as is it okay. What is d of non P dp by P ray okay, now please help me you got you got this integral P2 to P1 Tv dp by P is nothing but Tv bar into non P1 by P2 this 1 if you substitute here then you will get this scale thickness equation where the Tv bar is considered or factored in is it okay.

So, substituting for so substituting for this thing from equation 22 what do you get okay what can this be called H-bar very good. So, this is a average scale height, so I stand corrected so Z2 –Z1 is that H non P1 by P2 is also some form of hypsometric equation the correct form of the hypsometric equation is this okay this is the original undiluted hypsometric equation. It is also called the thickness equation because it gives an expression for the thickness of the geo-potential height okay.

With the scale height and the ratio of the 2 pressures it is a fundamental equation in atmospheric science all right.

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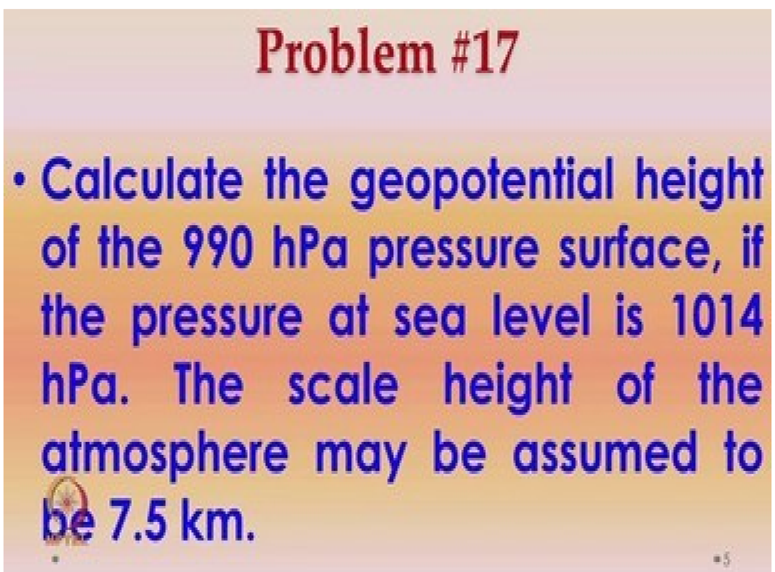

Problem 17, so let us move on to problem number 17 please take down this problem calculate the geo-potential height of the 990 hPa power pressure surface if the pressure at sea level is 1014 hPa calculate the geo-potential height of the 990 hPa power pressure surface if the pressure at sea level is 1014 hPa hectopascal.The scale height of the atmosphere may be assumed to be 7 and half kilometers.

Simple and so please apply the hypsometric equation when it says scale height know is it H or H bar? H bar very good so the H bar is 7.5 apply the hectomeric equation everything is given to you right. What is a Z dead sea level or very good the dead sea level is 0 all right.

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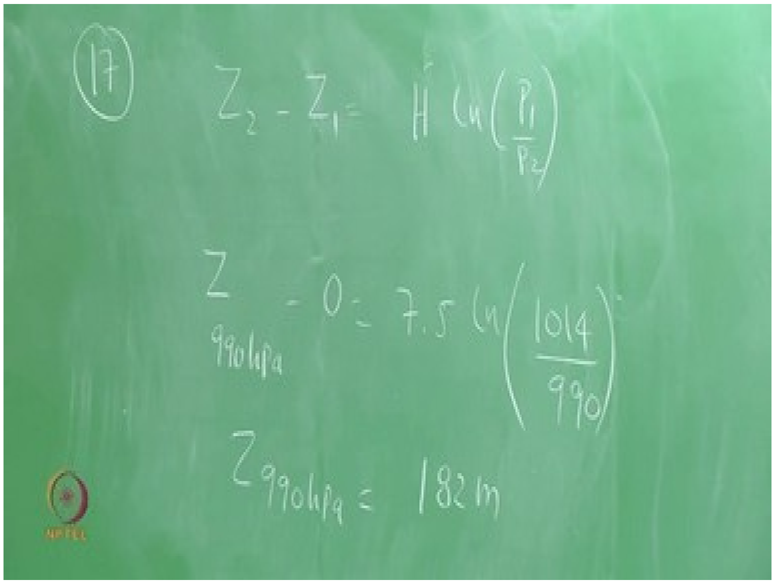

1014 by; how much is this 182 meters okay? So, we need not say pressure is 990 hPa I am talking about the cloud or something at 182 meters means it corresponds to pressure of 990 hPa. So, pressure and Z are mutually interchangeable that is the important thing in atmospheric science. So, the Z can be used as a proxy or okay Z can be used as a surrogate or a proxy for pressure. Because, do you using the hydrostatic equation you know the relationship between this.

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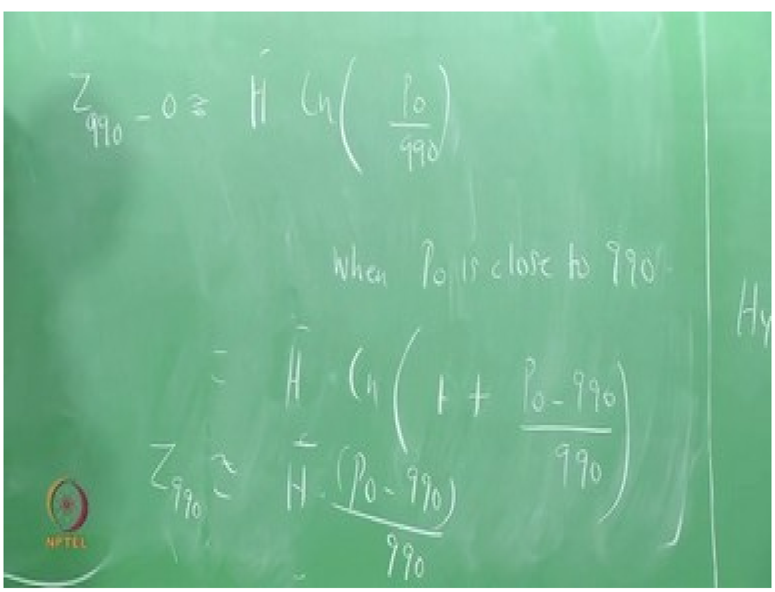

We can also do it however very because this P naught is very close to the P naught is very close to the 990, there is a; it is possible for you to, is this correct, what is lonna of 1+ write what lon at lon of 1 + X and X is much, much smaller than H is = this is not required if you have the calculator you can go ahead but I want to look at the asymptotic limit and look at what happens in; how much are you getting this way okay.

Before calculators were invented people would have used such methods like this in fact the calculator does that instead of stopping with the index it does an expansion the other thing is an expansion okay. So, this is a smart way of doing you may or may not agree with this statement but I think it is a smart way of doing it. So, around 180 meters is 990, so for 250 I can also have the question as at 300 meters Z, what will be the pressure okay. Or 900 hPa what will be the Z, so you can have questions you can go from Z to P or P to Z okay.

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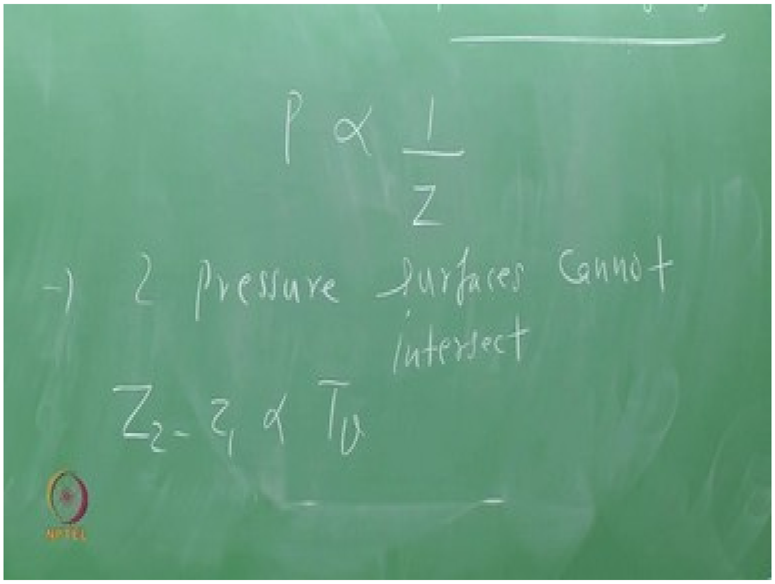

Now, we have to look at thicknesses and Heights of constant pressure surfaces is this fine okay. As height increases the pressure decreases, so P is inversely proportional to Z. So, pressure surfaces a pressure surface is an imaginary surface which connects all the points having the same pressure. So, no 2 pressure surfaces can intersect in the atmosphere is it okay. Just like 2 streamlines cannot intersect to pressure surfaces cannot intersect very important concept to precise pressure services also Z2 - Z1 is proportional to Tv correct.

As Tv X as Tv increases the air between the 2 pressure levels expands and therefore the layer becomes thicker or thinner? The layer becomes thicker. I come again as the temperature increases the air will expand so between 2 pressure levels the air will become thicker people who are frowning and who cannot digest what I am saying you will digest after we were good problem number 18 okay.

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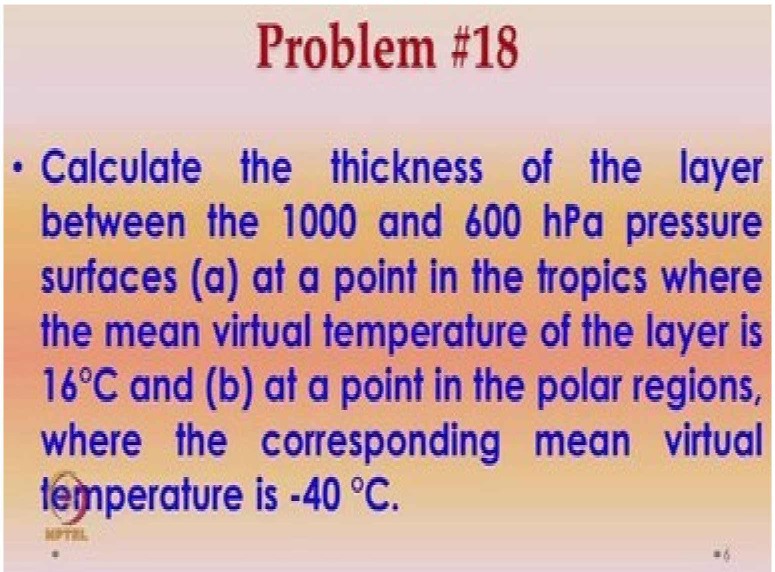

Yes, so this will be the last problem for today's class after we do this we are done okay please take down this problem calculate the thickness of the layer between the 1000 and 600 hPa pressure surfaces. I am giving you 2 situations one is the tropics where the mean virtual temperature of the layer is 16 degree centigrade and the part in the second one corresponds to the polar regions where the corresponding mean virtual temperature is - 40 degrees.

So, I dictate the problem fully calculate the thickness of the layer between the 1000 and 600 hPa pressure surfaces a at a point in the tropics where the mean virtual temperature of the layer is 16 degree centigrade and b at a point in the polar regions at a point in the polar regions where the corresponding mean virtual temperature is - 40 degree centigrade in both the cases P2 and P1 are 1000 and 600 they are the same. The funda here is the Tv is different work it up we will just take a couple of minutes.

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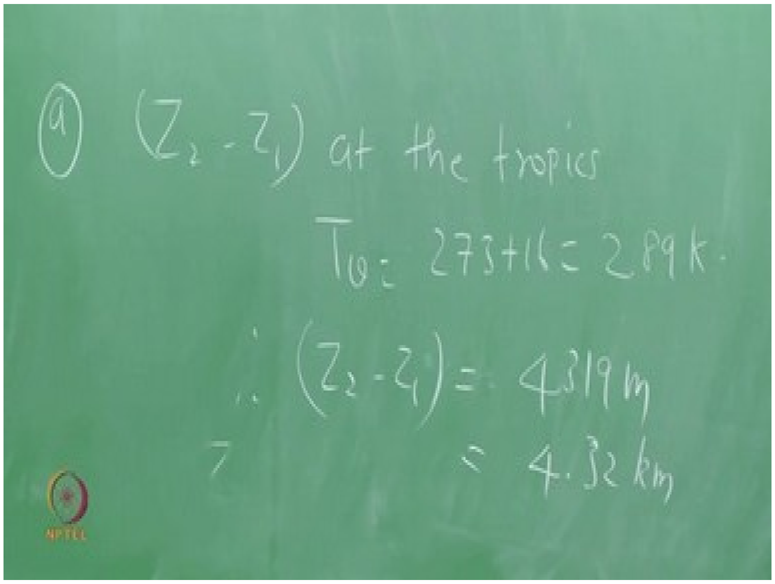

So, Z2 - Z1 g naught into lona variable yes Rd is 287, 9.81, P1 is 1000 okay. So, you have to do smart work this is a constant for both Part a and Part b so greater general relationship where Z2 - Z1 is a function only of Tv okay. So, Z2 - Z1 yes please help me what is this lon of 1000 by; what is value 14.94 very good okay. How much is this, this will be in kilometer ah do not forget everything is in kilometer I hope so, meter, so you are having meter then fine well stick to this. Z2 - Z1 is and now you are saying kilometer okay 4319 meter okay so, now let us do 4.32 kilometers okay, very good.

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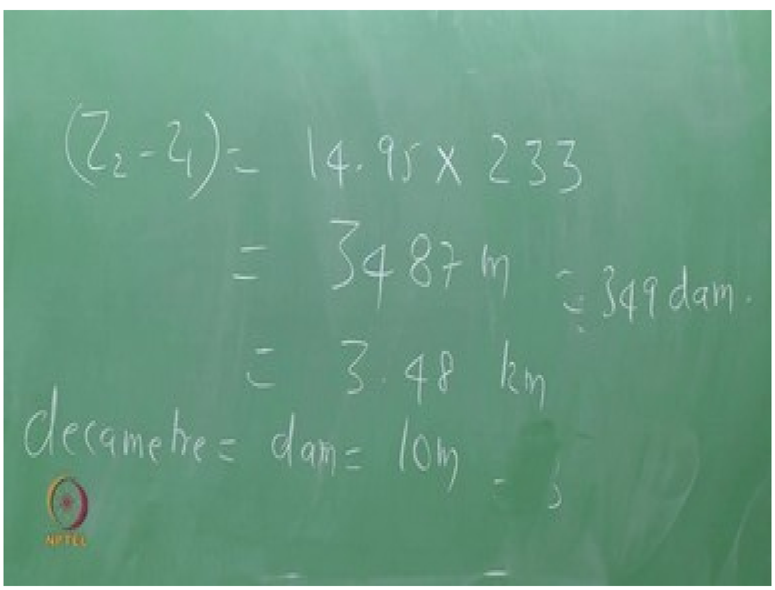

Now Z2 - Z1 at the poles 3000, 3487 okay there is one more unit which is used okay. So, that decameter also called damn what decameter is 10 meter, so this is 432 damn this is 349 damn, so now you believe that Z2 - Z1 is more in the tropics okay. For the same 2 pressure levels 1000 and 600 hPa compared to that in the polar region okay. So, this brings us to some sort of closure as far as the hypsometric equation is concerned.

So, next class I will just quickly go through derivation where reduction of pressure to sea level to handling of the formula and after that 15 to 20 minutes we will get on to the first law of thermodynamics okay CPC CVQ - W tilde you all those things and then we learn to proceed to something called lapse rate. What is lapse rate the rate of change of temperature with height that is very, very important in atmospheric science?

So, we first start out with what is called the adiabatic lapse rate we work out the adiabatic lapse rate which will help us in the problems.