Turbomachinery Aerodynamics

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Lecture No. # 09
Three Dimensional Flow Analysis in Axial Flow Compressor

We are talking about three-dimensional flows in actual flow compressors. In the last class, we had a look at various aspects of physics of the three-dimensional flow in actual flow compressors, and its various ramifications of the understanding of how actual flow compressor works, and how it impacts on the performance of actual flow compressor. In today's class, we will try to capture much of the three-dimensional flow in mathematical form. These mathematical forms are very useful in the sense that eventually, they would be used for various computational analysis purposes, which aids the design process, and in the process of this coupling between design and analysis, the design time is substantially cut down.

We do not have to go through a very costly experimental analysis to finalize the design. So, the design time has indeed been cut down by years with the help of these analytical tools with the help of CFD. And in today's class will try to capture some of the mathematical form of the physics of the flow that we have done in the last class. So, let us take a look at some of these issues that we are about to capture in mathematical form. So, today’s class is about three-dimensional flow analysis in actual flow compressors.

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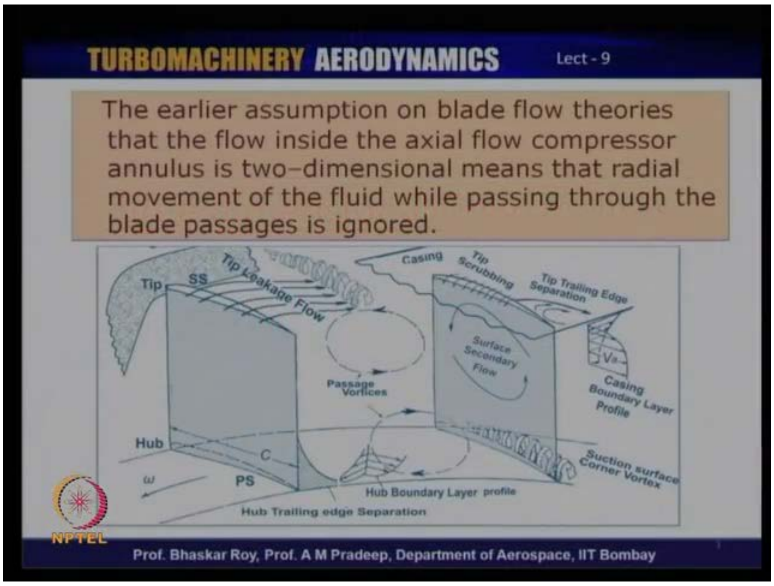

Now, as we have seen many of the blade theories are indeed actually based on two-dimensional understanding, and those two-dimensional understanding actually ignore the effect of three-dimensionality of flow while the fluid is passing through the blade passage. Now, the picture below we have seen in the last lecture and it gives us a fairly detailed idea about the three-dimensional nature of the flow. There are so many things going on inside the blades, specially inside the rotating a row of blades that it is really a not a right thing to do is ignore some of these three-dimensionalities; so, we will today try to capture some of these things in mathematical form. So, that much of it is actually included in the design and analysis process.

So, we will keep this picture in mind which we have done in the last class, and try to move forward to see what all things in some summary form in a overall form is captured in mathematical formation.

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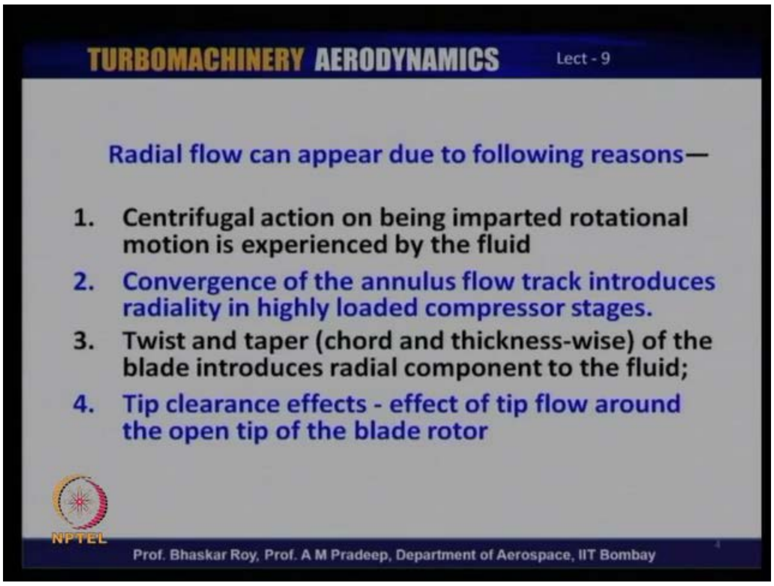

Now, we see that one of the things that is normally ignored or assume to be not present is radial component of the flow, and we had a quick look at a simple three-dimensional radial equilibrium, what it actually means is that the radial motion of the fluid particles inside the blade is sort of assumed to be or pretended to be not present at all. Now, we see what are the reasons why radial flow would almost invariably be present in actual flow compressors. Then, the first reason is a centrifugal action of the rotational motion which is passed on to the fluid, and this centrifugal action would invariably try to impart certain amount of radial flow to the fluid as it is passing through the blade specially the rotating blade.

Then the convergence of the annulus flow track, we will you know discuss a lot of things about the flow track little later in this lecture series. But, the flow track in the modern compressors is quite often converging type and especially in the highly loaded model aircraft, engine compressors and this kind of annulus flow track are geometry introduces the radiality anyway, then of course, we have the twist and the taper and various three-dimensional blade shapes, the solid body of the blade often has highly three-dimensional shape these days they are. In fact, becoming more and more three-dimensional and as a result of which are certain radial component of the fluid is invariably introduced into the fluid motion as it passes through the blades.

And then of course, we have the tip clearance effects. Now, the tip clearance effects we have studied in the last lectures, and we know that there is a substantial cross flow through the open tip of the rotor and this introduces a three-dimensional radial flow and definitely introduce a certain amount of radial component to the fluid flow.

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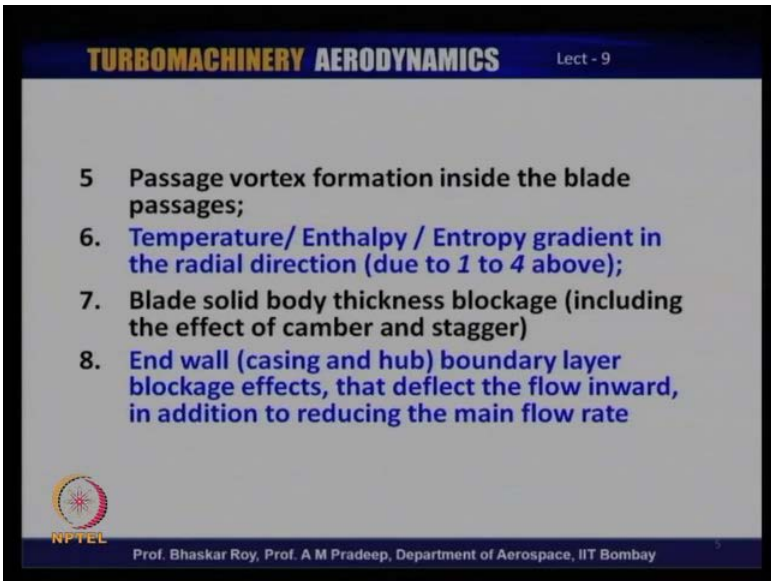

Then, if we look at the fact that you know in the first picture that you we looked at even today that a passage vortex is formed inside the blades. Now, this formation of passage vortex creates radial flow a strong component of radial flow in the main flow itself, which is passing through the blade passage. So, as a result that is another contribution from the various things that are happening to the introduction of radial flow. Then, through the blades quite often, we assume in our theories that the temperature and then the enthalpy and then entropy gradients are you know zero.

Now, that is an assumption in a real blade quite often, we will find that there is a certain gradient in the rear stages of a multistage axial flow compressors, we would find a temperature gradient from root to tip or even from blade to blade, there would be enthalpy gradient, we assume that the enthalpy is a uniform along the length of the blade from hub to tip, and we are also assume that the work done is constant from hub to tip in the earlier simple theories and then of course, there is a general assumption in various theories that we have propounded that the entropy gradient in the radial direction is absent; that means, there is no gradient.

Now, these are assumptions in reality some kind of gradient would invariably appear as the flow goes through the various stages, and those will have to be taken into account when you are analyzing fluid flow. Now, taking them into account means bringing in certain amount of thermodynamics and those thermodynamic issues would invariably be present in the real flow as mentioned specially in the latest stages of a actual flow compressor, where the temperature is higher, the flow has become more three-dimensional and as a result temperature, enthalpy and entropy gradient start appearing in the fluid flow passing through the blade passage.

Then, we have the blade solid body thickness which is a blockage; now, this includes that camber and the stagger you know more the camber higher the thickness, higher the stagger, this blockage to the main flow is likely to increase, usually the flow is basically an actual flow and the moment you have blade there, 8 solid blade it obviously has a blockage, and this blockage will change with these essentially with these three parameters, the thickness of the blade the camber and the stagger, now, you see we have already noticed that the thickness the camber and stagger, all three of them actually vary from root to the tip of a blade.

Now, if they are varying from root to the tip of the blade,. it it stands to reason that the blockage effect would also vary from root to the tip of the blade. And this variation would also then introduce on other aspect of radial flow or three-dimensionality into the flow. And then of course, we have the end wall boundary layers. And then, now these boundary layers are developed, because of the flow essentially being in a adverse pressure gradient in a compressor and these boundary layers over the casing and the hub internal surfaces actually create a certain amount of blockage, these blockage is a fluid mechanic blockage and this blockage often has a tendency to deflect the main flow in wards.

From the casing and from the hub and this inward deflection of course, introduces three-dimensionality, it introduces a radial flow in addition to the fact that it has a tendency this blockage has a tendency to reduce the main flow rate at in the actual flow rate that you would see would be less than what was assumed initially probably during the design. So, these are some of the effects of actual realistic flow on the formation of radial and three-dimensional flow features inside actual flow compressor blading.

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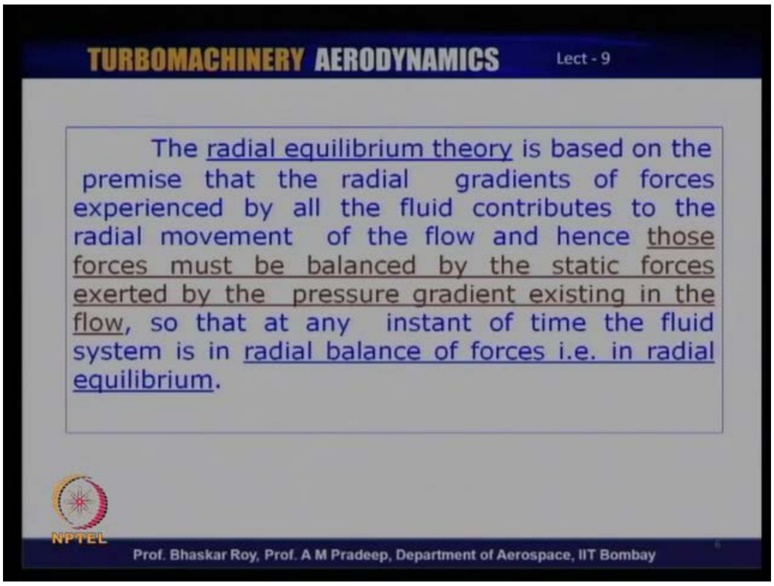

Now, we have created a radial equilibrium theory earlier a simpler version; now, this is based on the premise or assumption that the radial gradient of the forces which are experienced by the fluid, and we had taken a fluid element earlier of an appropriate shape and size and that a fluid element experiences radial gradients, and those things contribute to the radial movement of the flow, and if they are to be checked; that thing the radial movement of the flow or not to be allowed, it it becomes incumbent that those forces must be balanced by the static forces exerted by the pressure gradient which also created inside the flow.

So, that at any instant of time the fluid system or the fluid particle or the fluid element is in radial balance of forces that is in radial equilibrium. It is this balance of forces between the dynamic and the static forces that we would move forward take, forward today to create a more comprehensive radial equilibrium theory, much different from much more complex and much more involved than the simple radial equilibrium theory that we had created in the last class. So, this balance of forces is one of the issues that axial compressor, turbo machinery designers quite often need to do during the process of design and analysis.

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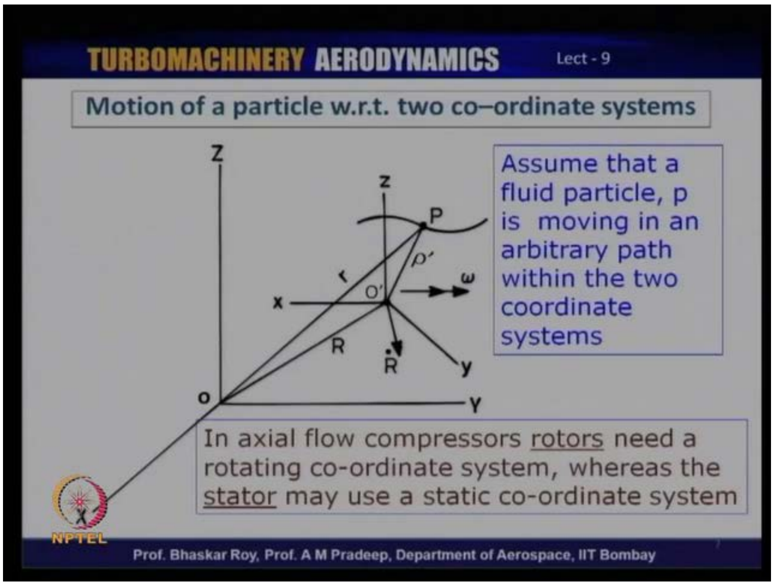

Now, let us try to lay down a few simple conditions mathematical and geometrical conditions that are going to help us in formulating the mathematical theory of three-dimensional flow. We assume that we have motion of a particle with respect to 2 coordinate systems, and this particle P is moving in an arbitrary path within the 2 coordinate systems. Now, the two coordinate systems are XYZ. And then with origin o and then x y and z with origin o prime, so, we have two origins and two coordinate systems and the small xyz coordinate system is a moving coordinate system.

In fact, with respect axial flow compressor, it would be a rotating coordinate system which means it is fixed on the rotor body. So, it is a it is a body fix coordinate system which rotates with the rotor whereas, we have capital XYZ which is a fixed coordinate system, and static coordinate system and the stated analysis could be done with respect to that coordinate system. But the rotor needs to be analyze with respect to the rotating or body fix coordinate system.

So, we do have 2 coordinate systems to deal with in this theoretical formulation, and we need to keep that in mind. The moving coordinate system is at a distance R from the origin of the fix coordinate system, and the particle P is at a distance rho dashed or rho prime from the moving coordinate system origin and is at a distance small r from the fix coordinate system origin. So, we have 3 distances over here. One is this coordinate system is at a distance capital R from the fix coordinate system, and then the particle p itself here a distance a rho dashed from the moving coordinate system origin and is at a distance small r from the origin of the fix coordinate system. So, this is how we have put together the particle that is moving in an arbitrary path.

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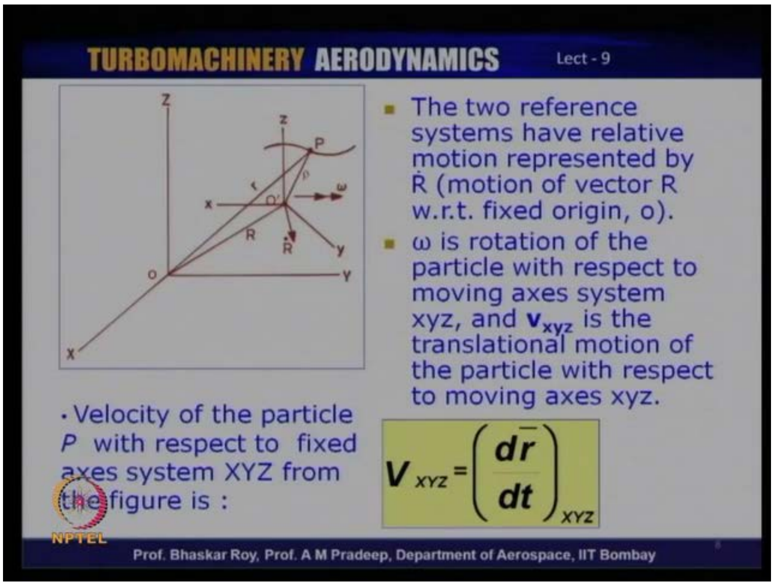

Now if we move forward, the two reference systems have relative motion which is represented by r bar r dash r bar, and that is it is the motion of the position vector r with respect to fix origin o, omega is the rotation of the particle with respect to the moving access system xyz and V xyz is the translational motion of the particle with respect to the moving it access system xyz. Hence, the velocity of the particle p with respect to the fixed access system capital XYZ based on this figure would be v, xyz equal to d r bar d t of xyz capital XYZ. So, this is the basic definition or description of the fluid particle that we are tracking in a motion involving 2 coordinate systems.

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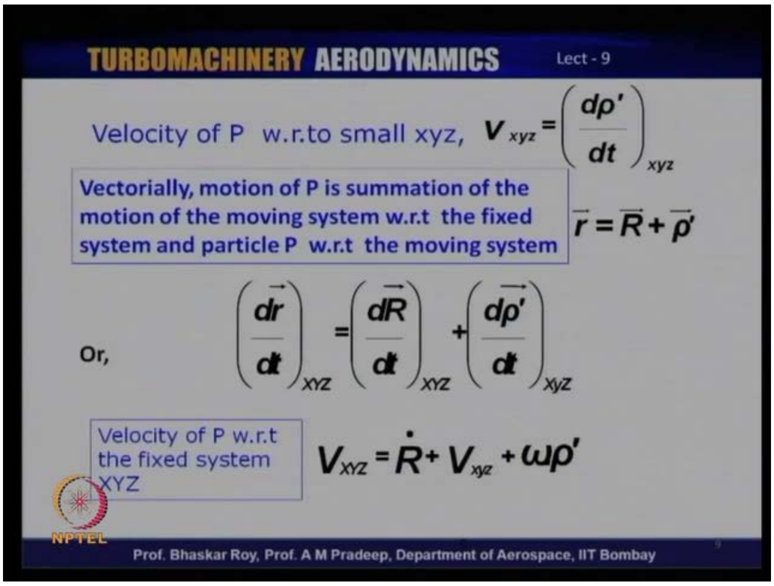

Now, velocity of the particle p with respect to small xyz that is the moving x y, z is V small xyz and that would be d rho dashed d t as per the diagram that we had given. Now, the vectorially; the motion of particle P is summation of the motion of the moving coordinate system with respect to the fix coordinate system, and the particle P with respect to the moving system. So, vectorially r bar would be equal to capital R bar which is the distance of the moving coordinate system to the fix coordinate system, and rho bar which is the distance of the particle with respect to the moving coordinate system.

So, from which we can say that d r bar d t, xyz that is capital XYZ would be equal to d r d capital R bar d t plus d rho dashed bar d t and these are of course, the velocities of the respective coordinate system. So, we can write that down that the velocity of p with respect to the fix system would then be capital V, X capital XYZ that would be equal to r dot plus v, xyz that is small xyz plus omega that is the angular velocity into rho dashed. So, that gives us the victorial summation of the velocity field that we have in front of us for the tracking of the fluid particle through the two coordinate systems.

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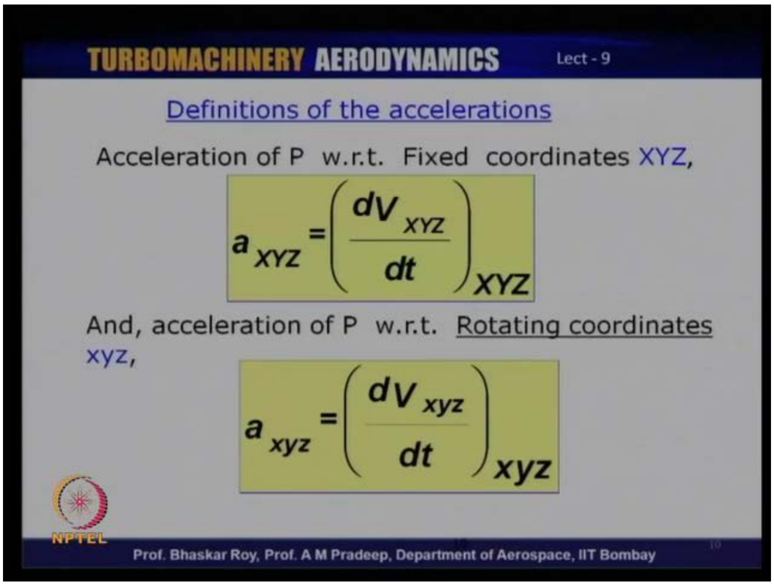

Now, if we define the accelerations, the acceleration of P with respect to fix coordinate system that is capital XYZ is given as a capital XY Z equal to d V capital XYZ d t with respect to xyz and the acceleration of p with respect to the rotating or moving coordinate system that is small xyz and that is a small xyz equal to d V small xyz d t with respect to the small xyz coordinate system. These are the two accelerations which we will be contending with, because we have seen before the acceleration is is what contributes to the creation of dynamic forces that is mass into acceleration. So, we need to write down what the acceleration terms are going to be for this three-dimensional fluid flow.

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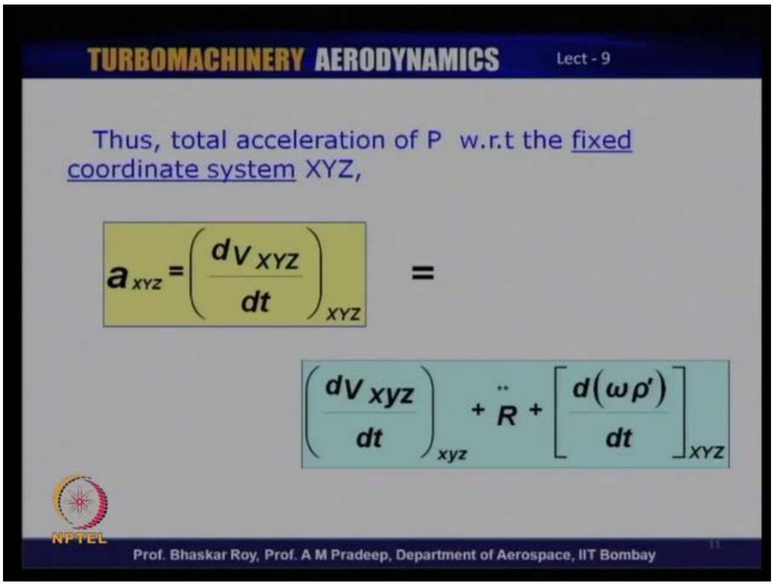

So, let us move forward, then we can write down that the total acceleration of P with respect to the fix coordinate system capital XYZ would be a capital XYZ and that is d V xyz d t with respect to the capital XYZ and that would then be equal to d V small xyz d t plus r double dot, this is the acceleration term the earlier one r dot was the velocity term of the position vector r and then d of omega rho dashed d t; now, this is of course, the due to the rotational component of the coordinate system is small xyz itself with respect to capital XYZ. So, this is acceleration term that we can write down with respect to the coordinate system that we have defined earlier.

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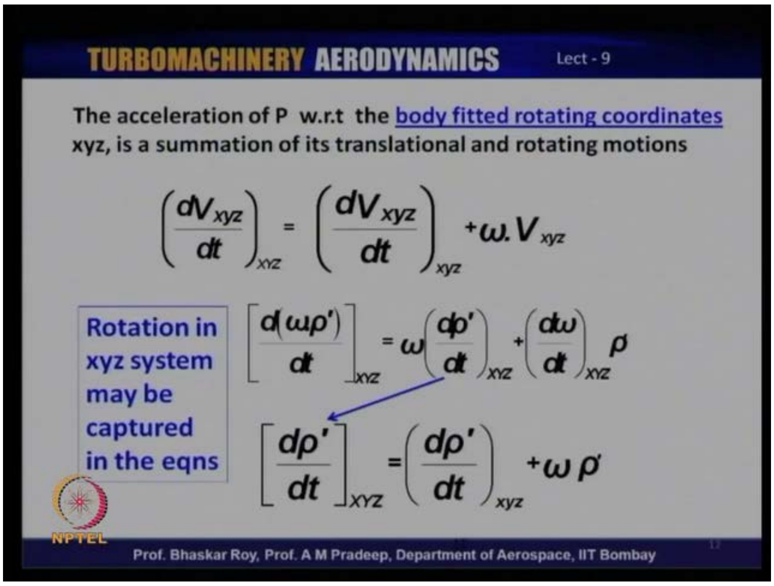

Now, the acceleration with respect to the body fitted rotating coordinate system small xyz. Now, this is a summation of its translation and rotating motion at the moment, we are assuming that it has translational motion and rotating motions; and this we can write down in terms of d V small xyz d t and that would be equal to our d V small xyz d t plus omega into V small xyz. Now, you see the rotating component is the the second term over here. Now, if we write that down over here, we see here d omega rho dashed d t xyz would be can be split up in two terms that is omega into d rho dashed d t and d rho d t into omega dashed.

Now in the first term d rho dashed d t which is a differential of the position vector rho dashed can be written down in terms of d rho of dashed d t is equal to d rho dashed d t into small that is of the small xyz plus omega into rho dashed. So, rotation in xyz can now be captured in the form of these equations with respect to the position vectors that we have defined before.

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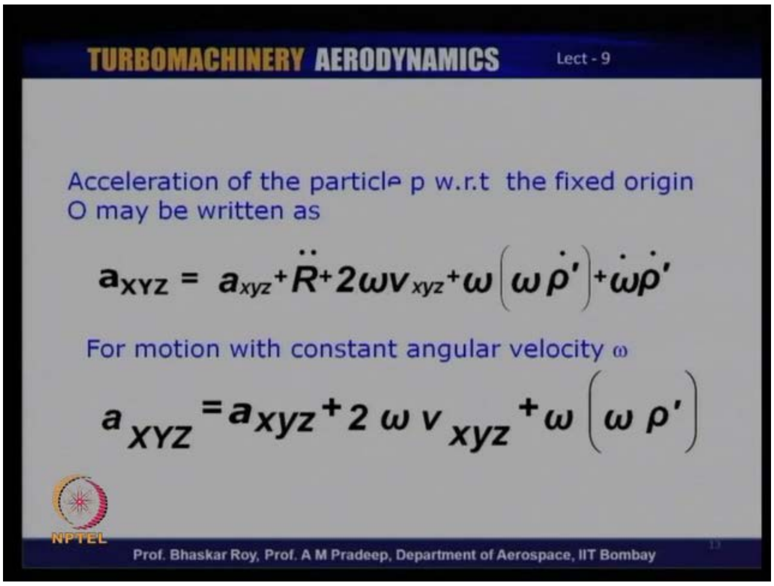

Now, acceleration of the particle P with respect to the fixed origin o may be written down, now we have seen that we need to write down the acceleration terms and these lines be edited in terms of a xyz that will be equal to a small xyz plus r double dot plus twice of omega into V xyz plus omega into omega into a rho dashed dot plus rho dashed rho dot plus omega dot omega dot into rho dash dot. Now, if we assume, now this is the full the first gives the full equation that we get of the acceleration term with respect to the fix coordinate system.

Now, if we assume that the angular velocity omega is constant, and it does not change then the rho omega dot term would of course, immediately go and as a result of which we get a little simpler a version of that acceleration term and that is a capital XYZ and that would be equal to a small xyz plus twice omega V xyz plus omega into omega rho dashed. So, this is the terminology that we finally, get for acceleration assuming that the flow is a constant angular velocity which is very fair, because most of the time the axial flow compressors are rotating with constant angular velocity except when they are in transition during acceleration or deceleration of the machine.

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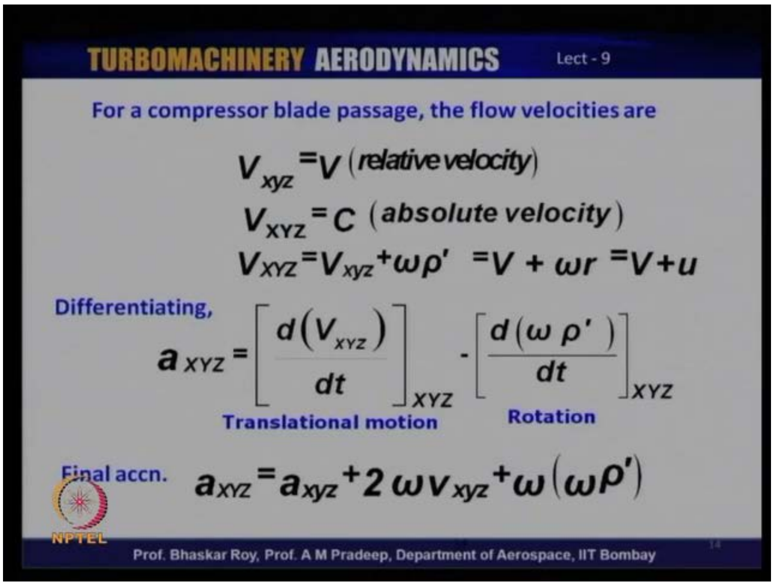

Now, if we look at for a compressor blade passage, the flow velocities that we are looking at are V small, xyz and now this is the relative velocity you remember, we had two velocities that we had defined one was relative velocity, and another is the absolute velocity. Now, we can relate those two known velocities with respect to what we are doing right now today. So, V small xyz is the relative velocity and V capital XYZ is the absolute velocity C. We have seen earlier that sometimes the relative velocity is the larger component, and that is typically at the inlet to a rotor sometimes the absolute velocities the larger component and that could be true at the exit of the rotor. So, V capital XYZ that is absolute velocity is of course, a summation of the other two vectors that is V small xyz plus omega rho dashed.

Now, this can be written in terms of V plus omega r that we had written earlier r being the radius at which the fluid particle is rotating with respect to the rotating coordinate system, and this earlier we had written down as u. So, V plus u then becomes the victorial summation which gives us C and this is what we had done earlier very quickly in our two-dimensional flow theories. So, we come back to what we had done in two-dimensional flow theories, in the sense we were nothing particularly wrong with them except that they were two-dimensional in their nature.

Now, if we differentiate this situation what we get is acceleration terms, and now we see that a capital XYZ would be equal to d V capital XYZ d t and then d omega rho dashed d t with respect to capital XYZ. Now, the first term then is the translational motion and the second term is due to the rotational motion. So, the both the terms would need to be taken into account for finding the total acceleration. And then the final acceleration then is a capital XYZ is equal to a small xyz and then twice of omega V xyz plus omega into omega rho dashed, now this is what we have done earlier if you remember in the last slide. So, we get back. So, this equation with constant angular velocity is what we again get back here using the concepts that we had used earlier in our two-dimensional flow theories.

So, we get the same acceleration term, a combination of acceleration with respect to rotating motion, and combination of the two and the translational motion. Now, the third term is often sometimes referred to has carioles force on carioles acceleration, and we have done that before also, and this is what typically comes out of a full mathematical formulation that we are attempting in this class today.

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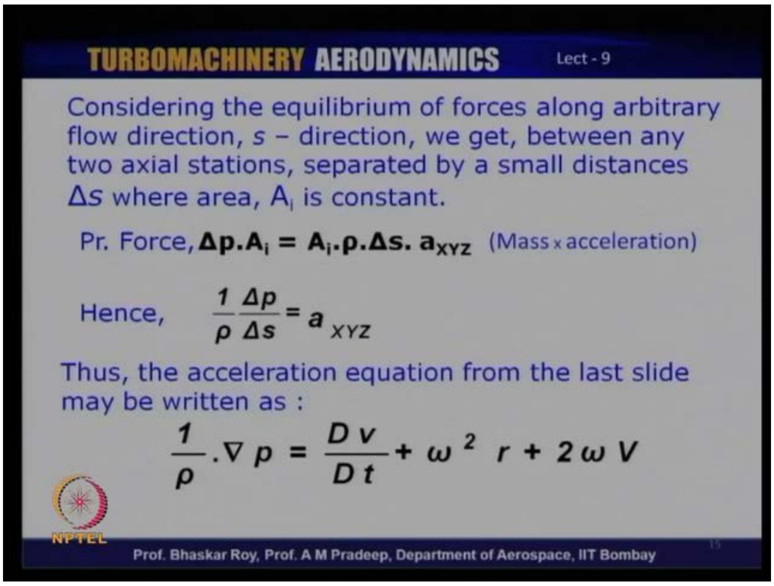

Now, if we consider that equilibrium of forces along arbitrary flow direction s. we get between any two actual stations separated by a small distance delta S where A i is constant, A i being that area at that particular station. So, if we take two stations of an axial flow compressor longitudinally. The during which we assume that A i is constant. A i being the area annulus area at that particular station. And we assume that is constant and in this longitudinal flow field which is a small distance delta S. And in this we try to track what is happening in the flow field in this small distance delta S that we have defined here.

Now, over this distance what we see here is the pressure force that is working, this is of course, the static pressure force delta p in to A i which is constant now, over this flow field, and then we have A i into rho, rho now is density please understand we have always used rho as density and that is why we used rho dashed earlier has a position vector. So, this rho is density and delta S which is the distance. So, A into this is the volume and rho is the density. So, that is the mass and this is acceleration. So, mass into acceleration is the force that is the dynamic force that has been created and this is the axial force that has been created by delta p and that is the pressure differential or pressure will change over this distance small distance delta S.

If we resolve this, from this what we get is one by rho delta p by delta S equal to acceleration a xyz. So, this is now the pressure gradient along the distance delta s. Now, we can write down that the acceleration equation from the last slide, now can be written down. In terms of this acceleration term can now be replaced by one by rho delta p, and then we get the D V D t plus omega square r position vector rho dashed is now replaced by r which we are familiar with r being the radial distance of a particle from the axis of rotation of the moving body that is the axial flow compressor access, and twice omega V which we had called sometimes in some books is referred to as cordials force.

So, this is what the equation we get now. So, acceleration term has now been replaced by pressure terminology or pressure term, pressure gradient along the distance s or [de\delta] delta s.

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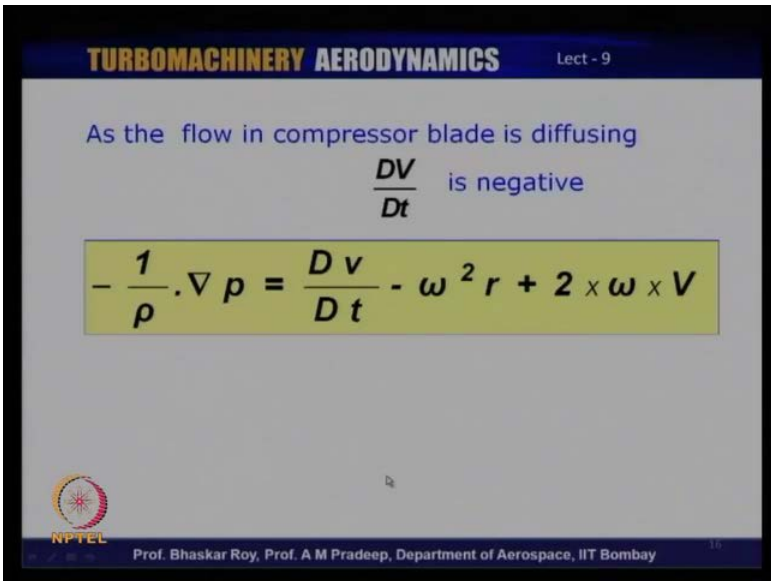

So, with this we can now write down that as the flow in compressor blade is a diffusing flow, we need to think that D V D t is likely to be negative; that means, the flow to the blade passage is actually decelerating flow. So, the D V D t is likely to be a negative in nature, and as a result of which the equation can be slightly recast as minus 1 by rho into delta p equal to a D V D t minus omega square r plus two twice (( )) two cross omega cross v. So, that is here in Victoria representation of the corialise force.

So, we get compressor blade; now, positioned signifying that D V D t is most likely to be negative in nature.

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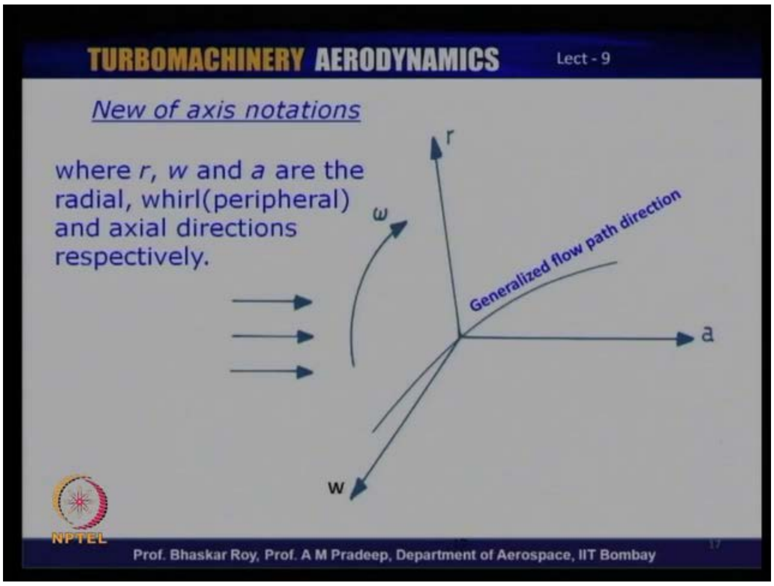

Now, this gives us a situation that we now need to look at representing the flow in a manner that gives us a generalized flow path of the particle, while the position of the particle is being tracked, it as its rotational motion captured in the form of omega, and then we have the three coordinate system which we are familiar with the radial coordinate system is r the axial coordinate system is a and a tangential or the peripheral coordinate system as w which we call the whirl component.

Now, this is what we have been doing earlier in our two-dimensional flow theories where we had the radial component, we had the actual component and we had the whirl component, and we call the three velocities for example, V a, V r, and V w. Now, V r had been neglected earlier in all two-dimensional flow theories, but we are going to bring that back here, and V w remains the whirl component or the peripheral component, and we would now go back to those terminologies which we had used earlier, and are somewhat familiar to us from the earlier theorizing that we have done.

So, as we can see there is nothing particularly different about the notation that we had used earlier, and the notation that we have introduced in today's lecture; you can find very easily the parallels between the two, and the equality between the two sets of notations. So, you may have to really just sit down and look at these notations and find that these things actually mean one and the same thing. So, with this we look at the new axis of notations system.

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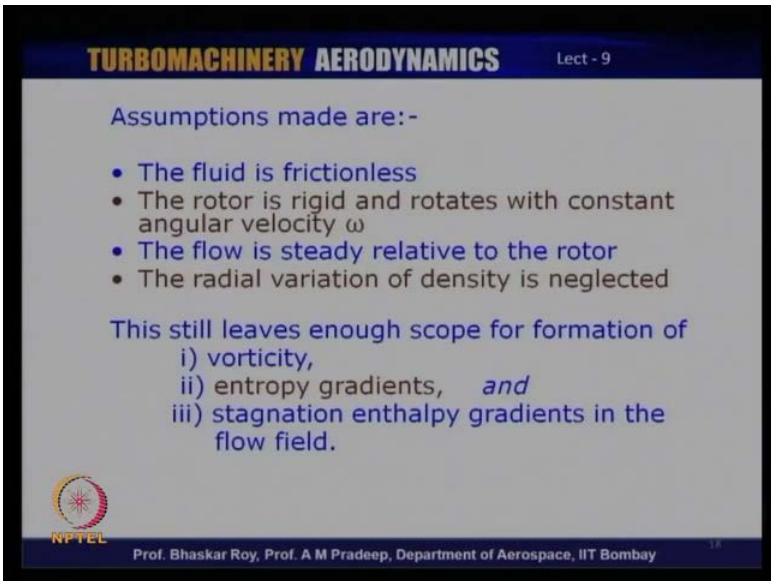

Now, the assumptions made here are that the fluid is frictionless, the rotor is rigid and rotates with constant angular velocity an assumption we made just a little while earlier; the flow is steady relative to the rotor, we assume that the flow is not experiencing any unsteadiness. We will formalize it in our mathematical formulation and the radial variation of density is neglected. Now, this is something which is an assumption in many flows that may not actually be true, but radial variation of density to begin with for design purposes, and for design analysis look that assumption is reasonable and a valid assumption and maybe proceeded with.

Now, this leaves is still enough scope for formation of viscosity, then entropy radiands and stagnation enthalpy gradients in the flow field from hub to the tip of a typical compressor blade. So, these gradients and the formation of the passage vortex, and the tip vortex, and the trailing edge vortex all these things would indeed bring in the three-dimensionality which actually impacts the real flow, and we are trying to see how much of these things can be captured a priory in a mathematical formulation, so that it can aid the design and analysis process of axial flow compressor. So, these are those assumption that we have to live within our mathematical formulation.

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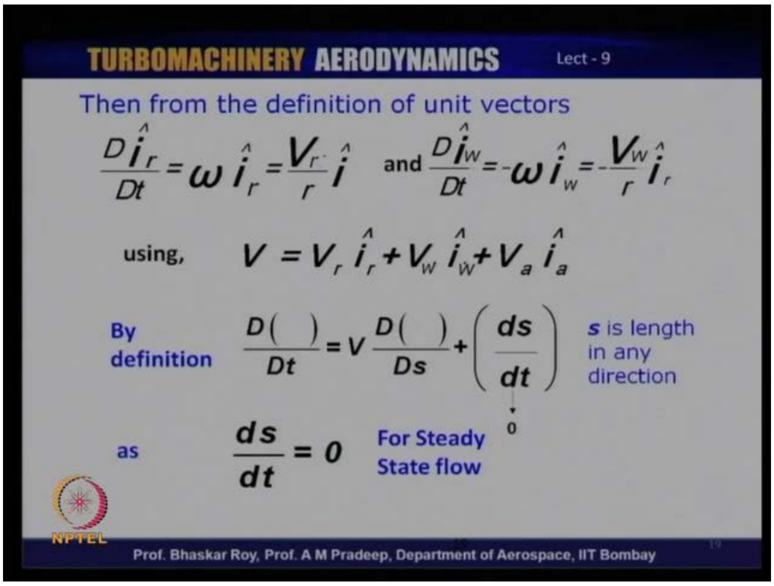

The other things we are going to bring in are the definition of the unit vectors which we have written down here are in terms of D i r i being the unit vector in the radial direction. And then D i w which is one in the whirl component or the whirl direction, so the overall V is a V r into unit vector i r V w into unit vector i w and V a into unit vector, i a those in part the directionality with which is the flow is proceeding in or the flow has three components in radial whirl component and the actual component.

The other thing we are in to do is look at the fact that a capital DDT is a total operator. Now, from your basic mathematics you know that the total operator can be split up in two terms, one is the operator with respect to the space coordinate where s is the length in any direction; so, the same term can be written down in terms of total operator DDS and that plus d s d t, now the d s d t of course, is the unsteady terminology, t being the time; now, as a result of which d s d t is an unsteady terminology if the flows are shown to be steady and at the moment we are assuming the flow to be steady, then the d s d t is equal to zero. Hence, that term would be neglected and as a result will be left with only the total operator with respect to the space of the distance s, and the with respect to time it is considered to be zero.

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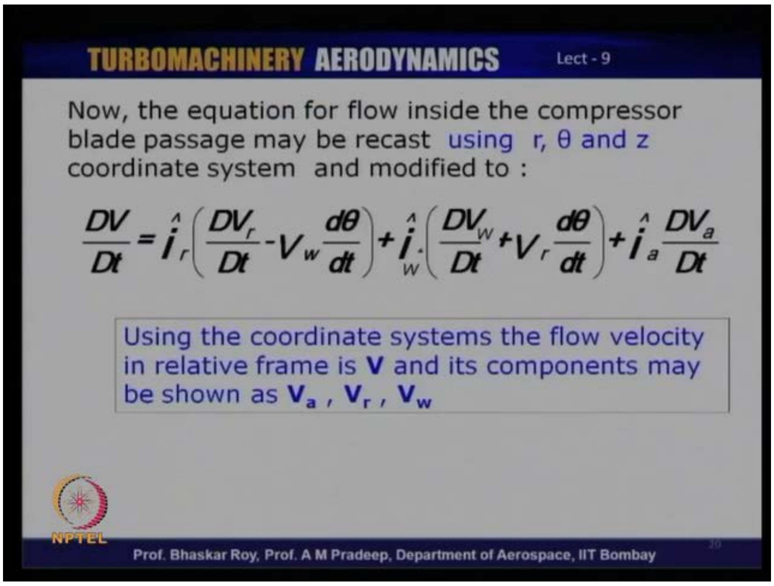

So, the equation for the flow inside a compressor, we would now like to recast using r theta z coordinate system, because simply because it is a more appropriate for a rotating coordinate system also, because of the fact that. There is a variation of parameter in the circumferential direction and from blade to blade, and this blade to blade circumferential variation is quite often an important variation. We know that the two surfaces of the blades are indeed actually dissimilar, one is a suction surface and on[e] one is a pressure surface.

So, theta variation from suction to pressure surface is quite often an important variation it sometimes it is caught a lot in highly loaded, and we shall we be know that that variation also changes from hub to the tip of the blade. So, variation from one blade surface to the other in the passage is an important variation, it everything varies the pressure varies, the velocity varies, the temperature varies, the enthalpy varies from one surface to the other in one passage. So, that can be tracked by variation of theta that is the angular circumferential variation from one surface to the other, the other is of course, from root to tip, and the third is in the z direction that is axial coordinate, axial distance.

So, if we use r theta z coordinate system, we see that DV Dt can be written down, now in terms of unit vector i r into D V r D t V w d theta d t plus unit vector i w DV w Dt plus V r d theta d t plus unit vector i a DV a Dt. Now, these coordinate systems and the flow velocities in the relative frame [weto\we] we always have considered as V v its components are being shown again as V a, V r and V w as we have done before in a 2-D flow theories, where we have used use V a and V w we did not use V r as at that time, we assume that V r does not exist. So, we see here that the acceleration term can be split up in three components.

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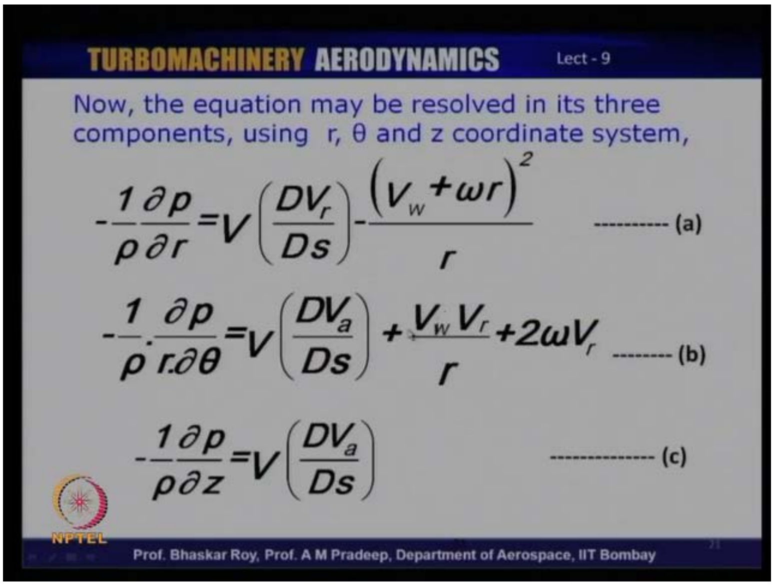

Now the equation may be resolved in its three components using the r, theta z coordinate system that we have brought in now; now the what it shows is that the three components are 1 by rho del p del r and that would be equal to V d V r d s minus V w plus omega r square by r that is equation S. Now, that is in the radial direction; so, the left hand term is the pressure gradient, and the right hand term is the dynamic acceleration terminology of the terms; the second equation is in the circumferential direction theta coordinate and that is 1 by rho del p del theta, 1 by rho 1 by r del p del theta. So, that is the pressure gradient in the theta direction and that would be equal to V into DV a DS plus V w into V r by r plus twice omega V r and that is the equation two that is in the circumferential direction.

In the actual direction it is 1 by rho del p del z equal to V DV a DS and that is in the actual direction. So, again the left hand term is the pressure gradient and the right hand term is the acceleration term, which gives rise to the dynamic or the kinematic forces. So, these are the three components of the equation that we had written down in the earlier slide, and three components are in the radial circumferential and in actual directions.

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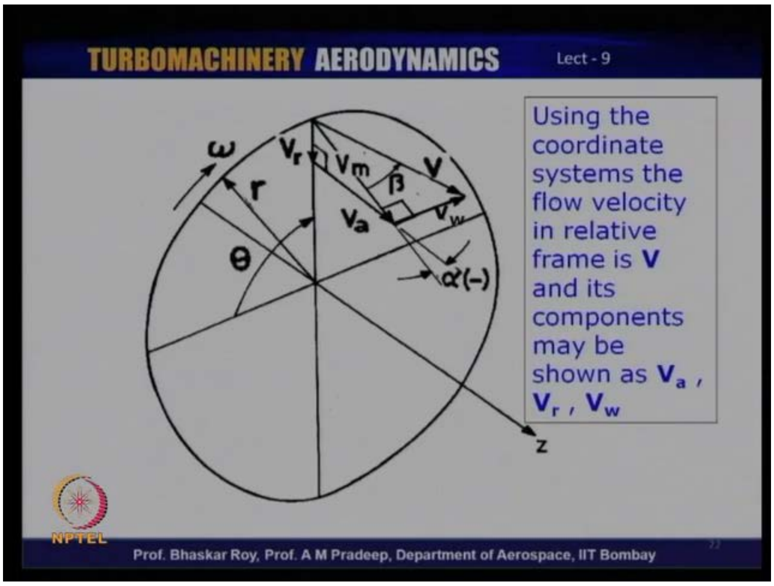

We can now denote these components in a very simple vector diagram, in which all the velocity components are shown here, we see the velocity velocity vector V r has now appeared, we are tracking fluid flow particle and this now has three components V r, V a which is actual velocity. We are looking at a particular plane on which we have the fluid particle, and this fluid particle is experiencing a rotating motion, omega it is at a distance r from the centre of rotation and it is at a [ang\angular] at a angular distance theta from the reference axis. So, at this position it has three velocity components V a that is axial velocity, V r that is the radial velocity, and V w that is the peripheral velocity.

A combination of all the three is of course, the overall relative velocity with respect to this coordinate system which write in the beginning in today's lecture, we have seen itself may be moving with respect to a fixed coordinate system. But let us look at this moving coordinate system analysis right now. Now, we see that a combination of V r which is the radial component and V a the axial component also gives a velocity which can be now described as V m, and these V m is velocity which is a resultant of V r and V a and is on a particular plane of this plane is a radial, and axial plane a combination of radial and axial plane. In this plane, which is perpendicular to the plane we are looking at which is the radial and peripheral plane and, V m is on that radial axial plane and this V m is referred to as meredional velocity.

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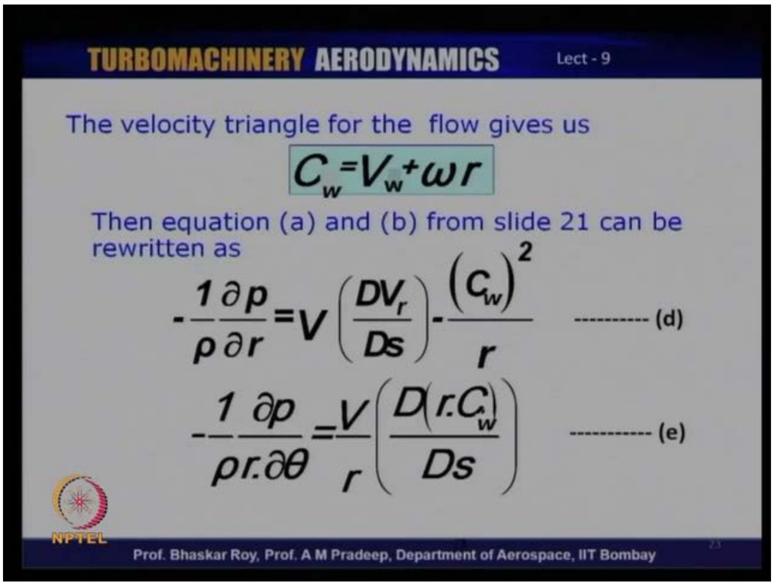

We will come to this definition of meredional velocity. So, the velocity triangle for this flow gives us that the C w would be equal to V w plus omega r, what we see now is that the velocity triangles which we are all familiar with actually gave as C w that is equal to the axial velocity equal to V w plus omega r omega being the angular velocity, and in the earlier coordinate system that we have just seen we get a V w which is the peripheral component a relative peripheral component with respect to the rotating coordinate system.

So, we come back to our C w which is vectorially has to be equal to V w plus omega, r in this coordinate system, you remember this whole thing is rotating or the particle is rotating with respect to this with angular velocity omega. So, this entire coordinate system is a rotating with angular velocity omega. Now, the equation a and b from the two slides earlier, we had three components from the slide 21, let us go back very quickly what we see is we have two components of the full equation in two directions the radial and the peripheral.

If we bring forth, then we have the radial component can be recast as one by minus one by rho d p d r equal to V into d V r D s minus C w square r by r and this rewrite as a equation d, and the second equation is now one by rho r into the del p del theta equal to V by r into D r C w D s. So, what we have done is we have replaced the V w is now with C w, because that is the axial velocity component that the flow is actually having a with respect to the fixed coordinate system. So, the equations have been slightly recast.

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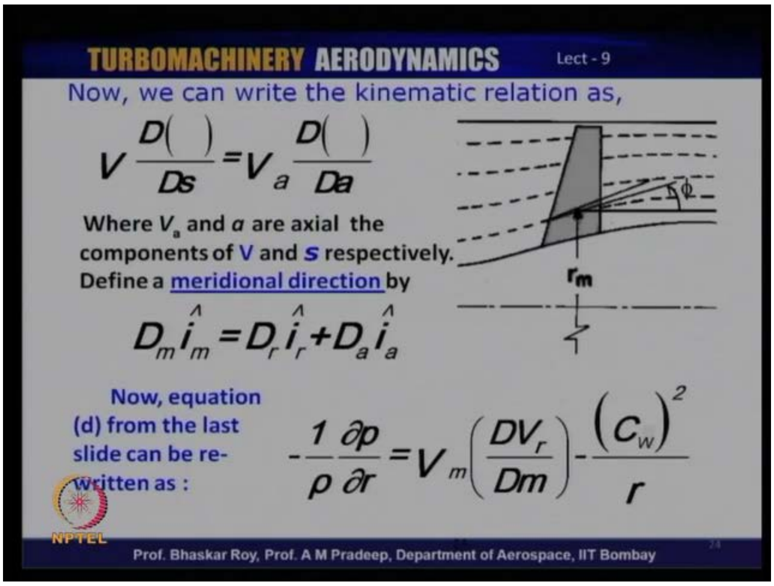

If we now look at the flow, we can now use the kinematic relations as V D capital operator D D S equal to V a capital operator D V a of any parameter where V a and a are the axial components of V and S. S being the distance and a is the axial distance and V a is the axial velocity and V o is the overall velocity. We can now define a meredional velocity to begin with let us define a meredional direction, and this meredional direction is D m into unit vector i m equal to D r into unit vector i r plus D a into unit vector i a and this is what I was talking about just a little while earlier that this meredional flow is in a plane that is composed of radial and axial plane.

So, in this plane we can have a meredional direction defined, and this meredional direction of a flow or a particular flow path a flow particle taking a particular path would have would create an angle phi with respect to the axial direction. So, this phi is the angle with which the meredional flow is proceeding in the meredional direction, which is not the same as axial direction. Now, the equation d from the last slide can be rewritten as minus 1 by rho del p del r equal to V m we have brought in the meredional velocity V m and that is d V r D m and d m is the meredional direction minus C w square by r.

Now, C w of course, we are familiar with is the whirl component of the absolute velocity are being the distance of the fluid particle with respect to the axial coordinate system. As a result of which we get a for motion again on the left hand side, we have the pressure gradient on the right hand side, we have the acceleration terms due to the dynamics of the flow.

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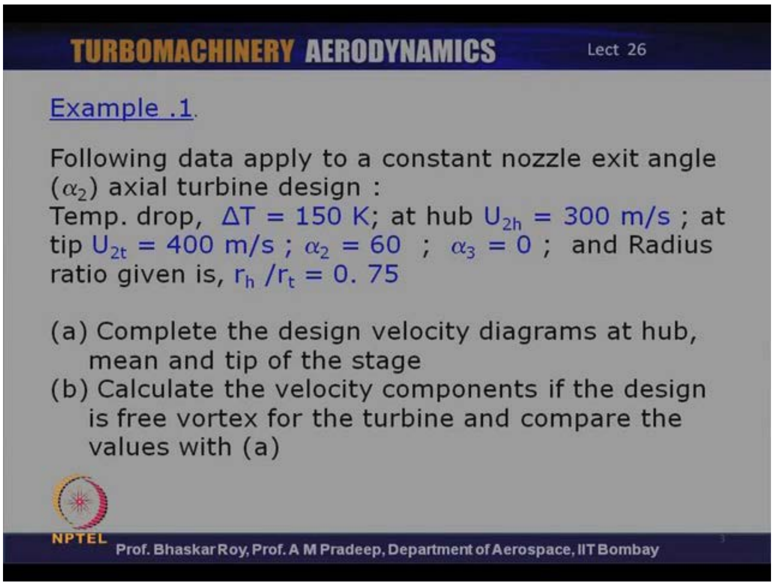

This gives us the fact that in the meredional direction, if we complete the definition of meredional direction 10 phi is V r by V z or V a and V is V of meredional direction, V m into sign phi and phi is defined here and as a result of which we can write down that the force balance equation only in the radial direction can be recast or rewritten as one by rho del p del del p del rho is equal to C w square by r minus V m square d sin phi d m equal to minus V m sin phi d V m D m. Now, this is the full equation in the radial direction. Now, by our definition V D D s the total operator is equal to V m D D m, this is mathematically approved and as a result of which we can replace with respect to the meredional velocity and using the meredional direction in this equation.

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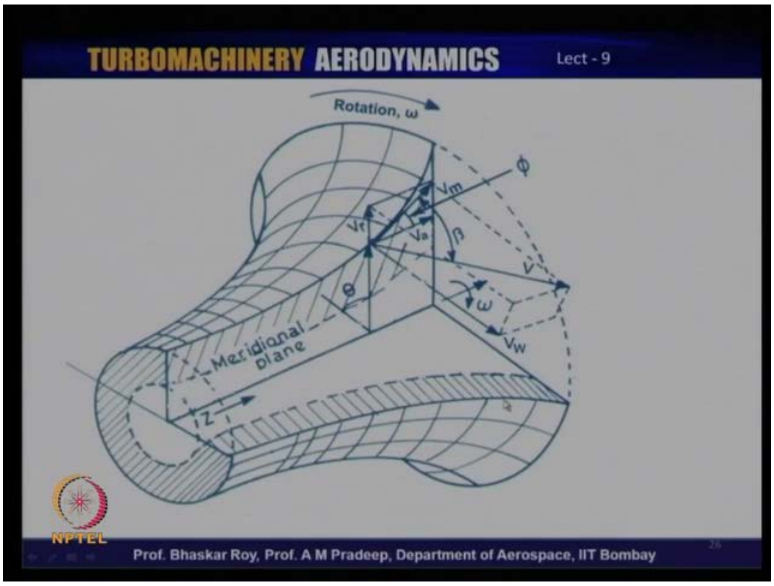

So, if we look at the flow as we [desc\describing] what we are describing is that we have other meredional plane over here now; and this is the direction of the fluid particle of fluid path given as phi, V r is the radial flow component, V a is the axial flow component and V m here is the meredional flow component in a fluid path. And this is what we are talking about an V w of course, remains the peripheral component or the whirl component that we have talked about which means this total V would be the full complement of all the three components put together omega of course, is the rotational angular velocity that the entire flow is experiencing with the rotating blade.

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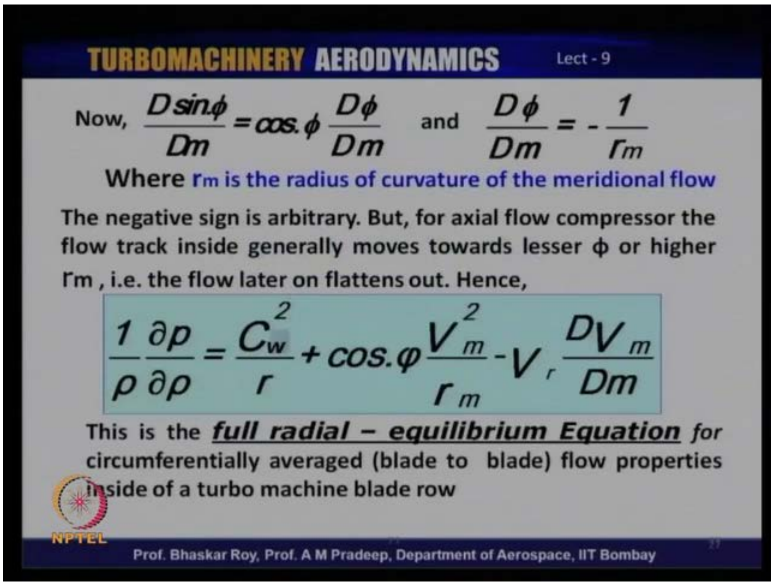

So, given this model, given this model we can now write down that D sin phi D m would then be equal to cos phi D phi D m, and D phi D m will be equal to minus one by r m. Now, minus one by r m is indeed the radius of curvature of the meridional flow, and this radius of curvature of the meridional flow is due to the fact that the meridional path may not be linear, it may not be a curved path and if it is a curved path the radius of curvature of that path. In the meridional direction would create a certain centrifugal action, and hence the meridional radius of curvature of the meridional flow is being brought into the picture.

Now, this negative sign is a somewhat arbitrary, but the axial flow compressor the flow track inside generally moves towards a lesser value of phi as it moves actually forward that is the r m becomes higher and higher which means the flow tends to become a later on flatter and flatter. So, it moves from a high r m to low r m as it moves forward through the blade passage. Hence, the full radial equilibrium that we are looking at now is 1 by rho del p del r equal to C w square by r plus cos phi into V m square by r m minus V r D V m D m. And this is the full radial equilibrium equation which in the say is a circumferential average from blade to blade flow properties inside a termination blade row.

Now, this radial equilibrium equation captures the all the components of the dynamics of the flow, in terms of all the acceleration terms that come about due to three-dimensionality of the flow. Now, this we can see is has a more terms than what we had seen in the simple radial equilibrium equation, we had done couple of classes earlier which tells us that more of the dynamics or kinematics of the flow have been captured in our mathematical formulation.

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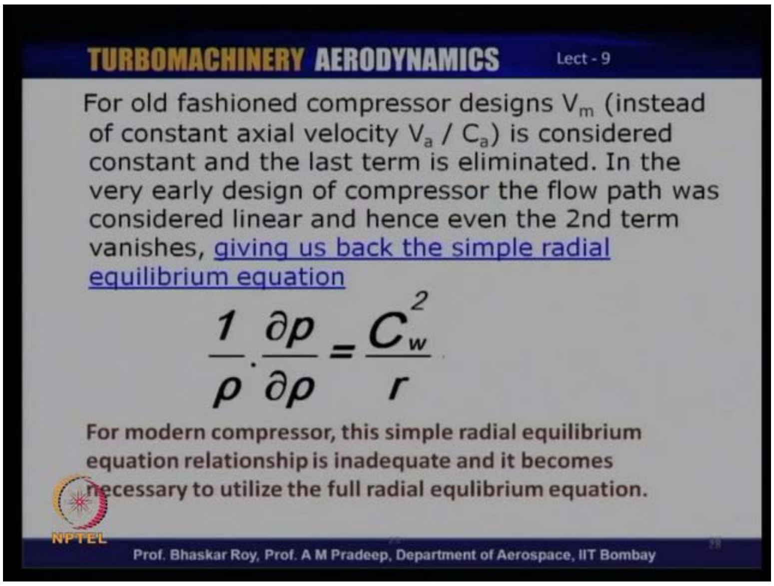

And this tells us that if we now simplify backwards; that means, if we consider that the flow is actually more or less actual then the last term is actually eliminated in the very early designs, it was a done thing that the flow path was considered linear, and the second term is also then neglected or it vanishes and which gives us back the simple radial equilibrium equation which we had formulated earlier in terms of one by rho del p del r equal to c w square by r.

Now, this was the simple radial equilibrium equation; what we need to look at now is a full radial equilibrium equation and we see now that a the simple radial equilibrium equation was somewhat inadequate, and it becomes necessary to utilize the full (( )) equilibrium in design and analysis of modern axial flow compressors.

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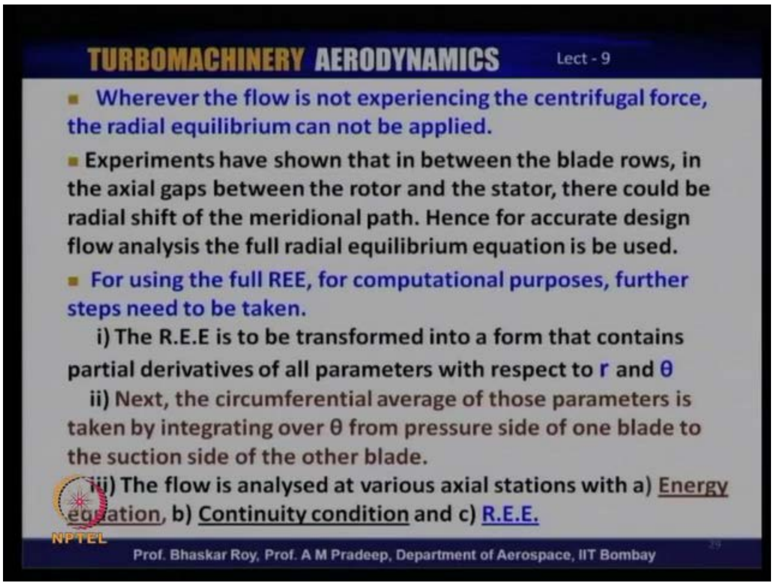

Now, if we go back and summarize some of the things that we have done, we can say that wherever the flow is not experiencing the centrifugal force the radial equilibrium cannot be applied, and with with this understanding it was at one time assume that radial equilibrium is not applicable between the rotor and the stator where no blades exist. However, experience experiments have shown that in between the blade rows in the axial gap between the rotor and the stator there could be a radial shift of the meridional path, and hence for accurate design analysis, radial equilibrium full radial equilibrium equation should be used in that flow path also between the rotor and the stator.

For the computational purpose is further steps need to be taken from this mathematical form, the radial equilibrium equation is transformed into a form that contains partial derivatives of all parameters with respect to radius and a peripheral coordinate theta, because all the parameters like pressure, and density and enthalpy are likely to vary from root to the tip of the blade and also likely to vary from blade to blade. So, this blade to blade variation and root to tip variation need to be captured, I will in the next slide how the physical model or mathematical model attempts to capture it in its computational form.

The next circumferential average of those parameters is then integrated over the theta from the pressure side of one blade to the suction side of the other blade. And then the flow is analyzed and various axial stations with energy equation, continuity equation and the radial equilibrium equation. So, you need all three of them to analyze the flow.

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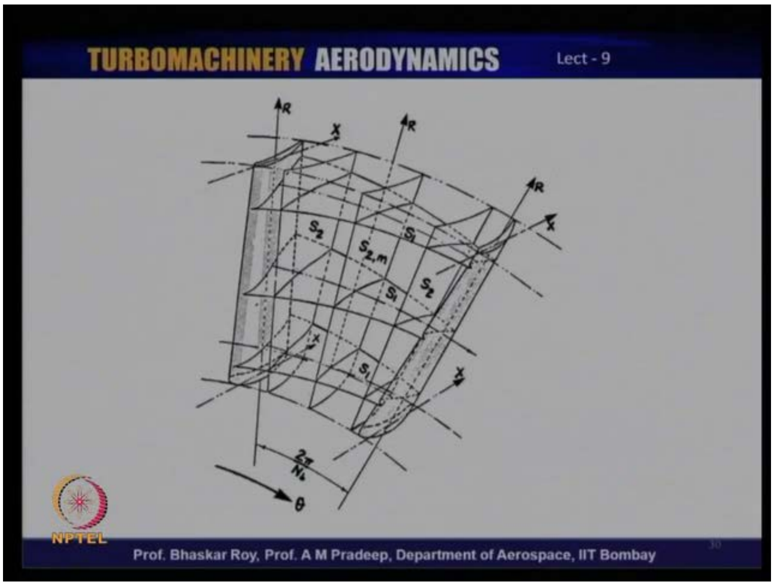

Now, this is what I was talking about that, you have a model in which you have two blades in between the two blades, you need to create these surfaces, these are known as S 1 surfaces which if you very S 1 from root to the tip of the blade, you have the variation of various parameters from root to the tip of the blade, base two surfaces from blade to blade that is from pressure surface to suction surface of the other blade. So, if you take the variation of parameters on is two surface, and then vary them circumferentially in the theta direction from one surface to the other, you have the variation of all the parameters in the circumferential direction.

After you integrate on the S two surface and on the S one surface you get integrated parameters of all the parameters. So, this model has been used by the computational people to transform radial equilibrium equation into a computational tool for analyzing the flow through action flow compressors.

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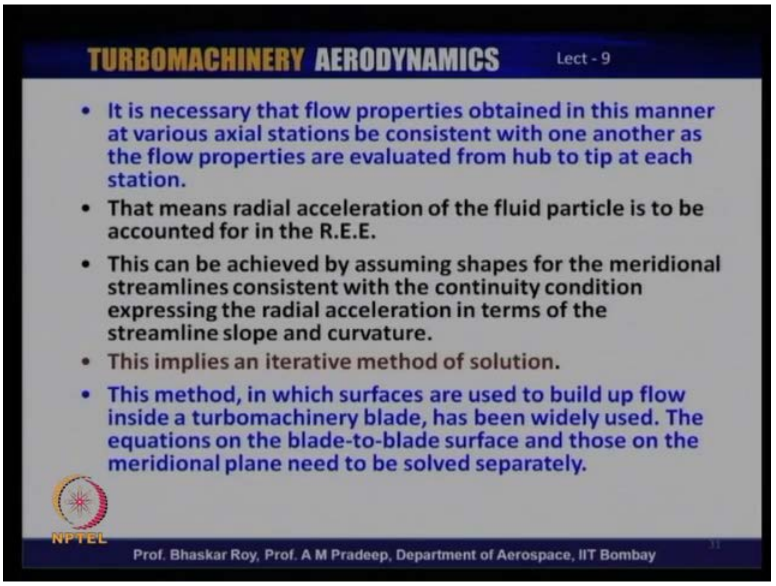

It is necessary that the flow properties obtained in this manner and various actual stations be consistent with one another as the flow properties are evaluated from up to the tip at each station, that mean that the radial acceleration of the fluid particle is to be accounted for through radial equilibrium equation, which is the additional equation, you need to use in addition to the energy equation and the continuity equation or continuity condition for analyzing flow through action flow compressors. Now, this can be achieved by assuming shapes for the meridional streamlines. So, you need to assume the shapes of the meridional streamlines, consistent with the continuity condition, which expresses the radial acceleration in terms of the streamlined slow and the streamlined curvature. So, you got to find the meridional path of the meridional streamlined along which you can apply the radial equilibrium equation that meridional path can be found by using the continuity condition along the meridional path.

This of course, implies that unique may need to hydrate the method of solution, because paraphrasing to begin with you may not know the meridional path, you may not know that streamlines that create the meridional path. This method in which surfaces are used to build up the flow inside to missionary blade has been widely used and this wide you widely used formulation is being used for last twenty twenty five years for analyzing flow through action flow compressor from blade to blade surface and those on the meridional plane, and they need to be solve separately to account for all aspects of the three-dimensionality of the flow through axial flow compressors.

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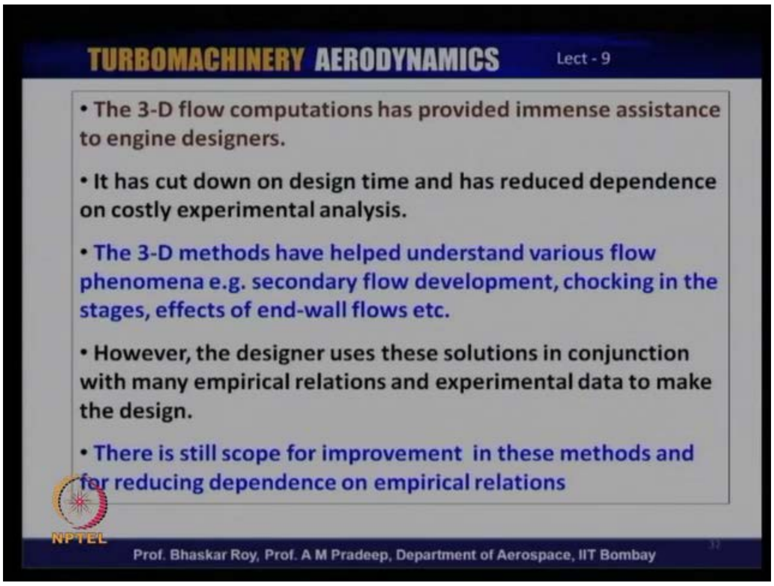

The 3-D flow contributions has provided huge assistance to the Indian designers of the compressor design is more specifically, it is cut down the design time and it has reduced dependence on costly experimental analysis, experimental analysis making the blade testing it in the real is a very costly business and time consuming business; competition is cut it down substantially. The 3-D methods have helped understand various flow phenomena such as the secondary flow development, choking in the stages, as an when the occur aspects end of wall flows, not to speak of problems like stalling, and those can be then sorted out in the design itself, before the blade is actually fabricated.

However the designers use these solutions quite often in conjunction, even today in conjunction with empirical relations specially the losses that occur are very difficult to capture in CFD, and hence quite often empirical relations do again come back into picture, and experimental data to account for some of the losses, and thereby predict the performance of the design action flow compressor blade, which means that there is still a scope for improvement of these methods. So that the empiricism, the empirical formulations are reduced further to the minimum, some of it may have to be done in the last stage of the design before the blades are actually fabricated and tested; however the dependence on the empirical formula can be reduced, if this is the method is taking forward. So that more and more of the three-dimensionality that we have seen right in the first slide is captured in the formulation.

So, today's class we have done lots of attempt to capture a lots of three-dimensionality in the mathematical form and finally, we have got a full radial equilibrium equation, that is substantially different and for more complex in Bob Anderson radial equilibrium equation that we had done earlier. This allows us to go forward and do an analysis of a modern action flow compressor and eight in the process of the design of this modern axial flow compressor. So, that brings us to the end of today's lecture of creating a mathematical formulation of flow through a three-dimensional flow through axial flow compressor. In the next class, we will look at and tent to solve some of the problems of simple 3-D flow theory is that we have done earlier, and we will try to use those theories to solve some of the problems, use the free vortex theory and other vortex theory to solve some of the typical problems, and show how the numbers actually come up in solution of axial flow compressors.