Turbomachinery Aerodynamics
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Lecture No. # 07

Three Dimensional Flow Analysis Radial Equilibrium Concept

In the last class, we had an introduction to the three dimensional flow features in an axial flow compression machine. In the last class we of course, talked more about the physics of the three dimensional flow, how the three dimensional flow really develops inside the blade passage even if the flow coming in is relatively uniform and then of course, it goes out with a full three dimensional flow feature. In today's class, we will try to capture some of it in simple mathematical forms. Later on of course, we will look at a little more comprehensive mathematical form to capture the three dimensionality of the flow inside actual flow compressor machine, more notably inside the rotor which is of course, the rotating machine.

Now, as we have seen, the flow inside the blade rows develops three dimensionality. It may not have three dimensionality to begin with, especially in the first stage of an axial flow compressor. In the subsequent stages, it normally goes in with a certain amount of three dimensionality and further three dimensionality may develop depending on the stage design, depending on the how the stages are stacked and then again if you have a multi spool axial flow compressor arrangement. In a multi spool arrangement, the flow going from the low pressure spool to the high pressure spool of a gastro vine engine quite often has a small intervening duct in between which often is designed to straighten out the flow; that means, the flow going into the high pressure spool is somewhat straightened out or made uniform going into the first stage of the high pressure or H p spool. However, it immediately again starts developing large amount of three dimensionality as it goes through successive stages.

So, three dimensionality of flow is a part and parcel of the modern axial flow compressor. The basic theory that we are going to cover today was indeed developed quite some time back, little more than fifty years back and at time, the compressors were not very highly loaded. They were rather lightly loaded and as a result of which certain very simple mathematical formulations were often sufficient to represent what is happening and as I mentioned, later on in this lecture course, we will have another lecture in which we will try to capture a little more modern version of axial flow compressor three dimensionality. But in today's class, we will look at some what the simpler version of the three dimensional flow in simple mathematical form.

So, let us take a look at what are the features of three dimensional flow and how do we go about capturing it in simple mathematical form.

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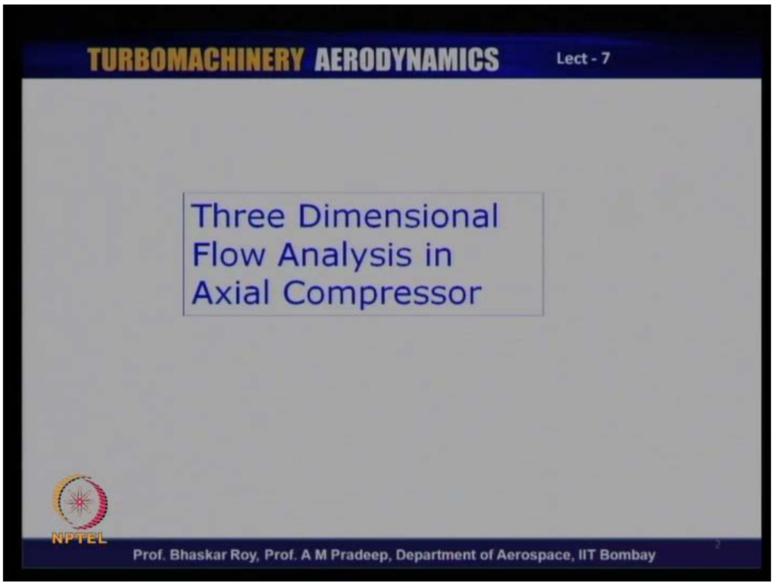

The three dimensional flow features in an axial flow compressor. It is understood that the flow we are talking about is air and not any other fluid. For the movement, we will stick to the fact that we are dealing with air and this air is as you know is very light. So, number of the assumptions can be made with respect to flow of air through axial flow compressors and we will also make those assumptions before we set forth on the mathematical formulations. But we will take a quick look again at the physics of the flow and exactly how do we formulate the mathematical form.

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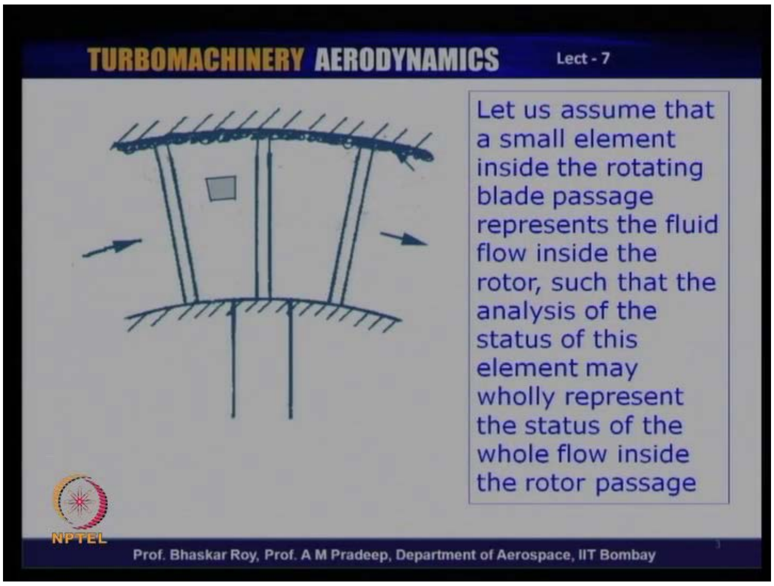

If we take a simple axial flow compressor representation, we have the blades and those blades let us say are rotating in certain directions and that direction is given by the arrows. Some are when these rotating blade passages, we can say we have a fluid element and this fluid element is a representative of the flow that is flowing in this entire passage. We have discussed this passage and what happens inside this passage in some detail in the last class. So, much of that will of course, be at the back of our mind when we go forward and hence we say that this fluid element, a small element is representative of the fluid flow inside this passage. We have seen that this passage consist of one side you have let us pressure surface, another side you have a section surface, on the top you have casing and the bottom you have hub. So, it is bounded by four dissimilar surfaces and so, we have a fluid element inside a bounded passage of four dissimilar surfaces; however, we assume that this fluid element is representative of the status of the entire flow inside the rotor passage. So, when we analyze what is happening to this element, essentially we are trying to capture what is happening inside the entire passage.

So, we will have a look at this elemental fluid or the fluid element and see what happens to this fluid element as a representation of the entire fluid passage and try to figure out how this fluid passages flow or its path through the blade passage can be captured in simple mathematical form. The next thing we would like to remember is that this fluid element is also captured inside the curve blade passage between two blades.

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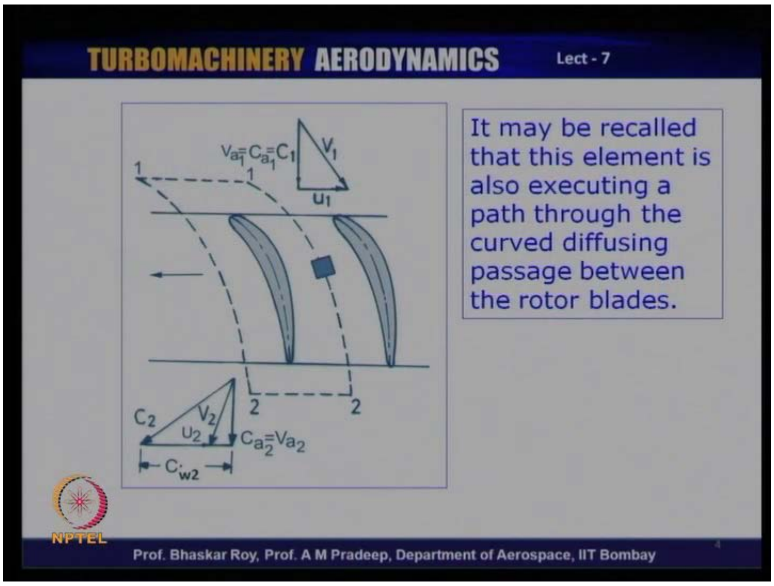

Now the two blades as we just talked about has this is a section surface and this is your pressure surface and as a result the passage in between is actually a slightly defusing passage and of course, it is a curved passage. So, let us say that this fluid element is situated on the mean of this passage which as we have discussed before is indeed actually this curved line is indeed parallel to the camber of the airfoil that is been used here. So, some are in the middle. Let us say that we have a fluid element that is captured and is been subject to certain analysis.

You remember that in the cascade analysis that has been done before; we have a vector diagram in front of the blade. Let us say this is a rotating blade. So, direction of rotation is shown over here and in front, we have a vector diagram which shows all the relative and the absolute velocities coming into the blade and of course at the exit side, we have the absolute and the relative velocities and ` of course, the tangential components shown here. This is what you have done already in the cascade analysis and this particular blade passage is actually part of the domain that is been figured out enclosing one particular blade and this bounded surface is of course, holding the fluid element in the mid passage representing the fluid flow through this blade passage.

So, this is the blade element that we will be talking about and we will try to see how this blade element behaves under certain assumptions or under certain circumstances which we will set forth right in the beginning. So, let us take a look at what are these assumptions.

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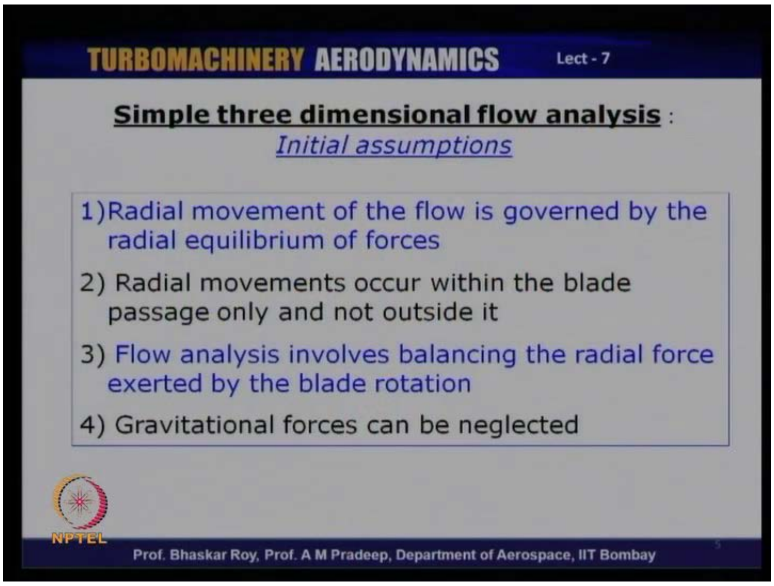

Now these assumptions are created essentially to create what we call a simple three dimensional flow analysis. As I mentioned, we will go into a little more comprehensive three dimensional flow analysis later in this lecture series. Now the initial assumptions are: the radial movement of the flow is governed by the radial equilibrium of forces. Now that essentially means that the fluid element which is captured there actually is in equilibrium in the radial direction; that means, the fluid element which has been captured inside the blade passage has a certain balance of forces in the radial direction and this balance of forces is what keeps the fluid element moving more or less in the actual direction and this is what is meant by the first assumption; that means, radial movement of the flow is governed by the radial equilibrium of forces. The forces are balanced any radial equilibrium must necessarily be in consonants with passage shape that is provided. So, the forces are adequately balanced not to create any radial movement of the fluid element.

So, the radial movement occurs when the blade passage is creating a certain radial flow. For example, we shall see later on that in the modern axial flow compressor, the blade passage because of the high compression ratio. The blade passage is often somewhat converging in nature and that converging passage either the tip is converging or the hub is converging. As a result of this converging, automatically certain fluid element near either the hub or near the tip does acquire a radial component of the flow. Now that is part of the blade passage shape and not part of the fluid mechanic forces. So, at the movement, we are assuming that the fluid mechanic forces; the fluid that is held inside the two blades, that fluid is in equilibrium of forces in the radial direction. So, in the radial direction, all kinds of forces are to be balanced. Now this is the first assumption that we will make that the fluid will be in radial equilibrium.

The other thing is that the gravitational forces can be neglected. Now as I mentioned, we are dealing with air. Air is a very light fluid. So, we can say that, for all practical purposes the gravitational force can be neglected and we can proceed without taking into account the g force which in a very heavy fluid or heavy liquid would indeed be an important issue. But as far as we are concerned, we are going to neglect the gravitational force. Let us take a look at details of the fluid elements.

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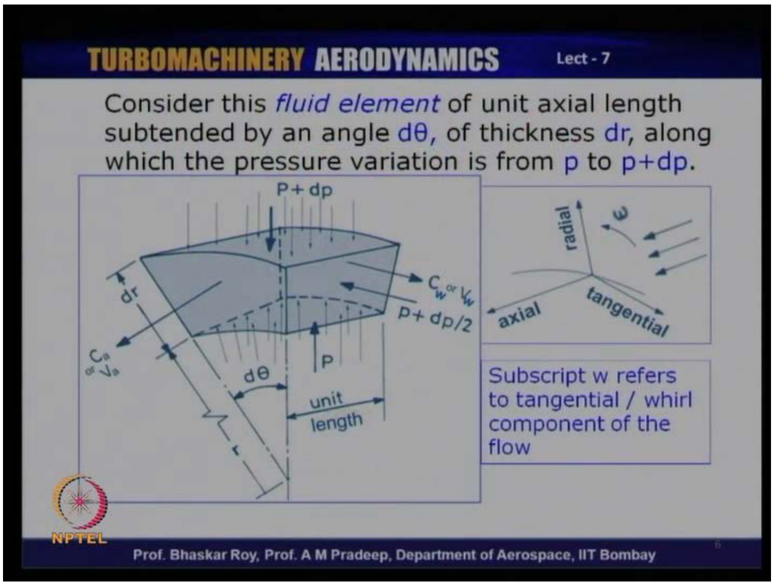

If we look at this fluid element, we can see that it is been created in a manner such that it fits into the blade passage and hence it has a small angle of which is shown by the angle d theta. So, it is not rectangular or a cubic element. It is an element that actually creates an angle d theta from the centre of rotation, but it has a unit length. So, the length of the fluid element is considered to be unity and it has a depth, radial depth of d r along which the fluid element is created and this fluid element is at a distance r from the centre of rotation of the axial flow compressor. So, it at a distance r and it itself has a small radial length of d r and an actual length of unity and circumferential depth of d theta subtended by angle d theta. So, that would be r d theta. So, that would be the width over here.

Now, we say that, if we us this co-ordinate system in which this direction is the actual direction, the upward is of course, the radial direction and the direction in perpendicular to the actual is the tangential direction. So, in this direction, we show that the fluid has a tangential fluid velocity typically given by C w if it a stator blade or V w if it is a rotor blade and the forward velocity is normally given by C a or V a and that is a actual velocity and the forces or the pressure; the fluid pressure that is acting on the surfaces is shown here.

Now on the bottom side, the fluid pressure is said to be or assumed to be p and it increases the pressure through the depth of the fluid element which is d r and on the top surface, the fluid pressure is p plus d p. So, the change of pressure from here to here is d p and on the sides it has therefore, an average pressure of p plus d p by 2. So, that is the pressure that is acting on the two surfaces; on this surface as well as on the surface on the other side and as a result of which these other pressures acting on the fluid element from all its bounded surfaces of the element and this fluid is in motion. It has a motion which is predominantly axial, given by the actual velocity C a or V a and it may or quite often has velocity C w or V w and the subscript w refers to the tangential or what in many books is referred to as whirled component. That is how w comes in. So, it is a whirled component of the flow given normally by C w or V w depending on whether it is stator or rotor.

We shall be mostly talking about the flow through the rotor which means the rotational speed of the blades that contain the fluid element would also be brought into the picture. So, this is the fluid element that would indeed be subject to analysis in this simple mathematical formulation. So, let us try to remember that we are analyzing a fluid element which has pressure on all its bounded surfaces and this static pressure which is acting on this fluid element on all its surfaces has to deal with the dynamic forces due to the motion of the fluid particle or the fluid element which is shown by the velocity C a or V a or C w or V w. So, this is a fluid element which we shall subject to mathematical analysis.

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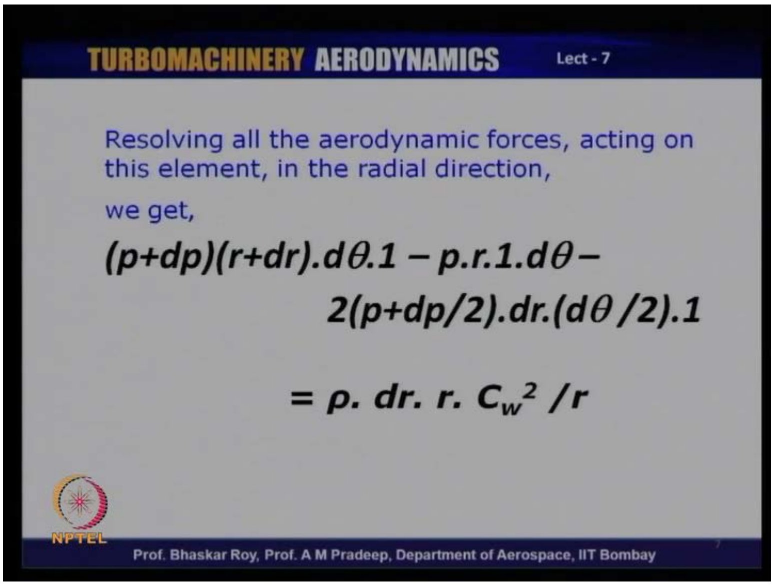

Now if you resolve all the aerodynamic forces and we are dealing essentially with only aerodynamic or fluid mechanic forces and we are not dealing with any of the mechanical issues in this is machine, we are restricting ourselves essentially to the aerodynamic issues. So, we will deal with only the aerodynamic forces and the aerodynamic forces that are acting on this element and we say that the aerodynamic forces that are acting in the radial direction because we had set forth right in the beginning, that we would like to achieve radial balance of forces and hence we take the radial component of all the forces that are acting.

Now, the first term here is the static force that is acting on the top surface of the fluid element which is p plus d p is the pressure and hence that multiplied by the area and that area is r plus d r into d theta into 1 which is the unit length. So, that is the area and this is the pressure acting. So, this whole thing is the force that is acting on the top surface of the fluid element.

Now this force is counted by the force acting on the bottom surface which is simply p into r 1 d theta; r 1 d theta being the area of the bottom surface and p is the pressure acting on the bottom surface. On the other hand, we have seen the two side phases also experience pressure which we have said is the average is p plus d p by 2 and hence and there are a two surfaces; one on this side, one on the other side and hence that is again multiplied by the area d r d theta by 2 into 1; that is area.

So, the component of this is d theta by 2 and that essentially gives us the area of the fluid element which is acting in the radial direction.

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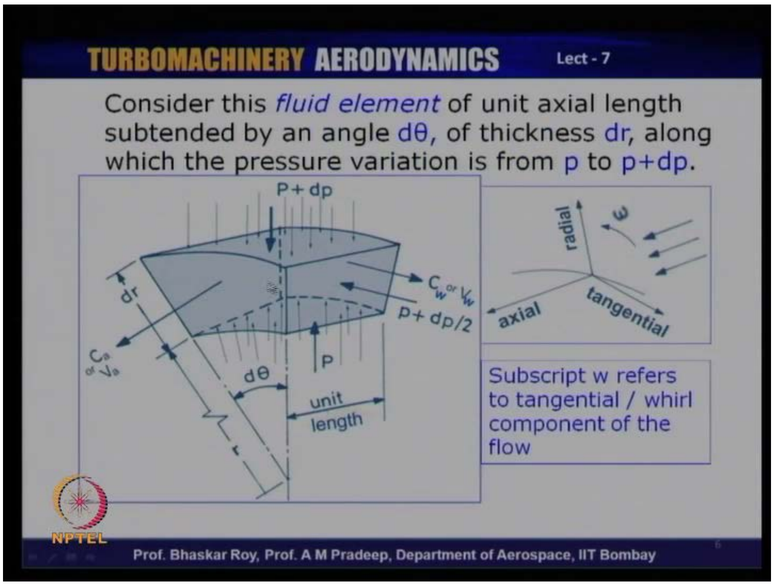

Now, if you look at the other surfaces; this so called front surface and the so called back surface, they are essentially parallel to each other and hence they are vertical and hence there is no radial component of those they in those surfaces are themselves radial and hence they do not contribute any more to the radial balance of forces. They are indeed already in the radial plane. So, if we take these static forces, they are to be balanced by the dynamic force which is the centrifugal force and that is the C w square by r is the acceleration in the centrifugal direction or the radial direction and row d r r is of course, the volume of the fluid element, a row of course, is the density and this and 1 of course, is the unit length. So, this gives us the volume and this is into the density, that is a mass and this is of course, the acceleration in the radial direction. So, that is the force due to the motion of the fluid element in the blade passage which is executing radial force due to acceleration in the tangential direction.

There is no force due to the motion in the actual direction because that is actually said to be in the purely in the actual direction whereas, the one in the tangential direction or whirled component as we call it, would indeed have a radial component of acceleration and that actually provides the dynamic force due to the motion of the fluid element and that has to be balanced by the static forces that are acting all over the fluid element.

So, the static forces and the dynamic force due to the motion are to be balanced and this balance is what we are talking about. This has to be balanced to ensure that this fluid element is in equilibrium of forces so that due to unbalance of this static and dynamic forces, there is no motion that accrues to the fluid element. So, fluid element is not experience any radial motion due to unbalance of these forces; that is the static forces and the dynamic forces. They are to be held in balance by design, by intent of the blade that is created for axial flow compressors. Let us move forward and let us see what happens to this equation.

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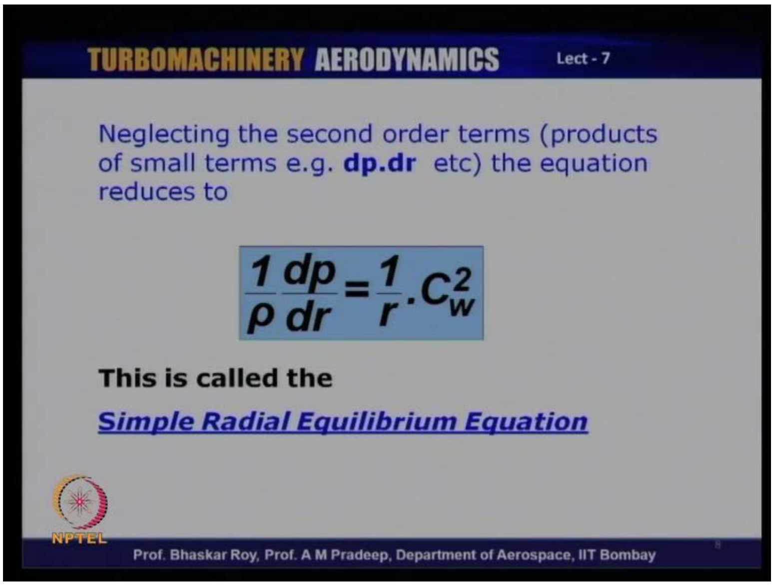

If we look at this equation, we will see that it has a large number of terms that are second order or even higher order which means they are products of small terms like d p d r or d p d theta or d r d theta which are to be neglected. So, the combination of these terms, so, if you look at this in the first, we will have product of d p d r or product of d p d theta or d r d theta; these terms are to be neglected as small quantities.

On the other hand, p r 1 d theta gets balanced by a term which is created here that is p r d theta and they would neglect each other. So, the result is that we arrive at a very simple equation and that is 1 by row d p d r equal to 1 by r C w square. C w square of course, is the square of the whirled component of the velocity of the fluid element. Now this is called the simple radial equilibrium equation. In many of the books that you may study, you will probably find that this is also simply referred to as radial equilibrium equation or sometimes radial equilibrium condition. So, many of the text books that you may look at in axial flow compressors or in a chapter of axial flow compressor, you might find this equation referred to as radial equilibrium equation or radial equilibrium condition. This is born out of the simple balance of forces of the fluid element that we have just looked at. Now this is a radial equilibrium of forces that produces this simple radial equilibrium equation.

Let us go forward and see how we can make use of this radial equilibrium equation for the sake of analysis and design of axial flow compressor flow.

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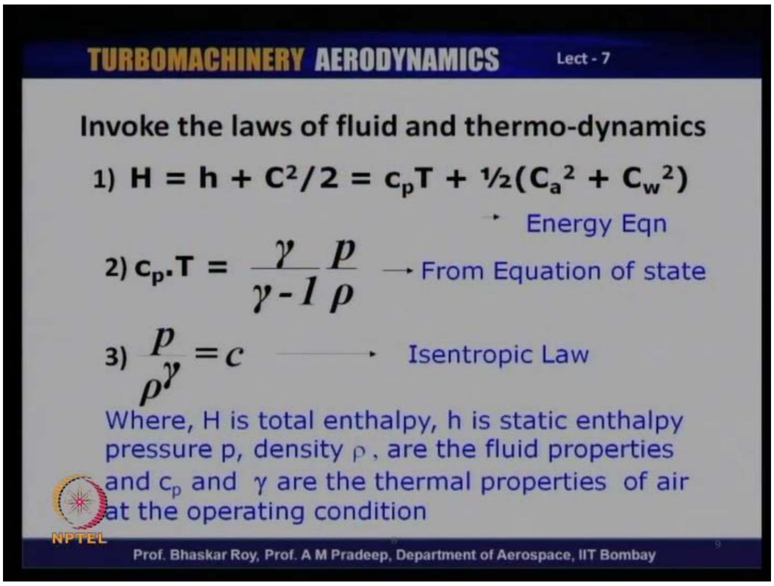

If we look at the fact that this flow is experiencing a number of motions, we capture these motions typically in let us say, three equations which are born out of the basic fluid mechanics and thermodynamics of the flow. The first one is the energy equation which is the total enthalpy is equal to static enthalpy plus the kinetic energy which is captured over here and this c square is the absolute velocity. We can split it into two components; the actual component and the whirl component and we are neglecting because we have assumed that the fluid is in radial balance of forces. Hence there is no C r. If the fluid is not in radial balance of forces, this C over here would indeed have a plus C r square which is the radial component of velocities.

So, at the movement we have assumed that that does not exist. There is no radial motion of the fluid element. C p t of course, is the static enthalpy of the particular condition of the state at which the fluid is in. The second equation comes from the equation of state which is of course, as you know p equal to row r t and that produces a situation where this can be written down, derived essentially from the equation of state and the third equation that we will invoke here is the is the isentropic law; that is p divided by row to the power gamma equal to constant this comes from the isentropic law. We are assuming indeed that the flow through the axial flow compressor is prima facie isentropic in nature. There is no heat transfer in or out of the flow during the process and hence it is essentially to begin with, considered an isentropic process and hence it allows us to prima facie use the isentropic law.

So, these terms and notations are explained over here that each is the total enthalpy, small H is the static enthalpy, pressure is p, density is row which are the fluid properties or the properties of the state and then of course, the thermo dynamic or thermal properties C p and gamma of the air under the given operating condition.

So, we invoke these fundamental laws of fluid mechanics and thermo dynamics on the energy equation that we have looked at and let us see where this transposition takes us to.

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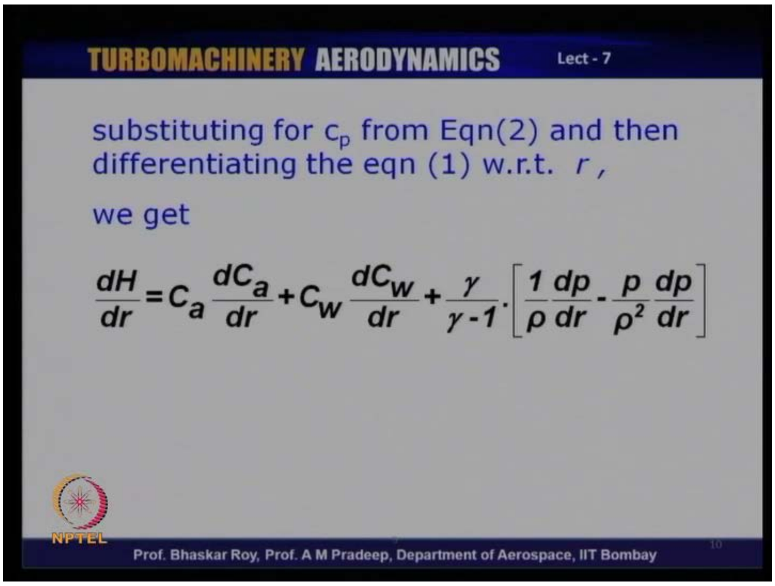

So, invoking these laws of fluid mechanics and thermo dynamics and then if you substitute them what we do is, we substitute this second equation to begin with C p t in the first equation and get a new version of the energy equation by differentiating with respect to r. Now, as I mentioned earlier, we are dealing with what is happening in the radial direction or balance of forces, balance of energy in the radial direction. That is what one of our prime concern at this moment and hence we differentiate equation one with reference to r and that gives us d H d r equal to C a into d C a d r plus c w into d C W d r plus gamma by gamma minus 1 into 1 by row d p d r minus p by row square into d p d r.

So, this is what we get by differentiating, first we replace C p t from here and then we can say that this as a first term and this goes into the second term and then H plus that whole thing is differentiated with reference to r and if we do that this is what we get in terms of the variation of each parameter in the radial direction.

So, this is the enthalpy variation in the radial direction. This is the C a variation in the radial direction, this is the C w variation in the radial direction and this is the p variation pressure variation in the radial direction and this is the density variation in the radial direction. Now this energy equation indeed captures variation of all the important parameters in the radial direction. If you do have access to or you have means to actually calculate the variation of each of these parameters in the radial direction realistically, you can pluck them in here and solve this equation.

Quite often to begin with, you may not have access to the variation in the radial direction of each of these parameters. These are the important parameters; the enthalpy variation, the axial velocity variation, the whirl component variation, the pressure variation and the density variation. Now in a modern axial flow compressor, I might as well mentioned here, in a modern axial flow compressor, it is entirely possible that all the terms on the right hand side would indeed have a tendency to vary in the radial direction and some of the terms may vary quite a lot quite significantly certainly not negligible. With all those variations, can we still keep the variation of enthalpy in the radial direction negligible or to the minimum? That is the question that the axial flow designer or the initial designer analyst would have to contend with and decide to begin with.

So, these are the issues that a typical designer analyst, we will have to decide during the process of the design itself before the designing the blade shapes. So, let us look at the simplest possible thing because as I mentioned, we are doing the simplest possible radial analysis today.

So, we will look at it in the simple possible fashion. So, we say that in this equation, we shall consider the variations of these parameters probably in the most simple way.

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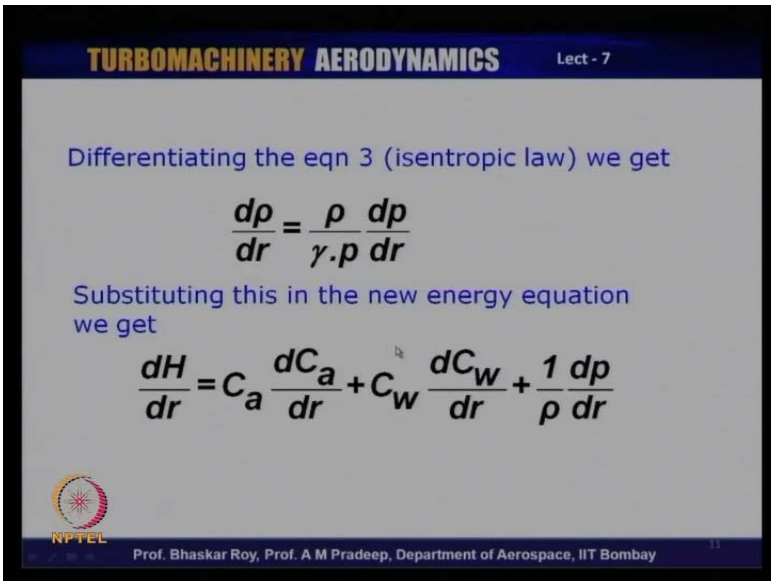

Let us look at what can be done. To begin with, if we differentiate the equation three that was given earlier, that is the isentropic law, in the radial direction again, we get a variation of density in the radial direction and that comes out, in terms of the pressure in the radial direction with reference to the other terms. If we substitute that, the energy equation takes the form d H d r equal to C a into d C a d r plus C w into d C w d r plus 1 by row into d p d r.

So, this is of course, a similar version compared to this one which has a little more variations. So, those variations have been simplified out by invoking the isentropic law and that is what isentropic law allows us to do.

So, this is the first simplification that the flow is isentropic and by invoking by isentropic flow, we have simplified the situation; that is the third term and we are able to put this third term here as 1 by row d p d r. Now this tells us that we have now three variations; the axial velocity in the radial direction, the whirl component in the radial direction and a static pressure in the radial direction and as a result, it assumed that to begin with, density is sort of assumed to be more or less constant. We will do that formally as we go along. Now in the modern axial flow compressor, we will do later on. We shall see that in the first stage, it is possible that the C a variation in the radial direction could be quite uniform to begin with, but in the subsequent stages we shall see later on in fact, we have look at in the earlier last lecture that the C a profile going into the stages can vary substantially in the radial direction. So, that variation of C a would need to be invoked here appropriately to tell you what is happening of the radial balance of forces of the static and the dynamic forces.

So, those are realties that an analyst will have to bring in as and when those realistic numbers or realistic values are indeed made available to him. That may happen in the second round of analyses or the third round of analysis when the realistic values start coming through from the first cut design or analysis. Now let us take a look at what happens to this equation which we have simplified already. If we look at it, we can now invoke the radial equilibrium equation or the simple radial equilibrium equation which was of course, 1 by row d p d r equal to 1 by r into c w square.

Now this can be now be substituted in the energy equation which we are put in the last slide and in the third term, it is a direct substitution and it is not involve a great deal of problem and if we do that, the equation takes the form now d h d r equal to c a into d C a by d r plus c w into d C W d r plus C W square by r. Now C W square by r of course, is the acceleration term which we had talked about earlier and hence we what we see is that all the three are essentially representative of the dynamics of the fluid and the statics of the fluid have essentially now been eliminated from the energy equation.

Now this of cause tell us that the variation of d C a d r and d C d w d r are the prime forces which would have to be brought into the focus if we are to get more realistic analysis, but that as I mentioned, we shall do in one of the lectures later on.

Now, all this together tells us what should be the variation of enthalpy in the radial direction because this enthalpy is what is going is to undergo change as the flow goes through an axial flow compressor because an axial compressor remember is a working machine and this working machine has to work. When it works, the enthalpy of the fluid would change. The very purpose of doing the work is to change the enthalpy or capital h which is the total enthalpy would indeed change. What we are looking at the left hand term is the variation of total enthalpy; that is static plus to dynamic in the radial direction from root to the tip of the blade. How does it change? That is what or how should it be allowed to change? That is what we are contending with at this moment. How should the enthalpy change occur as it passes through the blade from root to the tip of the blade.

So, this is something which the designer analyst would have to decide a priori before he sets forth his design and creation of axial flow compressor. Later on of course, it will have to be analyzed what happens to it under various operating conditions, but those are the issues we should deal with a little later in this lecture series.

So, let us take a look at the most simplified version; that means, we say to begin with, the radial variation of enthalpy is to be deliberately held 0; that means enthalpy in the radial direction would essentially remain constant.

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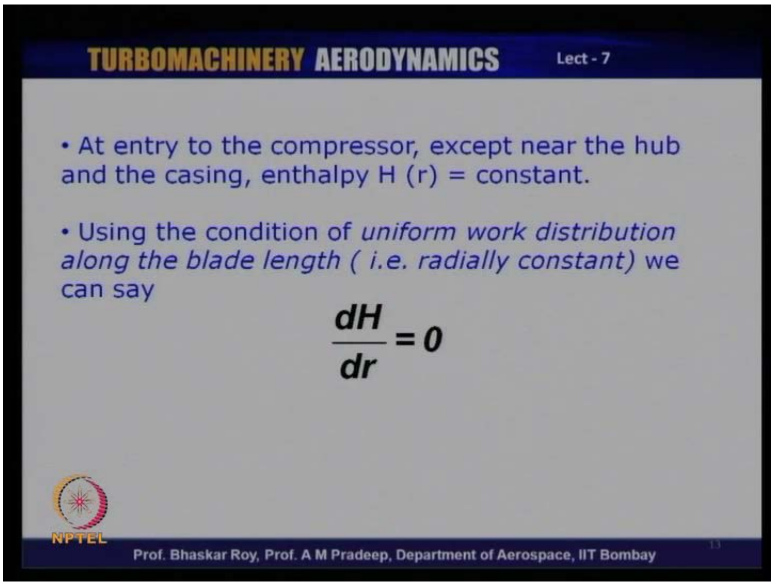

If we do that, we see that the flow actually has a constant enthalpy characteristic and hence the work done distribution also; that means, d H along the length of the blade from root to tip in the radial direction we can say is 0. So, d H d r is to be held 0. This is a deliberate decision that is been taken for let us say, a simplified axial flow compressor design and analysis.

So, that is the first thing we do. We simplify the whole thing and say that the left hand term d H d r is being held 0; that means, the work done is uniform from the root to the tip along the blade length in the radial direction. So, this allows us a certain freedom or certain simplification; that means the work done from root to tip is to be held constant.

So, somehow we have to manage to do equal amount of work at all the radial section from root to the tip of the blade. How do you do that? That is a separate issue, we will come to that and we will do that a little in the next lecture, but today we just make an assumption that the work done is to be held constant along the length of the blade. So, that is the first assumption. That gives us the left hand term of the energy equation now goes 0.

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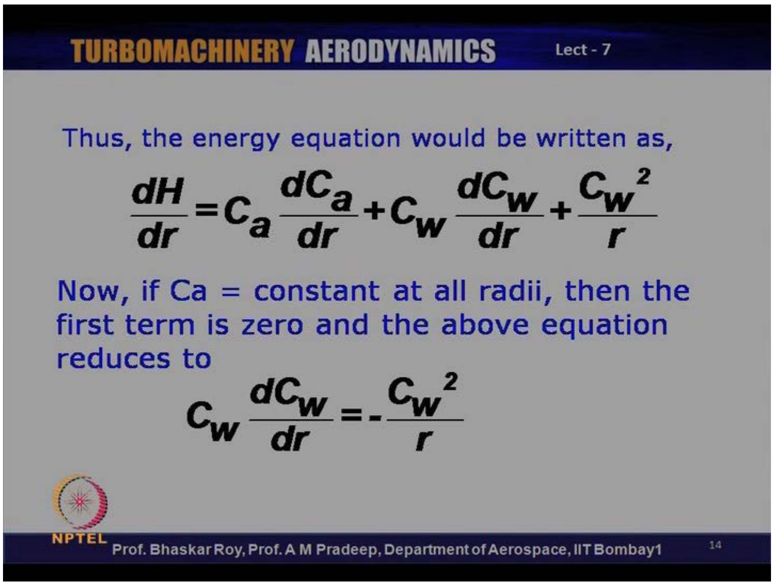

Let us see what happens to that. So, we have. So, we have brought it on the inside. So, d C a d r C w d C w d r and C w square by r is equal to 0 and if C a is held constant already here, then the first term also goes 0 so; that means, the constancy of c a is now to be invoked and if we do that, the first term goes away and we are left with only this term and this term is taken to the other side. So, we have minus C w square by r and hence this becomes now the energy equation or the governing equation of the fluid element that we are looking at.

So, we have simplified at three levels, we have said that d H d r is equal to 0. We are also now saying that c a is held constant. So, d C a d r is also 0 and then we are left with only two terms now and we take the acceleration term to the right hand side. So, it becomes minus C W square by r and as a result of which, we get a very simple situation where we can say that the equation becomes d C W d r equal to minus d r by r. Now this is the simplified version subsequent to number of assumption that we have made. Now remember, those are assumption, simplifying assumptions we are dealing with rather simplified axial flow compressor in which number of assumption have been made, the density is held constant, the gravity has been neglected. We have assumed that the work done or the enthalpy is held constant from root to the tip of the blade along the length of the blade. We are also assuming that while flowing through the blade passage right from the entry to the exits, C a is held constant. It remain constant from root to the tip of the blade at the entry as well as the exist and through the blade passage and when we put all these assumption put together, a simplified fluid mechanic equation that comes through from the energy equation. These are derived with all those assumptions and we get a very simple equation d C w d r equal to minus d r by r.

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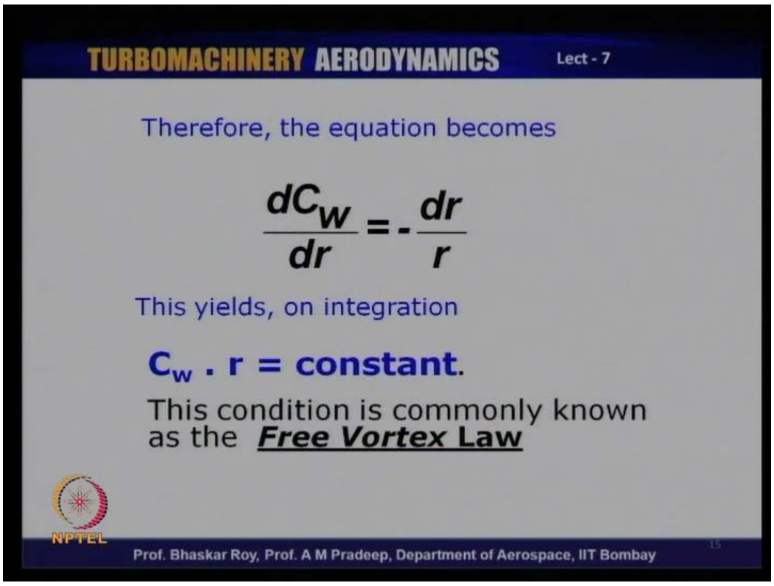

Now this on integration, yields a very simple relation which is C w r equal to constant. Now this of course, is a very famous equation, you would probably see it in many of the books that you may look at and this condition is very commonly known as free vortex law. This is a free vortex law which the axial flow compressor designers arrived at a little more than fifty years back and allow them to design axial flow compressors invoking those simplification. Remember we had a large number of simplifications; thermo dynamic, fluid mechanic, gravity is neglected, density is kind of held constant which is true for you know low pressure compressors which are not very high speed.

So, comparatively low speed. So, all those things taken into account, the reality of the flow has been somewhat simplified and we have taken a fluid element in which certain fluid static forces and dynamic forces were allowed to hold forth along its surfaces. As a result, the balance of those forces in the radial direction finally, tells us that we can say that whirl component c w into r is equal to constant along the length of the blade.

Now, this is a very simple, but very important and useful relation. It tells us what should be the nature of variation of C w along the length of the blade. Otherwise there is no other way you can find out what should be the value of C w at any radius of the blade from root to the tip. C a we have allowed it to be constant, but how about C w? We have no way of figuring out what the value of the C w should be unless we set forth a law like this. We need a law like this to figure out what should be the value of C w and of course, if you do not have C w, you cannot go forward or analyze your axial flow compressor.

So, this is a very important law which was arrived at by the pioneers of the axial flow compressor and it allowed them to actually design and analyze the axial flow compressor (( )) in very simple fashion, but that is all right, but without this we cannot go forward.

So, C w r tells us that with the increase of radius, the value of C w can reduce. So, at the tip, the value of C w would be small. At the hub the value of C w W world be somewhat large. So, did the product of c w and r is constant from root to tip and this is what is called the free vortex law. Now let me explain very briefly, why it is called a free vortex law.

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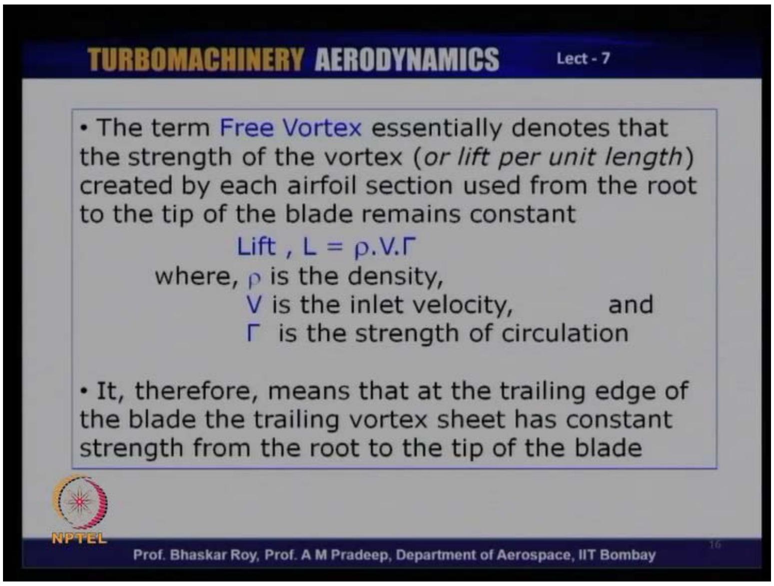

The term free vortex actually comes from the fact that it denotes in some way the strength of the free vortex which as you all know from basic fluid mechanics or aerodynamics, essentially represents the lift per unit length of the aerofoil sections used at that particular radial station.

So, at any radial station, you are deploying normally aerofoil. You have discussed this in the earlier lectures in some detail and these aero foils are essentially lift producing entities. Each of those aero foils creates lift. How do they create lift? They create lift from aerodynamic understanding or aerodynamic theory by creating a vortex system around itself and this vortex system has strength of its own, without the strength it cannot create lift.

So, lift is essentially a direct result of the strength of the vortex system if the vortex system is 0, the lift also would be 0. So, strength of this vortex system is now to be held constant from root to the tip of the blade and hence it is called free vortex so that all the vortex from root to the tip of the blade are of the same strength. Now if you write down in terms of lift from the basic aerodynamics that all of you have done, normally it is written as lift per unit length and that is l equal to row V and a capital of omega and that is a row is density, V is in the inlet velocity or the velocity of the fluid and capital omega is the length of the circulation and this tells us that if the strength of the circulation is held constant, if the density is constant and if the inlet velocity is constant, the lift produced would also be constant from the root to the tip of the blade.

Now, the strength of the circulation is what we are talking about and that is what gives rise to the vortex and this is to be held constant and hence the term was coined as I said more than 50 years back, free vortex; that means, it is free of any change of vortex strength from root to the tip of the blade. This also means that the twilling edge of the blade which typically produces a twilling edge vortex sheet coming out of the blade or any three dimensional body made up of aero foils or lifting elements creates a twilling edge vortex sheet and this vortex sheet would have constant strength from root to the tip or hub to the tip of the blade.

So, it will not have any variation or strength along the length of the blade. Now this is what is meant by free vortex law or free vortex condition and this allows us to create a simple blades which are of immense use to begin with and this free vortex law was one of the first things that needed to be created to analyze and design axial flow compressor rotor blades.

So, today we have set forth a very simply relation born out of the basic energy equation and deployment of the basic thermo dynamic conditions invoking the isentropic law and making a number of assumptions; all of them put together finally, has given us two varies simply things: one is the radial equilibrium equation or what at the moment what we call as simple radial equilibrium equation and consequent to that, we get the free vortex law. This free vortex is something we will go forward with and in the next class, we will look at this free vortex law as a design principle and how various design principles have been created based on this free vortex law and it is derivatives of various laws which are simply called vortex laws again derived from the word free vortex, a terminological free vortex and various vortex laws have been created there off which tells us how the compressor should be designed and further analyzed. So, this is what we will be doing in the next class.

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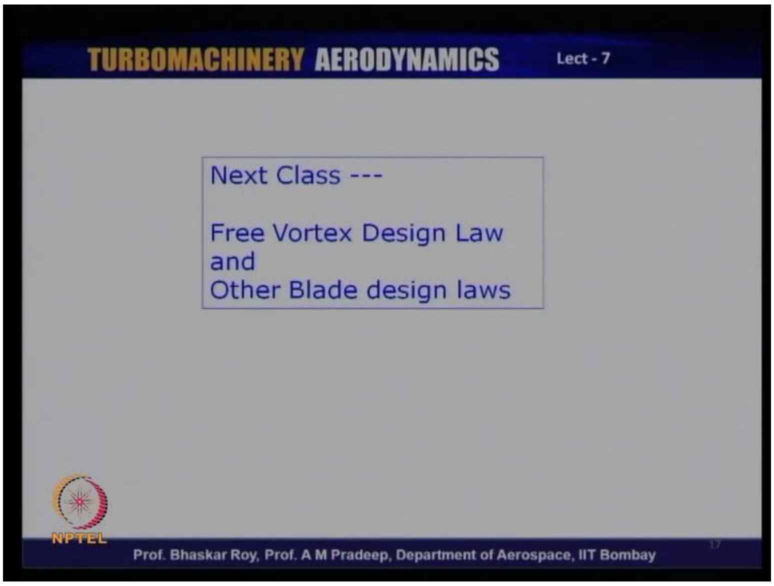

We will be looking at free vortex law as a design law and the other blade design laws which are derived from those free vortex laws and how do they help us essentially in analyzing the axial flow compressor blades. Later on, we will talk about the various design principles.

So, in the next lecture, we will follow it up with the design laws which govern the axial flow compressor, flow through the axial flow compressors.