Indian Institute of Science

Photonic Integrated Circuits

Lecture – 04
Vector Modes

T. Srinivas
Department of Electronics Communication Engineering
Indian Institute of Science, Bangalore

NPTEL Online Certification Course

So in this discussion we will consider vectored modes.

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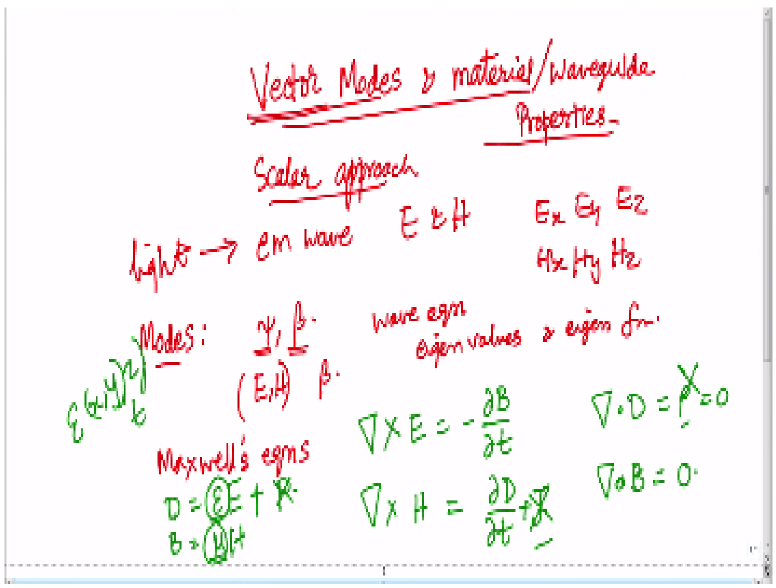

And other material properties, material or waveguide properties, when we started the optical wavelength theory we consider the scalar modes that is would, we would like to quickly we wanted to quickly get into the properties of optical waveguides and so we take a scale outwards to simplify the mathematical complexity and so on but actually as we know the light is an electromagnetic wave, consisting of electric fields and magnetic fields and all its components let Eva e z + HX h y +h z.

So we want to have a complete picture of the light propagation in integrated optical circuits we need to consider light as an electromagnetic wave and we need to take what can we call as a vector scratch which considers all the components, so before we proceed further that way we will have to define what are the modes, we have already seen in the scalar balls that a mode is pattern of four field along with this propagation constant. we have mentioned that the board's the mode field is the mode field and the propagation constant are obtained from the wave equation we use the scalar wave equation and they are obtained as the wave equation of the scalar wave equation they are obtained as the ideal values and ideal functions so the scalar equation in the same manner.

We can consider the vector box as the vector field components E and H along with the propagation constant, so we need to have a composite vector field to define the what not modes this is particularly important when we are looking at the total properties of the wavelets and devices and want to manipulate the electromagnetic fields and use all those effects the starting point for this analysis is from Maxwell's equations and so select or magnitude equations the one how electric and magnetic fields are related and interact with the medium.

So as we know we have the Maxwell’s equations in various forms in differential form with an express Maxwell’s equations as follows Δ*equals Δ V by Δ T and Δ cross H = del dot d equal ΔD/ΔT to the charge density or equal to zero if is uniform medium and here also you have the current density which can be ignored if there are no free carrier currents and also you have the magnetic field since you will be equal to zero is all four Maxwell’s equations which will be useful to study the properties of light waves in integrated optical devices of course, we have ignored the charges and currents which can be incorporated if necessary into maybe a discussion complete.

And also as we know on D and D and the Fields and D and H fields are related for the constitutive relations as follows because epsilon knot e and if there are material in a properties polarization of the medium can be included otherwise it is not necessary and you have the be given in terms of Hs nu H and these are the electric permittivity and magnetic permittivity of the medium in which the light is cooperating if medium is uniform this material constants material is description parameters describe individual are constant but as in thecae of optical waveguides if there is variation with respect to the fields.

Which have V and H to be functions of the point where we are observing the fields so they can be dependent on the space also the propagation direction sometimes and also the time they can be time getting also so we need to have a brief discussion about the nature of the materials that we use for integrated.

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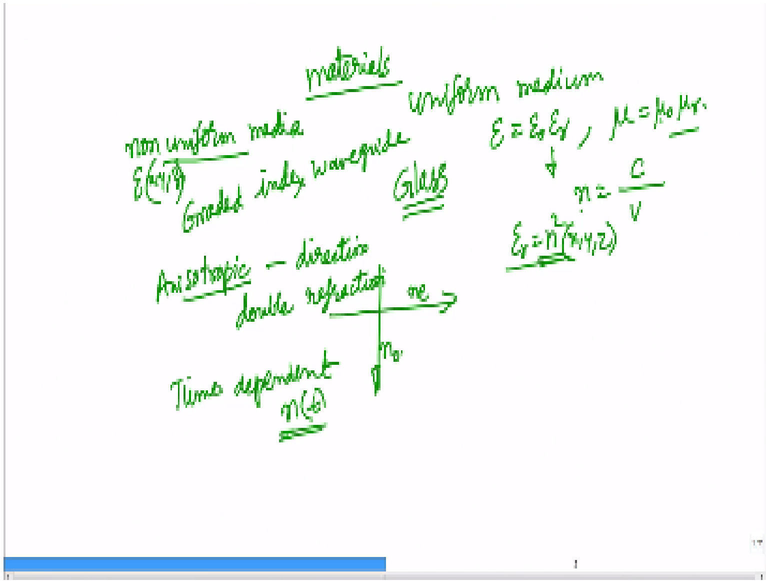

So we usually when we study the electromagnetic phenomenon in the medium the first simplest medium that will turn think of is a uniform medium where we can express the electric fields in 93 so simply in the constants are simply epsilon 0 0*multiplied by relative dielectric constant or the magnetic permeability can be simply expressed 0* universe is imagining the medium is non-magnetic the young equals mu zero so this is chaos and we can also relate this epsilon R with the refractive index as we know the refractive index is defined in terms of the velocity of the light in being in doing some space by the velocity of light in the medium.

So we can say by considering the frequency and wavelength of operation we can express young interns of epsilon R as epsilon R is square because it depends on the operating frequency but at a given frequency we can express the relative dielectric constant in terms of the defect next profile so the media properties are varying with different points at various points we can say that the flux index is a function of the medium point where we are observing and like in the case of scattering problems we have the fact rates of the medium varying everywhere.

So we can also think of materials like this is a typical example for a material with uniformly correct index you see in glass and there are materials which have got non-uniform media, that means that properties of the media braiding with the different coordinates and in particular as an example of non-uniform media you can have a graded index wave it so, let us distinguish media which is non-uniform non info media has got epsilon varying with X Y Z X so at respective X or the benefit constant varies with X Y Z T also we have media which are called anisotropic which means the properties of the medium depending on the direction in the crystal.

So in one particular direction you have one different Felix and in the other direction you have other 4 X 8 Typically in the bulk crystals we call them as ordinary refractive index and extra 30 L u x mix typical example of anisotropic medium is a calcite crystal where if you observe from a particular angle the it is possible for light to come in different directions and you see what can be called as a double refraction double diffraction is a manifestation of an isotropy of the medium similarly you can also think of media whose properties can depend on the type so time-dependent properties also are important for example electro-optic modulator we would like to change the properties of medium based and in that case we need to use them as a function of time also.

So these are the various combinations or various material properties that we come across so let us make a comment on the refractive index of the optical waves.

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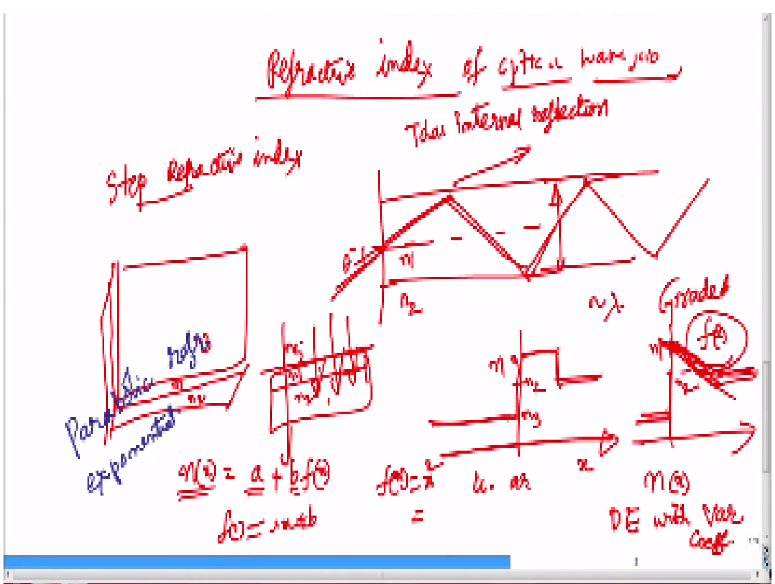

The fact MINIX of optical wave weights of we will make it more specific to optical waveguide optical waveguide are typically formed to have different refractive index in different regions so that light is confined within particular pages the bulk optics is a simplest in the simplest cases we know about throat healed panel deflection which is a means to explain light propagation thick wavelets suppose you have a wave way like this it is quite thick so that we need not use the wave optics we can study these properties by considering light propagating at different angles in wave guide.

So suppose you have bill fraction X Y n1 getting beaten into in the regional side and if you launch a ray with particular angles possible from the light to their ref fact ft at this point and refract at this point also and beyond a certain critical angle it can reflect back to the same medium and get propagated so this geometrical description of light is useful to understand then the thickness of the wave which is very large and of course when the dimensions become smaller and smaller of the orders of wavelength of light the geometrical approach is not useful and we need to go to wave approach.

And the earlier discussions we have seen that how using scale are clearly of the light propagation, so now let us get back to the various infracted configurations that are possible so two important factors considerations that we can identify or call the step refract index Which we have considered in the last two discussions and where the refracts disk changes abruptly from one medium together medium it is a simple example you can consider an optical waveguide waves.

Record a thin layer of another medium of a higher refractive index compared to the substrate to create and above the 8so N 1 n 2 and n 3 substrate with the filaments create and the Frederick's inconstant in each of these regions such as frack index of the waveguides called the step refractive index and the wave guides are called the step index wavelengths and we can express with you can plot the refractive index and a function of the distance let us say this will be this is the acceleration we can say X as a function of the refractive index as a function of the X That a factor X be expressed for example if you have a certain X of N 1 at this point and n 2 in the substrate and say n3 in the covers region.

So we have refractive index profile case like this so this is called stepped aside for Similarly when you diffuse impurities into the substrate negative 8 we can obtain a refract wave which is gradually varying from the top surface to the bottom so we can express the refractive index variation as follows to what accost so in the cover it is yet again and you have a refractive index N 1 at the surface which is maximum and towards the surface T 3 n 2 is the surface of the substrate for X 1 X and the refraction it changes gradually.

So this sort of profile is called a graded refractive index provided so mathematically conceptually there is no difficulty with treating step index or graded index but only thing is we should note that refractive index as a function of X will result in the wave equation and the differential equations with variable coefficients differential equations with variable coefficients, so we have talked about the steps of our index and graded different win ix.

So the gradient effect miss could be of several types for example the function that that can be used to present can be a linear function of parabolic or exponential on I think it is possible to construct it is necessary and possible to construct the refraction profile as a function of X to use it in formal mathematical purposes for example we can fit a curve which is can be obtained with these function f of X can be obtained by the experimental measurements of the diffusion process we can use a function of the form in to be a + PDF of X to determine what is a function of this for example f of X could be a function of X^2.

So given a configuration a variation like this we can find out the values of a and B to find out how the refraction actually varies with the distance so you can have a linear variation of the diffraction X f of X equal to say ax + B you can also have a parabolic reflector if Equinox variation or exponential and whatever you like and on the similar lines.

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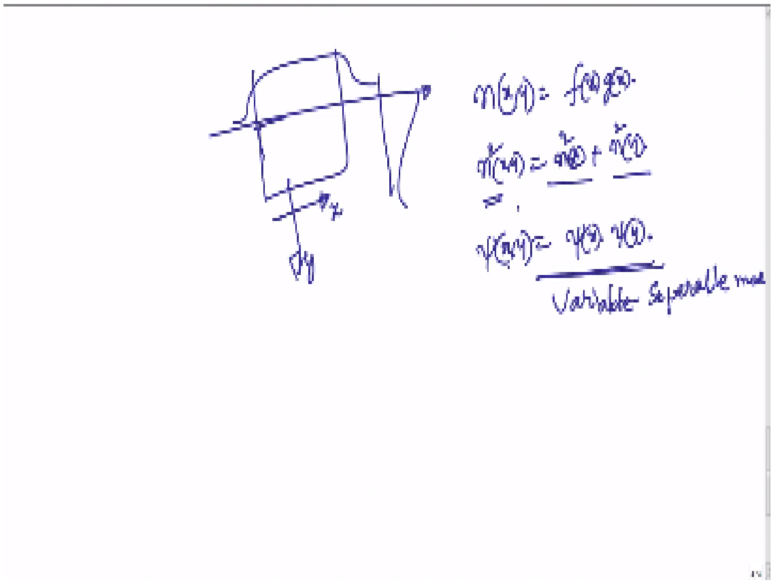

Along the bit direction also if you have a channel wavelength we have are re fractive index radiation that can be with the depth. and also you can also have a refractive index variation with the width. so you can have a graded radiation in both X as well as Y direction suppose this is a let me call it as a x direction and the depth is Y direction you have a refractive index which is where in both X as well as Y.

And sometimes you can express for mathematical purposes that you can separate out the variables f of X into Go f X in particular when you when you can express the refractive index as a sum of these variables X comma Y is squared of X + n square of Phi or even of XY e square X + n + Phi because the squares of the refractive indices are including to the wave equation this is more convenient form and when we express the refraction issues like this it is also possible to express the mode field profile in terms of the in functions variations along the X direction and variation along the Y direction.

This is popularly this method of analysis in differential equation is properly known as splitting the function in two variables product of two functions radiation of parameter or let us call this variable separately we did not bring it also make a note about the types of modes that can existing optical wave .

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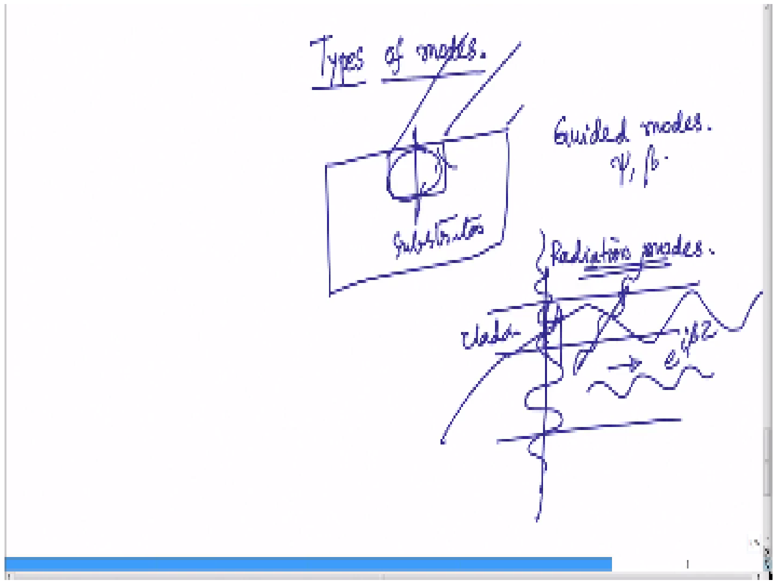

Waiters we have noted that most of the light has to be guided with indeed a weight if you have a wavelike this we would expect most of the light to be guides in this and we have seen that collide could extend outside into the substrate and into the ACE in to a. So when the light is confined mostly to the guiding region these can be called as guided mode for the wave modes which consists of field lead her vision film region as well as the cladding region entire thing constitutes one mode and is characterized by is mode field and the crop based constant similarly when the light leaks out of this optical waveguides know what are called the radiation modes.

Radiation modes of the box which have got a field variation which is which represents the power leaking out of the optical group their base as a simple example you have the oscillatory field within the guiding region and also an oscillatory field outside the state increasing as we know that the light propagates as E power I beta Z along the propagation direction combining with the radiation the periodic variation along the set direction and periodic variation along the in perpendicular direction we can say that the light is propagating in this direction.

The combination of wave vectors along the right direction and along the transverse direction could lead to a wave which is propagating at an angle this can be used this can be considered to be the percentage radiation modes switch most other modes which click out of the optical rivets several terminologies are used for example the radius modes that lead to a substrate or called the substrate modes which are confined to the cladding regions if there is an additional occurring yes the light could be confined to cladding regions so they can be called as ac lading modes and so on.

And so forth finally we would like to do a simple exercise to create the vector modes will be scalar modes.

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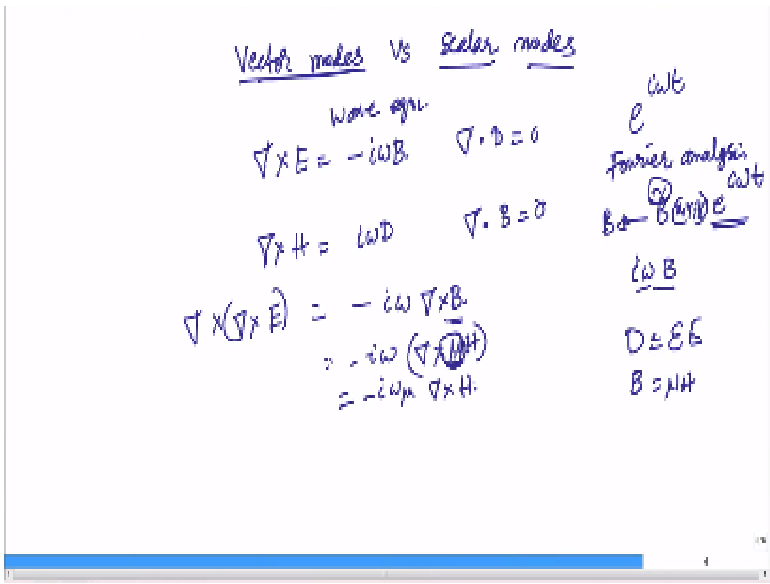

We have noted that Maxwell's equations represent the entire set of electric and magnetic fields of a like safe so it impossibility is known and possible that by manipulating the Maxwell's equations we can derive what can be called as awake equation let us quickly revisit the derivation of the wave equation the bring out an important difference of the modes of an optical wave rates.

So let us consider starting from del *=s + σ V by σ T and Δ* H σ U T /σ u T and the dot products del cross D =to 0 and Δ* B Δ dot be = ω to 0 so I am considering a uniform medium and of course the time variation also could be reduced by taking a power I Ω T as the time duration of the field which implies that we are doing it for your analysis so all the electromagnetic fields are replaced by D for their coefficients so this V will be replaced ,B, the X Z* by E power I Ω T so the variation switch respect ω could determine could be linked by replacing the /ω.

So this results in fields which are in turn generation will be I Ω multiplied by D so for simplicity we do not use the same notation and replace with I Ω PI only got a connection already up B and three oh my god D so this is like all the faces growing right sides about there is no time variation here so we can say that these represent time varying fields or the harmonic fields the fields are harmonic so in order to get the better wave equation.

Let us take the curl of the first equation and expand so - ω I my going to ω *by using constitutive relations we can replace PMT B= to epsilon a and B =l *MU H so + IΩ * Δ l * H so – I ω connect up you know assuming this is non-negative medium I got removed and the magnetic variation negative filter negative constant media properties or constant we can remove the and replace with it Δ *H. So this can be replaced by karl theorem ∇^2 E+ ∇.E or ∇.E ∇(∇_oE) so ∇^2 E+ ∇.E =∇.E so in this expression -iω^2 μ£E so you replaced with only with electric field with this expression.

So entire course we might have few assumptions that medium is uniform and the electric field is not coupled to the magnetic field so you can say that because this is further defined of this equation for example I can do ∇.D with replaced with ∇.£E and of course now if £E is constant the medium properties are constant you can mode of the but as we know for grade in waves in many problems £E not a constant so you cannot removed.

And we will just replace as using the product rule we can replace this expression ∇.E µltiplied £ +E.∇£ into the bracket anyway so you can use equal zero for this equation so we can say that ∇.E equals E. ∇£/£ with minus sign. So I can replace ∇^2 E this one because minus I can say replace with ∇^2 E is that so this becomes -1 and so one third thing is one and we can replace this entire thing with ω^2 by noting down that the velocity is a function of the frequency and wavelength we can replace this with another square £E and also we will £0*n^2.

So ω^2μ£0*n^2 so this entire thing will be placed with $k0^2$ where if you say μ£ is the velocity, velocity is 1/μ£0 we can get this equal to 2ρ/ λT is equal to 2ρ/ λ or k0 so this becomes just $k0^2$ n^2 filtered by E so I will replace this here and – sign is here so this come + or there is a - sign here and - sign is here there is a + sign here so by improper manipulation please check plus $k0^2$ n^2 of E on this side and this we need to put to the other side.

And because of already - sign we have used up when it was the set ∇ of ∇.E and we have replaced it with E.0 and ∇.E=-E. ∇£ and of course we have a mode so we have considered the wave equations which are of the form $\nabla^2$of E+ $k0^2$E=0 in a uniform medium we would like to note that if the medium is not uniform on the right hand side we have a factor which is depending on ∇£/£ this is an important feature of which will be useful for integrated optical wave weights dealing with graded index bits.

We have a simplification when the for the step index media by using the boundary conditions because each of the regions is uniform we can assume the issue we can use this wave equation for each of the regions and then match the boundary so that is what we have done in the scale at approach but when we would have to take the vectors approach we cannot do that and we can say that the mode fields are coupled for example EX, EY, EZ interpolated by this expression.

And if you do a similar exercise with the ∇XH we will observe that it is possible to note that E and H fields also are interrelated depending on the medium properties E&H fields are interrelated and we get fully coupled set of equations that means there are six equations this equals this is equivalent to three wave equations and similarly we have three equations with respect to H.

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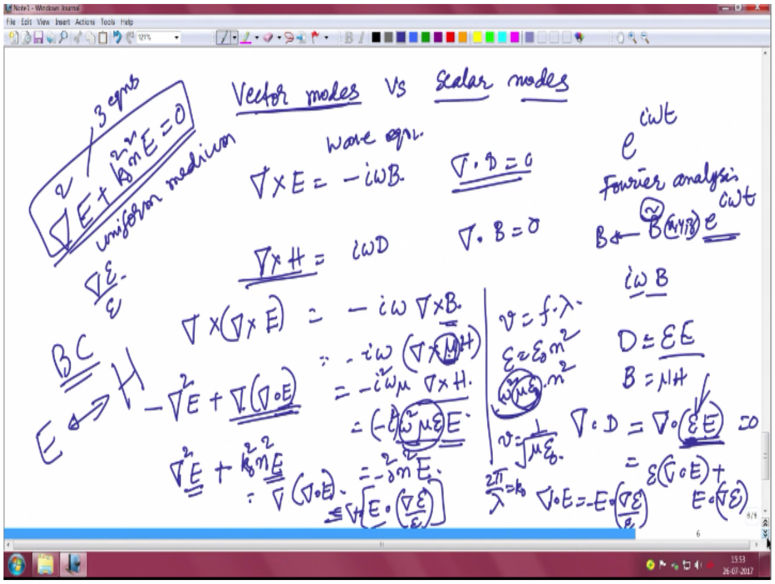

So all the six could be coupled due to the factors like this is the important feature that we should note which is not term existing for the uniform area so finally we would like to do a small exercise where we can separate out the mode fields of a slab they work by starting with the Maxwell's equations for a slab defect we can separate out the vector modes into TE modes and TM modes.

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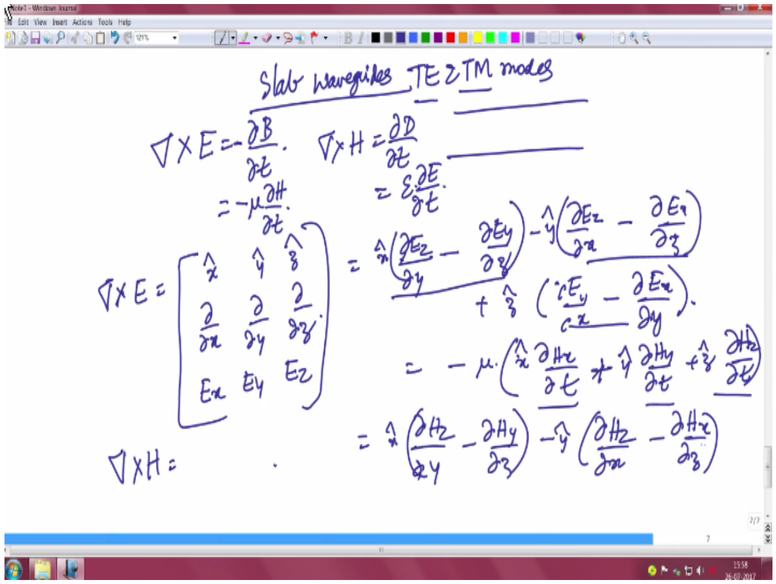

So this is a slab defect so that to bring out the vectorial nature of the modes of an optical wave which in use the example of a slab wavelet and note how of TE and TM mode fields are represented which components of that you can make the fields exist for keywords which camp in TM modes can be obtained in a very simple manner so once again starting from the Maxwell's equations.

I will once again use ∇XE=DB/Dt ∇XH= DD/Dt minus W and will replace with the 40th time harmonic field –µ*∇H/∇T and ∇H also to place it so that we don't we have only H field £*d/dt quickly let us expand the fields using the matrix approach we can write ∇XE as the matrix form by using the X component Y component unit vectors and said competitiveness vector into D/DX d/dy and d/dz and Ex Ey and Ez this how we can evaluate ∇XE.

And we get X into X component to Ez by dy-d/Ez similarly = or -, -y component into y component d/Ez by Dx-Ex/dz+z component dE/dx- dEx/dy and this has to be equated to –µ* because now there is no cross we can simply say X component into Dx /Dt also you know that's so we can equate the X component Y component and Z component separately.

And similarly the same exercise for ∇XE XH using matrix we can write it budget without doing by looking at the symmetry and replacing with electric fields and magnetic fields and of course we have the right hand side that is a positive sign and we can express it quickly as ∇ H that by ∇ Y- ∇X Y/∇ Z and y component ∇ X and ∇ X because that Plus that component and then

EH.EHy/∇X- ∇μX/ ∇ which is equal to £*x-component into d DX/ DZ and Y component divided by E.

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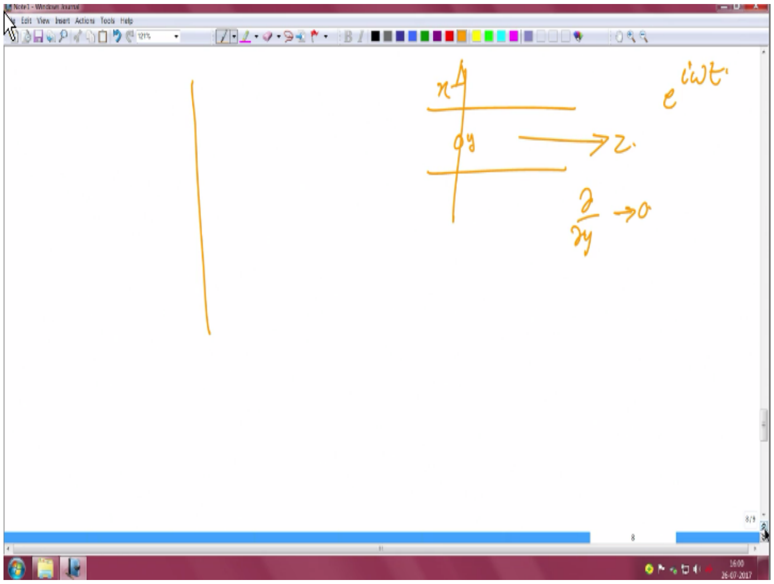

So this is T is this is T into distinguish champlain is it that plus that component in two digits so now we can equate x into components and then express before that will make a simplification saying that the field is only waiting along the X direction and the population is around is Z direction and time variation is taken as a power I ωT may be simplification when you can reduce all these fields into two.

So when we replace because the field is not varying along the y direction we can say that D/dy components do not exist and so we can replace the fields for example X component D is it by dy is zero because very, very soon we are not doing and we can Z variation we can replace with I become into Ey $e^{i\beta}$ sector Iωt-β Z variation or minus Iβ variation similarly DX also can be replaced with βX component and Y component here and see also Y component can be ∇.

Similarly all the ten components can be replaced with Pω variation I ω Hy and I ω X similarly here also I ωEX Ey Ez and Z components we need to take X components this X component also goes and give it component also goes and the second one is XZ third one is XZ so by now properly equating we get minus Iβ ey equals I ωH why it is or not I ωHy X component of Ey related to H component of X.

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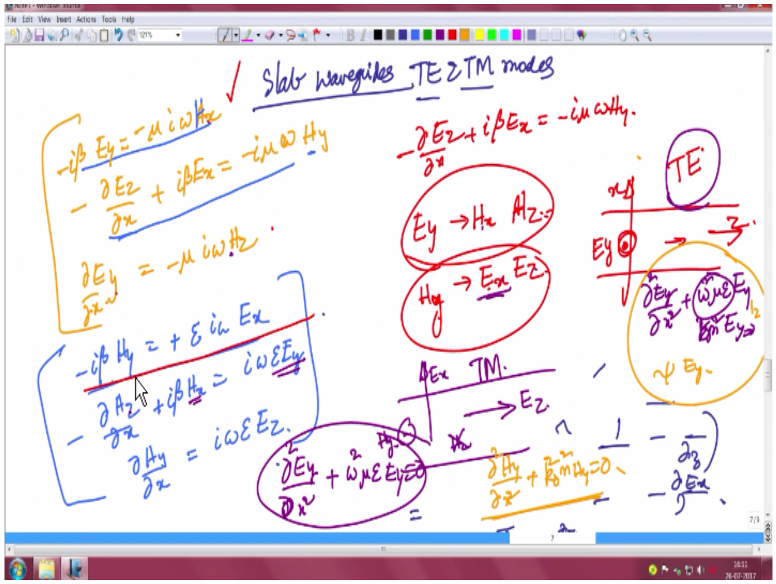

So okay let us quickly ahead then we will go back Iβ/Ey will delete something so that we can make use of this space so we will write for the one set of fields and replace itwith other set of fields for ease of operation for example we have iβ Ey equals minus μ I ω H X this one equation we have another one and for y minus Ez by EX so plus I β X should be greater to the Y components less Iω minus μ I minus μ i μ ω XY this is the another equation.

And for other one Ey/Ex Z component should be equal to minus μiω that is it these are the three equations that we will get from the first set of equation similarly you can get from the second set also we can write without eliminated – now we get a table that is mean difference and - goes minus I β into Hy I will use a different color so that I will use a blue color equal excellent- I β into Hy minus μ£ plus and I ωEx.

Similarly that is already here and my summarize class will have made it yes - ∇H/ D DX- the respect that means I β plus Iβ*Hx equals £ into I_O and finally one more d HX which is that components X Y by B X equals from I ω X £ I ω £ Ez so these are two distinct sets of equations if we I think we it is so that we can use this top equation so let us look at the equations and look at overall picture of what, what is happening with XYZ components of electric and magnetic fields.

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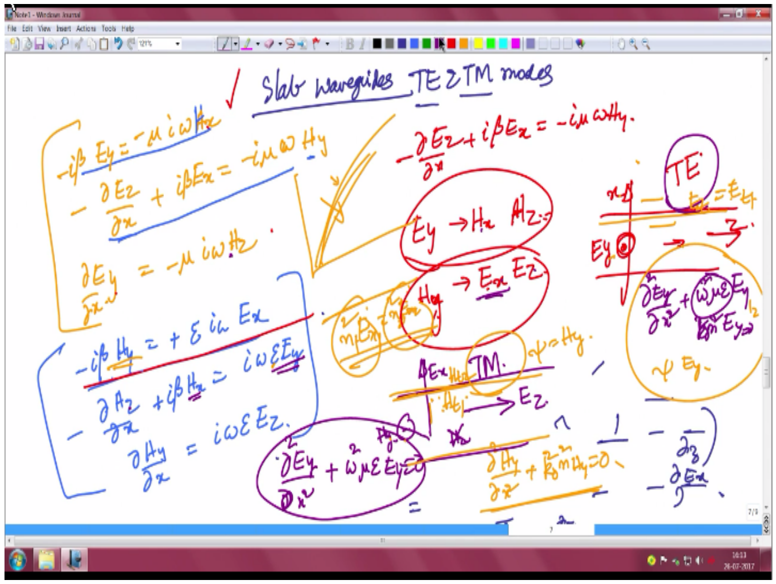

So this equation to present is a relation between Ey and headed and this equation has got a derivative with respect to X that is the important part so we can replace the say for example if you want to have only one particular components you can replace with Ez and Ey with respect to X is also there here so we can eliminate say let us try to eliminate Hy so Hy can be eliminated by using this expression so let us combine these equations so De/dx+iβ*Dx+Hy combined with missing by total play skip to E that and we are not be careful we will see that Ey component is related to Hx component and Ey component is related to a third component also.

So this is a group Ey component is related to Hx and Hy this equation and Ey component to Hx and Hz similarly Hx component can be related to the Ex component and Ez component so the conclusion is that we can group the fields in terms of Ey/Hz and Hx is Ex Hy component is also Ex and Ez so these are the equations could be split up distinctively into two groups one containing given a check side effect and other containing Hy exact physical geometry what we have.

And if we say that the propagation direction is Z we can say that the fields which are and this is the X direction you have the Levi's Ey field is in this direction and there is no easier component so these could be called as TE modes so TE modes have got field component along the plane of the way similarly if you look at the other equations we can say that the hy component or usually when you describe the state for the modes with respect to the electric fields.

So you have an electric field along the this is X direction along the X direction and this set of propagation of components also this but HY is in the plane of the wave length so this and there is no Hz component so we can call this as TM modes we can have TE modes having Ey Ez when TM modes having hy Dx and we can use the other relationship to eliminate the other variables for example Hz variation is there here and you can replace it helps and variation.

So you have ∇^2 Ey/ D by Dx^2 related to once again Ey is there here so you can you might be able to get Ey and if you replace with it set you can get mega square μ £ into Ey you get the difference equations are only for Ey so in the case of for the TE modes we can get d squared dy by DX^2 plus another square μ£ is we can replace with $K0^2$ and Ey component and similarly for the TM modes we can replace with ∇^2 Xy/∇X^2+k0^2 of x by equal to 0.

So one important feature that we also here is that we have the scalar wave equation except the sign is replaced by dy and in this case we have the same form of the equations that we use for scalar approach where P is replaced with hy so by properly interpreting the wave function side we can use the same scale of theory that we described earlier for TE modes as well as TM modes by properly noting which are the components existing one more important point to be noted is the boundary conditions.

The boundary conditions are expressed in terms of the tangential components of the electric field and normal components of the magnetic field so we need to use et1 equal to et2 where t1 and t2 are face in the film region with guiding region so in this case Ey is tangential so we can quickly Ey1T/2 which is exactly same as the scale of across that we have taken without mentioning about the electric and magnetic fields in the case of TM modes the electric field is not at the boundary the electric field is not continuous.

We don't have a field which is just along the boundary but you have a magnetic field which is tangent component so Ht 1 should be equal to Ht 2 so H components in the TM mode can be replaced with key components and so you have for example if you observe it at this point we have N 1 Ht 1 there is a hy this is the component hy, hy component here expressed in terms of e x component so the boundary condition reduces to n1 square e t1 or ex1 to n2square e x2 so by modifying the boundary condition to include the refractive index on both sides.

We can get we can use across that we notated four scalar mode except that the boundary conditions architect so this leads to a slightly more improved form of the dispersion equation for TM modes so the β is different and so the vb curve or the all the values are slightly shifted for TM modes if you see for the TE modes this one can be for TM mode so the, the dispersion curve for the TM modes is slightly should kill compared to the tables so this is a typical because in the case of more complicated ways mathematics could be complicated but this the scalar apposite we have taken is quite useful to study the vector modes particularly when there is no coupling between fields so thank you very much.