

**Indian Institute of Science**

**Design of Photovoltaic Systems**

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**NPTEL Online Certification Course**

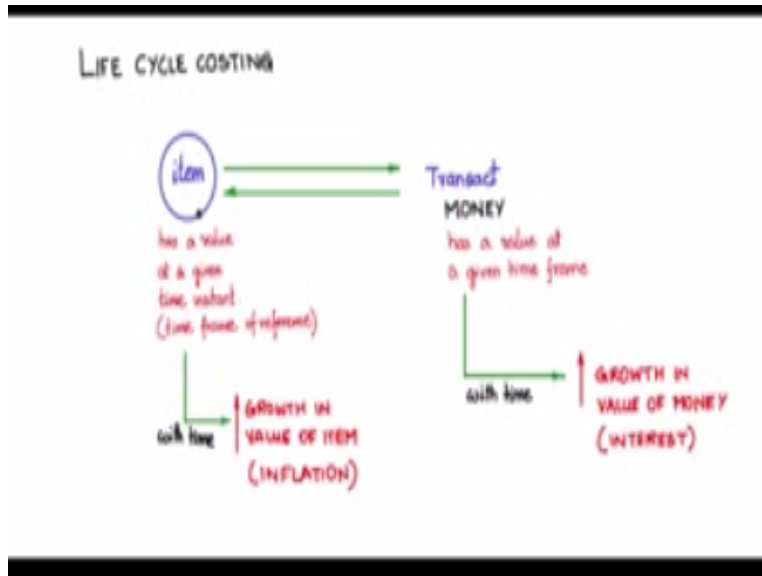
We shall now discuss on a topic that is not very much related to the electrical engineering aspects that we have been discussing till now. However this is none the less a very important topic especially if you want compare the bench mark, reference the many systems that we have we discussing. You see in a photo world types in the systems take for example the PV modules the PV panels, they have the lifetime for around 20 to 25 years.

Take for example batteries they may last for around 5 years which means that you may have to replace the batteries sets in between. Take the electronics, electronics may also have a large life span. So photolytic system contains many sub systems and the sub systems have different life spans. So when you have comparable systems how do you bench mark them? How do you compare them?

What is the value of the particular system what is the value for the money and the value for the system? What are the economics for the various systems and how do the variables play and how do you obtain the unit cause? Meaning that if it is water pumping system, what is the cost of the unit of the water? If it is the electricity generation system PV connected to the grid what is the cost of the 1 unit of the electricity?

So how do you arrive at that, so for that you have to construct the lifetime of all the sub components, the replacement, maintenance, all these issues start coming into the picture. So this is that point that we would like to address it is from a engineering perspective that we will look at it, it is not exactly to arrive at the exact cost more from the draft economical analysis, so that we would be able to compare many systems and that is what we would come to do in this topic of life cycle costing.

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Consider an item, now this item as a value and this value is related to time, so it has the value at a given time instant or we could say at the time frame of reference, it could now, it could tomorrow, it could be 10 years from now but the value of the item is coupled directly with time. Now this item is either sold, so there is the transaction where you will buy this item or sell this item and the medium through which you will buy or sell would be using this money as an item, money as the medium.

So the money is also having a value and that value again is time dependent. Now with time the value of the money can grow. So there is the growth in the value of the money with time, of course the growth can be positive or negative but most of the time it is positive as time progress the value of the money increases and that is what we interest. So there is an interest, interest rate which can be used for calculating for how the value of the money is grown.

Likewise an item also with time its value then increase or decrease there is growth or change in the value of the item and it is called inflation. So let us look at this growth profile of this item and money and try to bring in that relationship and use that relationship to measure the lifecycle cost.

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## GROWTH PROFILES

1. Linear growth
2. Compound growth (discrete compounding)
3. Exponential growth (continuous compounding)

### 1. LINEAR GROWTH

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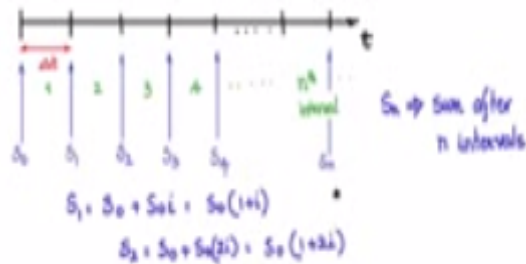
The interest and the inflation have various growth profiles, so one can model it with different growth profiles. One simple way is the linear growth another way is called the compound growth, 3<sup>rd</sup> way exponential growth. The compound growth very popular it is also called the discrete compound growth and the exponential growth is called the continuous compound growth. We will see this relationship come above 1<sup>st</sup> consider the linear growth.

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## 1. LINEAR GROWTH

### SIMPLE INTEREST

$i$  = rate of interest for an interval of time  $\Delta t$  ( $\Delta t = 1$  year)  
 $n$  = number of time intervals,  $\Delta t$  under consideration  
 (number of years under consideration)



Simple interest is the typical example of linear growth, there the key variable is  $i$  which is called the rate of interest,  $n$  is the number of the intervals. Now the rate of interest for an interval of time  $\Delta t$  and  $n$  is variable associated with the number of time intervals  $\Delta t$  under consideration. So if you say  $\Delta t$  the time interval is 1 year which is what most commonly used in banking.

Number of years under consideration  $n$  will number of years, you can visualize this with timeline, let me draw the time line  $t$  x axis and I will put several equates tick marks like this to represent the intervals, this intervals is  $\Delta t$  and I will this one as interval 1, interval 2, interval 3, interval 4 and so on, this is interval  $n$ . now there is another variable that I will introduce this is called the variable  $s$ , now let us I had this point at the beginning of the interval of ending of the previous interval, I will mark it as  $s_0$ .

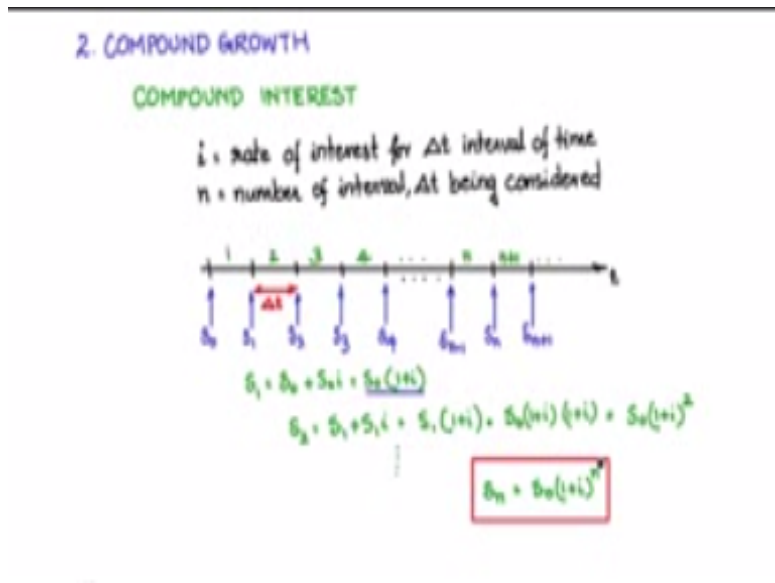
At the end of the 1<sup>st</sup> interval I will call it as  $s_1$  and the end of the 2<sup>nd</sup> interval is called  $s_2$ , then the 3<sup>rd</sup> interval  $s_3$ ,  $s_4$  so on at the end of  $n$ th interval it will be  $s_n$ . Now what is the meaning of  $s_n$ ?  $s_n$  is sum of the  $n$ th interval of time that as elapsed. So if  $s_0$  was the sum of at the end of the 0<sup>th</sup> interval this is the sum that you had the start beginning after  $n$  intervals over lapsed, the sum at the end is  $s_n$  so that is called the sum of the  $n$ th interval.

If each interval is 1 year than it would be sum after  $n$  year, now  $s_1$  is given by the relationship linear relationship,  $s_0 + s_0 \times i$ , where  $i$  is the rate of the interest for the interval. So  $\Delta t$  is the interval and it is just one interval  $s_0 + s_0 \times i$  which is  $s_0 \times 1 + i$ . now  $s_2$  is  $s_0 + s_0 \times 2$  intervals over lapsed, so  $2 \times i$ , so that is  $s_0 \times 1 + 2i$  and if I move it up.  $s_n$  would be  $s_0 + s_0 \times ni$  it will  $s_0 \times$

$1 + ni$ . So this is relationship that you would get for the sum after  $n$  intervals or time periods have elapsed.

With the rate of interest  $i$  and with the starting value of  $s_0$ , this is turned as the simple interest equation.

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Let us now discuss compound growth an example of this of course is the compound interest may be aware with the compound interest formula. Here to write simple interest there is variable  $I$  which represents the rate of interest for  $\Delta t$  interval of time,  $n$  is the number of intervals  $\Delta t$  intervals that are been consider. Now let us visualize again like before on a timeline and I will mark equates ticks marks like this and each interval here is what I will call  $\Delta t$ .

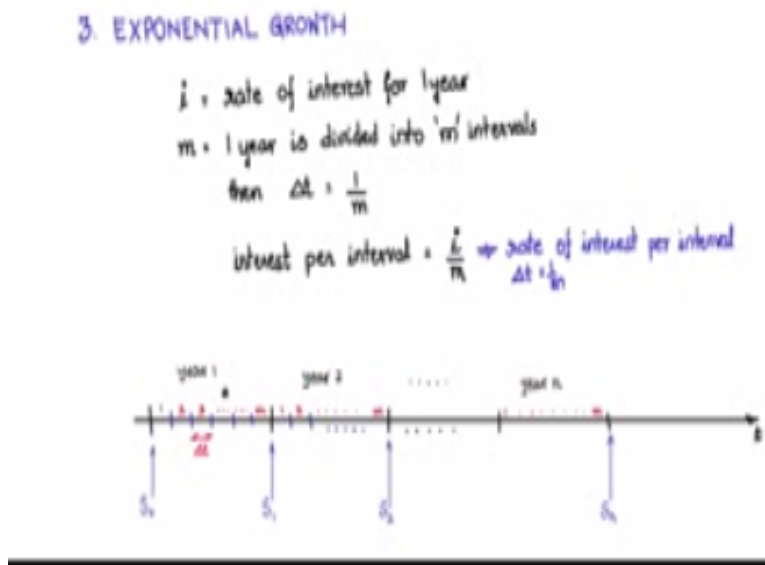
Now that will be interval 1, interval 2, interval 3, interval 4 and now this is interval  $n$ , so this is  $n + 1$ . So let me take this at this point, you have  $s_0$ , the sum the staring sum at the end of the  $0^{\text{th}}$  interval or the beginning of the  $1^{\text{st}}$  interval. Now at the end of the  $1^{\text{st}}$  interval I have sum  $s_1$  at the  $2^{\text{nd}}$  interval sum  $s_2$  at the end of  $3^{\text{rd}}$  interval sum  $s_3$ , sum  $s_4$  and at the  $n^{\text{th}}$  interval it is  $s_n$ , at this point it will be sum  $s - 1$  and here sum  $s + 1$ .

So now let us try to find out what is  $s_1$ , now  $s_1 = s_0 + s_0 \times i$  this is framed like in simple interest or linear growth case and then it is  $= s_0 \times 1 + i$ , now  $s_2$  is  $s_1 + s_1 \times i$ , 1 step application. So for

$s_2, s_1$  previous so if you do one step application  $s_1 + s_1 \times i$ , and  $s_1$  we actually know  $s_0 \times 1 + i$ , so it is  $(s_0 \times 1 + i)^2$ . So in the same way  $s_n$  will be  $s_0 \times i^n$ .

So this equation is the compound interest equation, so if  $s_0$  is string value start and  $i$  is rate of interest for each interval,  $n$  is number of intervals so if  $\Delta t$  is 1 year  $n$  will be number of years. So if you begin with  $s_0$  the end of  $n$ th year the sum would have grown to  $s_0 + i^n$ .

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Now let us consider the exponential growth model, so in exponential growth model basically they would expand the compound interest model in this fashion here let  $i$  be the rate of interest for 1 year and  $m$  be the number of intervals 1 year it is divide into. So note the slide difference in the definition,  $i$  is the rate of interest for 1 year, whole year and  $m$  is the number of intervals that 1 year would be divided into. Then  $\Delta t$  is nothing but  $1\text{year}/m$  and the interest in the interval would be  $i/m$ .

So this is the rate of interest per interval, now let me draw the time line as before now I am going to mark of equates space ticks on the timeline. Now each of these spaces will represent the year, so this is  $m_1$ , this is  $m_2$  so on this is year  $n$  and each year is divide into  $m$  intervals, smaller intervals like this, so on. And then the time period between these intervals is  $\Delta t$  which is  $1/m$ . now I can say this is sum interval 1,2,3,4 so on upto  $m$ .

And the 2<sup>nd</sup> year is also divided upto m intervals, mth is also divided into m intervals, each of the  $\Delta t$ . Now at the beginning of year 1 we will have  $s_0$ , at the end of year 1 we have  $s_1$ , at the end of year 2,  $s_2$  and the end of year n it is  $s_n$ , here we are interested in finding what is  $s_n$ .

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Apply compound interest model

$$S_n = S_0 \left(1 + \frac{i}{m}\right)^{mn} \quad \text{discrete compounding}$$

Let  $x = \frac{i}{m}$ , then  $m = \frac{i}{x}$  and

$$S_n = S_0 (1+x)^{\frac{i}{x}n}$$

As  $m \rightarrow \infty$ ,  $x \rightarrow 0$

$$\lim_{x \rightarrow 0} S_n = \lim_{x \rightarrow 0} S_0 \left(1+x\right)^{\frac{i}{x}n} = S_0 \cdot \left(\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}}\right)^{in} = S_0 \cdot e^{in}$$

$$S_n = S_0 e^{in} \quad \text{continuous compounding}$$

Now let us apply the compound interest model  $s_n = s_0$ , the starting  $1 + i$ , was the original compound interest model here the interest per interval is  $i/m$ , so  $1 = i/m$  and how many intervals are there, there are n years and each year has m intervals, so  $m \times n$  intervals. So this would be the discrete compounding formula. Now let us  $i/m$  to x some other variable  $x = i/m$  and then what is m?

$M = i/x$ , now substituting you have  $s_n = s_0$ ,  $1 + x \times n/x$ , so now in this as m turns to infinity, x will tend 0. So let us apply the limit as x tends to 0 for this equation,  $s_n$  will limit as x tends to 0,  $(s_0 \times 1 + x^{1/x})^{1 \times n}$ . So this is actually split into  $1/x \times n$ , so that is come about, so this can be written as  $s_0 \times \lim_{x \rightarrow 0} (1+x)^{1/x}$ . now this limit tends to  $(1+x)^{1/x}$  is the definition of e, so this is defined as e.

Therefore I can write it as  $s_0 \times e^{in}$  is the exponential growth model, it is also continuous compounding because discrete intervals because number of intervals as tended to infinity, so it is called continuous compounding.

