Indian Institute of Science

Photonic Integrated Circuits

Lecture – 02
Optical Waveguide Theory- symmetric Waveguides

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NPTEL Online Certification Course

So we will start the course with optical wave guide theory.

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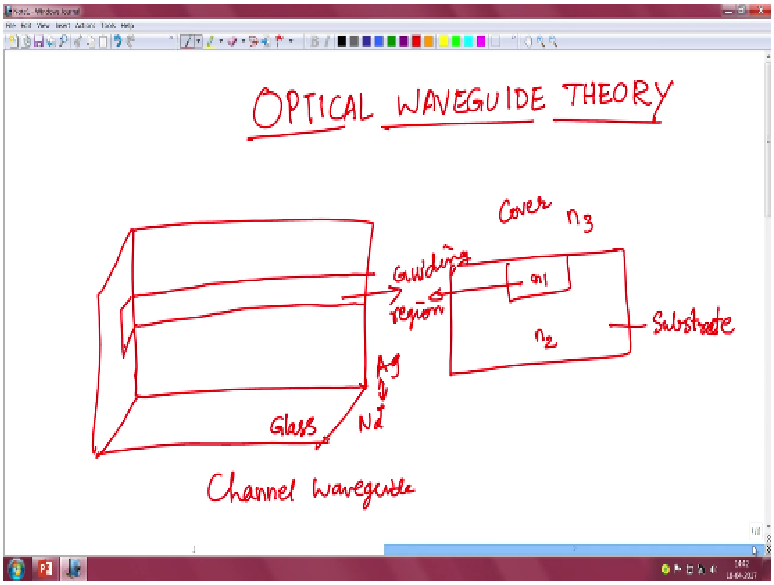

Optical waveguides are the basic elements for integrated optics so light propagation optical wave guides is important to understand their properties they design and so on so there are many types of optical waveguide like slab waveguides ridge waveguides channel waveguides because there are also optical fibers we need to interface optical fibers with integrated optical devices, so we need to understand their properties so the mathematics of this is quite simple and we will start with the basic self optical wave theory and proceed to understand these properties.

So a typical integrated optical waveguides looks like this you have a substrate of some material like glass and using lithography which we will see later on we can create regions where the refractive index is high like this is a typical optical waveguide called channel wave guide the cross-section of this looks like this the higher refractive index region here in the guiding region and lower refractive index in the substrate and in the covered regions, so this is called the subset this is the guiding region and of course the covers are rarer.

So this is called a channel waveguide typically this can be formed on a glass sub state by ion exchange into it as the typical glass consist of sodium ions you can exchange sodium ions with silver ions to create higher refractive index it suppose you create the whole surface of a sub state.

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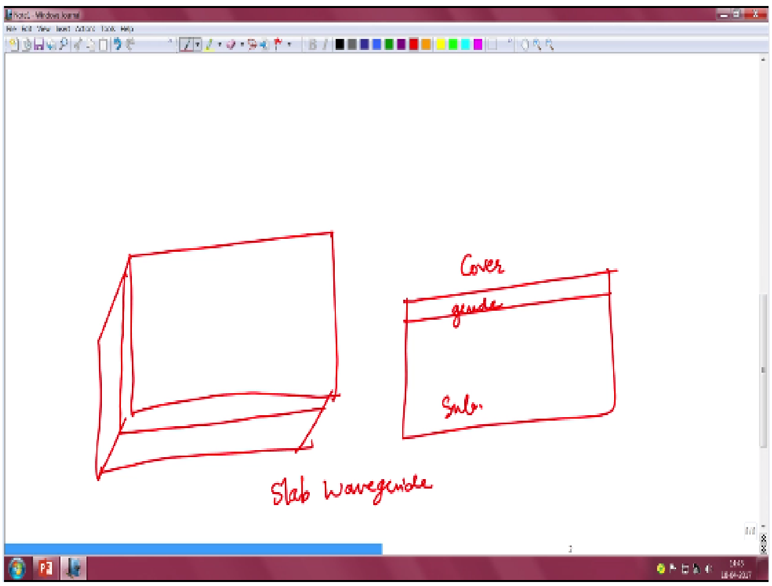

Suppose you take a glass slide and make hole of the top surface as the waveguide then this is called the slab waveguide sub state guide and cover is called ass slab waveguide so here the whole tops whole top surface is the waveguide.

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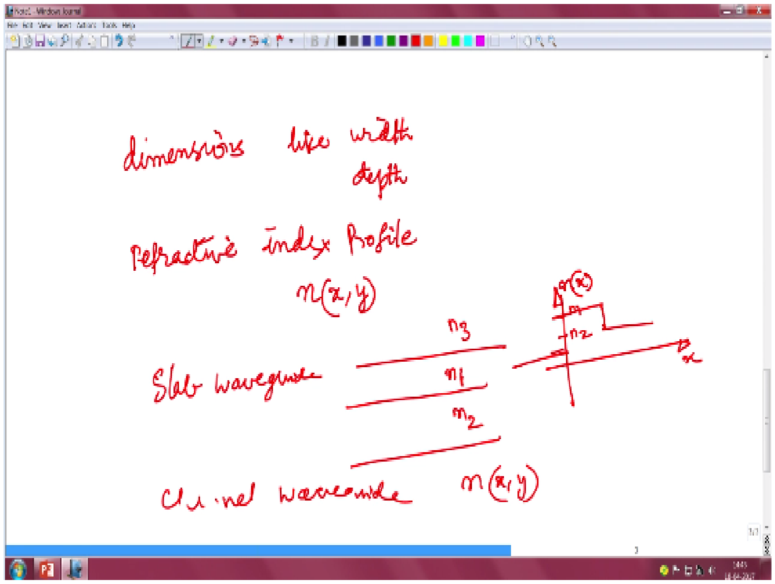

So how do you characterize these waveguide is an important question what are the factors what are the structural parameters which will define an optical waveguide and it is a design, so the most important property of course is the dimension like width then the depth and so on and also optically the most important parameter is called the refractive index profile as you have absorbed the refractive index varies with the dimensions and we can represent it using n(x , y) where x and y are the coordinate axis as an example for a slab waveguide we can say suppose it is can be drawn like this.

And you can say the refractive index is n1 in the guiding region and to in the sub state and n3 in the cover and we can draw the refractive index profile as follows if this is the x axis this is the n is a function of x and it is called the refractive index of n1 in the guiding region and n2 in the sub state region and n3 outside refractive index profile looks something like this, so refractive index profile is variation of the refractive index with dimensions in the case of a channel waveguide we can represent we can to present the refractive index n in terms of to coordinate variables. So these are transverse cross section what I hope shown.

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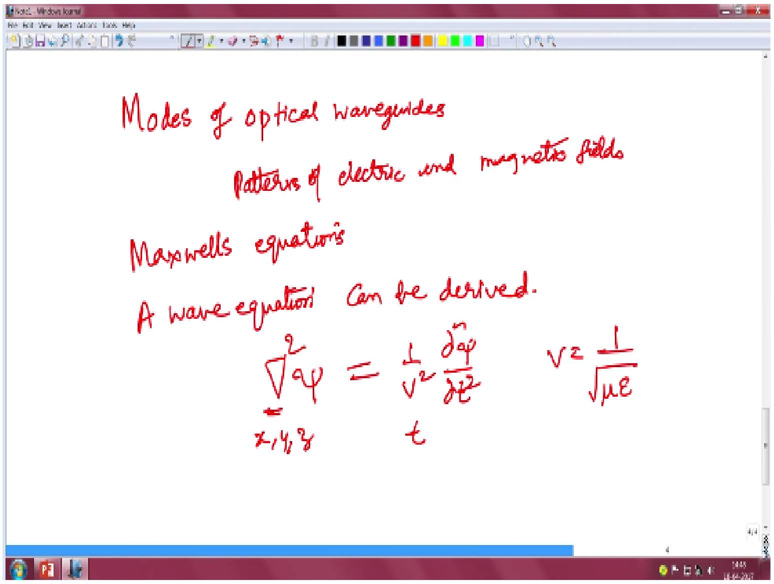

Now before going into the properties of optical waveguides we need to understand what are called the modes of optical waveguides in microwaves and other branches of physics you must have come across modes of the waveguides or propagation, so mode of an optical waveguide can be defined as a pattern of electric and magnetic fields, we know the light is electromagnetic wave and has got electric and magnetic fields which are which are governed by Maxwell's equations, so the propagation of light in optical waveguides is also governed by Maxwell's equations we need not go into the Maxwell's equations right now.

But we know that a wave equation can be derived from the Maxwell's equations, so the typical form of a wave equation looks like this, so where ψ is the wave function representing any of the fields electric or magnetic field components and V is the speed of the light in the waveguide so this is called the time-dependent form and ∇^2 is the operator which contains transverse components x, y, z or in any other coordinate system that we choose, so we can say that a wave equation is the result of Maxwell's equations which can be used to study the propagation of optical waveguides.

So there are several parameters with us like x, y, z and the structural parameters go in the form of key velocity which can be expressed in terms of the medium properties by the electrical permeability and permittivity and electric permittivity of the medium properties.

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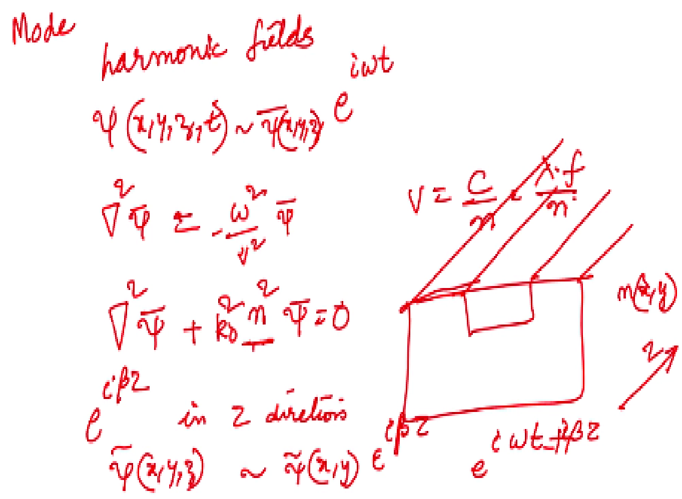

So a mode of an optical waveguide can be defined as a pattern of electric and magnetic fields which can be obtained from the wave equation and we talk about the wave propagation in terms of harmonic fields, if ψ is the wave function dependent on x, y, z and t we can study the properties of the wave weights in terms of a harmonic field defined as having a single frequency. So we can call them as monochromatic waves of a certain given frequency ωt, so this is equivalent to saying that we are going to study the Fourier components of the field.

So if we express ψ as ψx, y, z say let me put a bar about is to say that this is only a coefficient $ei^{\psi t}$ we can reduce the wave equation to $\nabla^2 \psi = \omega^2 / v^2$ I am differentiating this twice and then substituting and ψ bar we can also express we can also express the velocity in terms of frequency into the wave length and reduce this equation further to let me put velocity is C / n where n is the refractive index of the medium and the wavelength multiplied by the frequency will give the velocity divided by n and after a little bit of manipulation we can reduce this to $\psi^2 + k0^2 n^2 \psi = 0$.

So the refractive index profile is appearing in the wave equation as follows this is called the Helmholtz equation which does not contain a time variation this is for the harmonic fields, so we are talking about the wave guides which have a cross section I will draw one thing the waveguide geometry, so you have a refractive index which is a function x, y only and along the propagation direction the refractive index is not varying, so let us say that the propagation is in terms in the z direction and we can define a mode which varies as $ei^{\psi t}$ $ei^{\beta z\ t}$ in the Z direction.

So I can replace once again this ψ in terms of okay Let me go to X, Y, Z has going to ψ I will use another symbol ψ~(x, y) $e^{ißz}$. So we can define a mode as a component which varies as $e^{iωt} \pm ei^{ßz}$ so with this we can reduce the wave equation as follows.

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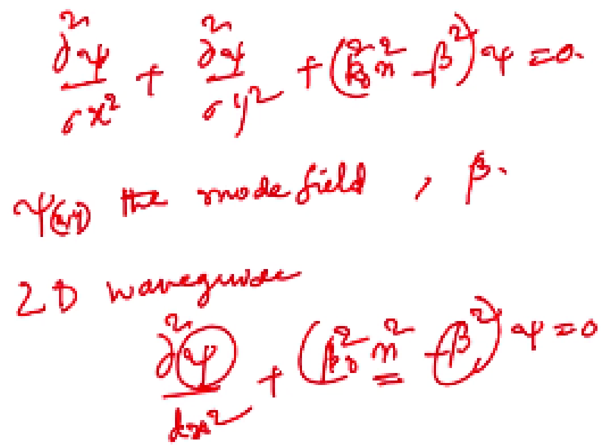

So I am using the same notation ignoring the ~ and bar place the above ψ , so the ψ represents the mode field and along with the ß the Ø characterizes the mode so the mode has got a field pattern and the propagation constant and I have shown the two-dimensional form x^2 and y^2 for a channel waveguide both the components and if we are talking about a 2d two dimensional waveguide like a slab waveguide.

We can ignore one of the variations like Y then for a 2D waveguide we can reduce this to d^2 ψ/ dx^2 + ($k0^2$ n^2 - $ß^2$) ψ = 0 just remind that n is the structural parameters which varies with the X or Y, so the problem is to solve for the wave function as well as the propagation constant simultaneously, so let us take an example and try to solve this for a typical example.

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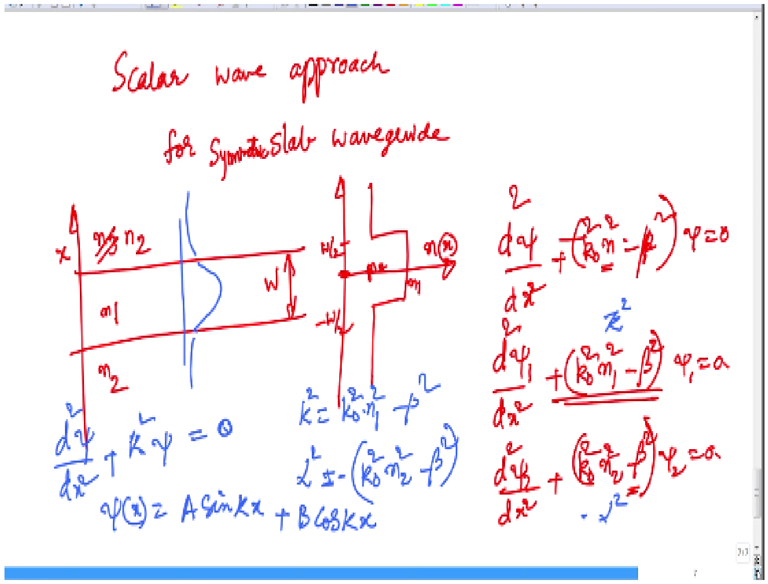

Before that as we know the electromagnetic fields inside the optical waveguide have has many components X component Y component Z component and so on and electric fields EX, EY, EZ and HX, HY, HZ in principle all the components can exist and can be important but it is possible to reduce the study of optical wave guide to a single component sometimes if you are not interested in the polarization issues etc.

The basic properties of optical wave weights can be studied using what can be called as a scalar theory, where only one component is important and all the other components could be derived from the given component we will go into the details later on but let us start with the scalar approach and see the properties of waveguides and relate them to the vector properties where polarization and other properties will be important.

For this let us consider the slab waveguide that we have discussed earlier, for slab waveguide, so the slab waveguide the geometry is like this where you have refractive n1 in the guiding region n2 in the sub state region and n3 in the cover region and I will put the axis X and I will take a specific case of a symmetric slab waveguide, a slab waveguide can be called symmetric when the super state also has a refractive index n 2.

And what is expected of this before we derive let us look at what is expected of this as a solution, so the wave equation are first of all let me write the refractive index profile I will draw it the same axis that we have chosen to represent the waveguide and let me take the origin as the middle and the refractive index profile could be drawn like this n(x) this X direction this is the let me say the width of the waveguide is W width or depth and from –w/2 to +w/2.

We have a refractive index N1 and outside is a refractive index n2 this is geometry and this is a refractive index profile and the corresponding wave equation turns out to be $d^2 \psi/dx^2 + k0^2 n^2 - ß^2$) $\psi = 0$ our problem is to find ψ and ß similarly, we should remember that N 1 this contains the refractive index profile n on their which have got a refractive index n1 in the core and n2 in the cladding.

We can also separate out these we can write these equations differently for core as well as cladding and then later on match we can say D $\psi1/ dx^2$ in the core region the field and K 0^2 where refractive index is n1 and ß and $\psi1 = 0$ and in the cladding region $D^2 \psi2/ dx^2 + k0^2 n2^2 – ß^2 (\psi2) = 0$, so we have written the we have split the wave equation the core region and the cladding region.

But we should remind that this ψ is a composite function having some value in the core and another value in the cladding and refractive index also having a different values the core and cladding but the ß the propagation constant belongs to the entire waveguide and wave propagation , so we can the form of the equation is familiar suppose we replace the constant now the N 1 N 2 are constants separating as K and α.

Let me define as K^2 and α^2 where K^2 can be defined as $K0^2 N1^2 - ß^2$ okay I do not need to go and also α^2 being defined in terms of the constant line there I will define it as a negative parameter I will explain why it is o case $k0^2$ and $n^2 - ß^2$ so if the equation inside the core is $d^2 x^2 /dx^2 + k0^2$ and are plus $k^2 \psi = 0$ then we know this is the question for a simple harmonic motion and we know the solutions in terms of the trigonometric functions.

ψ (x) as A sin ox + B cos kx so this representation field which is oscillating along the x direction we can write similarly the typical solution for the cladding region and we see that instead of K there is an α and if α is a positive then we can expect an oscillating solutions at the core and cladding also and if we choose α to be negative or if we choose ß such that K 0 and $n2^2 < ß^2$ than then α is negative and we can represent a field in the cladding in terms of the exponential phase expansion rising and exponentially decaying fields. So this is a typical form of the mode field profile.

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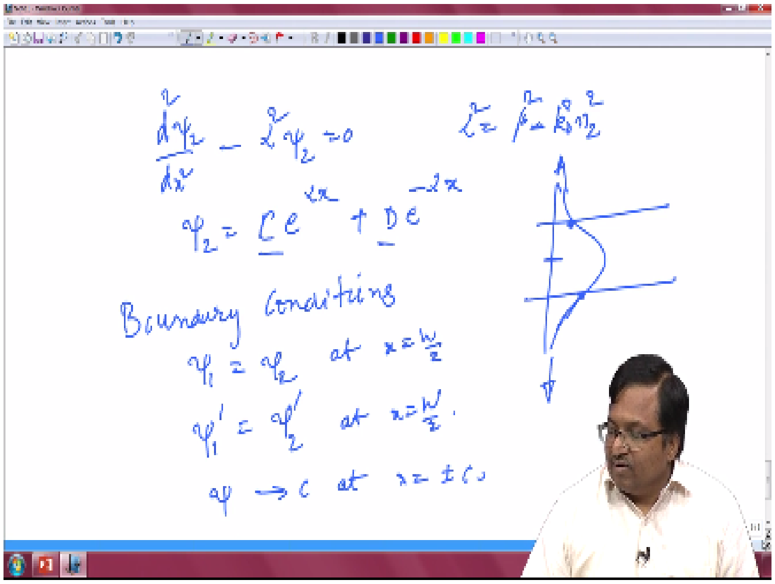

And we can express this is suppose $d^2 \psi /dx^2$ this is ψ 2 in the cladding region - α^2 x ψ 2 = 0 Now with α of the negative a α being defined in terms of $ß^2 – k0^2\ n2^2$ so now we know the solutions of this equation in terms of the exponential functions ψ 2 is $Ce^{\alpha x} + De^{-\alpha x}$ so we have these solutions to be wave equations as oscillating fields inside the core region and exponentially rising or decaying fields in the cladding region.

We can take a physical explanation saying that the fields of an optical wave at the desirable form of the fields of an optical waveguide as oscillating fields which gradually decay to infinity and choosing the constants will be an important problem now, so if you look at the wave from waveguide and the equations we can say the field should be continuous at the boundary. So now we use the boundary conditions to boundary conditions.

To choose the parameters or the arbitrary constants barring any equations, so we have a field which is trigonometric inside record is in an exponentially decaying or reducing in the cladding region, we can give a rigorous mathematical explanation or meaning for this problem for the shape or the fields that we choose but for understanding purposes let us now choose a very tentative approach and say that the fields are required to go to zero at infinity and have an oscillating form given by the trigonometric functions in this region.

So and also the boundary conditions imply that the fields should be continuous at the boundaries. So let us force that the field $\psi_1 = \psi_2$ at x equals W/2 and on the other point and the other side also.

Similarly we have many constants in the arbitrary constants in the solutions, so we need more boundary conditions to evaluate the further constants, let us force one more boundary conditions saying that the derivatives of the fields are continuous at X=W/2 and the fields go to infinity four fields go to 0 at x equals ∞.

So these conditions which invalid the infields and remain working that the way we need to solve for the data and the mode field.

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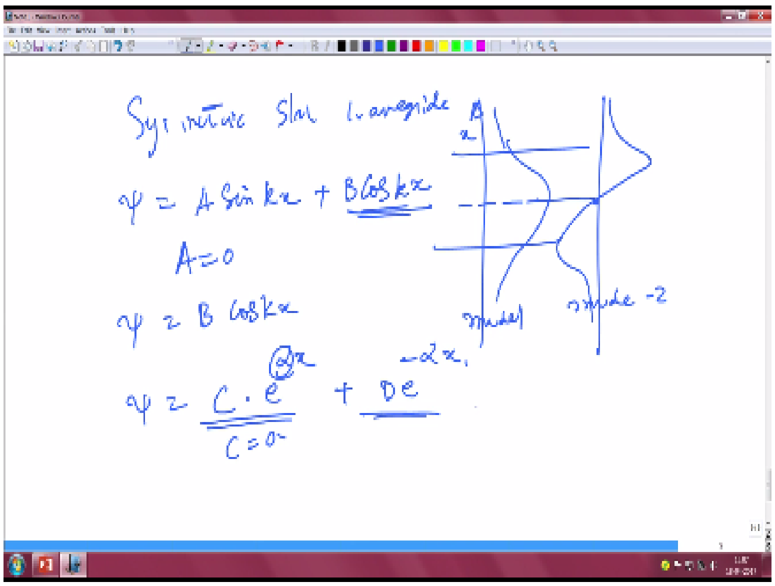

Now in the case of asymmetric slab wave guide, the case of symmetric slap wave guide we can expect two types of modes from the symmetry of the geometry we can expect that the field for X greater than 0 and X less than 0 or symmetric or anti-symmetric, so the field can be symmetric like this or anti-symmetric like this and only if it is enough if we solve the equations for X greater than 0.

So by looking at the solutions that we have ψ=A sin kx+B cos kx for the case of mode 1 we can say that it is like a cosine field and for the sake of for the second mode they say that the field is like sin, to cook any process let us take it symmetric mode with cos is about fields is a code region and say that the by choosing constant A=0 we can force it to be symmetric. We can be done more rigorously more mathematically, so let us choose ψ to be B cos kx in the code region.

Similarly if we look at the field in the cladding region where we have ψ= C.$e^{\alpha x}$ and D.$e^{-\alpha x}$ we have chosen the feeling to be going to 0 at ∞. So if we take α is a positive quantity so the field you should go to should decay the cladding if you choose only this part of the field. In other words we can put C = 0 for this and choose the field to be D e.$^{-\alpha x}$ in the cladding region.

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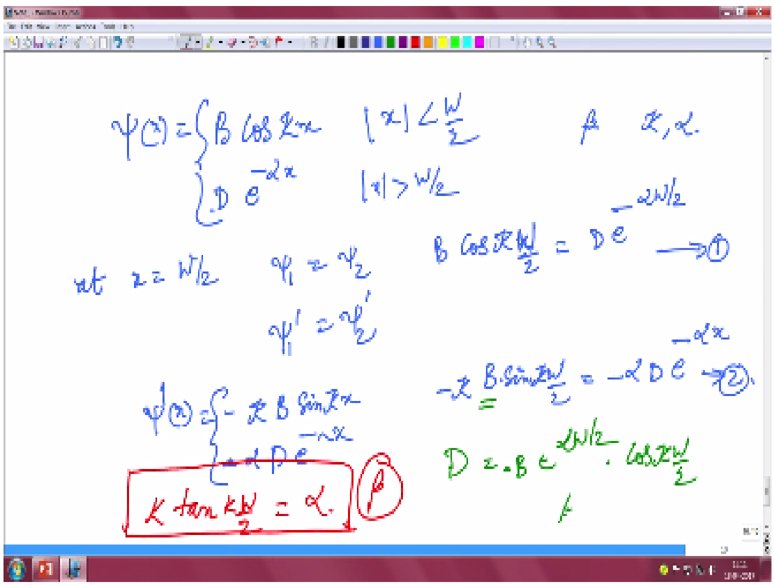

So we have the ψ(x) is B cos kx for x less than W/2 and D $e^{-\alpha x}$ for x greater than W/2, now let us apply the boundary conditions to find out the exact form of the field and once again to remind you that we need to find out the β which is their embedded inside the K and α in some complicated way. So at x=W/2 ψ_1=ψ_2 implies B cos κx or xl replaced by W/2= D $e^{-\alpha w/2}$ and once again at x= W/2 if I force ψ_1'= ψ_2'1 we get let us derive a differentiate first and then apply the boundary condition.

So ψ'x is κ.B sinκx and –α D$e^{-\alpha x}$ so we have the by matching the boundaries we have –κ.B.sinκ.W/2=-α.D.$e^{-\alpha x}$ so this equation 1, and this is equation 2 to solve for the arbitrary constants BD and so on. So from the first equation I can say, I can find out one constant D to be equal to $e^{\alpha W/2}$.cos κW/2 E we can use up the first equation to find out to eliminate one arbitrary constant D. We can use the second equation along with the first equation to eliminate the other arbitrary constant D.

But one o four choice one of our intentions is to find the beta which is embedded inside the α and K so we will use instead of eliminating the arbitrary constant we will use it to find out the expression for the β, so by looking at these equations we observe that on the left hand side and right hand side we have the arbitrary constants B and D, so we can eliminate this B and D by dividing one equation by other equation. So by doing so we get and dividing the equation 2 by equation1 we get K tan kW/2= α this can be called as the characteristic equation to define or to obtain the value of the propagation constant β.

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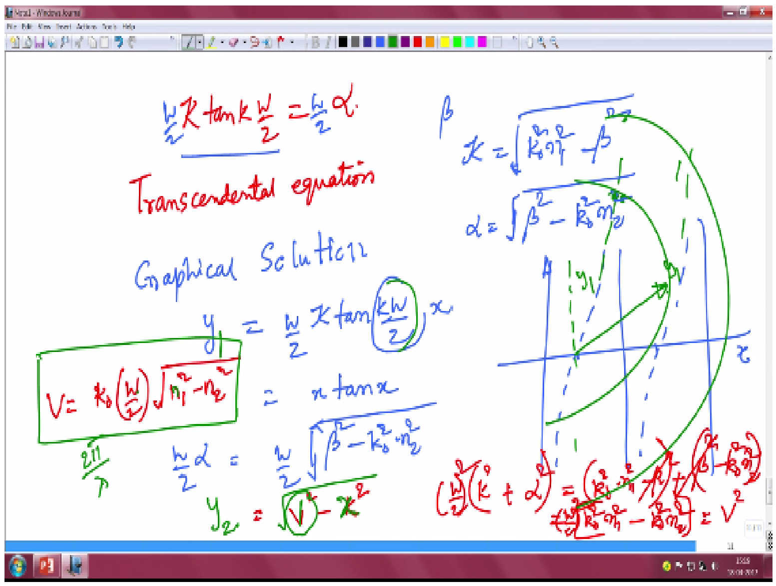

So this equation is a transcendental equation where the constant the required constant β is inside the constant k, $k_0^2$and 12-$^{\beta 2}$ and α, β^2-k_0^2 and 2^2 so it is not easy to solve this equation directly but by programming is a MATLAB or any numerical software we can obtain the values of β. Before we go further let us interpret this equation and how we can solve this graphically, graphical solution of this equation can be obtained by looking at the left hand side and right hand side and the geometrical figures that they represent.

So the expression K_{10} KW/2 let us say is equals to y and if we represent the independent variable by x let us say this is x and I will multiply the equation with W/2 on both sides to get so we can say this is x tanx this is of the form x tanx, we know the form of tan function and X multiplied by tan x also would look somehow similar these are form of the tan functions or I would use a different fine, even though I use the x for the other variable.
Similarly the right hand side of the equation if we look at the definitions of if on K can be written in terms of K again, so W/2 α can be written in terms of W/2 in terms of K as follows. $\beta^2 - k_0^2 n_2^2$

and if I add k^2 and α^2 I get $(k_0^2\ n_1^2 - \beta^2) + (\beta^2 - k_0^2 n_2^2)$ so the $+\beta$ and $-\beta$ can cancel and say that $k_0^2\ n_1^2 - k_0^2\ n_2^2$ or $k_0^2 . n_1^2 - n_2^2$ so I can say and multiplying by W/2 on both sides square of it I guess and representing this by some constant v^2.

I can say this is v^2 -K^2 where the constant V is defined in terms of K0 ^{2} x W /2 now square can be removed W /2 √ N 1 square - n 2square, so this is called the V parameters of the wavelet. With a slight modification you can say this represents three parameters containing the row bed with the refractive index contrast and through K 0 which = 2π/λ will represent the operating frequency.

So the constant α if we are representing XKW /2 as X we can say this = X^2 v square- X ^{2}, so we can say that this if we say the first curve is y 1 the second curve is y 2 then these are curves y s the this represents a circle of radius V. The right-hand side of the characteristic equation represents a circle of radius V and depending on the value of V the point of intersection of these two curves representative solution that us how we can obtain the solution B equation graphically draw.

The graph of left hand side and right hand side with normalized parameters like X and then find the point of intersection and from the value of the solution the X we can find out what is value of β so from this expression we have said that K W/ 2 as X, so once we got the value of x we can find out the value of β. That is how we can obtain the value of the β, so we observe that we observe that the radius of this circle is depending on the parameter V.

So V is the parameter which will define how many points of intersection we have if the release is bigger this will intersect the left hand side curve at more points and we can say there are more equation more solutions if it intersects only one curve representing the left side you can say that there is only one solution and we have the single-mode wave rate. So we can design a single-mode wave rate by choosing the parameters of the wave red light wavered with the operating wavelength and the refractive index contrast.
As I mentioned earlier we can solve this equation directly using a MATLAB and you can find out what are the various solutions that we can get.

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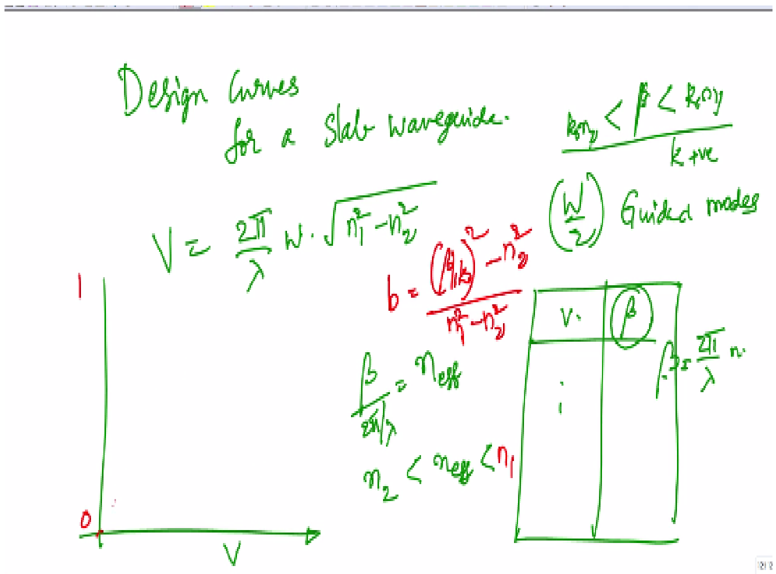

Now we let us discuss little bit about the design curves for a slab wave grade. We said that the design parameter V is1/K0 x width x√n ^{2} - n ^{2} 2. I will replace the width parameters earlier we have W /2 in the equations later on when we generalize the theory to estimate X laborite we see that there is a more general solution and W Z parameters or before going for that I will go back and say that we can also think of symmetric modes.

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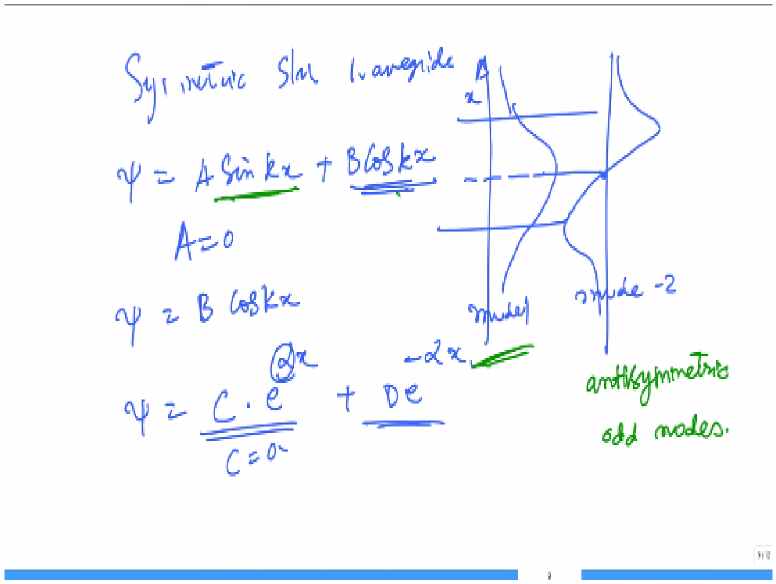

See we have called these are asymmetric modes and these are symmetric modes, so we have solved for a symmetric mode all the characteristic equation. So we can also solve do the exercise for anti symmetric or our modes of odd mode by choosing sine kx instead of cos kx. We may end up in having cot θ instead of tan θ, so the design curves are generally drawn as a β as a function of with how the propagation constant varies with the design the waveguide parameters.

So we can form a table of values of the V versus propagation constants and then plot them as a two-dimensional plot V on one axis and a bit other axis. At this point the propagation constant can also be normalized by saying that the above theory we have found that β should be we should lie between should be ˂K 0n1 for K to be positive and should be more than K0 x α to be negative.

So K is positive if you threw this and α is also positive with you, so this range of values of β has to be chosen for what can be called as guided modes. Guided modes are the one which are got the constantly nature in the grading region and the exponentially decaying the cladding region. So these are the ones which are used to guide the lights inside the waveguide and we can say that if we choose the value of θ like this the light in the mode is confined when it is a wavelength region.

So this β is normalized with respect to the refractive XS as follows. β has got the dimensions in the free space with a has body dimensions of 2 ϕ / λ into the fraction X, so if we choose if we solve for a particular β and divide /2 ϕ / λ, so it has got the dimension of a refractive index, but as we see from the expression that brittle ice between K0 and N1 and we can expect this n to be line between K0n2 and K0n1.

So and we can call it as an effective refractive index of the wave vector, so this effective refractive index of the waveguide lies between the referred to basis of the substrate and the guiding region, and a normalized propagation constant can be defined as follows. As a B which is β /K 0 ^{2} - n 2^{2}/N 1 - n 2^{2} later we will explore the logic how the logic behind the definition. So if you choose the β like this we can say that n effective is like between n 2 and CN1and so the β will lie between 0 and 1.

The plots looked like this for the fundamental mode the for whatever the value of the V parameters there is always a solution, so we can say the fundamental mode always exists and starts at origin and as we increase the wave that will tour the flatness contrast or the frequency you can say that it is a accurate towards some particular value looks like this. For the next mode we can see from the characteristic equation.

You can see from the characteristic equation that we there are multiple modes and the second one a pair softer $\pi/2$, so if we say that the odd modes are in between you can say the cutoff of the modes to be π and it starts at this point. So this is at typical nature of the curves representing the propagation constant with operating wave length and D will get bit. So we will take up the discussion about the wave grades further, when we go to the next exercise on the asymmetric slab will gates.