Indian Institute of Science

Photonic Integrated Circuits

Lecture – 03
Optical Waveguide Theory- ASymmetric Waveguides

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NPTEL Online Certification Course

Today we shall discuss about asymmetric slab waveguide.

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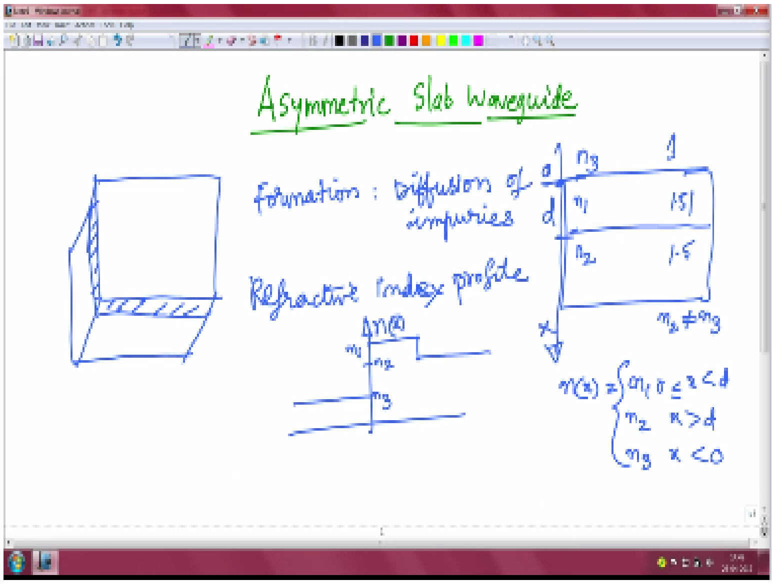

And its analysis and the properties, so earlier we have discussed about the symmetric slab waveguide where we considered the configuration as follows. The cross-section n1 is the guide refractive index n2 is the refractive index of the substrate and cover also has got the fracture index n2. In the case of asymmetric slab waveguide we have a configuration wave the cover refractive index is different n3.

So this also forms a very important waveguide in integrated optics. So we can say that the structure the three-dimensional structure of an asymmetric slab waveguide will look like this, we can say that if we take a substrate and make the whole top surface with the different refractive index we get an asymmetric slab waveguide, of course cross-section as we noted is like this. For example, we can form this by diffusing just as an example we can say that the formation can perform by diffusing computer piece into a glass substrate as an example.

And also we are considering only the step refractive index profile. So the refractive index profile of an asymmetric slab looks like this I would knock the coordinate systems, coordinate axis and I will take the positive x-axis down, and this point as zero, and the depth is small D, and so on. So for X lying between 0 and D refractive index is n1 and equal to n2 for X > D, and is n3 for X < 0, X can be set over the refractive index profiles.

And we can graphically we can represent it as follows n(X), so in the covering regions when X<0 it is very small some value of n3 and in the guiding region you can say the test code while Fn1 and in the substrate it has got the value of n2. Typically this is a glass waveguide you know that the refractive index is around 1.5 of the glass and by doping with some impurities then increase the refracted by a photon percent or so slightly higher than the substrate.

And if outside is difficult one but the typical values. So this is called the guiding layer, the substrate, and the covers, the three layers, three different layers. And tentatively we will say that n2 need not be equal to n3, and usually for convenience we can also say that n2 is more than n3. So it has got many interesting features like the cutoff of the modes like is not guided beyond a certain wavelength.

And the analysis of course is straight forward extension from the top symmetric slab waveguide and we shall consider as calor approach like what we did earlier for the symmetric slab waveguide and solve for the modes of this structure and the expression for the propagation .

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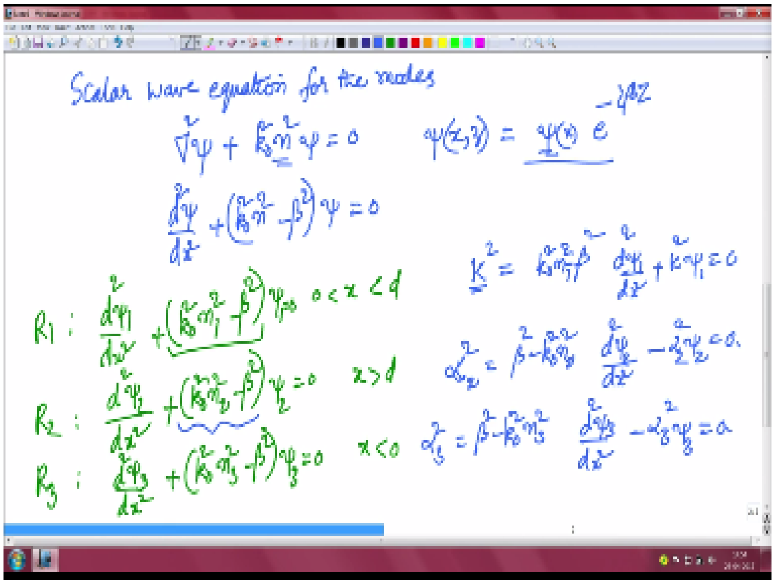

We can start the scalar wave equation for the modes that is like this where l is the refractive index profile, n is a function of X and as defined earlier the definition being, definition modes as they go as exponential $e^{-i\beta z}$ will say that the modes are of the form βz where β is the propagation constant and size mode field profile. And for simplicity I am using the same symbol for both and by the context you know whether we are talking about the that varies also along with the mode field or not.

So with this form for Ψ this can be reduced to $(dx^2+k0^2\ n^2-\beta^2)\ \Psi = 0$. So we can express, we can speak this differential equation into the three portions $\Psi 1$, $\Psi 2$, $\Psi 3$ in each of these regions. So in each of these regions we have the following form of the differential equations $d^2\Psi 1/dx^2 + (k0^2 n1^2-\beta^2)\Psi 1$ in the region 1 that is defined by x=0 to d and $d^2\Psi 2/dx^2 + (k0^2 n2^2-\beta^2)\Psi 2= 0$ for X>d the substrate and $d^2\Psi 3/dx^2 + k0^2 n3^2-\beta^2)\Psi 3= 0$ the cover region for X we can also call them as region 1, 2, 3, region 1, region 2 and region 3.

So we can define variables to put it in a simplified form and we know that the guided mode fields have the form of oscillatory nature in the guiding region, then exponentially decaying regions in the cladding and the cover regions. So we can define this as a constant Kn1 we call as a constant we are talking about this step refractive index wave guide, and we can call it as a K^2 or k^2 with $k0^2 n1^2-\beta^2$ and we can call this constant and remind that K is positive if $\beta > k0n1$ and so we can replace this equation with $d^2\Psi 1/dx^2 + k^2\Psi 1 = 0$ as the differential equation region 1.

α_2 let us define as a positive quantity by β^2 - $k_0^2 n_2^2$ so that the equation becomes an exponentially decaying field – $\alpha^2\Psi_2 = 0$ and in the region 3 also let us define by $\beta^2 - k_0^2 n_3^2$ so that the differential equation becomes $d^2\Psi_3/dx^2$ – this is α_2 and this is $\alpha_3^2 x \Psi_3=0$.

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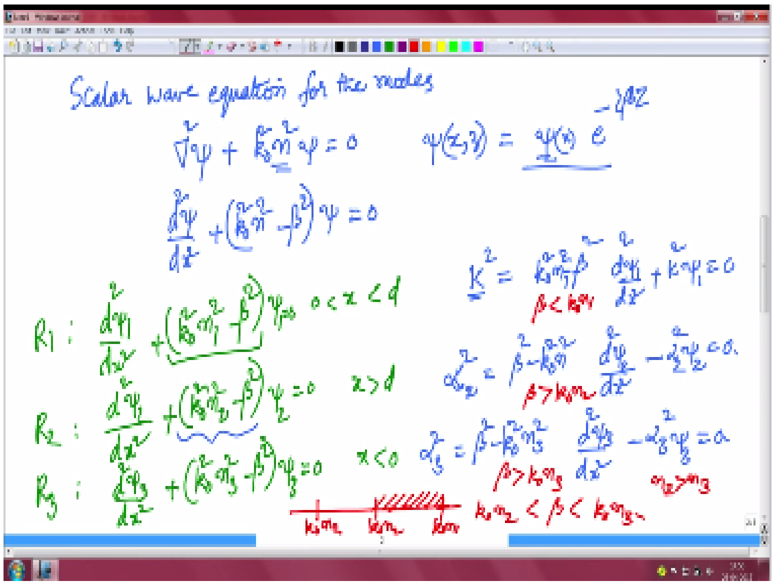

Let us choose the solutions based on the arguments that we have just now discussed that the guided mode feels into the oscillates in the guiding region and became the substrate and cover regions. So this will be possible if we choose β to be lying between the values of k0 so this implies if β should be less than k0n1 and β should be more than k0n2 and β should be more than k0n3.

So just we said that n2 > n3 just as a convention because n2 or n3 which I think will be greater or lesser so if we say that β lies between k0 and 2 and k0 and 3 you can say that we have a guided mode field. We can also represent it by in number line for β as follows k0n1 or n3, k0n3 and k0n2 and k0 n1. So this region corresponds to the guided mode fields for β. Of course as we have seen for the symmetric slab waveguide it can take only discrete values and even single value. So that we have a single mode wave guide.

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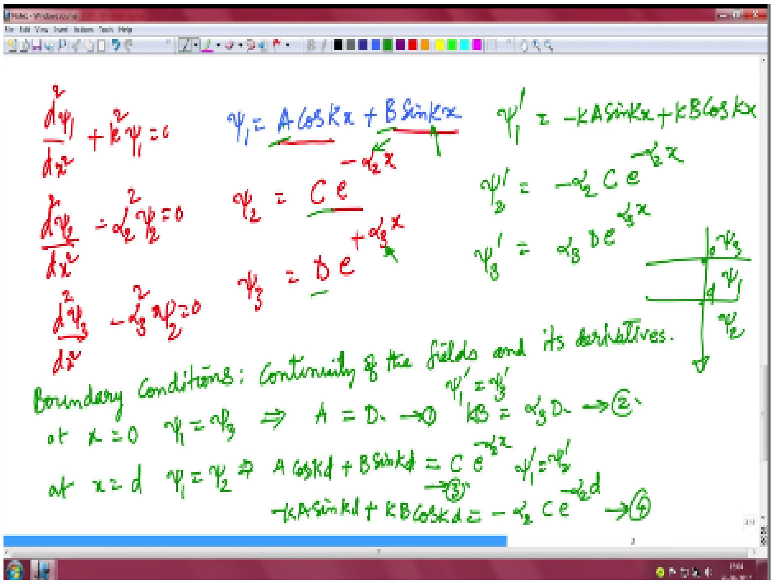

Great, now let us look at the solution of the fields in each of the regions. So in the region where $d^2\Psi 1/dx^2+k^2\Psi 1=0$ we have $\Psi 1$ = ACoskx + BSinkx because we know this is the equation for a constant coefficients of the type of simple harmonic motion and trigonometric functions are the solution of this. Similarly in the region $2d^2\Psi 2/dx^2-\alpha 2^2 (\Psi 2) = 0$ the solution is of form $\Psi 2 = Ce^{-\alpha 2x}$, this is oscillatory here and exponentially decaying here. So we will not consider the field which is exponentially rising in this region.

And we will ignore this function and so we will not put $e^{-+\alpha 3\alpha 2x}$ so we take only exponential decaying field as a solution. So this also implies that we are taking the boundary condition at ∞that the field is going to 0 at x = ∞. So we have taken this as the origin and we see that D similarly for the region 3 $d^{2\psi}3$ d^2 cycle dx^2 - α3 square ψ2 equal to 0 gives ψ3 as b^e + α3 x in this region also we want with 0 and this dx is negative in this side we can choose the function which has got a positive value so as x goes to get with this course we get to be 0 so we should note that we need both Cos and Sin terms as a solution of this differential equation we cannot split these modes into symmetric modes.

Anti-symmetric modes as we did for this matrix slab wavelength because of the asymmetry involved so we will also need the derivatives of the fields to the continuous at the boundaries so I will quickly find the derivative s of these also write here ψ - can be given as - KA and K is color 7 – to be - KASin Kx + kB and Cos KX the derivative and derivative of ψ2 - α2 C $e^{\alpha 2}$ x in release ψ 3derivative is α3 D^e α3x our goal is to find the arbitrary constants A B C and D and also the propagation constant β which is embedded in all these coefficients in all this constant so let us apply the boundary condition we have seen that the boundary conditions reduce to continue the of the fields.

And it is derivative across the boundaries so at x equal to 0 we have I will always need this so this is 0 this is D and this is ψ1here ψ2 here and ψ3 here at x = 0 that is this point ψ1= ψ3 which gives ψ1 and ψ3 when you put x = 0 here A = D similarly the derivative of the field at this point let the called as equation number one and the derivative of the field also should be continuous at this point so we will call It as substituting x = 0 here KB = α3 KB = α3 into D.

Let us call this as an equation similarly at x = d that is at this point the guiding layers and subsets interface ψ1 = ψ2 gives KCos Kd+ B is Sin Kd = $C^{e\ -\ \alpha2}$ x this is one equation and technically this has been obtained by technique derivatives ψ1’ = ψ3’ similarly by taking ψ1’= ψ2’ we get this and this okay I will write here - KA Sin Kd + KB Cos Kd = - $\alpha2C^{e-\alpha2}$ and four equations we have my arbitrary constants ABCD and β.

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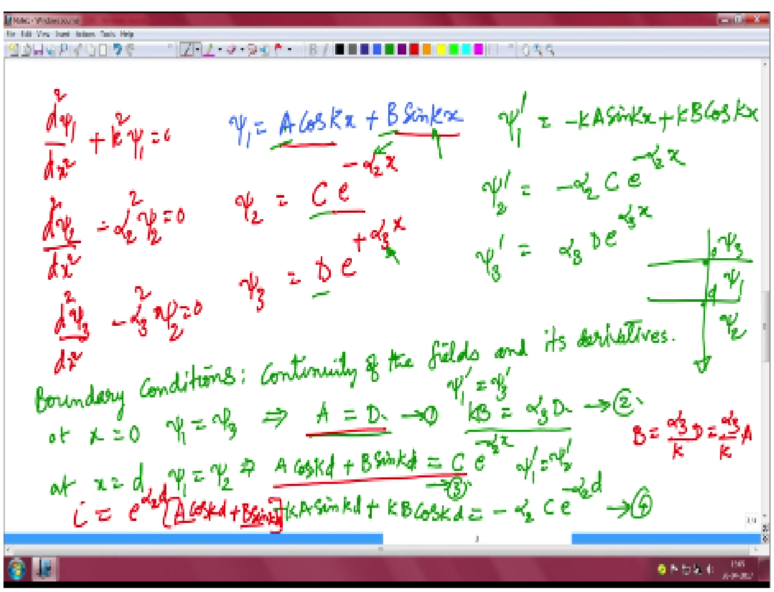

So let us eliminate some variables so from this we have for the first equation we already got D + A and from the second equation will get D = I will use the same board and see D = K into B by α3 or otherwise will I will use it for finding out B. B = α3 by K into D and since we have found from first that B equal say α3 by K into a these are the two constants we have eliminated and by using third expression equation we can get an expression for CA, so I can write C as $e^{\alpha 2d}$ and put it onto this side S Cos cause Kd + B Sin Kd which we can replace for constants K and B using the earlier expression and write C as follows.

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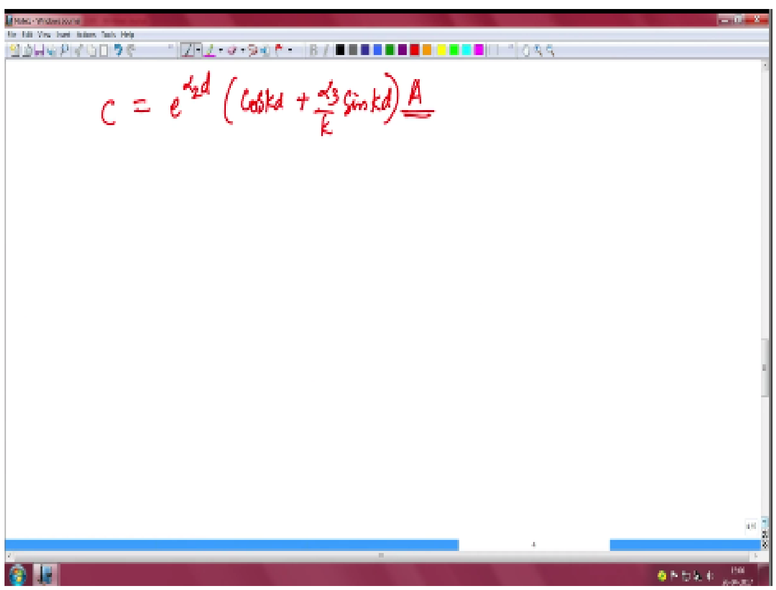

C = $e^{\alpha 2d}$ Cos Kd + is α3 by Kda and put a all set we are not concerned with the actual value of here so we will not try to evaluate right now and in fact when we are computing the mode fields etc. We can normalize it saying that the power is equal to 1 and value at here, so we can use the fourth equation to evaluate rather than evaluate any other coefficient so by looking at the equations like what we did for the symmetric slab wave weight.

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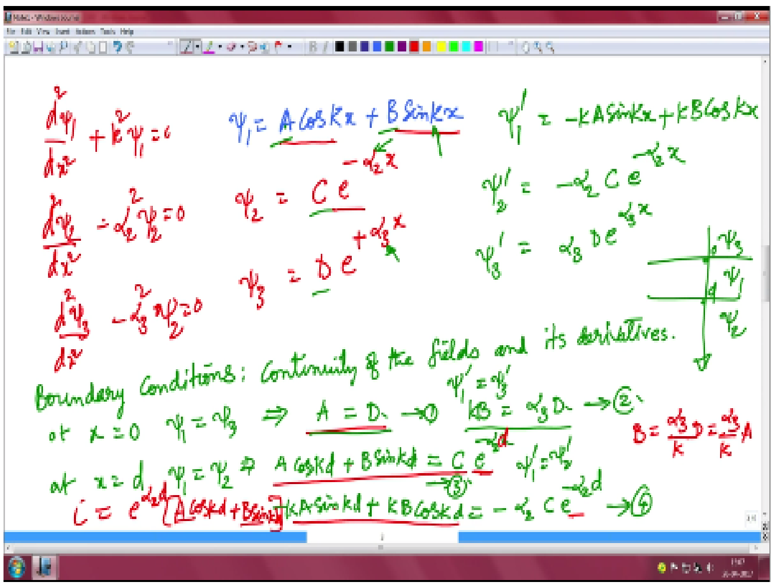

We note that between these two equations we know that we can eliminate these exponential terms the safety will eliminate these exponential terms by using by dividing one or the other so I will just divide for equation 4 divided by question 3.

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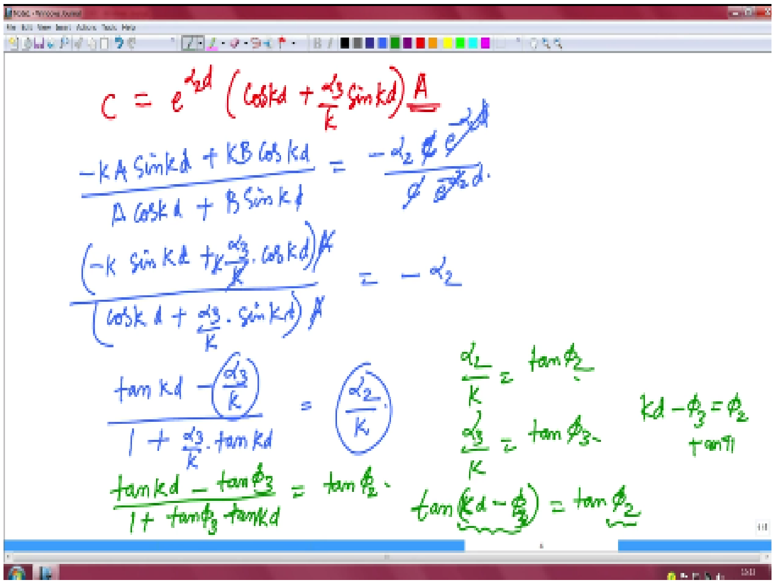

So we can express we can say of speaker – KA in Sin Kd + KB Cos Kd / A Cos Kd + B Sin Kd which will be equal to - α2 into C into e $^{-\alpha 2d}$ divided by C e $^{-\alpha 2d}$ so we can cancel out all the common factors oh yeah α to the base here and also we replace B with its value in terms of a so we get – K Sin Kd + is for what is B α3 by K to the α3 by K only one case they are here and A is Cos Kd and say I will take out common similarly the denominator I w take out A common and say Cos Kd + α3 by K the Sin Kd I mean on the way out this means α2 this K can be canceled out so I would like to simplify this expression by using the geometric properties of the trigonometric functions.

So let me cancel out this α also so that there is a positive sin for this Sin and so I can and also I will divide this row by K so that you have α3 by K so I put it in form of this so I will divide by Cos also so I will try to get something like this Sin over Cos - α3 by K and Sin is dead here one say here 1 + α3 by K into 10kd equals so I have divided the numerator by K I will also divide this side also by K this – n has gone with the other - n so α2 by K so let me represent this factor and this factor in terms of tan function let me define 5 2 equals 10 α3/ k α2 by K and n53 is factors okay let me put in the form of tanning is.

So that okay and put α2 by K as and Tan ϕ 2 and α3 by K as a Tan ϕ53 so that we have 10 K - tan ϕ3 1 + which is 2 which is and ϕ3 to tan Kd = tan φ 2, so the left side can be written as tan

KD -φ3 and the right side is tan φ 2, so we can create tan KD - φ 2 φ 3 with φ 2 and add an arbitrary periodic constant Empire. So the solution can be presented as KD - φ3 = φ 2 + M φ noting that tan function is periodic with a period φ.

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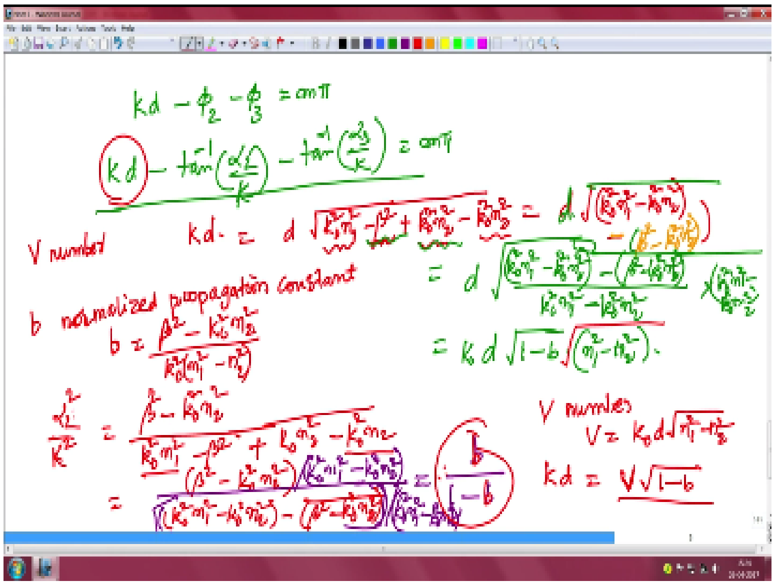

The words we have got a solution like KD - φ 2 - φ 3 = M φ replacing fat φ 2 and φ 3 with tan inverse α 2 / K and α 3 / K will get KD - tan inverse α 3 / K = F yes, so we can say that this expression represents the equation for our par constant propagation constant β which is there inside this K α this can be further simplified we can put this in a very neat form amenable for computation by manipulating some of these constructs let me define a few constants that will simplifies.

I will define a V number of the wave guide as KD, there I will get the value of K as $k_0{}^2$ $n^2 – β^2$ and $n_1{}^2$ according to the definition let us go back and see what is K so we have to find K as $K_0{}^2$ $N_1{}^2 – β^2$ I will simplify this by defining an arbitrary constant the normalized propagation constant this can be called as a V number and to the percent the data I will define a normalized propagation constant B.

I will simplify that later on bs $β^2$ - $k_0{}^2$ n^2 and 2^2 / $k_0{}^2$ and $N_1{}^2$ - $N_2{}^2$ we will get to the logic of this definition in a minute. But before going for this let us simplify this further to write it in terms of B and the $N_1{}^2$ - $N_2{}^2$, so I will add and subtract $k_0{}^2$ n^2 by combining $β^2$ and $k_0{}^2$ n^2 I can get d√ $k_0{}^2$

and N_1^{2} - k_0^2 n^2 as 1 - k_0^2 n^2 - β^2. So I have taken this and this is the first term and this and this is the second term which gives us okay let me divide this term and multiply.

D x k_0^2 N_1^2 - k_0^2 N_2^2 – β^2, I am trying to bring in this form. So that I can write it in terms of B, I will divide by k_0^2 N_1^2 - k_0^2 N_2^2 x k_0^2 N_1^2 - k_0^2 N_2^2, so with a note that this is equal to 1 and this is equal to small b, this is β^2 - k_0^2 n^2. So this can be written as d√ 1- b and K_0 I will put it over here and N_1^2 - N_2^2. So I will define the v parameters to be as k_0 to D x √ of course this √ for this and N_1^2 - N_2^2.

So that the fact of K x D is V parameter multiplied by 1 –B, I am starting with each of this term this K_0D is v parameter Y nicely, let us look at Tan inverse α2 / K 2 α 2 / K so α 2 / K then be written as follows in terms of V number and b. So we have got a V number definition to be like this, so α2 / K it can be expressed as β^2 - k_0^2 N_2^2 according to the decision of this / k definition is k_0^2 N_1^2 - β^2.

So I will add and subtract so we need to get to β^{2} - k_0^2 N_2^2 and denominators k_0^2 N_1^2 - N_2^2, so we have got N_1^2 only so we will add k_0 N_2^2 and subtract k_0^2 N_2^2 of course this is equal to α_2^2 / k^2 not α_2 / K. So will Club mean k_0 N_1^2 – k_0 N_2^2 and P'^2 - k_0^2 N_2^2 and β^{2} - k_0^2 N_2^2. So we can write this as equal to β^2 - k_0^2 N_2^2/ k_0^2 N_1^2 – k_0^2 N_2^2 one term - k_0 N_1^2 – β x k_0^2 N_2^2 β^{2} - k_0^2 N_2^2 , so by dividing like the numerator and denominator by k_0^2 N_1^2 – k_0^2 N_2^2 to connect this entire expression K, K_2^2 /K^2 as D / 1 – b here also I will divided by k_0^2 N_1^2 – k_0^2 N_2^2.

So this entire expression is reducing it to β / 1 - p B / 1 – p, so $\alpha_2{}^2$ / k^2 is written in terms of is equal to B / 1 - P only one constant B you observe that the exercise that we are doing is to eliminate as many constants as possible to appear in this equation for the propagation constant β you have or α_2 / K you have only one parameter B, similarly look at α_3 / K.

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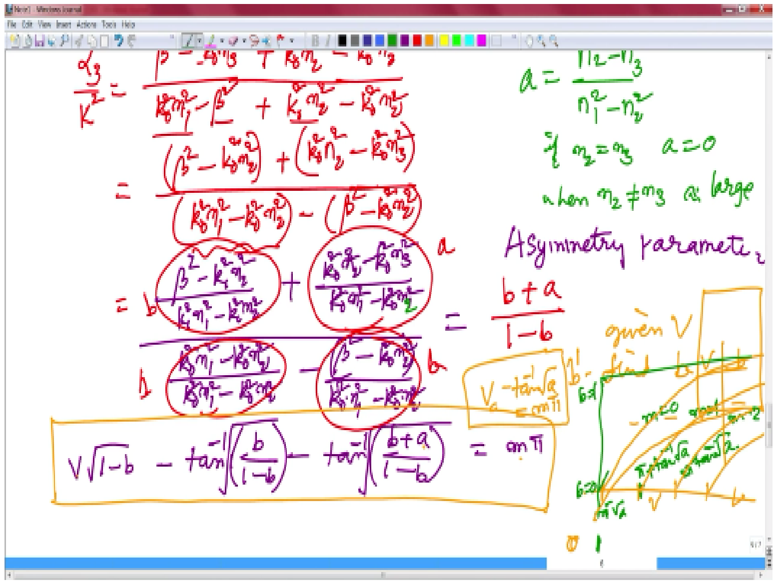

We can now quickly write by definition of α_3 β^2 $-k_0^2$ N_3^2 / k_0^2 N_1^2 – β^2, so once again like what we did earlier we can manipulate this and get α_3 / K once again α $d^{2/}$ K^2 in terms of small but here instead of n_2 we have n_3 what definition of V has got $n_{2.}$ So let us convert this into n_2 by adding and subtracting k_0^2 N_2^2 adding and subtracting squares k_0^2 N_2^2, so here we have K_0 n_1^2 and so we can add and subtract like what we did for the previous case put in at k_0^2 N_2^2 $\alpha_{3.}$
So because we need n_2 / n_2 - k_0^2 N_2^2 which usesβ^2k02 and two 2s here and zeros class speed is quite clean into ^{2}9x + we just got magic a Judas has come here this is k0 ^{2} and ^{2} 2d k 0 2d + 32 similarly the below is k0 2 1 k0 ^{2e} in $2s^2$ and bigger ^{2}_ $k0^2$ into ^{2}this+ is combined with this and this combined.

Now we can once again /by 0 ^{2} N 1^2 K 0 ^{22} s^2 bag ideas so we observe that object let me write afresh so this /by $beta^2$ K 0 ^{2}into ^{2}/by K 0 ^{2}N $1s^2$ K 0 ^{2}n 2 s^2 I am dividing numerator as well as denominator by K 0 ^{2}N 1 ^{2} K 0 2d n 2 K 0^2d n 3 2 / by K 0_2 1^2 K 0 s^2 N 1 2 denominator also I'll divide similarly K0 ^{2} N 1 ^{2} when we divide it becomes 1 okay let me write and then divide k0 2 n 1 2 $k0^2$ n 2 ^{2} / by K 0 ^{2} and n 2 k0 2 I* ^{2}s and the other part bigger 2ly Zk0 ^{2} 22 ^{2} / by K 02 n 1 ^{2} k0 2 n 22 now this is b.

And this is also the normalized propagation constant small d this of course is 1 and we will call this as some other arbitrary parameter a so this reduces to D + a / by 1 P let us define the parameter a as I can cancel out K 0 so getting here and say n 2 n3 2 / by N 1 2 within Dint 2s and this can be interpreted as and a parameter depending only on the fractal efficient 1 & 2 &

3and since we have n 2 n 3 the numerator with a note that is n 2 = 3 this parameter a = 0 thus for symmetric slab is like the parameter equal to 0 .

So and also when n 2 by n 2is not equal to n 3 since usually the refractive index difference / N 12 n 2 is very small you can say that the value of a is very large so the value of ranges from zero for the symmetric wave beds and very large for various imaging pivots so we can say that the a represents asymmetry parameter you can listen to the potent asymmetry parameters .

With these we can rewrite our equation for most of these estimated class favorite characteristic mode for estimating to slap their weight as the √1 B tan inverse D by 1 D tan inverse B + a by 1 B 2 vs. = M pesos we have a reduced key equation in terms of very few variable we have the variable the V numbers the normalized propagation constant the estimated parameters and the mode index so more index can be taken to be zero from the fundamental mode one for the next mode two for next mode and so on this equation cannot be solved directly in closed form but by using computer programs like MATLAB or Mathematical.

You can easily evaluate all these variables or the these constants given a B number we can find the normalized propagation constant by using complete programs of course we should be specific to only some asymmetry parameter and some mode index .

So let me draw and you can interpret the modes of the waveguide and before going further we can discuss about the nature of the modes so the Number on the x axis and d on the y axis and of course before you do that we conform a table of values of D was has bema programming or whatever grade and we can note that we get curves similar to what we obtained for symmetric slab we have good but there is a difference will note that so we can expect a curve that is like this because B lies between0 and 1 and gradually reaches a maximum value.

And the next one will start at this point and so on so we note that the cutoff values that is a equal to zero or join the equal to zero and when we substitute B into this we obtain the expression V = V cut off at = zero and this tan inverse 0 is 0and here a is makes and tan inverse v a is M pi this is expression for cut off so every curve starts at tan inverse duty in the case of symmetric waivered equal to zero and there is no cut off and E values will start at this point .

I will draw different color to illustrate so the value this is so these are all craftsman inverse v K and this is Phi +tan inverse rotate this is 2pi + tan inverse v a and so on that means every mode has got a cut off and for whom and this is more index M = 0 = 1 M = 2 and so on so we ‘retrying to observe the range of values of these curves they do these fall for Avery symmetric wave with a 0 and j0v sees m pi so that means the first one will started is will draw I thinks this might be I think I'll have torte be equal to 0 and be equal to 1 so this is starting at 0 of the elder in refresh .

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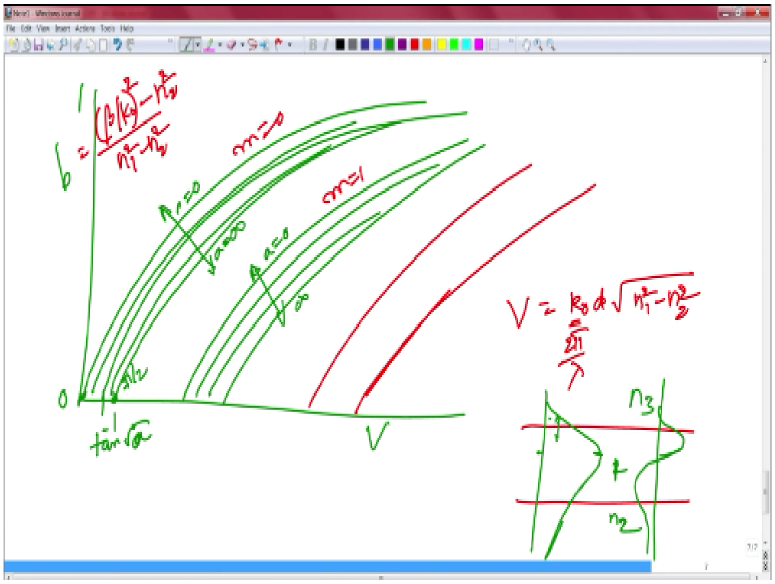

We number unless progress constant B = zero B equal to 1 they say cut off day and I am try to observe the range of values of these curves so for symmetric wavelength they start here and for highly asymmetric very wide is very large tan inverse very large quantities PI by 2 so it can go up to PIby 2 so this is 0 this is PI by 2 so all the curves correspond to say this is the fundamental mode all the curves corresponding the fundamental modes for various values of high symmetries.

Will fall in equal to 0 here and equal to infinity at a correct similarly the next modes will start at 0 + PI and +PI by 2 this is the range of values forth second modes all of the million following this is more values of less energy parameters from 0 to infinity and BC for m equal to 0 and this is our m equal to 1 and so on and so forth this in computer programs you can plot these and get an exercise of the babies.

Once again I would like to interpret what is the V parameter we have got k 0d the 2 v of N 12 n 2 so the way that we number is a parameter representing here you have the 2 pi by lambda because it in the frequency or the thickness of the guiding region or the refractive index context of today so within ultimate wave we as represented normalized frequency or normal sickness or the normal is fraction decks contrast be also once again .

We said that it is normalized with respect to beta is representing beta finally we can look at the modes nature of the modes n2 n1 n2 and n3 we observe that the contrast is small and so we can expect the Alpha 3 to be very large here and gradually we can say we can expect that it is not coinciding it middle but somewhere is offset towards the ACM and alpha 2 is small.

So we can expect longer train so there is an asymmetry of the expansion tale as well as the oscillate speed by similar arguments we can note the properties of the next modes so we have taken a scalar approach we can relate what all we have done with vector moves like modes TM modes actor modified is approach slightly to the present the vector modes more accurately thank you very much.