

Indian Institute of Science

Design of Photovoltaic Systems

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NPTEL Online Certification Course

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Numerical solution for Colebrook-white formula

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left(\frac{\epsilon/d}{3.7} + \frac{2.51}{Re \sqrt{f}} \right)$$

$$x = \frac{1}{\sqrt{f}}, \quad a = \frac{\epsilon/d}{3.7}, \quad b = \frac{2.51}{Re}$$

$$x = -2 \log_{10} (a + bx)$$

$$q(x)$$

Let us now try to find numerical solution for the Colebrook white formula, first we will look at the theory and then we will use the solution that we right away the solution that we arrive at, we will use that equation in octave and then execute that and try to find the friction factor. So we know the Colebrook white equation it is like this, you are now familiar with this.

Now $1/\sqrt{f}$ let us replace it with one symbol, ϵ/d Rockne's ratio /3.7 we will replace it with one more symbol, $2.51/Reynolds$ numbers, we will replace it with one symbol for a given Reynolds number and $V1$ reference index this potion and this $2.51/Reynolds$ will be a constant so let us say $x = 1/\sqrt{f}$ we will use that symbol for $1/\sqrt{f}$ and then we will use the symbol a for $\epsilon/d3.7$ and we will use the symbol b for $2.51/Reynolds$ number so now this core group white formula becomes simple to read $x = -2 \log_{10} (a+bx)$.

So this is the equation for which we need to find the $\sqrt{\quad}$ and find the value of x now let us say g of $x = x +$ this we will take $x +$ this one to the left hand side which will make it 0 so that is the function g of x $x + 2 \log$ based and $a + bx$ and we know that this should become 0 and that is why we put it as equal to 0 so we will try to solve this $g(x)$ equation either relatively and find a value of x such that $g(x)$ is 0 so that is what we want to do and for that there are really methods and techniques we will use the Newton codes techniques for solving this iterative relationship.

So consider this equation $g(x) = x + 2 \log_{10} a + bx = 0$ $\partial g / \partial x$ partial derivative of g with respect to x will give you g' which is equal to $1 + 2b/a + bx$, now $g(x)$ evaluated at k will be $g(x_k)$ is equal to Δ change in x and multiplied by the slope $\partial g / \partial x$ so this is evaluated at $x = x_k$, so this is the Newton quotes rule this will give you an evaluation of g at x_k , now $\partial \Delta x$ is nothing but $x_k - x_{k+1}$, so previous iteration minus the next present iteration minus in next iteration value so this into g prime evaluated at X_k .

So now this equation if you rearrange them you will get a $X_{k+1} = X_k - g$ evaluated at X_k / g prime evaluated at X_k so this would become our iterative equation in an more generic sense now we will replace this equation for g and g prime here so $X_{k+1} = X_k - g$ is nothing but this part of the equation $X_k + 2 \log_{10} a + b X_k$ evaluated at $X_k /$ that portion divided by g prime which is this portion with X evaluated $X_k (1 + 2b/a + b X_k)$.

So this is the entire iterative equation let us do some simplification on simplifying we get $x_{k+1} = 2 \sqrt{x_k - 2, a + b x_k} \log_{10} a + b x_k / a + b x_k + 2b$. So this is the entire simplified iterative equation that we would like to numerically compute. Now what should be the starting value of x_k initial? So that I will take it as $1/\sqrt{f}$ and that f is $1/\sqrt{0.1}$, f is nothing but 0.1 what I have taken. How do we arrive at this value? If you look at the moody chart if x axis is nothing but fiction factor so I have taken an high value of fiction factor 0.1 here so from here it should come down and boil down to the proper value of fiction factor for different numbers so it should convert.

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$$\begin{aligned}
 q(x) &= x + 2 \log_{10}(a+bx) = 0 \\
 \frac{\partial q}{\partial x} &= q' = 1 + \frac{2b}{a+bx} \\
 q(x_k) &= \Delta x \cdot \left. \frac{\partial q}{\partial x} \right|_{x_k} \\
 &= (x_k - x_{k+1}) \cdot q' \Big|_{x_k} \\
 \boxed{x_{k+1} &= x_k - \frac{q(x_k)}{q' \Big|_{x_k}}
 \end{aligned}$$

$$\begin{aligned}
 x_{k+1} &= x_k - \left\{ \frac{x_k + 2 \log_{10}(a+bx_k)}{1 + \frac{2b}{a+bx_k}} \right\} \\
 \text{Simplifying} \\
 \boxed{x_{k+1} &= \frac{2b x_k - 2(a+bx_k) \log_{10}(a+bx_k)}{(a+bx_k) + 2b}
 \end{aligned}$$

iteration equation

$$x_{\text{initial}} = \frac{1}{\sqrt{0.1}} \text{ inch}$$

So the initial value guess is from the moody chart and you can use this value $1/\sqrt{0.1}$ for any value of number and roughness ratio generally from that value it will convert to the final resulting value which is desirable for which value the g_x or g_k will become 0 let us now implement this equation in octave and obtain the value of x and then from x the value of fiction factor because fiction factor and x are related by $x = 1/\sqrt{f}$ and $f = 1/x^2$ after we finished the iteration we will we would have found out x_{k+1} for which g_{k+1} is almost equal to 0 and using that x_{k+1} factor so that will be the final value of fiction factor which we will use it for plugging into the formula to obtain the fiction loss head.