Indian Institute of Science

Design of Photovoltaic Systems

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NPTEL Online Certification Course

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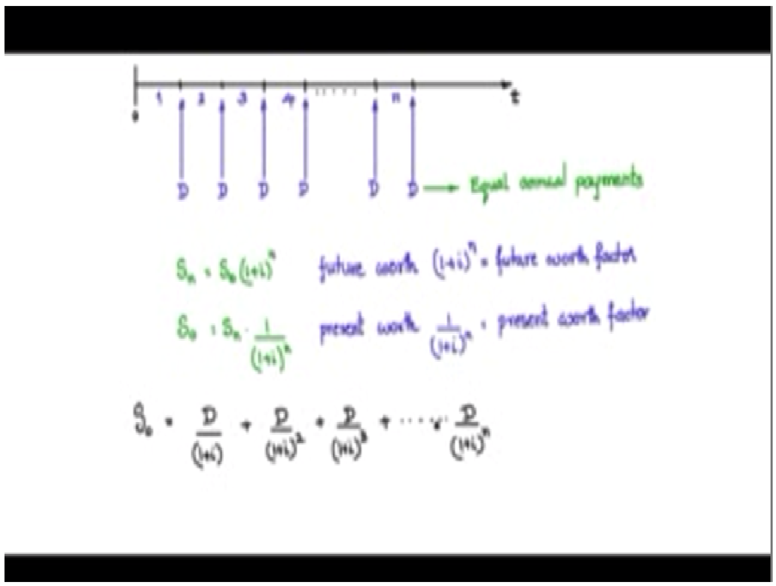

Let us now look at the topic annual payment and present worth factor. This is a popular topic common topic which you would encounter commonly when we are paying installments for items that we purchase, monthly installments, annual installments, equal annual installments. So how do we calculate the present words for such cases. Consider the timeline once again, now timeline there is a start point and then I am going to mark ticks at equal spaced intervals.

So the space between the ticks are supposed to represent the years. So let us say this time interval is one year, year one this time interval is year two, year three, year four so on up to year n. Now at the end of each year the end of every year I am going to pay an amount T, then for the second year pay an amount D, third year you pay an amount D, at the end of fourth year also similarly.

So till the end of the n^{th} year you are paying the same amount D. Now what we need to calculate what is the work at this time frame, see it is like transformation occurring across different time frames. So let us say this is our time frame of reference now today. So one year later whatever the value of D what is its present to earth, two years later whatever the value of D what is present to earth so on.

After three years what is its present work after n, what is its present to earth, you bring it all to the same time frame, then there you can add. So then there we algebraically add them and then you will get the equivalent value for the present work. So what is the principle that we are going to follow. So this D are the equal annual payments that one makes. Now Sn we know this equation Sn we will use the compound interest formula Sn that is the value of the money after the n^{th} year is $S0(1+I^n)$ where I is the rate of interest.

So this is the future worth and I+1 to the power of n is called the future worth factor. So S0 is the present value worth $1+I^n$ is the future worth factor. Now suppose we know what is the amount at the n^{th} at the P at the n^{th} time can you calculate the present worth of that. So it is like saying that what is S0 the given Sn is known. So it is just that it is equal to now $Sn(1/1+I^n)$, so this one goes down.

So $1/1+I^n$ is called the present worth factor S0 is now representing the present worth of the future value SM and $1/1+I^n$ is present worth factor. Now let us apply this to this problem and see what is the present words at this time frame which is not the present time frame. Now S0 the present worth here of this value D which has been a payment that is made out of the first year would be $B/+1+I^1$ plus at the end of the second year you make another payment D which is 1 the present worth here would be $T/1+I^2$ as I am using this equation.

Then the present worth for D at the end of 3rd year will be $D/1+I^3$ so on if I keep doing the installment D which is supposed to have been paid at the end of the nth year if it is reflected back to the present it will be $D/1+I^n$. So all the reflections to the present time frame are added up and that would give you the present worth taking all the installments together
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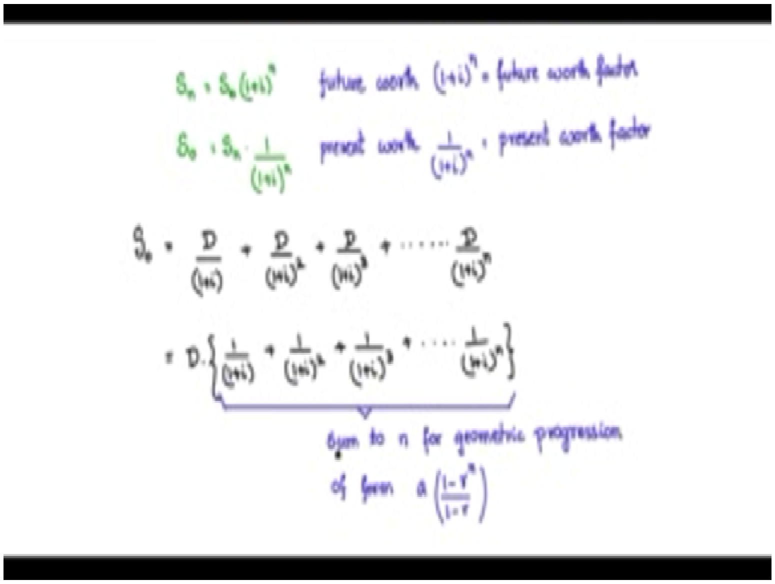

Now simplifying it you have D(1/1+I^1+1/I^2+1+I^3+ so on 1/1+I^n) now if you look at this pattern you see that this is a geometric progression, and this is the sum to n terms of a geometric progression and geometric progression is well known and understood in sum to n terms it is of the form A(1-r^n/1-r) sum to n terms where A is the first term, and r is the common factor which in this case is 1/1+I every term with respect to the previous term is different by a common factor 1/1+I. So if I apply these to this formula I will get the sum to n terms of the geometric progression, let us do that.

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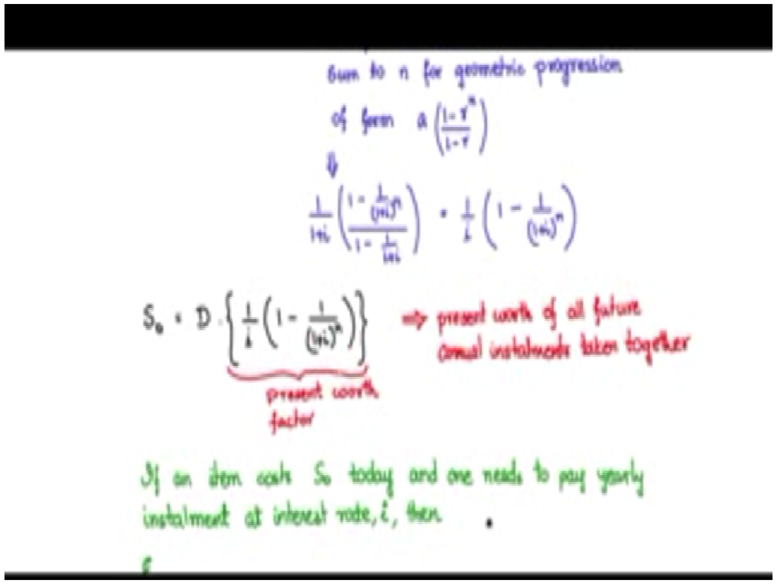

So I will put a 1/1+I, $1\text{-}1/1\text{+}I^n$ that is $1\text{-}r^n/1\text{-}r$ which is 1-1, 1/1+I. So this on simplifying will lead to $1/1\text{-}1/1\text{+}I^n$. So if you substitute this for the sum here then we will have S0 the present worth is equal to $D(1/n(1\text{-}1/1\text{+}I^n)$) and this is the present worth of all future annual installments up to n^{th} year taken together. And this factor D is the equal annual installment this factor year is called the present worth factor.

So if an item cost S0 today and one needs to pay yearly installments for the next n years at an interest of I interest rate of I.

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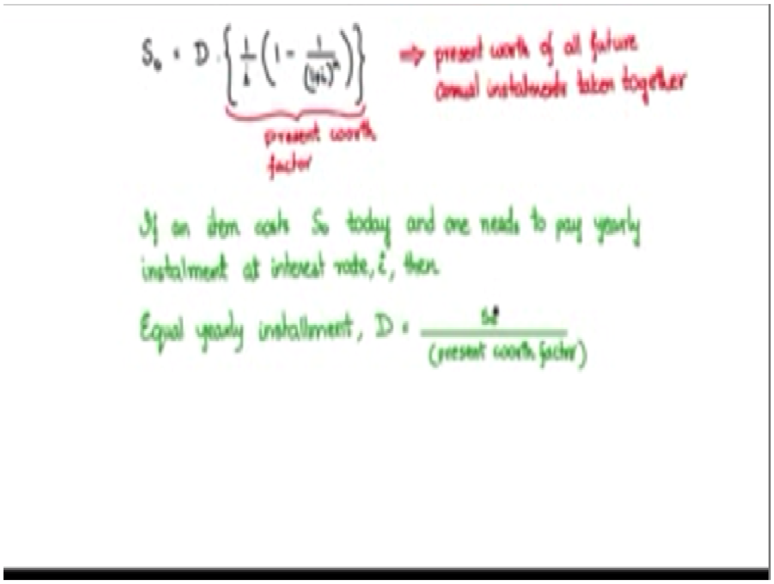

Then the equal yearly installments D is nothing but S0 by the present worth factor, this will give you the amount D that has to be paid every year at the end of every year till the end of n^{th} year. So that it has a present worth of S0.

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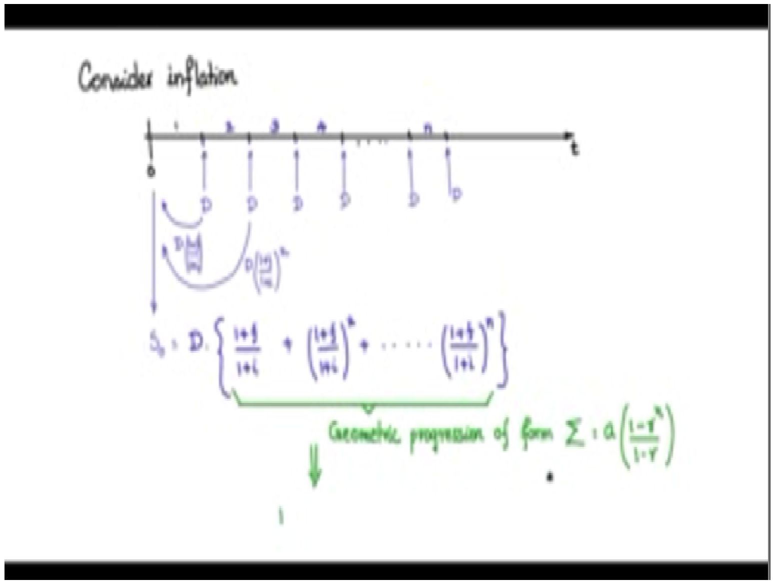

This principle can be extended to consider even inflation, we saw earlier that we know how to get the present worth with inflation included, inflation is having the symbol F. So let us say in this time line where each interval is one year this is year 1, year 2, year 3, year 4, year n and you are paying equal annual installments DDDD at the end of every year, and we are interested to find what is the present worth today.

What are the present worth of all the future installments equal future installments annual installments at one would be. So this can be reflected to the present time and the present worth is $D1 + F/1+I$ the present worth for the second year end the D value would be $D1+(F/1+I)^2$ so on. So it would be equal to $D(1+ F/1+I+1+F/1+I)^2$ so on $(1+F+1+I)^n$, so this again is a geometric progression. And it is of the form σ or n term is A to $1-r^{n/r}$.

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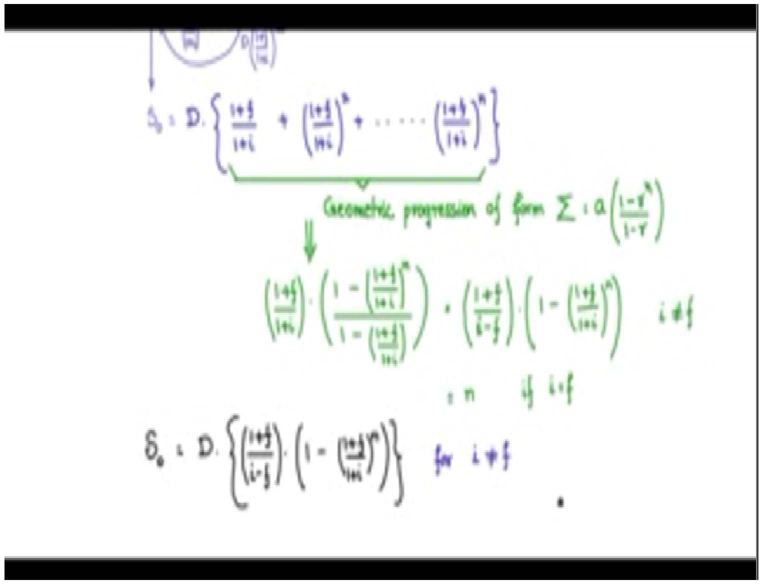

And this can be written as A is 1+F/1+I which is the first term R is the common factor $(1+F/1+I)^n$ (1-r). So this can be simplified as 1+F/I-F because there is an I-F there is a chance of this being 0 if I=F, so this equation will not be valid for I=F $(1-1+F)^n$. So this is the present worth factor if I is not equal to F, if I=F each term will be 1, 1, 1, 1 add it up for the N terms, so you will get value N.

So this would be for I=F so if you include that into this equation D into the present worth factor 1+F/ minus of all those things, this is for I not equal to F.

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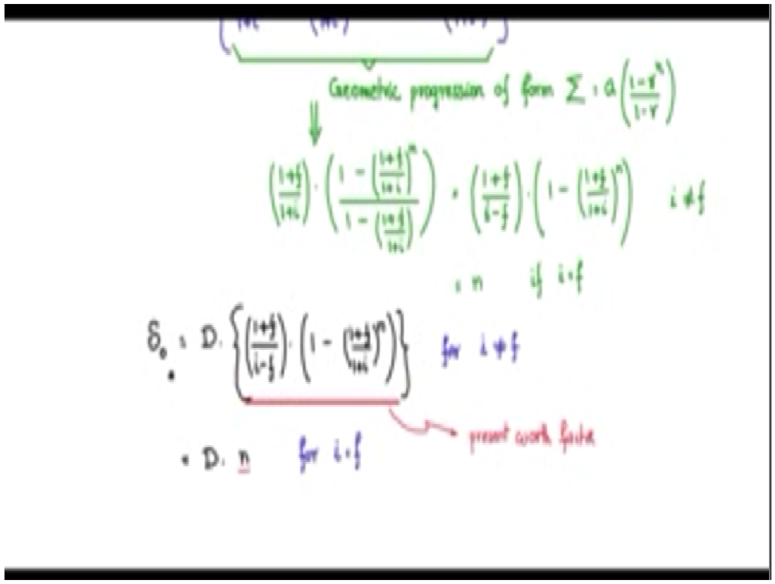

And is equal to D(n for I=F) this is the present worth factor, and for I=F the present worth factor is n So in this way with inflation included also you can find what are the equal annual future installments that needs to be done for an item that is costing S0 today.