Indian Institute of Science

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Photonic Integrated Circuits

Lecture – 06
Directional Coupler and Coupled Mode Theory

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NPTEL Online Certification Course

Today we will discuss about directional couplers.

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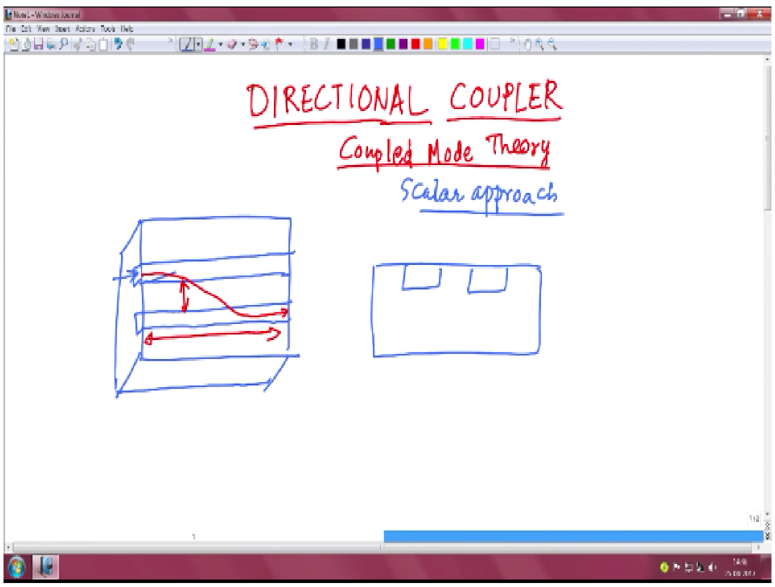

The method called coupled mode theory that will be used to study the properties of directional couplers, directional coupler is a very useful device in integrating operators. So the structure is like this, there are two wave weights which are very, very close together, the cross section can be expressed like this. So when light is launched into the first wave light it is possible to couple or transfer light from first wave light into the second wave light.

So we can change the parameters like the spacing of the waveguides or the length of interaction or the refractive index Δn between the waveguide in the substrate and so on to get different properties of this structure. So in order to study the properties of this we use what is called the coupled mode theory. So basically to address basically we have to solve the scalar wave with pressure corresponding to this refractive index profile corresponding to its directional coupler.

So that is all the wave equations are the field distributions and of course we have the refractive index profile to be more complicated than what we have seen earlier. So let us consider the 2D version of this directional coupler. So we have been following a scalar approach. So the extension to the vectors is of course possible and straight forward but more complicated.

So let us consider a two-dimensional form of this that is going to plus. So recall that we can use the effective index method to convert from the 3D geometry that is given to a 2D configuration.

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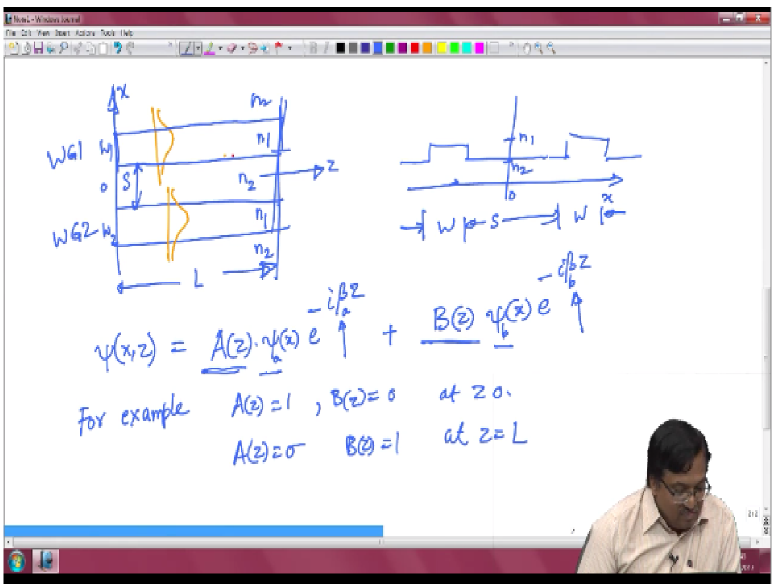

So the 2D configuration is like this. So we have two waveguides it is very, very close together we have waveguide one and waveguide two and this length is important as I said this length of interaction and we have the refractive index of the first waveguide as N1, waveguide second they can be different two different waveguides. So the width can actually be different and there is a spacing we will say the spacing to be S.

And ofcourse the waveguide width W1, waveguide width W2 the corresponding we know the properties of the individual waveguide, because these are symmetric slab waveguides, and we will assume that the propagation direction to present and the waveguides are aligned along the X direction this will be something that we can use. So the refractive index profile can bewritten as follows along the X direction, this X direction I will take the origin to be the middle of this structure.

And the refractive index is into the substrate. So we can write the refractive index profile as follows. Origin the spacing is the width W and so on. So this is N1 refractive index is N1 outside is N2 and of course it is zero. So it is possible to solve this problem as a multi-layer rivet problem by applying the boundary conditions etc, as we do for being symmetric slab waveguide and asymmetric slab waveguide. But of course is more complicated and writing the equations in each of these regions and matching the boundaries and manipulating the questions is quite involved.

And of course one of the purposes of choosing this problem is to introduce what is called acouple of mode theory which will solve this problem in a very simple manner, and of course a couple more theory has got many, many applications can be used for many other purposes as we will see later on. So in the scalar approach we express the mode field as a super position of individual waveguide modes as follows.

So it is assumed that the individual waveguide modes are known. So we can say that the individual waveguides have their mode fields known, because we can solve them as we have done earlier, we can solve for the propagation constants and the mode fields etc. So individual mode fields for the waveguides are known. So in scalar coupled mode theory we express the mode field, the entire mode field of the structure as follows, as a combination of the mode fields of individual waveguides.

Let us say $\psi(x,z)$ is the field that we are trying to evaluate and this is expressed as a linear combination as follows, $A(z)\psi_a(x)e^{-i\beta az}+A(z)\psi_b(x)e^{-i\beta bz}$, where ψa and ψb are the individual mode fields and β_a and β_b are the corresponding complication constants. So the coefficients A(z) and B(z) are Z dependent and represent the variation of the field and the exchange of power between the waveguides etc.

Just as an example this A=1 and B=0 at Z=0 it implies that ψ(z) is ψ(x) and the whole field this is the whole field the power of a whole field is concentrated only the first mode. Similarly at Z equal to at the end of the waveguide if you say that A(z)=0 and B(z)=1 at Z=L it implies that the whole power is in the second mode you can say that the whole power has transferred into the second mode if you get such A out.

So the crux of the problem boils down to evaluating A(z) and B(z) since we said that they are functions of Z, it implies that the power is exchanged between first waveguide and second wavegude, and we need to evaluate how they are exchanged. So let us solve a scalar waveguide equation and substitute for the mode field and evaluate.

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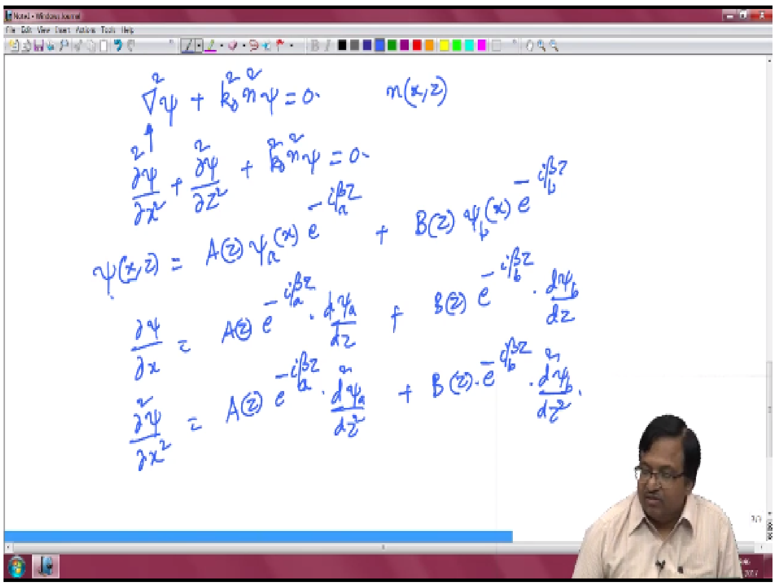

So the scalar wave equation for the whole field is given by $\nabla^2(\psi)+K0^2m^2\psi=0$ where n is the refractive index profile n, x, z. And the operator has X variation as well as Z variation. Now expressing the total field in terms of the individual mode fields we get ψx, z is A(z) $\psi_A(x)^{e\ bar\ -\ i\beta az}$ and B(z) ψB (x) e- $^{i\beta z}$ we are considering two arbiter wave guides which may have a different propagation constants.

So let us differentiate dψ/ d the equation constrains dψ/ dx in this by dz so we have to evaluate each of this so these have dψ is simple A(z) e $^{-\ i\beta az}$ x dψ / dz and B(z) ψb (x) or – $^{i\beta z}$ x dψb/ dz and the second derivative is also needed in the expression okay because ψ is a function of both x as well as z let us use $d^2\,\partial^{2/}\,\partial x^{2.}$ Next we need to the definite ψ with respect to z two times.

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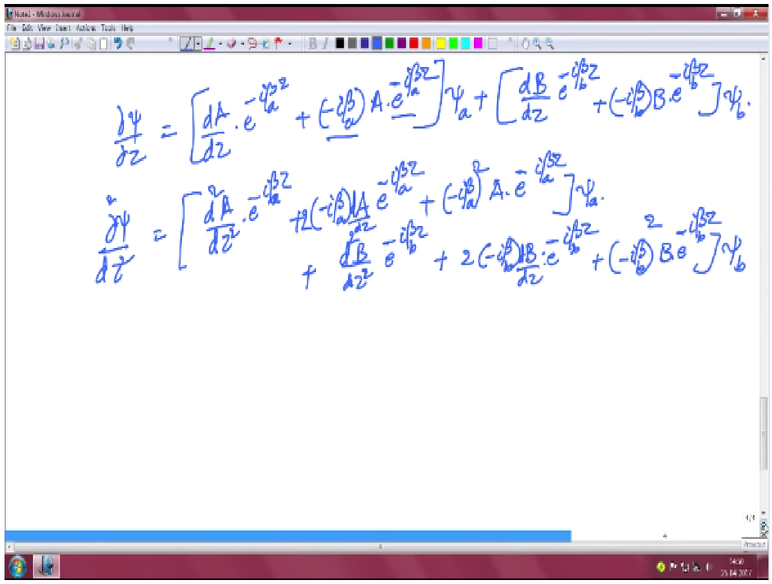

dψ/ dz x $e^{-i\beta az}$ + - βa x A x $e^{-i\beta az}$ so derivative of $e^{-i\beta az}$ results in the factor, so the entire thing multiple ψa similar expression for ψb now we need the next derivative of this $d^2\psi/ dz^2$ so second derivative with respect to a + retaining da / dz and differentiating this that means the same term so we can write it as $d(\psi a)^2$ x A $e^{- i\beta az}$ because already one terms is there we just say double of that + no this one there + differentiating once again this vector – $i\beta az^2$ x a $xA^{- i\beta az}$ + the corresponding factor with in terms of d d^2B/ dz^2 x $e-^{i\beta az}$ + 2 x – iβb x b just a minute please.

This is second derivate dA/ dz dB/ dz $e^{- i\beta bz}$ + - $i\beta b^2$ x b^{z} x $e-^{i\beta bz}$ of ψb cos I A as to be there here. So they are special for dz dz / dz^{2}

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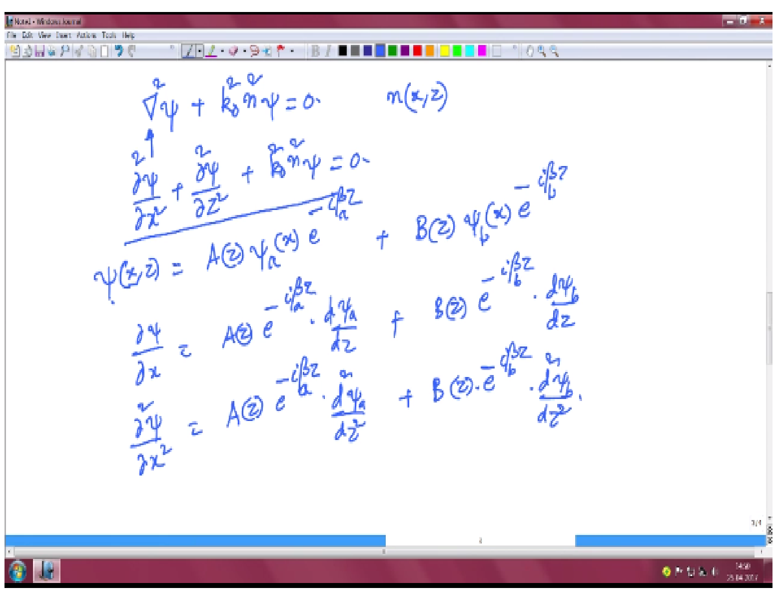

So just you can see the expression of ψ differentiating with respect to x so all these we need to put in this wave equation and simply.

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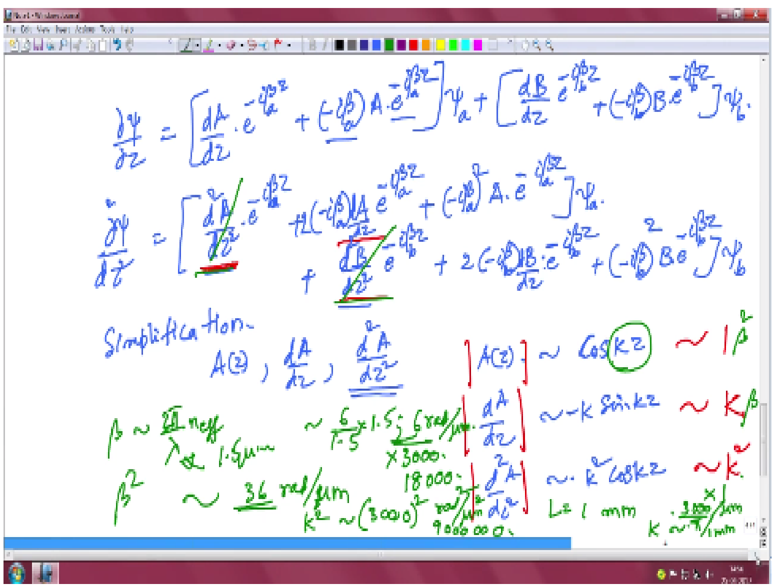

So before we go further we will try to see some of the terms can be neglected so we have the second derivatives and first derivates with respect to z so if we consider the factors A(z) dA/ dz and d^2 A/ dz^2 and how fast or slow the variation they vary we can neglect some terms for example if the variation is very slow compare to β we say that the second derivatives can be ignored, so just we will make a simple calculation and check whether they are really are they can really be neglected for that purpose let us consider some simple form for A(z).

Let me say that A(z) variations cos kZ this is just an example to test or check whether the second derivative can be neglected so if you say A(z) goes as cos Kz dA / dz we go as k times cos kZ k times sin kZ and d^2 A/ dz^2 will go as K^2 cos kZ so we will look at only the amplitude path and say that A goes has the unity and so I will put just the square of it goes as 1 or something so portion to k and the order of magnitude for this is k^2 and also if you look at the equations we multiply that we note that the second derivative is going this is going as one the first derivative is going as proportional to B.

And the second derivative is just not the let us look at once again the first the second derivative is going as one to do some other color for this, so this is multiplied by 1 the first derivative is multiplied by β and the second derivative is multiplied by β^2 so if you say that β is and just very rough calculation I am doing to check whether some of these things can be neglected β as you know is 2 β/ λ x n effective and λ is of the orders of wavelength of light is let me say communication wavelength 1.5microns.

So you have λ going as around 6/ 1.5 x refractive index also of the values if 1.5 so this is also order of 6 radians per micron and then β^2 if it is a expression like the constant β^2 so β^2 will go as say 36 radians per second per micrometer and then this is multiplied by the value of k suppose let us assume that the interaction length or of the odds of a millimeter suppose I have l of mm so the K x l think l the periodicity.

So we can say that case of the orders of typically periodic π / 1mm, so case of the orders of 3000, 300 per micro liter so okay so those are 3000 for my chemical it multiplies this β so we said the product β into case of the orders of 18000 radius square by mm^2 this is of the orders of 36 radiance per micrometer and then you have the second derivatives they are multiplied by K^2 so K^2 goes has 3000^2 radiant square / mm^2.

So this is of the order of 9 more followed by 1 2 3 4 5 6 so now we can guess what are the typical values involved and we can say that these second derivatives are very slow and can be neglected, so all these argument of typical calculation is to assess that the second derivatives could be neglected if the coupling is very slow.

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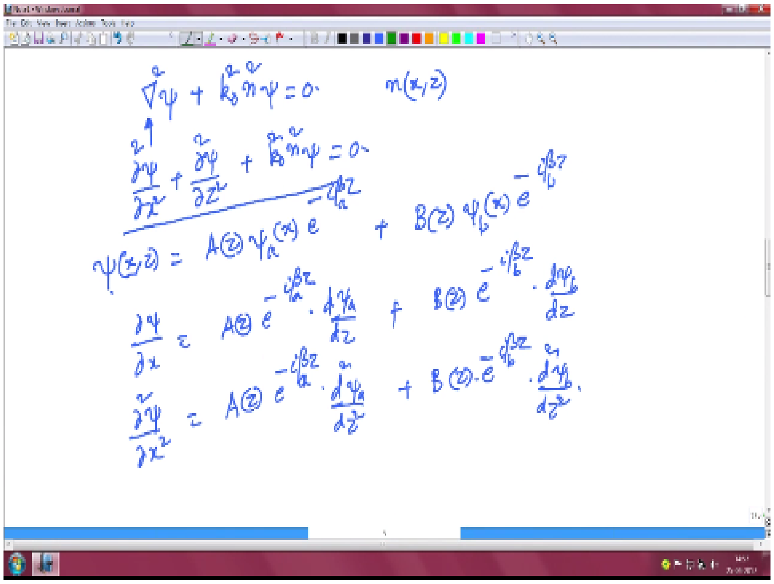

So will reduce this and then substitute to the personal wave equation and evaluate so this original equation to evaluate.

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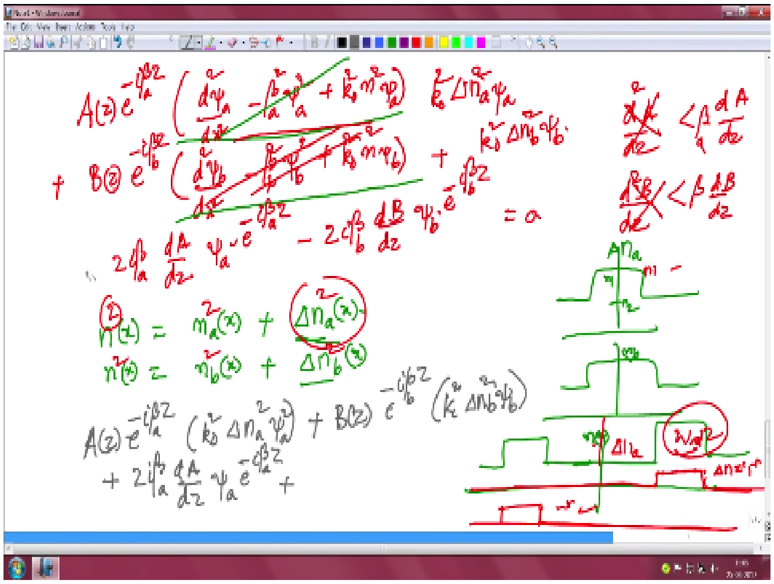

So the work we can take expression can be written like this and we substitute A of Z $^{e-\beta}$ x multiplying dψa dx2 - βa2 ψ2 + $K0^2$ m^2 of ψ and the other factor βz - I βbz d2ψb / dx2 - βb^2 into ψ2 + K02 m2 ψb2 ψb + 2ψba dA / dZ - 2ψ dB/ dZ – ψB e – iβ Bz = 0 I have broke the terms like that you can absorb an interpreter and take action on these each of these terms, so here as now we neglected d^2A / dZ^2 / d^2B / dz^2 compared to dA / dZ terms which over βA, so compared to them.

So the individual waveguide modes are grouped in these terms as you know dψA dZ of ψA and ψB satisfy individual wavelength modes which means that you can already different profile at this for the each of these modes fraction X profile and a let us call it na and na which is called n 1 here and n to add this one boundary nb and the actual profile is against the given here the difference, so we can say that the given refractive index profile n of x can be written as a combination of na and nb.

Like this n of x can be expressed as enough a point of x + Δna of x and n of x can also be simultaneously be written as nb of x multiplied by a perturbation Δnb of x, let me explain the terms na and nb, so this is given as fraction profile and if you say that this is the refractive extra hole for the any this extra term will be something like this, so the same scale I will put with Δna let me say this is n this Δna and will look something like this the place it has got a value of n is conditional output that I am only.

So if you say let, let we see as example we can say any one that the whole region into the cladding region this is $\Delta na = 1 - n2$ and 0 value all over, so the refractive index profile of Δna way will look like this all over the place it is equal to 0 and only in the area of nb this is the area of second wave the place of second layer alone at the Δ value equals, so similarly for nd let us say that n we will be looking like this, so the area of $na = n1 - n2$ and in all other places it is equal to 0 so this is called the perturbation to the na and this is basically called as perturbation to nb.

And it is occurring only in the region which is in the other then in the age of other waveguide, so look at this equation if instead of n you have in a look at observe that this was to 0 so instead of n we can replace to the Δna and then replace is interacting by Δna, so what we get is I simply put here in to not sit at different color but with a different color all these things I will replace by Δ na because I would like to again emphasize extra point that most of the time you have na^2 in your equation.

So I would say that I would represent the perturbation in terms of the squares of the refractive index rather than just any, so if I want to be placed this factors I can replace by Δna^2 see here k0 also of course is $a0^2$ similarly I will press this entire vector by $k0^2$ and use a different color $a0^2$ and Δnb^2 and ψB, so the final expression looks like this I will use a different color, plus the other remaining factors.

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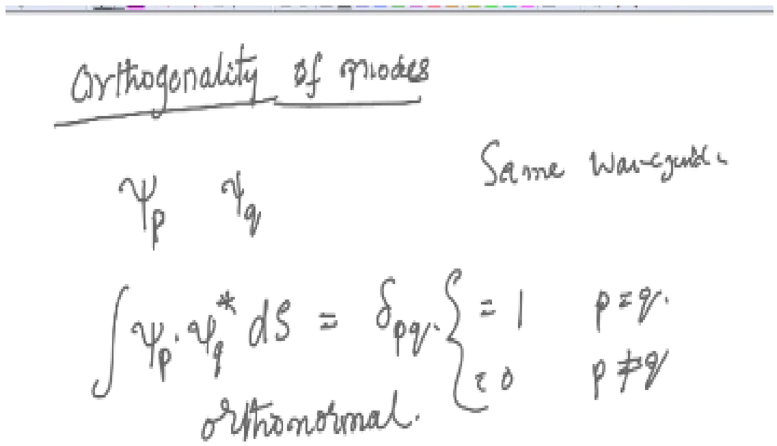

At this point I would like to introduce an idea called the, a clue that we take from what is called the orthogonality of modes. Orthogonality of a mod modes implies if you have a mode ψ_p and ψ_q corresponding to the same waveguide, ψ_p and ψ_q are two modes corresponding to the same waveguide then they are orthogonal meaning that the over large factor of ψ_p x ψ_q the complex conjugate start over the entire cross section equal to either 0 or 1 depending on these weather talk about the same modes or not.

For example if it is equal to 1 $\delta_{pq} = 1$, if P = q = 0, if $p \neq q$ which means if the modes are different then their overall factor is 0 this is called the orthogonality principle and of course most of the time the fields are orthogonal are normalized to 1 the Overall factor to be 1 so then they are called ortho normal, so the clue to take the overlap of the fields comes from the orthogonality of modes which has got a many, many applications and it is very general in nature for any arbitrary types of waveguides.

And so on and so forth but at this point in the in the study of the directional couplers and couple mode theory we take the clue that the overlap of fields can be very useful even though the orthogonality principle is not rigorously applicable to directional couplers because the modes ψ_A and ψ_B do not correspond to the modes of the same waveguide, so taking this clue we will multiply the first expression the expression that we have derived here.

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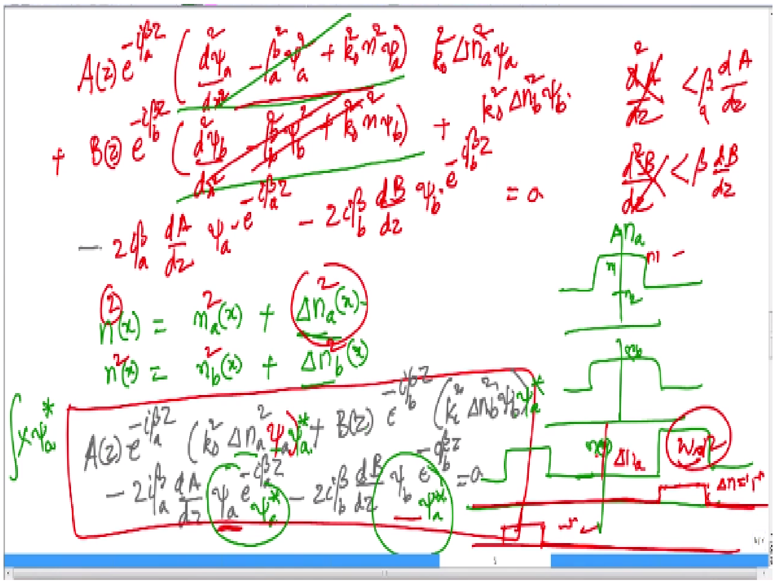

This expression is the one we are trying to manipulate and I will try to multiply this whole expression by here ψ_A or ψ_B and integrate over the entire cross section so that the terms which are crossed ψ_A overlap ψ_A x ψ_B etc will go to zero and the modes which are $\psi_{A\ x}\ \psi_A^*$ = 1 his seems some small errors there will change it is not square ψ_A only. So if you multiply this whole expression with ψ_A so the factors are this ψ_A here and there is a ψ_B here when you multiply with ψA this will go to zero and this is multiplied by $\psi_{A\ ,}\ \psi_{B.}$ So it is not going to be 0 let us see what comes out.

So I will rewrite this expression again or, so first step I will use a different color to multiply and then later on erase. So multiply by ψ_A and integrate ψ_A^* and integrate so this is ψ_A^* multiplication here this is ψ_A^* multiplication here and there is a ψ_A^* multiplications here and here also $\psi_A^{\ *}$ so we observe that here is ψ_A ψ_B combination and here is ψ_A ψ_B combination and we can apply the orthogonality here.

Whereas here we have Δ na ψ_A and ψ_A and the integral of this will not be equal to 0 because Δ na is multiplying and is also a function of X, similarly here ψ_B and ψ_A are there x NB and the integral of this, so this also not go to 0, we can say that we can approximate this factor to be 0 and the second derivative over B is going to be eliminated and we have an expression da/dz depending on e(z)and d(z) that is what I would let us right now.

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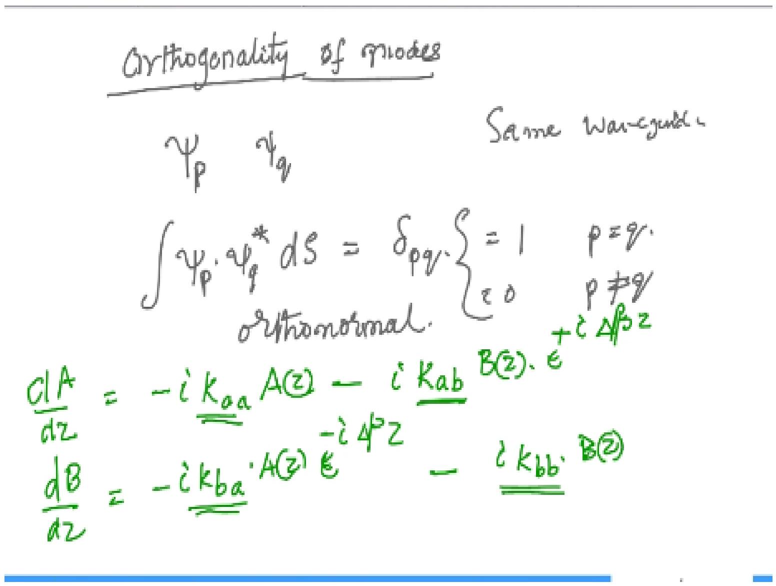

dA /dz written in terms of I will be simplifying and writing the final expression you can work out and then cross-check and of course I am using the two constants kaa and kab so that all the factors could be clubbed into that, the final form is correct into B(z) x $e^{-I\ \Delta\ ß}$ x cycle that means plus for this, so were okay let me write the second equation also and then we will interpret what are these factors.

And for the variation of the dB /dz will be given by $–IK_{ba}$ x A(Z) $e^{-I\ \Delta\ ß}$ x z – iK_{bb} x B(Z) as spend a little bit of time to understand this, the constant K_{aa} , Kab, K_{ba}, and K_{bb} contain the factors of overlap, so I will try to use the previous expression.

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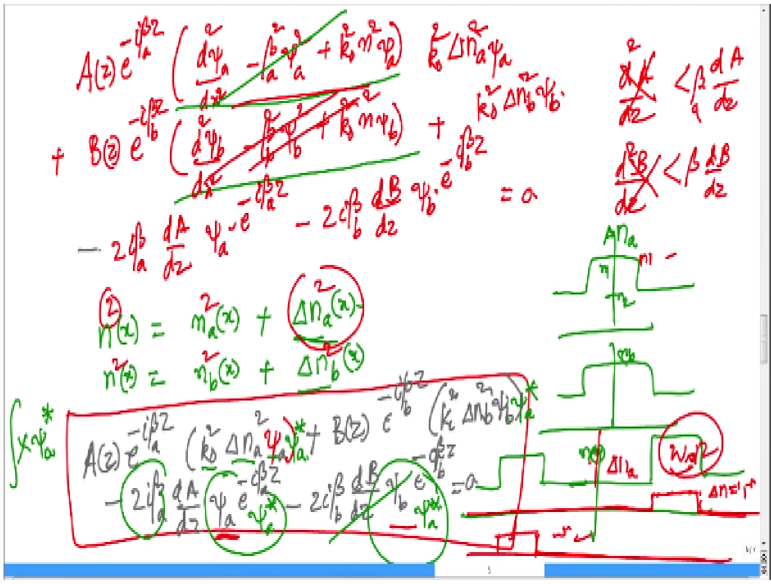

To manipulate and then show that once again so I will erase unnecessary fields and then try to manipulate this equation, so you have f(Z) x $K0^2$ and Δna^2 so and here we have 2I Δ ßA so we will do this goes to 0 and this is divided by this and you have I have multiplied throughout by $e^{R-I\ ßAZ}$ so this is will not appear at this point.

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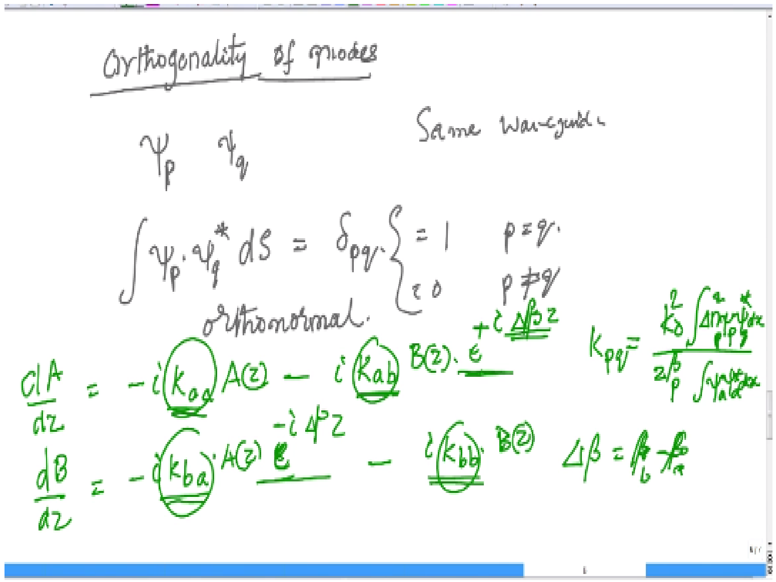

So the final equation is like this, so the K_{aa} or I will write a general expression K_{pq} let me say deposited in all p and q can b can be expressed like this, $k0^2/2ßp$ x you must have the integral with respect to Δ in a should be there in the expressions and ψ know is how we have been writing in terms of A and B so let me put in form of P and Q, Pψp, $ψq^*$ ds and of course the integral with respect to.

Or let me call it as X only for the time being dA /dz, Ψq dx, ψa, ψa* dx I hope this Is representing the and also will be minus, lot of minus and sub heading all the minus as could be complain to the constants, this corrector okay. So minus signs have added I is already we some used here so that is not necessary to be there and these are deposit, so these coupling coefficients so the so-called coupling coefficients are constants depending only on the refractive index profile.

The proper ways to the reflect index profile the waveguide dimensions and so on and so forth. And ß, Δß a factor Δß is what is used here, is ßB – ßA. Let us not involve plus it is, plus and the other plus is minus.

So these are quite general and is applicable to many types of units, let us simplify further taking a specific example. So I would say that the cup let us examine the nature of the constants kaa and kbb with respect to Kba and Kb.

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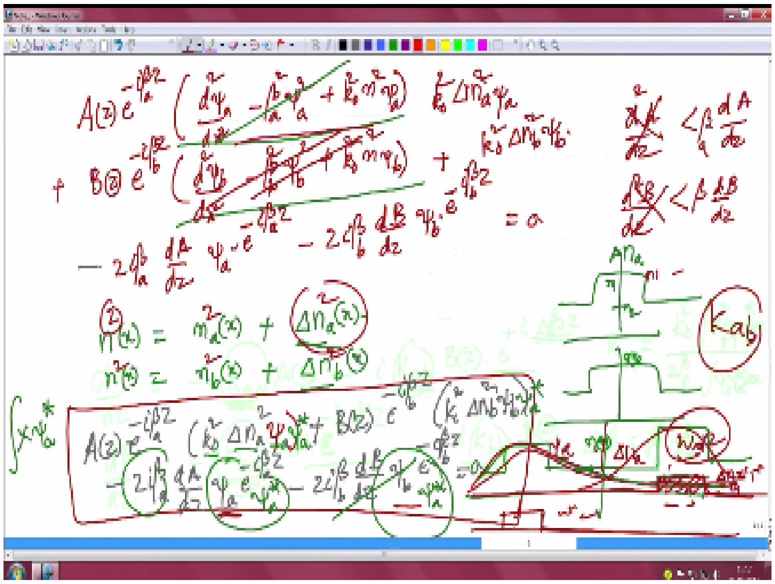

So we the Δ as I described earlier we are integrating over the region of perturbation only, ψ so in the case of ψ A we have the mode field ψ A, let me use some other color mode field ψ A the mode field once again one more ψ A because well multiplied there and the Δ na which is in this region, in contrast to that the constant K_{ab} is containing the mode field ψ_a the mode field ψ_b which is in this region and the perturbation Δ_{na} is so the product of Δ_{na}, ψ_a and ψ_b is what is appearing in K_{ab} the product of ψ_a which is very small here and considering ψ_a which is again small here and then the small quantities gets here.

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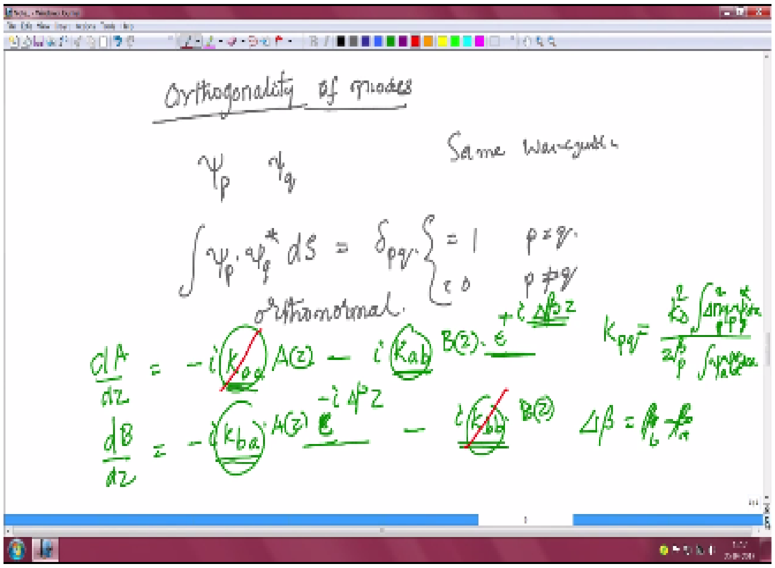

So we can argue that these coefficients are small compared to the other two coefficients and we get a very simple set of differential equations for the mode field.

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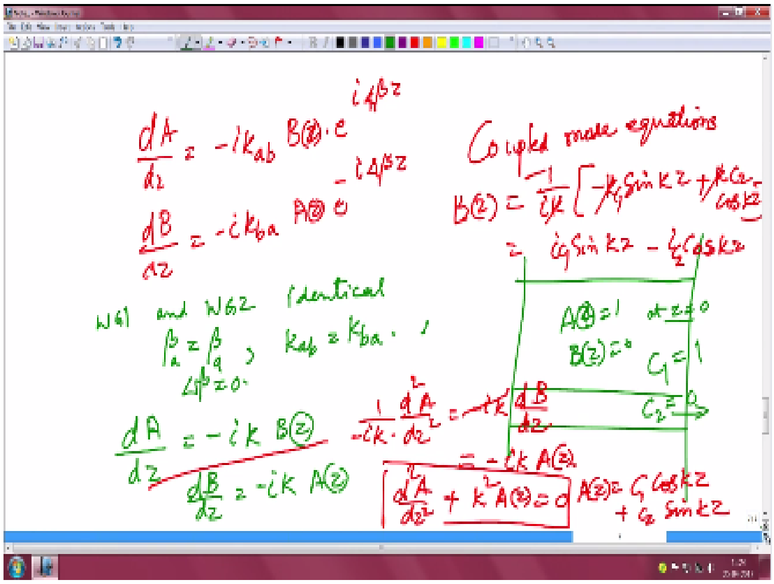

So I will quickly write again dA/dz is $-ik_{ab}B(z)$ and $e^{i\Delta\beta z}$, dB/dz is $-iK_{ba}$ A(z) $e^{-i\Delta\beta z}$ 1 is plus and 1 is minus whichever where you define the $\Delta\beta$, so these are called a coupled mode equations. We make several approximation please check your derivation and note all the places that we have approximated in the reasons for that. Let us apply this for a simple case of a directional coupler consisting of two wave guides.

I will not evaluate the coupling coefficients A and B right now, but let us see what happens to the solution when it comes, so when the wave guide 1 and wave guide 2 are identical we have a directional coupler consisting of two identical wave guides, so as I say you can call this as a symmetric wave guide directional couplers and check what happens when we launch like into the first wave guide and when you receive the light from the second wave guide.

So the wave guide 1 and wave guide 2 are identical you have $\beta_a=\beta_b$ and so $K_{ab}=K_{ba}$ and $\Delta\beta=0$, $\Delta\beta$ is 0, so you have to very simple form of the differential equations for the variation of the field injected to the wave guides, iK multiplied by B(z) and dB/dz equals let me use still use –iKA(z) these could be solved easily by differentiating the one expression between two the other. For example, I replace B into this so I will differentiate this with respect to β once and substitute. So you have $d^2A/dz^2=-iKdB/dz$ so –iK if I am come over the side and for the dB/dz I can replace with –iKA(z) and so we have d^2A/dz^2+ this K will go that side minus and $K^2A(z)=0$ this is a very simple equation which we know and the solution can be written in terms of cos and sine A(z)=C1 cos Kz+C2sin Kz where C1 and C2 are arbitrary constants. So we will quickly evaluate them by using the initial conditions.

So let us say the line like is launched in the first wave guide and not the second wave guide, so that of course we need also the expression for B which we can get from the first expression we can substitute for the A into this and get the expression for the eyelids where here. So B(z) can be given as -1/iK[db/dz] so –k sin kz Ci into that +K2kC2 cos kz this k's will cancel out with this and you get i will go to opposite and we make can it +i sinkz –i cos kz and this arbitrary constant still remain.

When we apply the initial condition so we can eliminate and get what are these A and B, let me quickly do using so let me say A(z)=0 A(z)=1 all the light is in first wave guide and we have z=0 as I mentioned in the beginning at that equal to 0, so substitute z=0 in this and A(z)=1 and we get C1=1. Similarly substitute we have z=0 in this expression and we will get C2=0 so we can say that the solution for this is.

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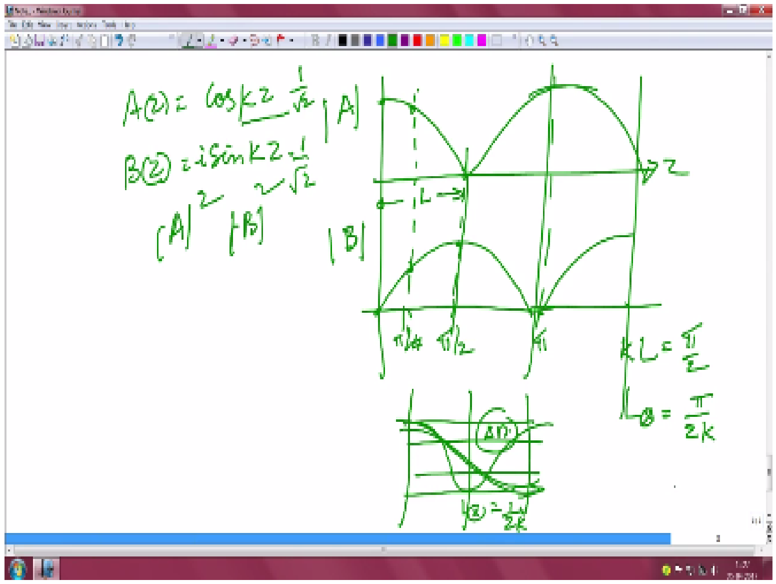

A(z)=cos kz and B(z) is –i sinkz so I plot the amplitudes to see what happens ,so let me this is z population direction here all the amplitudes on this side so this is going as a cos I will put only the magnitudes to illustrate is not a very and this goes has a sin and so on, so this is the magnitude of B and magnitude of A. From this we observe that when the field is maximum here the field is norm field no field in the second wave guide and the field when it goes to 0 it can go to 0 by a couple lengths called coupling length and the light.

So maximum this kind of wave guide, so by the use of this analysis and the study and the couple mode theory we can say that there is a periodic exchange of power between wave guides with the periodicity of pipe so is going back is the periodicity of pipe. If you choose the directional coupler length to be L which is equal to KxL if you choose it as equal to π/2 there is a π/2 point which is here the all the powers is esteeming into this.

So the coupling length necessary to first cross over can be called as π/2k so if you choose the length of the wave guides to be equal to π/2k then you can get the cross over state of the directional coupler, so all the part is going to this L and put the cross equals π/2k and of course you can observe many, many other things for example, it restores to the original situation after a length L equal to π.

And once again it L=3π/2 also you have cross over state and so on and so forth. There are several interesting features for example, which was the coupling length to be half of this π/4 for example then you know at this point the field is 1/√2 cos case that is 1/2 sin case it is also 1/√2 and you can say that the power is divided equally powers which are proportional to A^2 and B^2 can be power of representing the power and power is ½, so power is divided equally if you choose the length there cross by 2.

So in conclusion you can say that rational capacity is a very useful device I would just like to mention that in optical features we can change the Δ in the refractive index by using external fields and we can switch between the cross over state on the bar state by using external electric fields will come over to these after few more classes. So finally to summarize directional coupler is a device that can be used to exchange power between wave guides and there are many, many applications, thank you.