Turbomachinery Aerodynamics

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Lecture #08

Classical Blade Design Laws: Free Vortex and Other Laws

We are talking about three-dimensional flows. Three-dimensional flows in axial flow compressors, that is what we started talking about in the last class; and we solved some very simple examples, through our some very simple mathematical derivation; we arrived at a simple law, which is known as Free Vortex Law, which again comes out of a very simple radial equilibrium of forces, also simply called radial equilibrium condition or sometimes simple radial equilibrium equation.

Now, based on those laws, we will today discuss little more in detail various aspects of design laws of axial flow compressor blades. Now, those are derived from those Free Vortex Condition that we set forth in the last class. However, we will be moving forward from there; we will also see, what are the restrictions or you know conditions based on which Free Vortex was derived, and those restrictions would need to be overcome or you know got around in the modern axial flow compressor design.

So, we will try to look at, what the modern axial flow compressor designers are doing; we will of course, have a more detail discussion on blade design later on in this lecture series. We will have a full lecture on blade design principles, and blade design methodology later on. But today we will just set forth certain fundamental design principles, design laws starting with the Free Vortex Law, and try to put together a certain conditions, certain parameters and of course, a certain laws that the designers even today, start off with for designing a completely new axial flow compressor blades.

So, starting a completely new axial flow compressor blade, needs to be started in very simple way. We will… As, I said we will later on look at in more a comprehensive method; in which, how the modern compressors are designed, and in which we also used various modern saved techniques; and those things we will look at a little more comprehensively later on. Let us, take a look at what the design laws - the fundamental design laws, tell us. So, today's lecture is on three-dimensional fundamental design laws for axial flow compressor.

Now, this design laws of course, start off with the Free Vortex Law, which we set forth in the last class; we will look at this be Vortex Law, a little more in detail; and then we will see that it has number of restrictions, because it was set forth or derived based on certain simplifications. And we will have to see, how these simplifications can be got around, for more modern or other more specifically; more highly loaded, axial flow compressor blades. So, let us start off with what we ended with in the last class - that is the Free Vortex Law.

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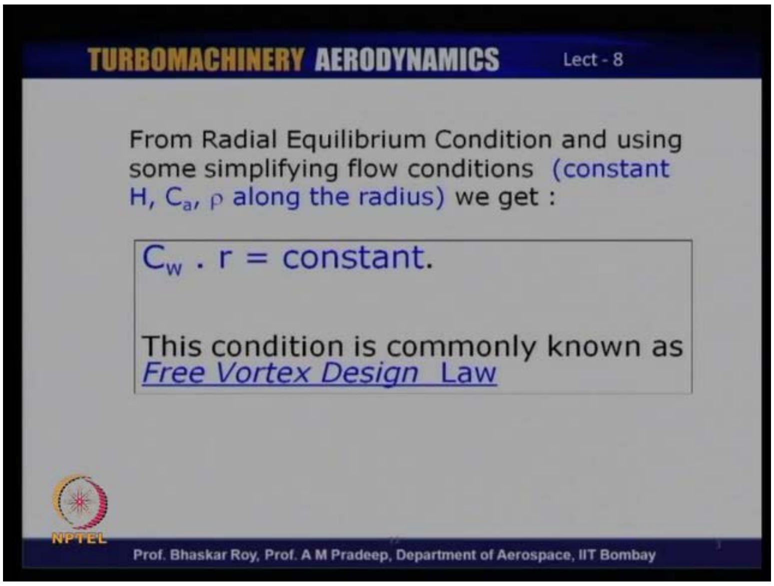

TURBOMACHINERY AERODYNAMICS Lect - 8

From Radial Equilibrium Condition and using some simplifying flow conditions (constant H, C_a, ρ along the radius) we get :

$C_w \cdot r$ = constant.

This condition is commonly known as *Free Vortex Design Law*

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Now you see, Free Vortex Law is based on what we had put together as radial equilibrium condition, and which had some simplifying flow conditions; these simplifying flow conditions, where that in the radial direction, total enthalpy H actual velocity, inlet velocity C a and a density are constant. Now, these are some of the simplifying condition that we had used along with the fact that the flow is isentropic; and as a result of, which using those additional thermodynamic conditions; we had found C w into r equal to constant. And this is the Free Vortex

Design Law, which people have been using, for you know almost 60 years, ever since, axial flow compressors made their mark in jet engines.

Now, this Free Vortex Law of course, means that C w into r means, as the flow goes from root section of the blade to the tip of the blade r is increasing; r is of course, the minimum at the root, and then it keeps on increasing right till the tip of the blade. Now, which essentially means that correspondingly C w would keep on decreasing; so, C w is maximum at the root and minimum at the tip, so that the product of the two C w and r is held constant, as per this Free Vortex Law. So, if we are using Free Vortex Law, as a guiding principle of the design, that blade is likely to produce flow, which would have that kind of a flow configuration in which C w, that is the whirl component of the flow would be decreasing from root to tip, proportional to the increase of the radius.

Now, you see when the flow comes into the blade C w is actually constant; that is C w 1 coming into the blade is constant. So, which means it acquires a different kinds of us C w characteristics, will characteristic as it goes to the blade. So, by the time it comes out at the rear of the rotor, because of the rotation and because of the free vortex design that has been used to create that rotor, that rotor based on free vortex design would create C w variation, proportional directly proportional to r in the radial direction.

So, that the product of C w and r is held constant. So, if you create a blade based on Free Vortex Design Law, you would get a flow in which C w varies inversely proportional to r with reference to change of radius. Now this is what happens, when you have a so called Free Vortex Design. Now, let us see, what are the ramifications of this Free Vortex Design?

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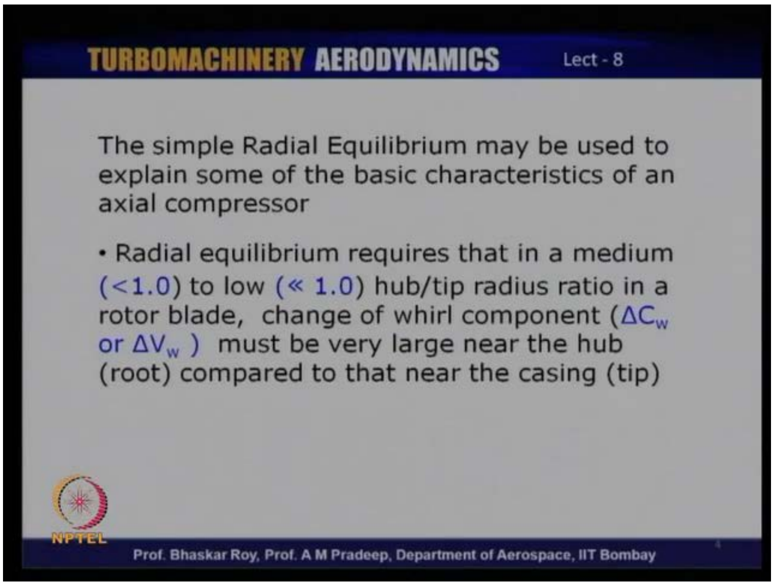

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The simple Radial Equilibrium may be used to explain some of the basic characteristics of an axial compressor

- Radial equilibrium requires that in a medium (<1.0) to low ($\ll 1.0$) hub/tip radius ratio in a rotor blade, change of whirl component (ΔC_w or ΔV_w) must be very large near the hub (root) compared to that near the casing (tip)

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What happens is that the radial equilibrium, which is used to explain some of the basic characteristics; the radial equilibrium requires that in a medium, which is define as less than 1 radius ratio, and low that is a much less than 1 hub to tip radius ratio in a rotor blade. The change of whirl component must be very large near the hub, compared to that near the casing; if you are blade is such that hub and tip are substantially separated from each other, because it has let us say low hub to tip ratio hub is small, and tip is far away and the different in r between the two is quite large; it essentially means that C w at hub is going to be very large, compare to that at the casing. That is what the Free Vortex Law, then tells us... So, C w are whirl component at the casing is indeed going to be very large.

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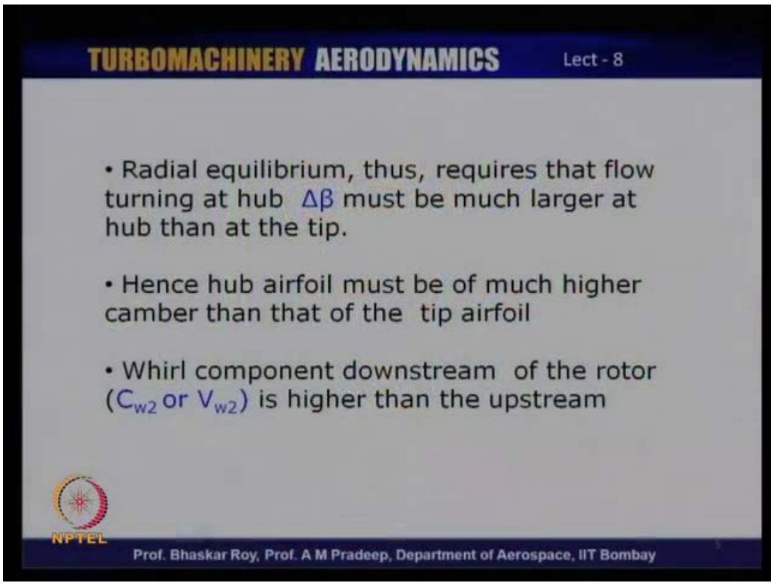

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- Radial equilibrium, thus, requires that flow turning at hub $\Delta\beta$ must be much larger at hub than at the tip.
- Hence hub airfoil must be of much higher camber than that of the tip airfoil
- Whirl component downstream of the rotor (C_{w2} or V_{w2}) is higher than the upstream

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Now, what does that mean; The C w… The Free Vortex also means that it has a radial equilibrium. So that, the flow turning at the hub, must be much larger than at the tip. So, which means, which is a corollary from which we can draw corollary, that the hub airfoil must be of a high camber than that of a tip airfoil. So, the tip airfoil is likely to be a flatter airfoil, low camber airfoil, and the hub airfoil would be a high camber airfoil. So, as you can see the camber would indeed vary from hub to tip, which mean you the essentially have different airfoils at the hub, and at various sections all the way up to the tip.

So, they are indeed, different airfoils not same airfoils. And then, thirdly the whirl component downstream of the rotor would be higher than the upstream. Now, the whirl component change of whirl component of course, is the work done. So, since work is been done, work is been put in, its stands to decent that the downstream C w or whirl component will be higher than the upstream one. Now, we have already seen that the whirl component varies along the radius, and that variations is at the downstream one, because upstream one depend on what the flow is coming in and in the first stage for example, it is likely to be constant from hub to tip.

So, the inlet whirl component is depended on how the flow is coming in, what kind of flow pattern or flow profile is coming in, the exit profile is decided by the blade design law.

So, if you have a Free Vortex Law, on which the blade has been design; the C w at the rear of the rotor blade. Firstly, it would be higher than the than at the front, that is at upstream one, because the work has been done. So, C w at the rear has to be higher than the C w one at the front; and then of course, C w varies from hub to the tip of the blade, as for the Free Vortex Law. So, these are the two things; that happens to the whirl component of the flow, as the flow goes to the blades. So, let is looks at what the other things are happening what does that means, if you have C w variation along the length of the blade.

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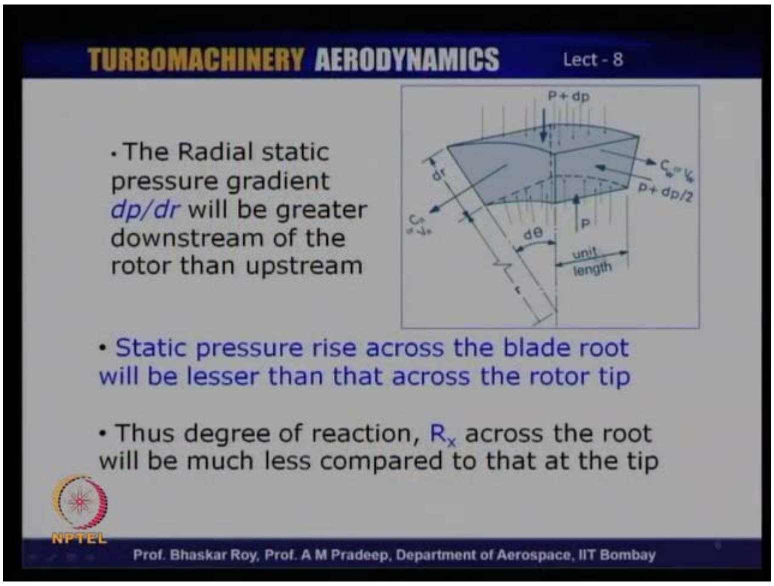

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- The Radial static pressure gradient *dp/dr* will be greater downstream of the rotor than upstream
- Static pressure rise across the blade root will be lesser than that across the rotor tip
- Thus degree of reaction, R_x across the root will be much less compared to that at the tip

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If you look at this picture, this is the picture we have used in the last class, and it tells you that, there is a balance of force, which is the balance of the static force P to P plus dp, which was balanced by the radial force created by the dynamics of the flow; and one of the major dynamics is the C w component, which is the whirl component with which the fluid particle is rotating with the blades.

And now, we see that somewhere in the blade C w is higher; somewhere in the blade C w is lower, which means wherever C w is indeed higher; it stands to reason that corresponding change of pressure would also be higher. So, that is what is said said down here, that the radial static pressure gradient dp, dr will be greater downstream of the rotor than upstream one.

So, as the rotor imparts energy into the fluid, the fluid coming out of the rotor, as it comes through the rotor blade passage C w would be higher, and in which case that value of C w would need a greater dp, dr or pressure gradient in the radial direction. We have also seen that the C w varies along the ring, length of the blades; so wherever C w is higher, it would correspondingly try to give stronger dp, dr gradient.

The static pressurize across the blade root will be lesser than across the rotor tip, this is also comes out of the Free Vortex Law. The work done at the tip is likely to be of higher older than at the where the u is much higher; and at the blade root and the value of u is much lower; and as a result the static pressurize would be much lesser at the root than at the tip. And then we come to the more important, very important parameter degree of reaction, which we have talk about before. And the degree of the reaction across the root will be much less compared to that at the tip; that is for Free Vortex Blade Design.

So, degree of reaction need the root is indeed going to be much lower; in fact, this is something which we will again come back later on when we discussed design; and we have we have already talked about a little, the fact that degree of reaction need the root can go indeed so low, to the extent that it can it can actually, tend to go below 0. That means it could tend to become negative. Now, negative degree of reaction is not a good idea, because it indicates at the flow at that station, at that radial position is indeed behaving like a turbine and not behaving like a compressor. So, in a compressor a negative degree of reaction indicates the flow is, flow and the rotor together is behaving like a turbine in that particular location.

So, negative of degree of reaction is to be avoided, but all costs if it has to work like a compressor. So, it has to be made 0; more than 0, minimum is 0; and it needs to be made a little more than 0 if possible. So, that near the hub under all operating conditions of the compressor, it never goes below 0. So, degree of reaction is another parameter, which we shall be discussing in this lecture today, and we shall see more and more; how it impacts on the design? In addition to the Vortex Laws a starting with the Free Vortex Law, which we were discussing right now. So, degree of reaction is other important parameter which will be discussing a little more in today lecture only.

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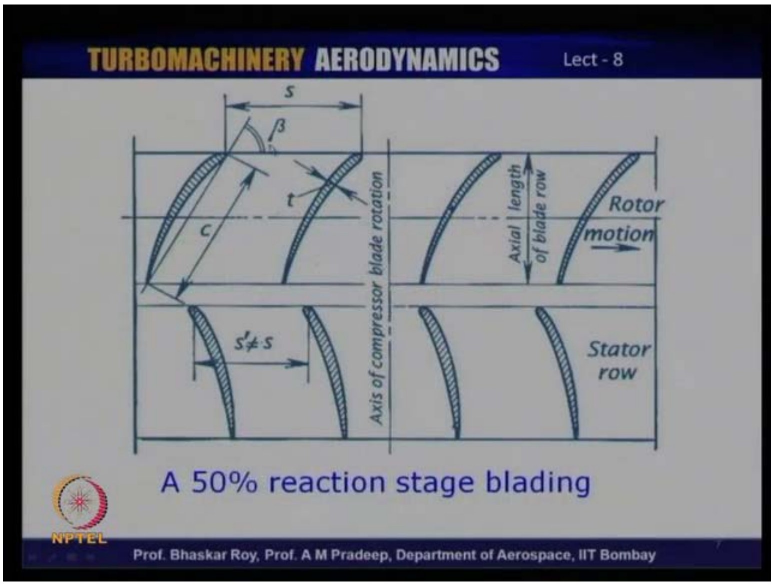

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A 50% reaction stage blading

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Now, if you take typical blade profile or stage design which uses 50 percent reaction - degree of reaction stage. What we see is the rotor and the stator are angled equally; that means, the flow coming into the rotor at an angle beta, and the flow coming into the stator at another angle alpha alpha two is a likely to be same. So, these two angles are same. So, the two blades the rotor and the stator are set at same angles.

Now, this is of course, to make sure that the flow coming to in to the rotor in relative frame, as the same angle as the flow going in to the stator in absolute frame. Now, this has been done during the earlier lectures. So, 50 percent degree of reaction stage blading, which gives as as we call it a symmetrical blading is a very preferred, and very popular design choice; for axial flow compressor design.

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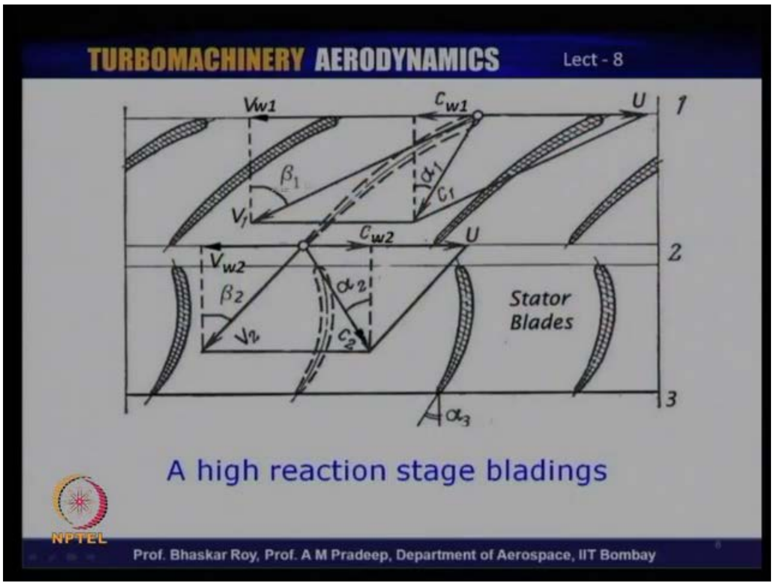

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A high reaction stage bladings

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The next preferred choice is a high reaction blading, which could be prediculous to a 100 percent reaction blading, which has been a preferred choice of designers in some parts of the world notably in Germany, where people have been designing 100 percent reaction blades for many many years. In fact, right from the beginning some people preferred to design blades with 100 percent reaction. Now, 100 percent reaction blade of course, gives completely different blade orientation and blade stagger.

So, that what you shown in this diagram for example, that the blades in the rotor are now highly staggered at a very high angle beta, compared to Let us, say 50 percent one, where the angle was moderate and this high angle comes out of the high reaction necessary; that means, the flow in the rotor would has would have to go through a high amount of diffusion and the flow in the stator, as we can see here, essentially is doing a turning job. It is not doing any diffusion at all. So, diffusion in the stator is 0; all the diffusion is occurring in the rotor, if it is 100 percent reaction.

So, and stator is just a turning wings; a set of turning wings. So, high reaction stage blading is another choice, and many years it was the 100 percent reaction blading, which many people have been using; and that is their design choice in a division two. Let us, say Free Vortex. Now, they the Free Vortex and the reaction choice are not in conflict with each other; essentially they are complimenting each other.

Free Vortex we have seen of course, that the reaction indeed varies from root to tip. So, what we are talking about is a value of reaction, somewhere in the middle of the blade or in the mid section of the blade, mid radius of the blade, where it could be let says 50 percent. On the other hand, some people have used a constant reaction blading, where the reaction is held constant. Now, that kind of blading is not free vortex design. So, Free Vortex Design will have reaction degree of reaction, varying from hub to the tip. In which case, when we say 50 percent blading, the mid radius of that blade has the 50 percent reaction; someone near the hub, it is very low that is a 0.5 at mid radius, at the hub it could be a nearly 0; may be less than 0.1; and near the tips it could be of the order of 0.8 or 0.9 which means 80 to 90 percent reaction blading. That means, near the tips the blade arrangement could be very similar to what we are looking at here, that this is an arrangement, which you could be seen near the tips.

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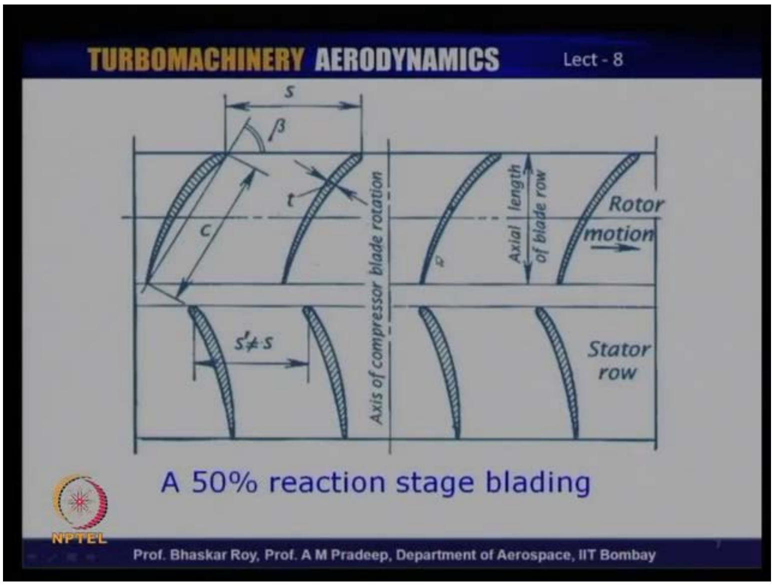

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A 50% reaction stage blading

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Whereas, this is an arrangement you could be seen near the mid section of the blade; and near the hub it could be nearly 0, so blades should be the rotor blades should be even more straightened out. So, from hub to tip the blades setting of the blade orientation would also change, because the reaction value is changing based on Free Vortex Design principle. So, this is the kind of blading, you would most likely to see in the other tip of the blade. So, these are the Free Vortex Design possibilities. Now, if you look at some of the other summary common that we would like to make.

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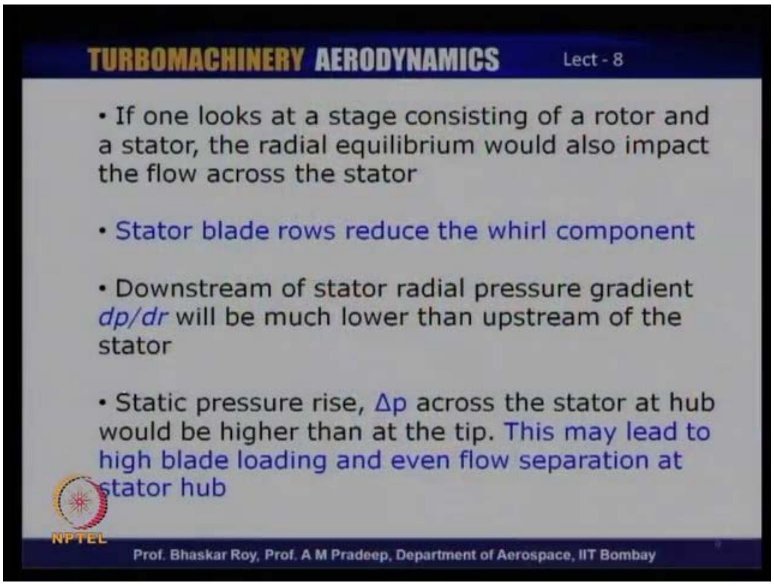

TURBOMACHINERY AERODYNAMICS Lect - 8

- If one looks at a stage consisting of a rotor and a stator, the radial equilibrium would also impact the flow across the stator
- Stator blade rows reduce the whirl component
- Downstream of stator radial pressure gradient *dp/dr* will be much lower than upstream of the stator
- Static pressure rise, Δp across the stator at hub would be higher than at the tip. This may lead to high blade loading and even flow separation at stator hub

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A stage consists of a rotor and a stator, if you have a radial equilibrium of forces balance in the rotor, it would impact the flow across the stator. Now, the stator blade, the rotor blade rows by virtue of the fact that they working, increase the whirl component the stator blade rows, would reduce the whirl component across its own row; across the entire length that is from a root to the tip of the blade; downstream of the stator the radial pressure gradient dp dr will be much lower than the upstream of the stator, we had seen exactly opposite happens in case of rotor.

The static pressure rise delta p across the stator, at the hub would be much higher than at the tip, in the rotor we had just seen it was exactly opposite; at the hub it was much less than at the tip, at the state across the stator it will be a high at the hub, and low at the tip. Now, this may lead to high blade loading near the hub sections, and even flow separation now increased blade loading, increase static pressure ratio; essentially means that the blade is aero dynamically loaded more. You are trying to get more out of it aero dynamically, and that is blade loading.

Now, if you are trying to achieve higher static pressure gradient, remember this is in adverse pressure gradient.

So, pressure is increasing from front to the rear of the blade; and this creates a situation that the flow gets loaded, the blade is loaded and the flow is now on the brink of separation. This is the problem; that if you increased the loading at anywhere, any section of the blade, whether it is the root, or the tip or the mean or any others section, if the blade gets aero dynamically loaded more than what it can withstand, the flow would indeed be on the brink of the separation, if it does separate the blade will get into stall. And this of course, could blow up into much bigger problem which we call surge.

So, these are the issues, which the blade designer would have to factor in, and they have, he has to take them into a count that under no operating condition of this axial flow compressor at any point, at any section of the blade, would ever be threading to the under stall or separation. That means, no where the blade loading should be more than what it can withstand; earlier, we had said forth parameters like diffusion factor as one of the blade loading parameters. So, we have to confirm to those limitations, to ensure during the design process that had no point of time, the blade is threatened with separation and stall, which as I said could lead to surge.

So, some of these issues would need to be contented with and as we seen now, the variation across the rotor is quiet often different; and is quiet often opposite to that of the nature of variation across the stator. Now, rotor and stator put together they complement each other; and they put together make up the whole stage. So, what happens in the rotor, and what happens in the stator are quiet often; opposite to each other.

And as, I just mention, it is also decided by the degree of reaction, if it is a 50 percent; the loading is 50 50 across the rotor and the stator; which normally happens at the mid radius of the blade or or mean passage of the blade from root to tip; and that is often equally shared between the rotor and the stator. Anywhere else, in the blade from the root to the tip, the share is unequal. Sometimes the rotor is loaded more, sometimes the stator is loaded more, aero dynamically; and this loading often carries an impending threat of separation and stall. So, the designer would need to keep all this in mind, while designing the blade. So, this is what setting for the basic design principles actually mean.

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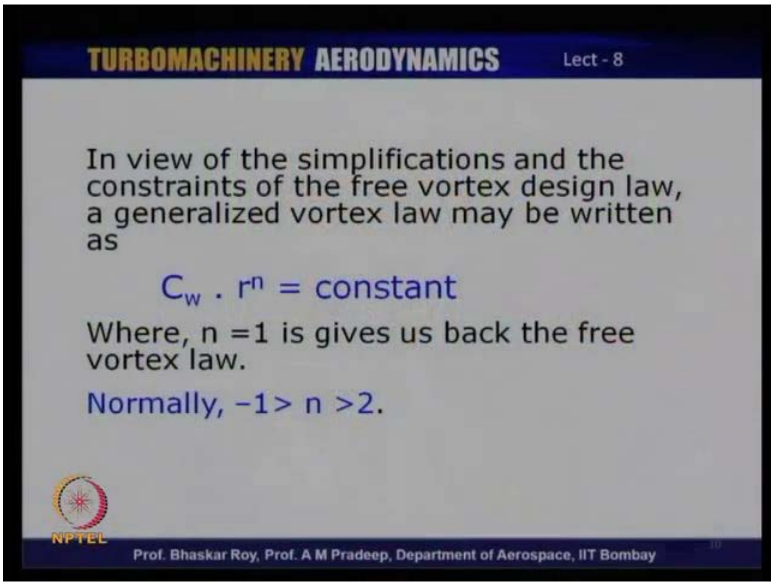

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In view of the simplifications and the constraints of the free vortex design law, a generalized vortex law may be written as

$$C_w \cdot r^n = \text{constant}$$

Where, n =1 is gives us back the free vortex law.

Normally, $-1 > n > 2$.

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Let us, move forward and see what happens, if you carry on with this design principles. The design principles that we have said forward C w into r. As, we see as a number of simplification and number of constrains. We just put together, all those constrains; it loads the blades differentially, and it loads the blade in a certain standard, one may says straitjacketed manner; if you do not like that strait jackets and most modern designers, do not like such straitjackets; they would like to break free from the Free Vortex Design Law.

And based on this consideration, a generalized Vortex Law has been put together, which reads C w into r to the power n is equal to constant; where, n equal to 1 gives us back the free vortex law. So, Free Vortex then becomes a one singularity case out of this generalized a vortex law. Now, this vortex law has been not been derived separately, it is simply up gradation of the free vortex law by using a value of n; which is sometimes could be other than 1.

Now, normally the value of n could be from minus 1 to 2, those other values people have used in the design. And, I have found their utility values; and we will see; what those values actually mean one of course, n equal to 1 means, we go back to free vortex, which we were discussing discussing in some detail.

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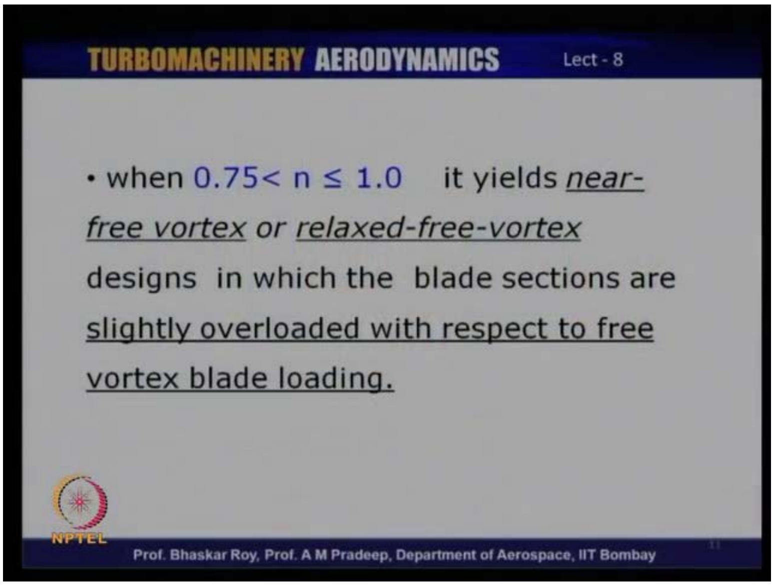

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- when $0.75 < n \leq 1.0$ it yields *near-free vortex* or *relaxed-free-vortex* designs in which the blade sections are slightly overloaded with respect to free vortex blade loading.

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Now, let is move forward and see, if the value of n is something other than 1. For example, it is possible that the value of n could be a little less than 1, and if it is somewhere between 0.75 and 1; this yields, what is often referred to as near-free vortex or simply relaxed-free-vortex design in which the blades sections are slightly over loaded with respect to the free vortex blade loading.

Now, free vortex blade loading for wide set an amount of blade loading, characteristics to the rotor and the stator blades; this relax free vortex where, n is less than 1, overloads the blades slightly, very slightly. So, that it is not threatened with separation or stall within the limits of the diffusion factor which we have discussed earlier.

And this slide over loading than allows the designer to have or create more pressure ratio across one single stage; that is of course the aim, that you try to create higher and higher pressure ratio across one single stage, and in a multi stage configuration; if you have higher pressure ratio - pressure ratio across each stage, you would indeed have less number of stages. So, that the overall intension.

Now, in the process of this you relax the Free Vortex Law; so, that each stage is now, slightly more loaded than the Free Vortex Design, and hence, it is doing a little more of pressurizing; and little more pressure ratio across one single stage design. So, this is one way of slightly over loading the blades. On the other hand, if you have blades in which the value of n is more than 1 which is one of the possibilities, the blades are under loaded with respect to the Free Vortex Design Law.

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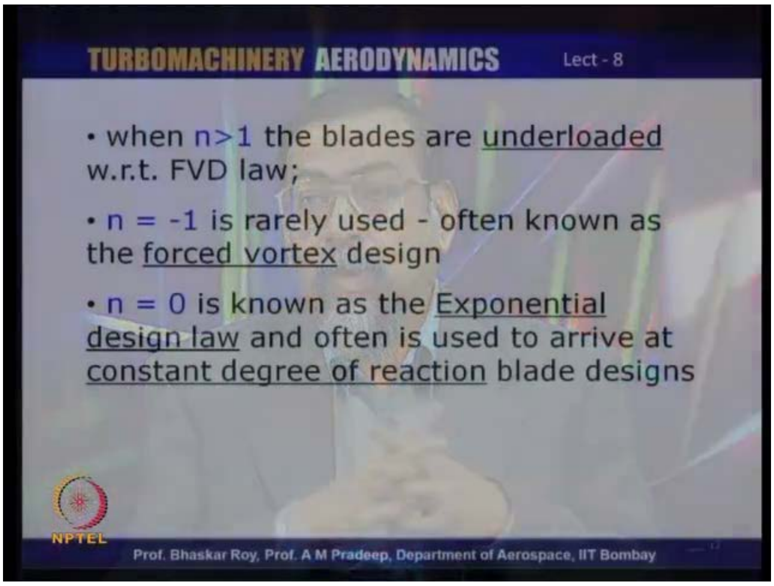

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- when n>1 the blades are underloaded w.r.t. FVD law;
- n = -1 is rarely used - often known as the forced vortex design
- n = 0 is known as the Exponential design law and often is used to arrive at constant degree of reaction blade designs

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So, moment the values of n used a more than 1, the blades are under loaded. Now, this is not entirely you know useless; you may like to under load the blades under certain operating conditions; and the reason is near the tip of the blade or near the hub of the blade, quite often the blades have to content with the case in boundary layer; and the hub boundary layer. Now, these two boundary layers interfere with the airfoil operation. And hence, the airfoil do not really operate like airfoils, as they should near the tips; and near the hub of the blades.

As a result of which you never get the full loading anyway, because of this three - dimensional flow nature especially near the hub and the tip. In which case, the blades are not going to give the same blade loading, same pressurization, near the tip and the hub; and as a result of which many designers now feel over the years, that there is no point loading the blades so much, near the tip and the hub; might as well under load them with respect to the free vortex loading, we are talking about, and may be the rest of the blade in between sections of the blade may be overloaded by using value of n less than 1.

So, that is one way of getting away from free vortex, the middle part of the blade is loaded more than the free vortex, the tip and the hub portions are under loaded by using a value of n more than 1. And, this combination is now not a free vortex design really, it is been relax; and this new blade now has a better characteristic; the tips are under loaded, and as a result it is expected that the tip flow would be less strong, because the strength of tip flow is dependent on the blade loading at the tip. If you are under loading, the flow across the tip would be of a lower strength, and and the tip will be less, the tip flow vortex will be lower strength, and hence the blade will hopefully be of a higher efficiency and hopefully of a better stall characteristics. So, these are the thoughts that are put together; in using values of n less than 1 and more than 1.

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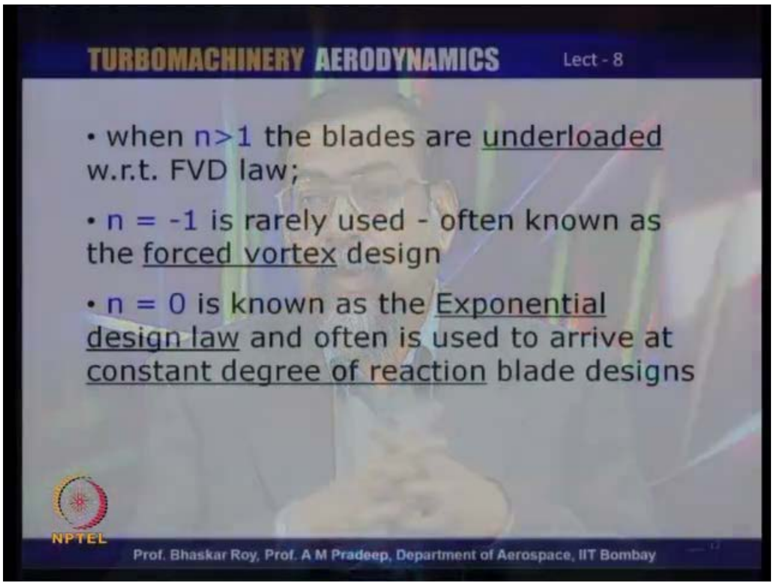

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Now, the other possibility is where, n could be minus 1; which has been use sometime, it is often called the force vortex design. The other possibility is where n is equal to 0, which is known as Exponential design law. If you use n equal to 0, you the see that C w is kind of constant; the relationship is C w into r to the power n; when n goes to 0, essentially C w is constant; and this is often referred to as Exponential design law. And this is often used to arrive at or it is a derivative of C w is equal to constant. It gives what is often, then can be called constant degree of reaction blade designs, which been the degree of reaction is now constant from root to the tip of the blade, tip of the stage.

Now, we have seen free vortex. And indeed its variants; the relax free vortex where n is slightly more or less than 1. The degree of reaction would vary from root to the tip of the blade. When using n equal to 0; the exponential law C w is constant, and hence the degree of reaction would be constant from root to the tip of blade, you can choose a degree of reaction now. And the early designers often used to choose, if they used a constant reaction blading design; either 50 percent or 100 percent, nothing in between for a long time people were using 50 percent reaction blading or 100 percent reaction blading, all the way from root to the tip of the blade.

However, model designers may like to do differently, and has we shall... We shall see as we go long; that the variation of degree of reaction is indeed an important issue, and modern designers do have relaxations of those things also along with the relaxation of free vortex law. So, both the free vortex law, and the degree of reaction variation is now relaxed. And it is done in a more control manner, and hence modern designers would like to call call that a control vortex law. So, that all the design is now under control; the variation of degree of reaction; and the variation of the vortex strength from root to the tip is done in a control manner, and modern designers would like to call that a control vortex designs.

So, we have seen that a number of possibilities do are there, in which you can actually, create the fundamental blade design - we are talking about fundamental blade design; the first cut blade design, when you are just creating a new blade, where they was nothing, and you are creating a new blade. Let us, move forward and then let us see what happens, if you use this kind of blade design laws or principles.

We have seen earlier that the blades have vertices created around, because they are made of airfoil sections; and those vertices have circulation, and the strength of circulation or variation of the strength of the circulation is one of the issues along length of the blade, which now we see depends on the reaction or degree of reaction.

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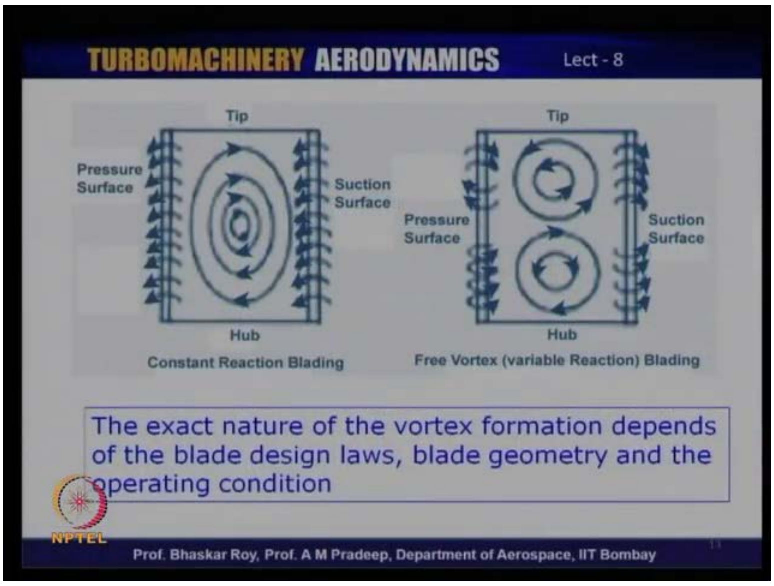

The exact nature of the vortex formation depends of the blade design laws, blade geometry and the operating condition

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If we have a Constant Reaction Blading, we shall see that the Vortex along the length of the blade would remain constant; and would would vary in a constant manner across the from one side to the other at the trailing edge. We are looking at it from the trailing edge. So, it will move from one side to the other side at the trailing edge, and you will have a more or less constant strength, along the length of the blade that is from hub to the tip.

On the other hand, if you have a Free Vortex, which is a Variable Reaction Blading. And depending on the reaction variation, depending on the Free Vortex Design Law that has been used; the strength of vortex would now be varying along the length, and this variation is from here in the diagram.

Now, what happens in a Free Vortex or in an Relax Free Vortex or the modern version of control Free Free Vortex Design; is that it gives a Variable Reaction Blading or Variable Degree of Reaction Blading. And as a result of which one can say that the nature of the vortex formation, and the strength of vortices from hub to the tip depends on the blade design laws, and the blade geometry and indeed the operating condition.

So, what happens is that the vortex formation from hub to the tip of the blade depends on design laws, that we are discussing; it depends on the blade geometry, and very importantly it depends on the operating condition. You see we have talked about before the blade is designed at a particular operating point, which is known as a design point. But the engine and the blades, the compressors would have to operate under various operating conditions, where the speed of rotation rpm, the mass flows or different from the design point.

So, the design operating point, if it is away from the design point would indeed impact on the vortex formation. The vortex formation depends on three things - design laws, blade geometry, and the operating condition; at which the blade is operating.

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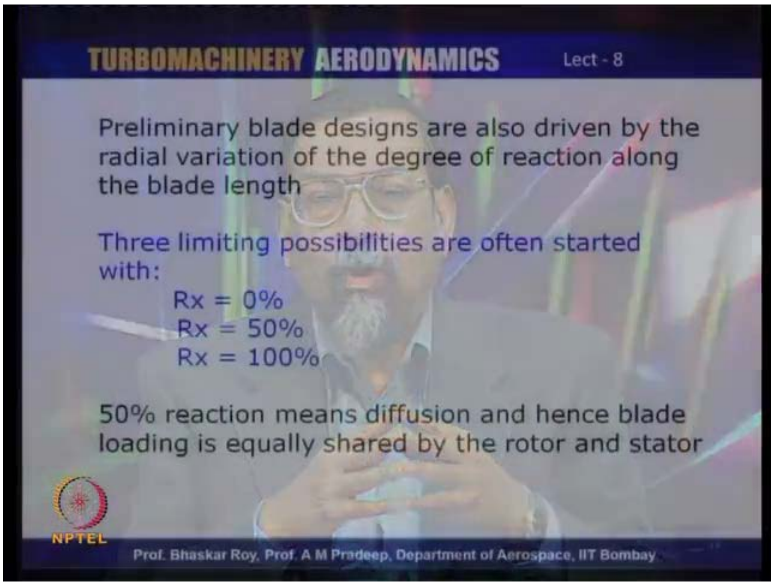

TURBOMACHINERY AERODYNAMICS Lect - 8

Preliminary blade designs are also driven by the radial variation of the degree of reaction along the blade length

Three limiting possibilities are often started with:

Rx = 0%
Rx = 50%
Rx = 100%

50% reaction means diffusion and hence blade loading is equally shared by the rotor and stator

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Now, the preliminary designs, also driven by what we have discussed as the degree of reaction, along the blade length. Now, the three limiting possibilities are often started with, when you are creating a first cut blade design; the three possibilities are - degree of reaction equal to 0 percent, degree of reaction equal to 50 percent that is the most popular one, and degree of reaction100 percent. Now, we look at these three possibilities: The 50 percent reaction blade essentially creates equal diffusion diffusion in the rotor and in the stator; that in the blade loading or equal between in the rotor and in the stator. And the diffusion is equally shared.

Now, the three limits that we are talking about 0 to 100 percent in a free vortex design. It is entirely possible that from the root to the tip of the blade, the degree of reaction would be varying from 0 to nearly 100; that means, different sections of the blade have different reactions. So, at the hub or near the root; it could be nearly 0, at the tip it could be nearly 100; someone in the middle its 50 percent. Now, that is a kind of variation, that is quite often people have used in the blade design; the other possibility which we talked about is that Constant Reaction Blading, where the entire blade has constant reaction; and in which case also three are possibilities - 0 percent, 50 percent, and 100 percent.

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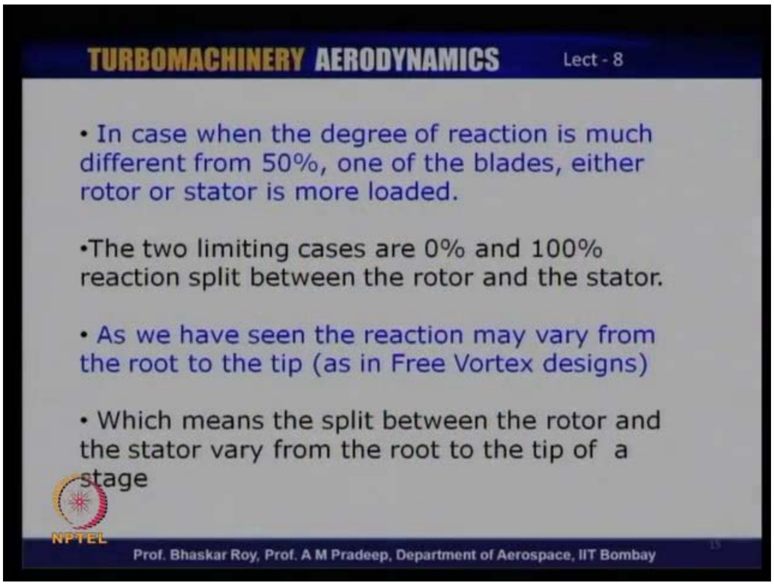

TURBOMACHINERY AERODYNAMICS Lect - 8

- In case when the degree of reaction is much different from 50%, one of the blades, either rotor or stator is more loaded.
- The two limiting cases are 0% and 100% reaction split between the rotor and the stator.
- As we have seen the reaction may vary from the root to the tip (as in Free Vortex designs)
- Which means the split between the rotor and the stator vary from the root to the tip of a stage

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Let us see, what that means actually? When you have a 50 percent Reaction Blading, the blades are equally loaded. When they are other than 50 percent, you have to remember that either the rotor or the stator is going to be more loaded. Now, the two limiting cases we discussed are 0 percent, and 100 percent reactions played between the rotor and the stator. And this reaction, now varies in free vortex or near free vortex would vary substantially from root to the tip of the blade, which means from the root to the tip of a stage, the split between the rotor and the stator would vary from the root to the tip of the blade; that means, the loading pattern would vary from the root to the tip of the blade of a rotor, and the loading pattern on the stator would vary in the opposite manner from the root to the tip of the blade.

Now, this is what exactly a free vortex or a near free vortex design would essentially yield. So, this variation from the tip root to the tip of free vortex kind of design is an important issue; we have to keep that in mind unless you are going for a Constant Reaction Blading.

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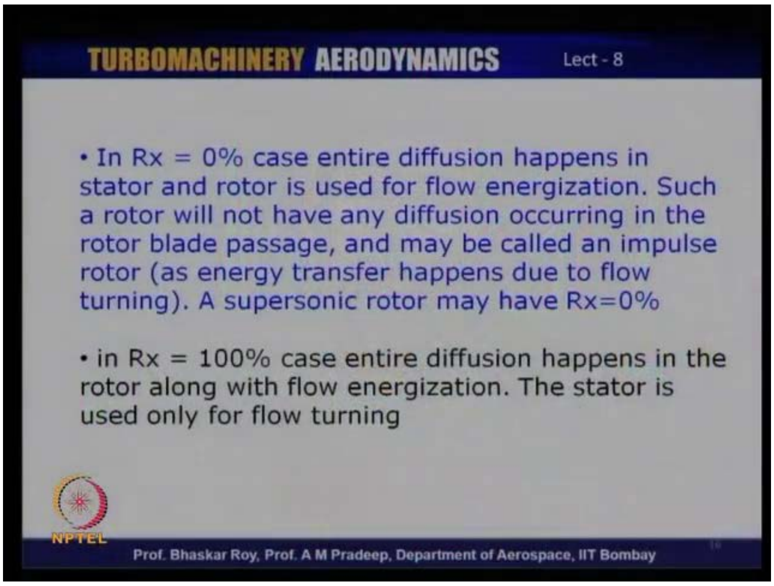

TURBOMACHINERY AERODYNAMICS Lect - 8

- In Rx = 0% case entire diffusion happens in stator and rotor is used for flow energization. Such a rotor will not have any diffusion occurring in the rotor blade passage, and may be called an impulse rotor (as energy transfer happens due to flow turning). A supersonic rotor may have Rx=0%
- in Rx = 100% case entire diffusion happens in the rotor along with flow energization. The stator is used only for flow turning

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If you have a reaction that is 0 percent, a limit; the entire diffusion happens in the stator. And that means, the rotor is not having any diffusion at all, hence the rotor is been used only for imparting work into the flow, energizing the flow, putting work into the passing flow; and such a rotor will not have any diffusion by design occurring in the rotor blade passage. Hence, such a blade may be called impulse rotor, very similar to the impulse turbine that you may have heard of. We shall we doing it in the turbine chapter later on, and hence the energy transfer happens due to the turning of the flow.

So, the turning of the flow essentially, is equated or responsible for the amount of energy transfer; and no diffusion is occurring in that particular rotor in which degree of reaction is 0 set forth as 0. Now, many supersonic rotors may have a degree of reaction, 0 percent. It is possible, that subsonic rotors may also have one of the possibilities is the supersonic rotor, where the flow is supersonic through the rotor blades, and during that lot of work transfer is accomplished. However, the diffusion is differed to the stator; mainly, because the rotor is busy transferring energy into the fluid. In case of a 100 percent blade design; it is exactly opposite. Rotor is now doing energy transfer, and energy conversion; 100 percent into pressure. So, energization and then pressurization 100 percentage occurring in the rotor, nothing is left for the stator. What is the job of the stator?

Stator essentially, turns the flow. Because in that kind of a design, it is most probable the stator will have a lot of turning to do; and as we have seen before, doing a lot of diffusion that is a diffusy passage, and turning the flow around a lot or two things which are of conflicting interest. They conflict each other; and a flow would refuse to do two things simultaneously; a lot of tuning, a lot of diffusion. Sometimes a lot of turning with a small bit of diffusion may be a difficult thing. In which case, the diffusion is completely dispensed with and the stator is asked to do only turning; and the entire diffusion is finished off in the rotor itself.

So, that is a 100 percent reaction blading. And as, I mention, there are some designers, notably got a few from Germany have preferred that kind of design. Where 100 percent energization and diffusion occur in the rotor and stator essentially, turns the flow for the next row of blades or for any other delivery purpose. So, these are the limits of various Reaction Bladings; that we have been talking about, and as I mentioned the variation of reaction is as important as the free vortex law or the vortex law, that we have brought forward for blade design purposes.

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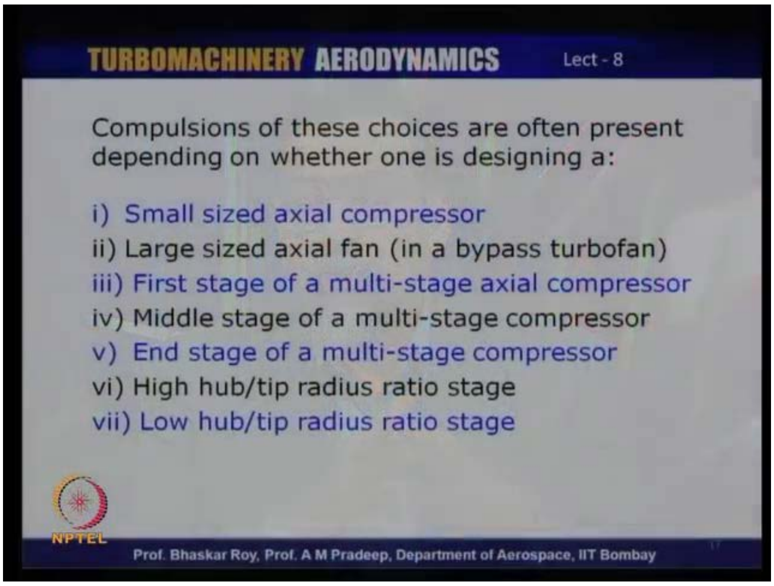

Compulsions of these choices are often present depending on whether one is designing a:

i) Small sized axial compressor
ii) Large sized axial fan (in a bypass turbofan)
iii) First stage of a multi-stage axial compressor
iv) Middle stage of a multi-stage compressor
v) End stage of a multi-stage compressor
vi) High hub/tip radius ratio stage
vii) Low hub/tip radius ratio stage

Now, let us look at some of the other issues, I mentioned that geometry is an important parameter. Let us, look at some of those issues. Now, if you have a blade, which is let us say a small size axial compressor, that will impact that design or it will tend to take the design in a different direction compared to a blade, which is a large size axial fan, as one can see in a bypass turbo fan. The blade design law of such a large sized axial fan, would be quiet different than that of a small sized axial compressor.

The first stage of a multi-stage axial flow compressor, which is likely to be a comparatively large size blade would be different, and would probably use different kind of design law combinations, compared to that of let us say a middle stage of a multi-stage or that of a end stage of a multi-stage compressor. And then again all of them put together, we can say that the particular blade would be either high hub to tip ratio or low hub to tip ratio stage.

High hub to tip ratio means that the different between the rotor and stator is very small; and the blade is set at a high radius, which is typical of end stages of a multistage compressor. Low hub to tip ratio on the other hand means, the hub is low, and the tip is far away and the different between the two is quite large and the ratio is small; the systifical of the first stage of a multi-stage compressor. As, I am saying that the design laws that you require to bring forward, to design these kind of stages are different from each other; you may like to use different combination of design laws, different combination of reactions or reaction variation to create these stages.

So, even if you have one single compressor consisting of multi-stages; each of those multi-stages of a multi-stage may be design as per different design laws. You do not use, same design laws for all the stages; you use quite often different design laws for the different stages of a multi-stage compressor. The other way of looking at the stages is, when you have blades which are often referred to as high aspect ratio or low aspect radio blades. Now, if you have a low hub to tip ratio blade; typically, you would probably looking at a blade which is something like this.

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Now, this blade you can see is a long blade; and you can see its quad is very small, which means its length to quad ratio is very high, and that is aspect ratio. So, aspect ratio is nothing but length to quad ratio, and this is a high aspect ratio blade; and this one can say also very confidently, that this is a low hub to tip ratio blade, rotor blade. And typically a low hub to tip ratio blade would tend to have a high aspect ratio blade, where the quad is small compared to the length of the blade.

On the other hand, if you take this blade, which is again a typical twisted blade, but as you can see it is a small blade, and its length is small compared to its quad. So, this is a low aspect ratio blade, comparatively low aspect ratio blade; and this could be used for middle stages of a multi-stage compressor. On the other hand, I will show you another blade, which is let us say, a low aspect ratio blade.

Now, this you can see the quad of the blade again a twisted blade, is almost equal to the length of the blade. Now, which essentially means that this is a blade with aspect ratio close to one; and this is the kind of blade, you might seen sometimes in the rear stages of a axial flow compressor, where the blades are indeed very small. We will discuss the effect of aspect ratio later on, and we shall see that more and more designers, on moving towards lower aspect ratio choices in the modern axial flow compressor design.

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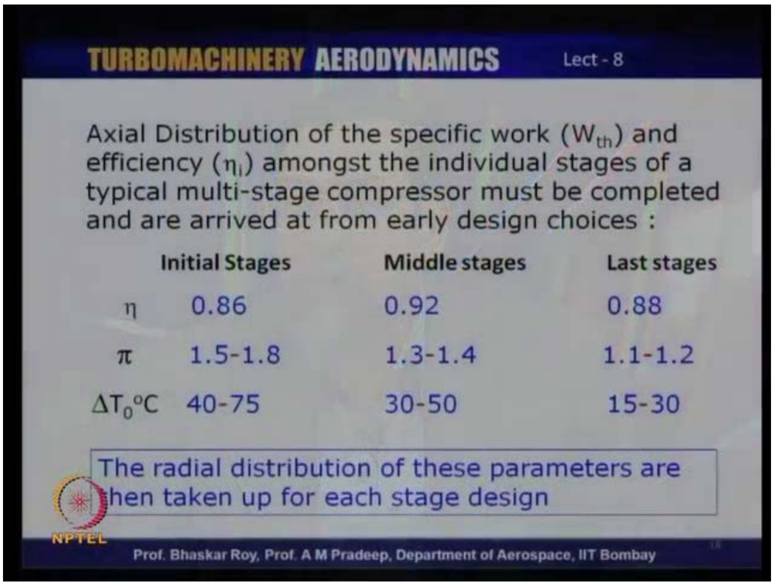

TURBOMACHINERY AERODYNAMICS Lect - 8

Axial Distribution of the specific work (W_{th}) and efficiency (η_i) amongst the individual stages of a typical multi-stage compressor must be completed and are arrived at from early design choices :

	Initial Stages	Middle stages	Last stages
η	0.86	0.92	0.88
π	1.5-1.8	1.3-1.4	1.1-1.2
ΔT_0°C	40-75	30-50	15-30

The radial distribution of these parameters are then taken up for each stage design

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Let us, look at a few numbers that you need to set forth for design of multi-stage axial flow compressor. In the initial stages, you would be looking for efficiencies which are likely to be a little lower, because the blades are big and high aspect ratio; and they often have a certain amount of distortion or non uniformity at the inlet. And hence, quiet often the penalty is paid in the in form of efficiency; in the middle stages those problems do not exist. So, the efficiency is going to be very high; and in the later stages the blades are very small, as a result of the smallest the three-dimensionality of the flow, impacts on the blade flow; and as a result efficiency again tips a little, to somewhat low values.

The pressure ratio of the initial stages, in spite of low efficiency is likely to be high, because you are operating at a low pressure, and low temperature; and due to the low temperature operation even with moderate amount of work input, you can get a very high pressure ratio. So, while design most designers like to accumulative very high pressure ratio in the early stages, because in the later stages you progressively get less and less pressure ratio to the extent that in the last stages, you get very low pressure ratios anyway; so by design, the designers like to put more work, now delta T 0 of course, is a measure of the work input; and as a result of which they like to put in more work to get more and more pressure ratio, which is easier to get at the initial stages. Because of the low temperature operation in a middle stages, you put middle amount of work and you get reasonable amount of pressurization; in the later stages, there is indeed no point putting very high work input, because the pressurized is not going to be very high anyway. So, that is a kind of first cut division of labor, who want me like to do across the stages in a multi-stage configuration.

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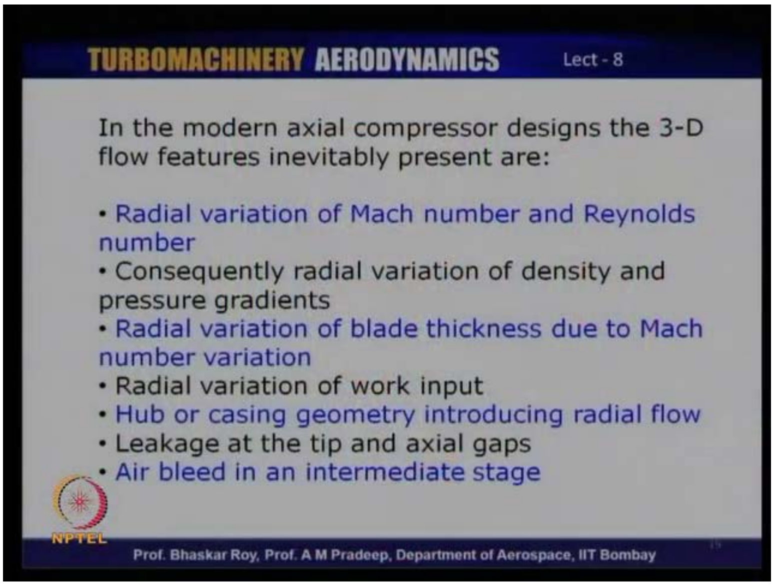

TURBOMACHINERY AERODYNAMICS Lect - 8

In the modern axial compressor designs the 3-D flow features inevitably present are:

- Radial variation of Mach number and Reynolds number
- Consequently radial variation of density and pressure gradients
- Radial variation of blade thickness due to Mach number variation
- Radial variation of work input
- Hub or casing geometry introducing radial flow
- Leakage at the tip and axial gaps
- Air bleed in an intermediate stage

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Following those configurations, you design the blades. So, the design of the blade would then be dependent on a number of 3-D flow features; so, these 3-D flow features are, set forth as you are going to have a radial variation of Mach number and Reynolds number. To the extend, the flow could move from subsonic near the root to supersonic near the tip, and the Reynolds number would also vary, and hence you would probably need to choose different kind of airfoils near the root than compare to that near the tips.

So, you have radial variation of density and pressure gradient. Even you are simple radial equilibrium condition that we had set forth, the balance of forces does give you notion that the radial variation of density, and pressure gradient would follow the variation of Mach number. Then consequently, the blade thickness also would vary from root to the tip due to the Mach number. And as, I was just mentioning that low Mach number near the root would warrant, low speed airfoils choices at the tip, high Mach number would warrant choices of thin airfoils, which are meant for high Mach number. If it is supersonic, he would need to choose supersonic airfoils.

We will discuss those, airfoil issues later on in this lecture series. As, we have seen the work input variation in a free vortex was considered constant from root to the tip. Now, we can see that, one can have a radial variation of work input from root to the tip in a control manner; if you are going for a control vortex design; that is breaking free from free vortex design. This means that the hub and casing geometry depending on the pressure ratio. If you have indeed pressure ratios of the order of 1.5 or so, your hub and casing is not going flat any more, they are going to be at an angle. And this hub and casing geometry then introduces a radial flow into the flow going through the blades.

So, this is introduce by the hub and casing; angle that is warranted by the high pressurize. And then of course, you have the leakage at the tip of the blade, which through the axial, and the axial gaps; which creates the tip vertices; and they introduce three-dimensionality, you have here blade due to various operations of the requirements of the engine or their craft, and those blades again take away flow from somewhere in the middle stages; and they introduces three-dimensionality intermediate stages. And then of course, all of them put together you have a secondary flow development; which is all of them put together.

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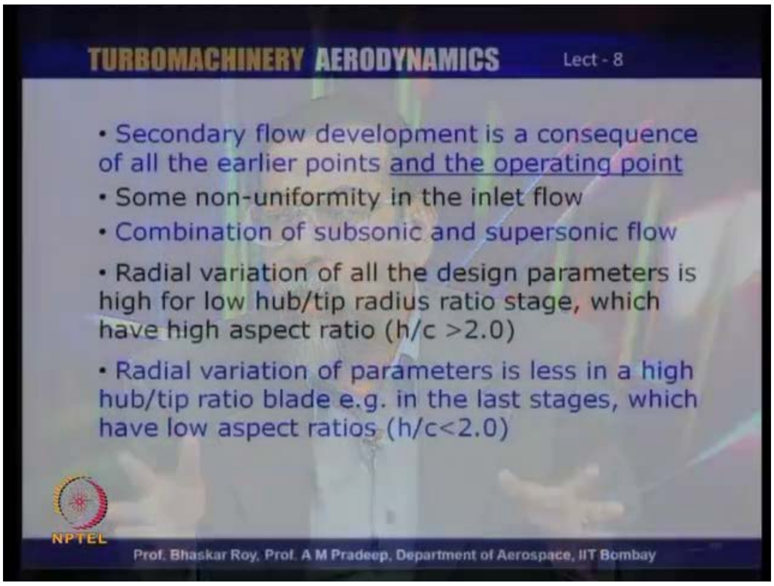

TURBOMACHINERY AERODYNAMICS Lect - 8

- Secondary flow development is a consequence of all the earlier points and the operating point
- Some non-uniformity in the inlet flow
- Combination of subsonic and supersonic flow
- Radial variation of all the design parameters is high for low hub/tip radius ratio stage, which have high aspect ratio (h/c >2.0)
- Radial variation of parameters is less in a high hub/tip ratio blade e.g. in the last stages, which have low aspect ratios (h/c<2.0)

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And also depends as, I mentioned earlier very strongly, it depends on the operating point. So, you may have a low secondary flow at the design point; at an half design operating condition, the secondary flow may be very strong. Then again it depends on the non uniformity of the inlet flow, as we have seen the flow becomes progressively more and more non uniform. As it goes through the stages, then you have combination of subsonic and supersonic flow; some are along the blades length you have shocks, some are you do not have shocks.

So, that produces a three-dimensionality all over again; and then the radial variation of all these design parameters is package together in a high or low hub tip ratio blade, which as I mention is given in a high aspect ratio blade, which I showed you on blade - where the aspect ratio was close to 6. So, anything higher than 2 is normally called higher split ratio. Whereas, the last blade, which I showed actually had a aspect ratio close to 1; and that is a low aspect ratio blade. So, radial variation of parameter is less, in a low aspect ratio blade, which is typically high hub to tip ratio blade and typical of the last stages of a multi-stage friction of a compressor.

So, we we see that we have number of issues; we have the vortex law, we have the variation of degree of reaction, we have the blade geometry blades are twisted, and you may like to control the twist with the control vortex design and then of course, you have the aspect ratio, which needs we chosen by the designer. So, the modern designers are going towards, somewhat lower aspect ratio compare to the first blade, which I showed which is a very old blade. So, that kind of blade is not used in the modern blade designs anymore. So, those are the choices, the blade geometry the variation of degree of reaction; and the vortex law. These are things that are put together, which the designers would like to create a new blade, package in which new blade creation can be started.

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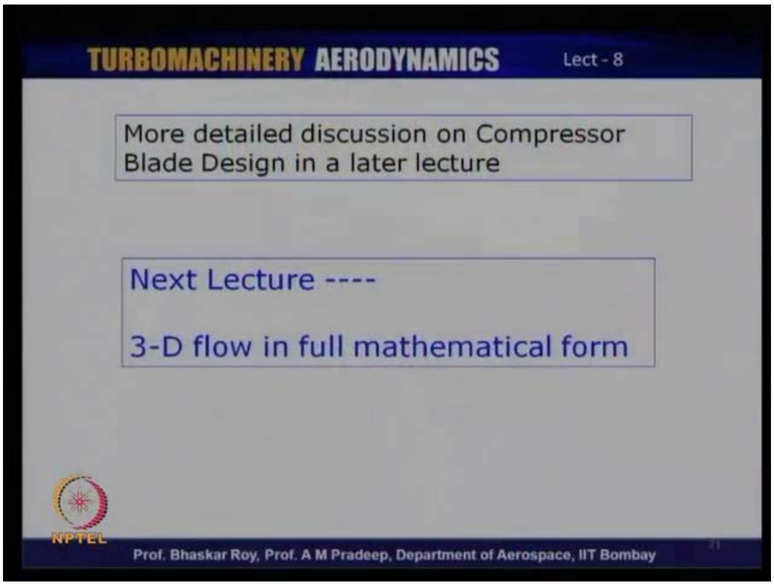

TURBOMACHINERY AERODYNAMICS Lect - 8

More detailed discussion on Compressor Blade Design in a later lecture

Next Lecture ----

3-D flow in full mathematical form

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We shall discuss, more of this compressor blade design in more detailed, in a later lecture. And we will bring all of them together in a design package in a step by step design methodology.

In a next lecture, we will look at a full mathematical form of the three-dimensional flow, which is what we are discussing in this lecture. In the last lecture and in today’s lecture, we will continue with the three-dimensional flow, and try to see whether we can have a mathematical formulation; a more comprehensive one then the free vortex radial equilibrium- simple radial equilibrium equation; that we had done. A more comprehensive one, that takes into a count many of the three-dimensionalities that we have talked about in today’s lecture; and we shall see whether we can capture all that, in it mathematical form that is want we will be doing in the next lectures.