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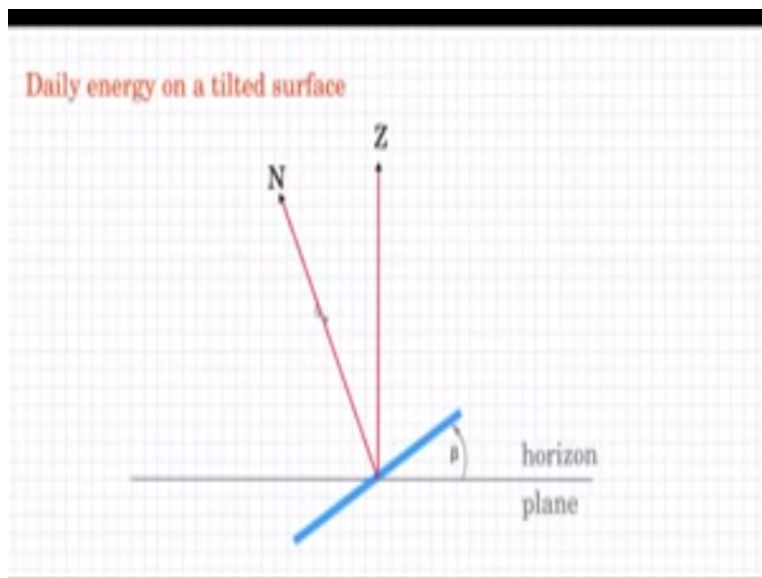
Design of Photovoltaic Systems

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NPTEL Online Certification Course

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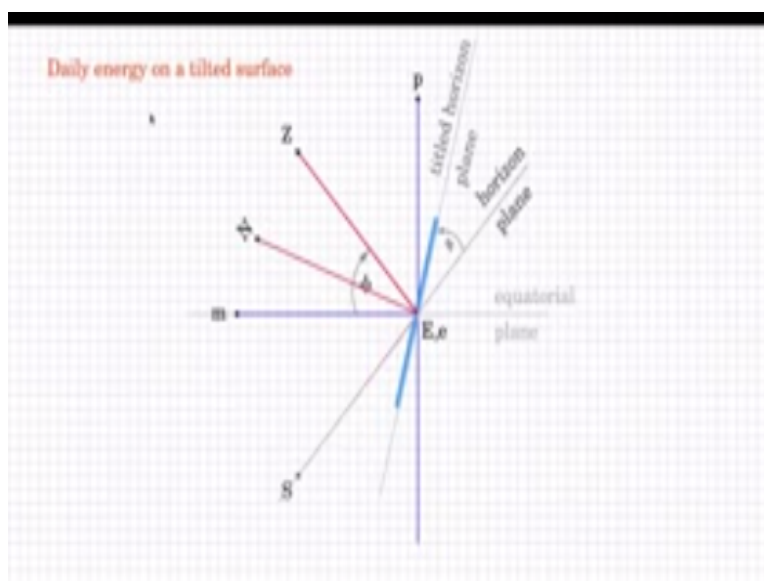
Let us now try to find out what is the daily energy instant on a tilted surface placed at a locality, so for that let us first draw the horizontal plane so this is a 2D graphics so the horizon plane is indicated in straight line and this is the horizon plane at some arbitrary locality on the surface of the earth. Now this axis is the normal axis normal to the horizon plane it goes vertically up and this is called the zenith axis Z this also we know.

Normally till now we used to have a horizontal flat plate collector something similar to this, and for this horizontal flat plate collector you were interested in finding the amount of the daily energy incident on this horizontal flat plate collector but in general normally the flat plate collector is not kept horizontal at the locality it is kept tilted at an angle at an appropriate angle which will collect the maximum amount of solar radiation.

So therefore if we tilt this horizontal flat plate collector at an angle β to the horizon plane so it will be tilted the horizontal flat plate collector will become a tilted flat plate collector with a angle of tilt β in such a way that the tilted collector tries to face the equator in the northern hemisphere it will try to face south and the southern hemisphere it will try to face north or in general you can always say the tilt is in such a way that the collector tries to face the equator.

I will now push and try to bring it down to the origin like this and also now let us draw a line normal to this tilted collector. So this line this axis named NN is the normal axis or a normal vector normal to this tilted flat plate collector which is tilted at an angle β to the horizon plane, now it is of interest that we try to find that insulation vector which is along this normal when we when the flatted collector was horizontal the normal and the zenith axis coincided and therefore you are interested in the insulation vector along the zenith axis but in actuality now in the case of a general tilted flat plate collector we must try to find the insulation vector along a normal to the flat plate collector that is along this N axis.

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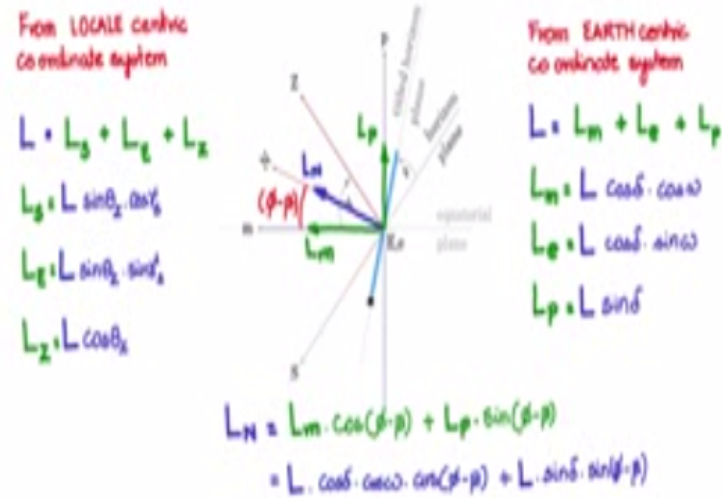
Now let me go back to the merged coordinate axis system recall that we have seen this merged coordinate axis system this is the horizon plane and with respect to the horizon plane you have the local centric coordinate system shown in red this is the south on the horizon plane this is the zenith axis and the one shown in blue is with respect to the equatorial plane that is earth centric coordinate system.

You have the meridian axis and the polar axis shown like this, now on to this let us merge the tilted flat plate collector to, now we have seen a tilted flat plate collector is like this we have the horizon plane at the locality and we have tilted the flat plate collector at an angle β with respect to the horizon plane and the plane along the tilted flat plate collector is called the tilted horizon plane and normal to the tilted flat plate collector you have this normal line or the normal axis N axis which is orthogonal to this tilted flat plate collector.

Now this one let us merge it with let us merge the origin with the already merged axis coordinate axis system like this, after placing it like this let us rotate it about the origin such that the horizon planes match like this so the local centric horizon plane is like this and with respect to the local centric horizon plane at an angle β you have the tilted flat plate collector and the plane along that is called the tilted horizon plane.

And the line normal to that is the normal axis and it is now important for us to find out the insulation that is incident along this normal axis, and later to find out the energy incident on this tilted flat plate collector. So remember this modified merged coordinate axis system and we will be using this to develop the equation for the energy falling on a tilted surface.

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As in the case of the energy on a horizontal flat plate collector we have from the local centric coordinate system the insulation given as the insulation along the south axis of the horizon plane insulation along the east +LZ along the zenith axis, so L as Le and Lz are given in terms of the zenith angle and the azimuthal angle. From the earth centric coordinate system the insulation is resolved into LM along the meridian axis along the E so meridian and along the polar axis.

And there given in terms of the declination and the other angle, now let us bring in the tilt also into the picture and let us see how we get the modified insulation along the normal to the tilted flat plate collector. So we have placed here brought in the modified merged coordinate axis system now this is the tilted flat plate collector tilted at an angle β with respect to the horizon plane and this is the tilted horizon plane.

And what is of interest is the insulation along this axis the normal axis or the N axis normal to the flat plate collector which is tilted, so therefore let us draw that and I will name that one as LN vector and there are a two vectors from the local coordinate from the earth centric coordinate system one is the LN and another is LP along the polar axis, so like as before we will use the same method and this is LP.

Now LP will reflect can be resolved onto this N axis LM can be resolved on to this N axis in the following manner knowing this angle now because this is β this will be β and therefore this will be $5 - \beta$ and LN is given by LM projected onto the N axis will be caused by $-\beta$ and LP projected onto the N axis will be $LP \sin \phi - \theta$, now we know LM is $L \cos \delta \cos \omega$ and LP is $L \sin \delta$ substituting we have earlier into $\cos \delta \cos \omega \cos(\phi - \beta) + L \sin \delta \sin(\phi - \beta)$.

Now this is the insulation equation for the insulation that is incident perpendicular to the tilted flat plate collector located at a locality which is at a latitude ϕ from the equator.

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$L = L_3 + L_2 + L_1$
 $L_3 = L \sin \theta_2 \cos \theta_1$
 $L_2 = L \sin \theta_2 \sin \theta_1$
 $L_1 = L \cos \theta_2$

$L = L_m + L_p + L_q$
 $L_m = L \cos \delta \cos \omega$
 $L_p = L \cos \delta \sin \omega$
 $L_q = L \sin \delta$

$L_H = L_m \cos(\beta - \theta) + L_p \sin(\beta - \theta)$
 $= L \cos \delta \cos \omega \cos(\beta - \theta) + L \sin \delta \sin(\beta - \theta)$

Daily energy incident on a tilted flat plate

$$H_{ot} = 2 \int_0^{\omega_{SR}} L_H \cdot d\omega$$

Now integration of this insulation L_N over the entire day will give the energy on a tilted surface, so therefore we can now find out the daily energy incident on a tilted flat plate and that is nothing but H we will now call it H_{ot} . H_o we had used the symbol for a horizontal plate energy on a horizontal plate. H_{ot} energy incident on a tilted flat plate and that is equal to 2 times integral 0 to ω_{SR} , ω_{SR} is the sun raised our angle.

Now I am going to slightly modify that it will be ω_{SRT} sun rise angle for a tilted plate. I will discuss that more in a shortly and the integration is for L_N with a integration parameter $D \omega$, ω is the our angle, now this would provide you with the daily energy incident on a tilted flat plate. Now let us perform this integration and then see what the relationship that we will get.

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Daily energy incident on a tilted flat plate

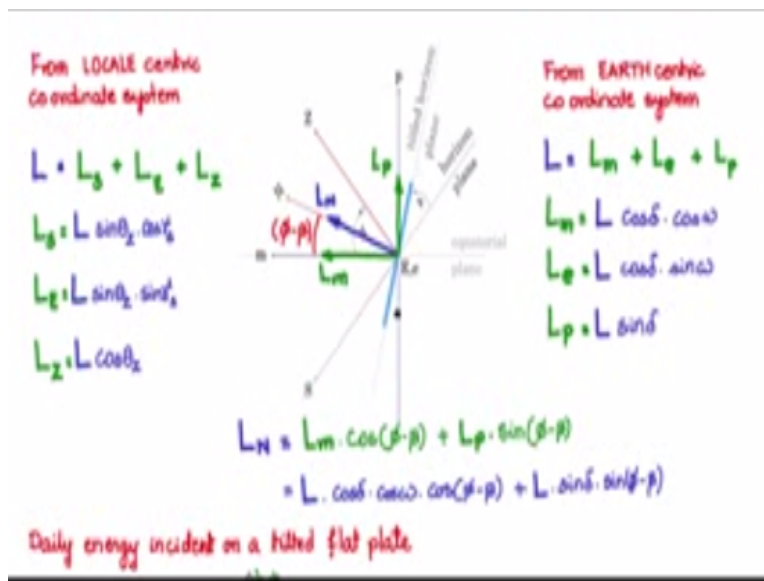
$$\begin{aligned}
 H_{OT} &= 2 \int_0^{\omega_{ST}} L_{NI} \cdot d\omega \quad \text{kWh/m}^2/\text{day} \\
 &= \frac{24 \cdot k \cdot LSC}{\pi} \int_0^{\omega_{ST}} (\cos \delta \cdot \cos \omega \cdot \cos(\phi - \beta) + \sin \delta \cdot \sin(\phi - \beta)) d\omega \quad \text{kWh/m}^2/\text{day} \\
 &= \frac{24 \cdot k \cdot LSC}{\pi} \left[\cos \delta \cdot \cos(\phi - \beta) \cdot \sin \omega_{ST} + \omega_{ST} \cdot \sin \delta \cdot \sin(\phi - \beta) \right] \\
 \omega_{ST} &= \cos^{-1}(-\tan \phi \cdot \tan \delta) \\
 \omega_{ST} &= \cos^{-1}(-\tan(\phi - \beta) \cdot \tan \delta) \\
 \omega_{ST} &=
 \end{aligned}$$

Now this H_{OT} has given here has the units of kilo watt radians per meter square per day because ω is an radians. Now to convert this into kilo watt hours per meter square per day like as we did for the H_0 case we will use the scale factor now you have 24 KLSC, LSC is the main solar constant by π integral of 0 to ω SRT and LN we can expand $\cos \delta \cos \omega \cos(\phi - \beta)$ which is the tilt angle + $\sin \delta \sin(\phi - \beta)$.

Now this would be the energy expressed in kilo watt hours per meter square per day because we have used the $12/\pi$ conversion factor. K and LSC are their usual meanings, so when you solve this you will have $\cos \delta$ and $\cos \omega \cos(\phi - \beta)$ are independent of our angle and $\cos \omega$ will become $\sin \omega$ SRT + ω SRT $\sin \delta \sin(\phi - \beta)$ now this is the energy incident on a tilted flat plate at located at latitude ϕ and having a tilt with respect to its local horizon with an angle β .

Now ω SR on with respect to the horizon of the place we know is given by $\cos^{-1} - \tan \phi \tan \delta$, now if you take the tilted horizon we take the tilted horizon and I will call that one as ω SR β the horizon that is tilted at an angle β which we saw in the profile of the merged coordinate system you will see it having a value $\cos^{-1}(-\tan(\phi - \beta) \tan \delta)$ now this would be ω sun rise our angle with respect to the tilted horizon we tilted an angle β with respect to the horizon of the place attracted to ϕ . ω SRT what is ω SRT, ω SRT is the sun rise our angle for the tilted flat plate should remember that the sun should be the horizon of both the horizon of the place and the horizon of the tilted plane.

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Here you have the horizon plane and you have the tilted horizon plane which is tilted at an angle β with respect to the horizon plane the sun should be above both the horizon for any effective LN that is normal insolation to fall on the tilted flat plate so not only is its sufficient at sun is above this horizon it should also be above this tilted horizon, so if we take for example this horizon we just having an our angle ω SR at some latitudes and some day of the year.

The sun may rise above this horizon first and the above this horizon next in the case of in some other cases you will see that the sun rises above the tilted horizon first and the horizon plane next depending upon whether it is on the northern hemisphere and the southern hemisphere, so therefore let us say the sun rises above the horizon of the locality then we call it as ω SR it has still not reason above the horizon of the tilted plane.

When it rises above the tilted plane you will see that the Ω SR would actually be smaller measured from the meridian axis than what it would have been per horizon plane therefore what we do is we take when minimum of the our angles as computed with respect to these two planes then we are safe are conservative we know that the that there will be an effective insolation falling on this if we consider that the sun has risen above both the horizon planes.

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Daily energy incident on a tilted flat plate

$$\begin{aligned}
 H_{\text{tot}} &= 2 \cdot \int_0^{\omega_{\text{SR}}} L_{\text{in}} \cdot d\omega \quad \text{kJ/m}^2/\text{day} \\
 &= \frac{24 \cdot k \cdot L_{\text{sc}}}{\pi} \int_0^{\omega_{\text{SR}}} (\cos \delta \cdot \cos \alpha \cdot \cos(\phi - \beta) + \sin \delta \cdot \sin(\phi - \beta)) d\omega \quad \text{kJ/m}^2/\text{day} \\
 &= \frac{24 \cdot k \cdot L_{\text{sc}}}{\pi} \left[\cos \delta \cdot \cos(\phi - \beta) \cdot \sin \omega_{\text{SR}} + \omega_{\text{SR}} \cdot \sin \delta \cdot \sin(\phi - \beta) \right] \\
 \omega_{\text{SR}} &= \cos^{-1}(-\tan \phi \cdot \tan \delta) \\
 \omega_{\text{SR}\beta} &= \cos^{-1}(-\tan(\phi - \beta) \cdot \tan \delta) \\
 \omega_{\text{SR}}^{\text{tilt}} &= \min \text{ of } (\omega_{\text{SR}}, \omega_{\text{SR}\beta})
 \end{aligned}$$

So therefore what we do for ω_{SRT} is to take the minimum of ω_{SR} value computed as above and the $\omega_{\text{SR}} \beta$ value computed as with a $\phi - \beta$ value for the latitude angle, so therefore whenever you use this relationship here for computing the daily energy instant or tilted flat plate the ω_{SRT} sun rise angle of the tilted sun rise our angle for the tilted flat plate collector use the minimum of the computation of ω_{SR} as our risen plane for that locality and $\omega_{\text{SR}} \beta$ for that locality if you place a flat plate which is tilted at an angle β if you computed like this that is for the tilted for risen plane that value. So these two values the minimum of them is what has to be assigned to ω_{SRT} .