Turbomachinery Aerodynamics

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Lecture No. # 5
Tutorial 1:
Two Dimensional Axial
Flow Compressors

Hello and welcome to lecture number 5 of this lecture series on turbomachinery aerodynamics. In the last few introductory lectures, we have been discussing about various aspects of turbomachines to begin within the first lecture; and starting the second lecture onwards, we were discussing about axial flow compressors. And we had some discussion quite detail discussion on the two dimensional aspects of axial compressors, and how is it that one can analyze an axial compressor, and from fundamental thermodynamic principals, we have looked that the thermodynamics of the compression process. We have discussed about the simplified versions of axial compressors that is the cascade, and how tests can be carried out on cascades, and what we can make use of or how we can make use of cascade data in terms of analysis.

So, this were some of the topics that we had discussed in the last 3 or 4 lectures, and so it is about time that we now try and utilize what we have learned in the last several lectures in trying to solve some problems. So, in that in mind I have configured a tutorial for you today, where in we are going to discuss about different problems, we will have a few example problems which I will try to solve it for you. Towards the end of the lecture, I will also have a few exercise problems for you to solve on your own, which you can solve based on our discussion during the last few lectures as well as our discussion in today's lecture.

So, today we are going to basically have a tutorial session and we will trying to solve few problems which are related to axial flow compressors.

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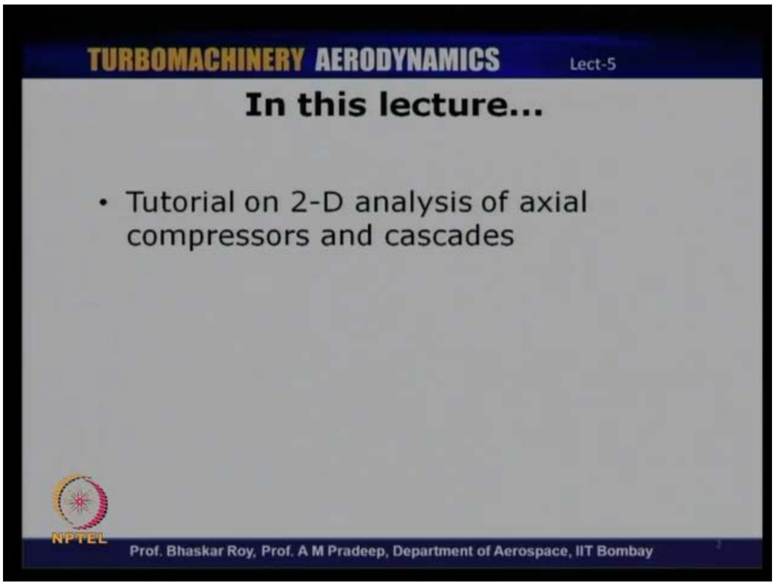

So, today's tutorial is going to be on axial flow compressors and which are also true for cascades, but we will be discussing only the two-dimensional analysis of axial compressors; we will not really be taking up 3-D analysis in today's tutorial. This will be taken up in subsequent tutorial after our discussion on 3-D flows in axial compressors. So, subsequent to this tutorial, we will have few lectures on three-dimensional flows and their analysis and then we will also have a tutorial from 3-D flow analysis.

So, let us take up the first problem which we have and let us see how we can go about solving this problem. So, the first problem statement is the following.

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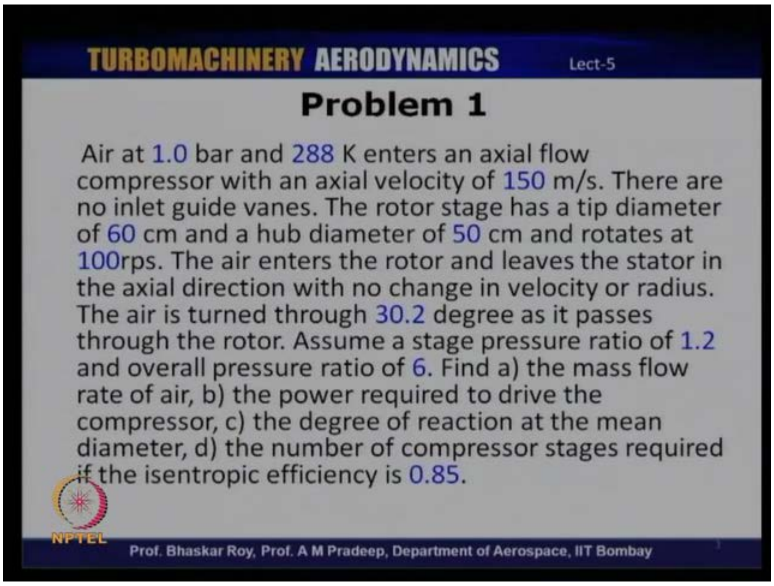

So, here we have an axial flow compressor where in air at 1 bar and 288 Kelvin enters an axial flow compressor with an axial velocity of 150 meters per second. There are no inlet guide vanes. The rotor stage has a tip diameter of 60 centimeter and a hub diameter of 50 centimeter and rotates at 100 revolutions per second. The air enters the rotor and leaves the stator in the axial direction with no change in velocity or radius. The air is turned through 30.2 degrees as it passes through the rotor. Assume a stage pressure ratio of 1.2 and overall pressure ratio of 6. Find the mass flow rate of air, part b is to find the power required to drive the compressor, part c is to find the degree of reaction at the mean diameter, and part d is to find the number of compressor stages required if the isentropic efficiency is 0.85.

So, in this question that you have been given now, if you look at the question or the problem statement carefully, there are several parameters or data which has already been provided to you. For example, the ambient conditions are given, the pressure ratio is given, and the efficiency is given, and the some information about the angle or at least the delta change in angle is also given to you. So, with this information you are required to find a few other parameters like the mass flow rate, the power required, the number of stages and so on.

So, the first step towards solving any problem in turbomachines is to start from the basics. And where does the basic begin? It begins with the velocity triangle. If you record when we had discussion on the velocity triangle, I had emphasize the fact that velocity triangles form a very important and part of the analysis of axial compressor, that is where the design of axial compressor begins with, and therefore it is extremely important that you understand the velocity of triangles and how velocity triangles are constructed because that that we required for any axial flow compressor analysis problem and that is the beginning or the fundamental.of starting.of analysis of axial compressor. So, that is why it is very important for you to realize the significance of the velocity triangles.

So, let us begin constructing the velocity triangle. So, for that how do we begin construction of the velocity triangle? Now we will take up the rotor first and what is given to you is that there are no inlet guide vanes and the inflow is axial. Now when you have an axial inflow and there are no inlet guide vanes it means that the inlet velocity or the absolute component of inlet velocity will be equal to the axial component itself, because there is no guide vane and the inlet flow is entering axially which means that the absolute component of velocity that is C 1 will be equal C a 1 which will be in the direction in the axial direction itself. And then we are given the blade speed, the rotational speed is given it is given as 100 revolutions for second which means that we can actually find out the blade speed U, because rotational speed is known and if the diameter is also calculated you can find the mean blade speed.

So, once you know C a or C 1 and U, you can construct the inlet velocity of the triangle which means that V 1 can be calculated from this. Similarly we move on to the exit. In the exit also we have some information given in terms of the angle with which flow terms, and since blade speed is a constant, and axial velocity is a constant from there we can actually construct the rotor exit velocity of triangle as well. So, if we do this kind of a construction, this is the kind of velocity triangle you are going to get.

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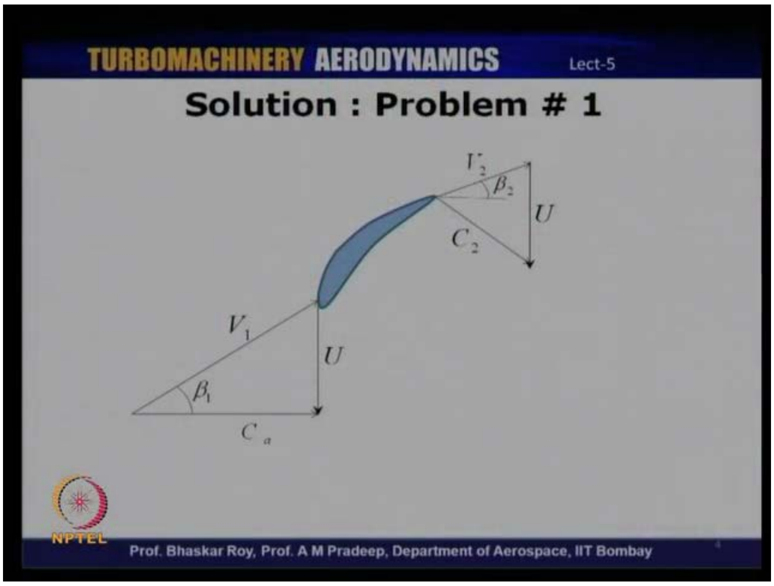

Now, let me explain this velocity triangle once again we already discussed this in the last few lectures, but we will take it up once again. So, what are the constituents of this velocity triangle? So, this velocity triangle here begins with at the inlet of the rotor this is U, this station which is basically station one is the inlet station or entry of the rotor. Exit of rotor of is denoted by this station and so, we have the blade speed which is U and I have denoted a direction for U as this. So, you might wonder how is that blade speed has a direction this way and why is it not that way.

The reason why it is this way is, because the blade moves in this direction, it is a compressor blade, and therefore a compressor blade has to rotate in this direction it does work on the flow. So, this is the blade speed U. C a which is also equal to C 1 is in the axial directions. So, this is the axial velocity which is equal to C 1 and the relative velocity therefore, is resultant of these 2 we get relative velocity V 1 as this. Therefore, the angle which the relative velocity makes with the axial direction is given by beta 1 since U is known, C a can be calculated we can calculate beta 1, and therefore V 1 can be know.

Now, what about the exit of the rotor? At the exit of the rotor, V 2 will still leave the rotor tangentially because that is the property of relative velocity that it has to leave the blade, enter the blade as well as leave the blade tangentially, and therefore if you draw a tangent to the chamber at the trailing edge you get V 2. U is known, you add V 2 plus U, we get the absolute velocity C 2. So, this is how we construct the exit velocity triangle. Angle which V 2 makes with the axial direction is given by beta 2.

So, this is the first step towards solving this kind of a problem that we construct the velocity triangle and that makes the problem solving a lot more easier because it is always possible to still solve a problem without having to… You know construct a velocity triangle and and still attempt to solve a problem it is still possible, but the chances of one making an error in calculation is very high and the fact that construction of a velocity triangle simplifies the problems substantially is enough reason that one should always attempt to solve such a problem with the fundamentals that is velocity of triangle.

So, once you get the velocity triangles right the chances of making error in subsequent calculations are reduced, and therefore I argue that when you attempt to solve such a problem, please make sure that you have the velocity triangle constructed and then continue to solve the problem. So, having constructed the velocity triangle here, now what we will do next is to take up the problem solving one by one. There are 4 different aspects which are to be found which are to be calculated in this problem. We will take them one by one in the process, we are also be calculating several other parameters.

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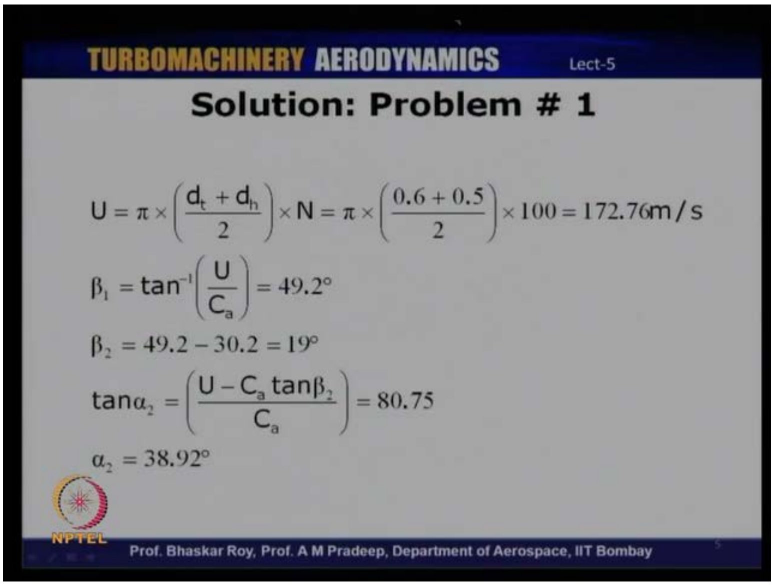

So, let us begin solving this part as well. So, what is given to us is the mean and we given the tip and hub diameters. The tip diameter is given as 60 centimeter and the hub diameter is given as 50 centimeter, rotational speed is given as 100 revolutions per second. So, based on this we can calculate the mean blade speed that is U mean which will be phi d and phi d mean by in to N and here d mean would be d tip plus d hub divided by 2. So, this gives the mean diameter this multiplied by the speed which is in revolutions per second.

Now in some problems you may be given in revolutions per minute in which will case we will have to convert into revolutions per second by dividing it by 60. So, the mean diameter is 0.6 plus 0.5 by 2 which is 0.55. It is multiplied by phi and the speed 100 will give us the blade speed as 172.76 meters per second. So, since C 1 or C a is also given to us we can easily find out the angle blade angle beta 1 which is tan inverse U by C a and that will come out to be 49.2 degrees.

Now it is given in the question that the flow is turned by 30 degrees as it passes 30.2 degrees as it passes the rotor which means that at the exit beta 2 would be equal to 49.2 minus 30.2, because the flow is turned by 30 degrees 30.2 degrees, and therefore at the exit the blade angle would be 49.2 minus 30.2 which is 19 degrees and then we again solve the velocity triangle at the outlet tan alpha 2. Let us go to the velocity triangle, alpha 2 is this angle. So, tan alpha 2 would be U which is this component minus what is left here which is given by C a tan beta 2 and why is it so, because the flow is turned by this angle and C a is this component, C a tan beta 2 is basically this component this part of it .

Therefore, U minus C a tan beta 2 is this and that basically is equal to this rest of it divided by C a will be tan alpha 2. So, beta 2 we already calculated, and therefore we can calculate alpha 2 as the 38.92 degrees. So, we have now solved both the velocity triangles, here we have only beta 1, at the outlet we have beta 2 and alpha 2 and so, we have all the angles required for the velocity triangle. So, once we calculate all the angles we calculated the inlet angle or blade angle beta 1, exit blade angle beta 2 as well as the exit absolute angle that is alpha 2.Having solved all these angles, it will now make it very simple for us to solve for the other components which are not known, and therefore calculate the parameters which we are required to find.

The first parameter which we will try to solve is the mass flow rate. Now mass flow rate as you know is equal to the product of density multiplied by the annulus area through which the mass flow rate is taking place multiplied by the axial velocity. So, row into the annulus area into C a is basically the mass flow rate. Out of this, C a is known, annulus area is known, because the diameters are given, d tip and d hub are given. So, square of the differences between the 2 divided by 4 multiplied by phi gives us the annulus area and this multiplied by density. Now density is something which we will need to find, because we will be given the ambient conditions, we can find the absolute with the density with which the flow is passing through the compressor, and therefore calculate the mass flow rate.

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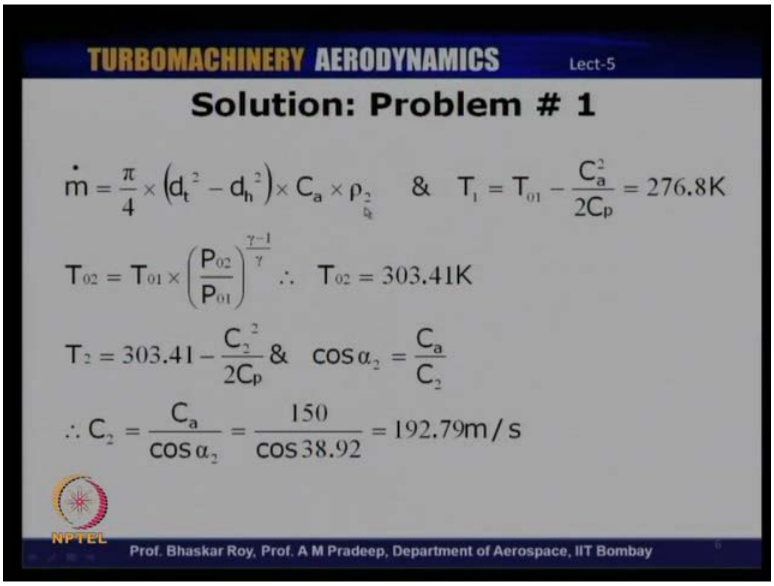

So, let us try to do that now… Now mass flow rate as I mentioned is phi by 4 times d t square minus d h square multiplied by C a into density row 2 or which even otherwise row 1 is also sufficient, we could use either row 2 or row 1, the mass flow rate does not change with that. So, since we have been given the temperatures, we can also find out the corresponding values at the exit of the compressor, and therefore T 1 which we will now calculate as T 0 1 minus C a square by 2 C p. T 0 1 is given already as 288, from there we get 276.8 Kelvin.

T 0 2 is equal to T 0 1 multiplied by P 0 2 by P 0 1 rise to gamma minus 1 by gamma this is; obviously, from the isentropic relations since the other parameters are known, the pressure ratio is also known, on the per stage pressure ratio is also given, we can calculate T 0 2 as 303.41 Kelvin. Therefore, T 2 is 303.41 minus C 2 square by 2 C p and so, here we need C 2 and C 2 is basically cos of C a divided by cos alpha 2 and C a is given as 150 meters per second, alpha 2 we already calculated as 38.92, and therefore C 2 is 150 divided by cos 38.92 that is 192.79 meters per second. So, once we have calculated C 2 we can calculate T 2, and therefore density and; obviously, the mass flow rate.

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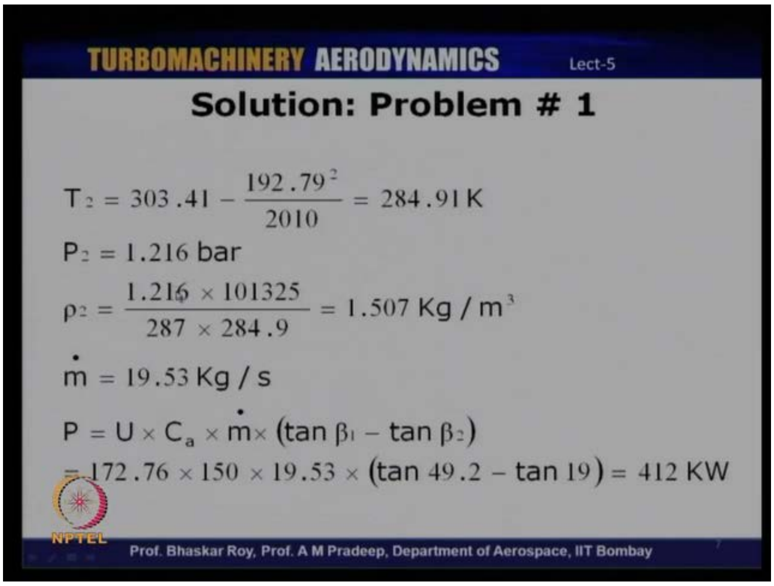

So, let us do that. Therefore, static temperature at the exit is T 2 which is 303 minus.41 minus 192.79 square divided by 2 into C p a that is 1005 that is 2010. So, this is 284.91 Kelvin. P 2 is the pressure ratio multiplied by inlet pressure, we get 1.216 bar. Therefore, row 2 is the pressure ratio which we have converted that into Pascals 1.216 into 101325 divide T by R t and R here is 287 into temperature 284.91. So, the density comes out to be 1.507 kilo grams per meter cube.

So, this if we substitute in the mass flow rate equation, we can calculate mass flow rate we get that as 19.53. So, if calculated mass flows, of course this is one of the ways of calculating mass flow we can calculate in different ways. If we calculate the inlet density you can still calculate the mass flow rate using the inlet density as well and the second component or the second aspect that you need to calculate is the power required to drive the compressor and how do you calculate the power required to calculate drive the compressor. Power required is basically the product of the blade speed multiplied by delta C w.

If you remember when you had done the analysis of the velocity triangles, I had mentioned that delta C w that is the tangential component of velocity is as significant parameter, because that is something which will determine the power that is required to drive such a compressor. Therefore U to multiplied by delta C w is the power required to drive the compressor. Alternatively you can use U times of m into C b into delta t or you can use the pressure ratio to calculate the power required, but the simpler way is to use the velocity of triangle which is U times delta C w.

So, if we look at the velocity triangle, let us go back to the velocity triangle now. So, what is the C w component? Here C w at the inlet is zero. Here there is no C w, whereas at the exit here we have a C w tangential component which is equal to this and so, from the velocity triangle, if we look at velocity triangle and try to simplify the delta C w is basically equal to C a times tan beta 1 minus tan beta 2. Therefore, mass flow rate multiplied by U into C w delta C w which is C a times tan beta 1 minus beta 2 is basically equal to the power required to drive the compressor. So, this is 172.76 which is U, C a is 150, mass flow rate is 19.53 multiplied by tan into beta 1 49.2 minus, tan beta 2 that is 19. So, this gives us the power required to drive the compressor that comes out to be 412 kilo watts.

The third component to find is the degree of reaction. Now if we recall our discussion on degree of reaction that is one of the performance parameters which are used extensively during design of axial compressors. Degree of reaction refers to the amount of pressure ratio that is developed by rotor as compare to the pressure ratio developed by a stage. So, that basically tells us, what is the total fraction of the power, pressure ratio that is developed by the rotor in relation to the entire stage itself.

So, degree of the reaction is something which is very significant in terms of the design of compressors because that comes up as a very significant parameter which will determine, whether the design is feasible or it has certain problem or not. So, degree of reaction, if we recall our discussion on that we had derived an equation for degree of reaction from the fundamental principals which was basically relating degree of reaction to 2 components. One is the axial velocity, then the blade speed and the angles, the blade angle at the inlet and exit. So, degree of reaction is something which you can determine by solving the velocity triangle, and therefore since we already have all the angles associated with this particular rotor, we can easily find out the velocity triangles, and therefore the degree of reaction.

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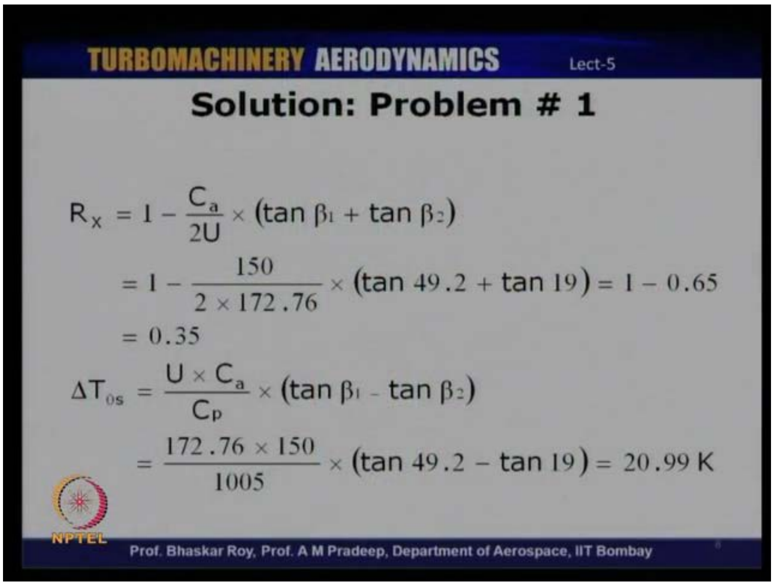

So, degree reaction in this case is 1 minus C a by two U multiplied by tan beta 1 plus tan beta 2. So, if substitute all this values which are already known to us we get 1 minus 150 divided by 2 into U which is 172.76 multiplied by tan beta 1 which is 49.2 plus tan 19. So, this is 1 minus 0.65 that is 0.35. So, here this degree reaction of 0.35 means that the rotor is responsible for 35 percent of the pressure ratio of a stage; that is the rest of pressure ratio that is 65 percent of the pressure rise is actually contributed by the stator. So, rotor contribute by the 35 percent in this particular case and of course, depending upon the design this fraction can be altered and that is why I said degree of reaction is a significant component or significant parameter which is used in design of axial compressors.

So, we have now solved three different aspects of this particular problem, mass flow rate, power required, and degree of the reaction and the last thing that we have to find out is the number of stages that this compressor will require if it has to develop a pressure ratio and certain isentropic efficiency. So, where we are going to solve the problem, this particular part is by first calculating the total temperature rise taking place over the entire compressor and then we will also calculate the total temperature rise in one stage. So, total temperature rise of the compressor divided by total temperature rise for one stage will give us the number of stages that are required for driving this of developing this kind of a pressure ratio. So, we will determine first the total temperature rise across the whole compressor then we will determine temperature rise across the one stage, ratio of that will gives us the number of stages.

So, for one particular stage the temperate rise across the stage that is T 0 s here represents the stage. So, delta T 0 for the stage is basically given by U times C a divided by C p into tan beta 1 minus tan beta 2 and where does this come from? Well this basically is equal to enthalpy rise. So, delta T not multiplied by C p is basically delta h not, which is enthalpy rise. So, enthalpy rise in one stage is equal to U times delta C w which is what is given by C a times tan beta 1 minus tan beta 2. So, from this we get since all this values are already known to us, once we substitute them we get 172.76. C a is given as 150, C p is 1005. This multiplied by tan beta 1 this is 49.2 tan 19, we get delta T not in one stage as as 20.99 Kelvin. That is this is the amount of pressure ratio temperature rise is taking place in one stage.

Now we can also find out the temperature rise taking place across the whole compressor and for that we will make use of definition of efficiency. Now in this case, the efficiency given as 85 percent and from fundamental thermodynamics you might recall isentropic efficiency of a compressor is basically equal to the difference in enthalpy for the ideal compression process divided by difference in enthalpy for the actual compression process and so, that is also expressed in times of temperature, you get T 0 2 prime minus T 0 divided by T 0 2 minus T 0 1. The numerator can be expressed in times of pressure ratio and the denominator is what we need.

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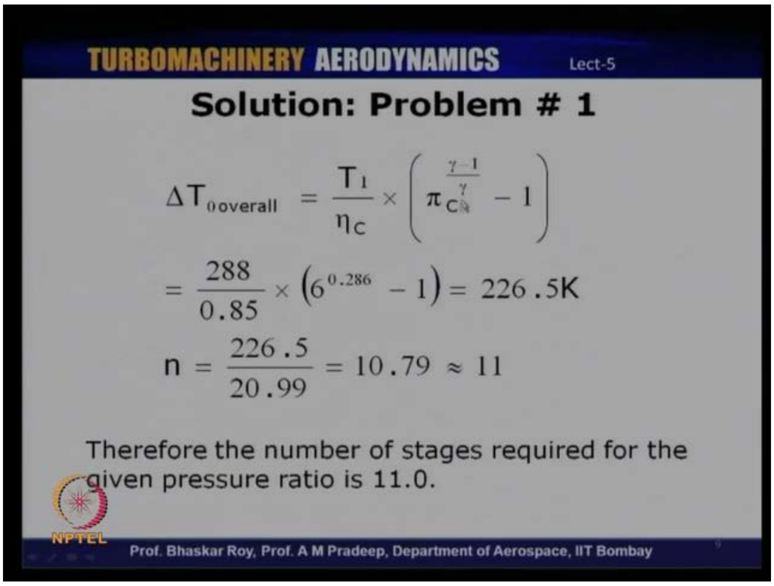

So, what we get is the delta T not overall as T 1 by efficiency, basically this comes from efficiency is equal to delta T not isentropic divided by delta T not overall, delta T not isentropic is also equal to T 1 into phi c which is compressor pressure ratio rise to gamma minus 1 by gamma minus 1. So, overall temperature rise is T 1 by isentropic efficiency multiplied by the pressure ratio phi c rise to gamma minus 1 by gamma minus 1. So, if you substitute all these values, we get delta T not overall has 226.5 Kelvin. So, this is the total temperature rise taking place across the whole compressor if it is developing a pressure ratio of 6.

Therefore, the number of stages for this compressor would be n which is 226.5 divided by 20.99 which is 10.79. This is approximately equal to 11. Therefore, this compressor requires 11 stages to develop a pressure ratio of 6 with an efficiency of 85 percent. So, this brings us to the end of the first exercise problem which we have solve today, which was basically of an axial compressor mean diameter analysis, we found out the mass flow rate, the power required and the number of stage as well as the degree of reaction.

Now what we have learnt from this problem and for from any other problem which we will solve is the significance of the velocity triangles. So, I am I keep emphasizing this every now and then because of the fact that in order to that one understands and develops the ability to analyze and design axial compressors it is essential that one understands the velocity triangles thoroughly, because without understanding velocity triangle it is nearly impossible for us to for anyone to analyze compressor from the fundamentals and also design compressors starting from the fundamental analysis.

Let us move on to the next problem now. Now next problem is again as I said on axial compressors and then we will take a look at what the problem statement is and how we can go about solving this problem.

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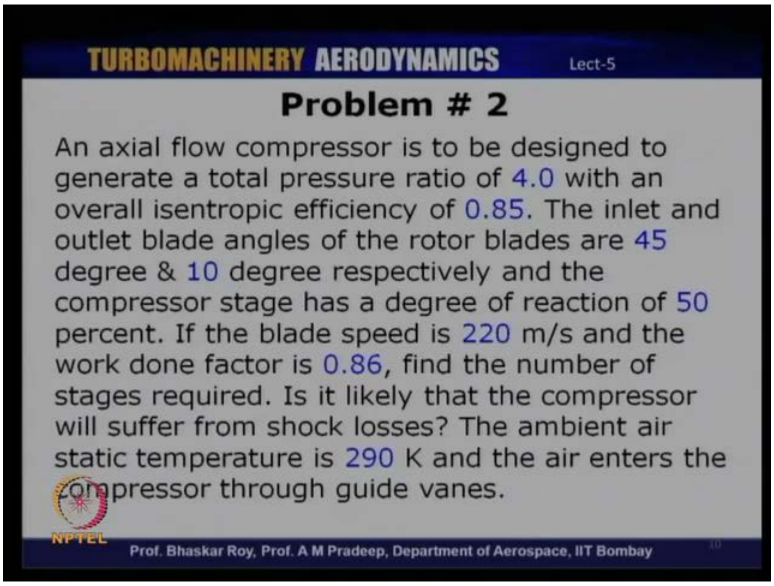

So, the next problem statement is the following. An axial compressor is to be designed to generate a pressure ratio of 4 with an overall isentropic efficiency of 0.85. The inlet and outlet blade angles of the rotor are 45 degrees and 10 degrees respectively and the compressor stage has a degree of reaction of 55 percent. If the blade speed is 220 meters per second and the work done factor is 0.86, find the number of stages required. Is it likely that the compressor will suffer from shock losses? The ambient air static temperature is 290 Kelvin and the air enters the compressor through guide vanes.

So, this is the statement of course, this is slightly different from the previous problem we have solved. one part of it is identical in the sense that we still need to find the number of stages required, but the second problem is quite different in the sense second part of the problem where we are required to find or determine if this compressor will suffer from any shock losses and you might probably notice that there is another aspect which has been described here are given as a data that is work done factor. So, we are not really discussed about work done factor in our discussions in the previous lectures. Now, what do you mean by work done factor? So, work done factor is basically parameter which is taking into account the aspect of blockage. Let me explain this in little more detail because you need to understand what work done factor is.

Now, in a multi stage axial compressor as we move from the inlet, all the way through the stages to the exit what happens is that there is a grow of boundary layer. You might have seen boundary layer development in a pipe flow right that is from the inlet of the pipe all the way to certain distance the boundary layer keeps becoming thicker and thicker till a point it reaches what is known as fully developed turbulent modular and boundary layer extends all the way to the midline or center line of the pipe. A very similar aspect happens here as well, there is a growth of boundary layer the difference here is of course, that there is also an annulus. So, there is a boundary layer growth on the casing, there is also boundary layer growth on the annulus. So, from the inlet to the exit, there is a successive increase in the boundary layer thickness and because of that the effective area that is available for this work generation or work done diminishes by a certain factor, because of this so called blockage which is created by the boundary layer.

So, work done factor is a parameter which is less than 1 in actual compressors, which will get multiplied by the actual work that the compressor is developing that is that much fraction of work which the compressors should have done is diminished, because of this blockage effect by the boundary layer. So, that is where the work done factor comes, we will make use of that in calculating the number of stages. So, in this particular problem again as we have done in the previous cases, we will begin with velocity triangle and we will see what this particular problem at hand is and how we can use at the data that is given to us to solve the velocity triangle and solve the problem in in this process.

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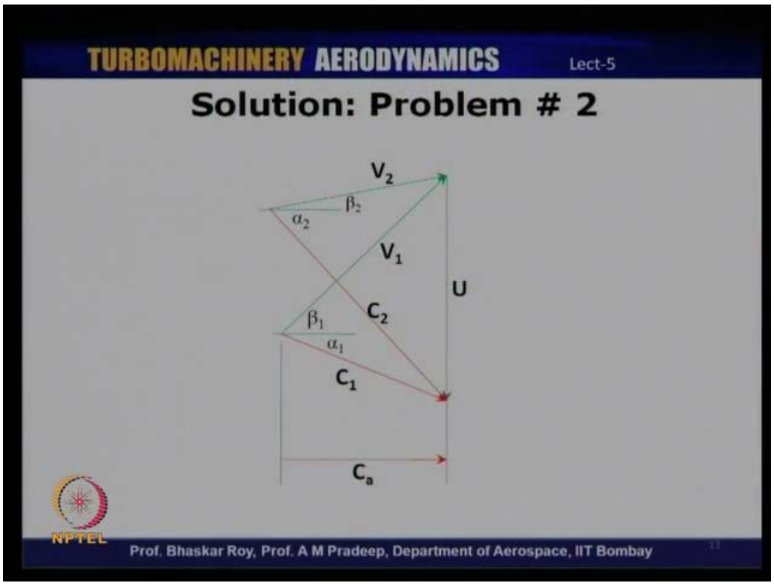

So, the velocity triangle for this case, I have combined the inlet and exit velocity triangles. So, at the inlet we have this velocity triangle which has an absolute velocity of C 1, relative velocity of V 1, angles of alpha 1 and beta 1. At the exit of the rotor, we have angles sorry velocities C 2 and V 2 with angles alpha 2 and beta 2 with the blade speed is U. Axial velocity is given by C a. Now in this question towards the end of the statement it is mentioned that the air enters the compressor through guide vanes. Now the effect of this guide vane is that there is an alpha 1 that is introduced, that is C 1 will no longer be exceeds.

Unlike the first problem which we solved, C 1 was indeed axial because it was it was mention that no guide vanes. Now guide vanes are provided to ensure that the flow enters the rotor in a certain direction and that is why the presence of guide vanes will cause a certain amount of alpha 1, and therefore C 1 is not axial and there is an angle which C 1 makes with the axial direction that is alpha 1. So, this is basically because of the effect or the presence of guide vanes, beta 1 is an angle which the relative velocity makes with the axial direction. C 2 makes an angle of alpha 2 to the axial direction and beta 2 is the angle which the relative velocity at the exit makes with the axial direction.

So, the first part of the problem let we go back to the statement. We need to find the number of stages required for which you have being given the angles, efficiency, pressure ratio, blade speed and the work done factors. So, all the data that you need for calculating the number of stages is already been provided to you and so, that makes it quite easy for us to solve this problem for the first part of it.

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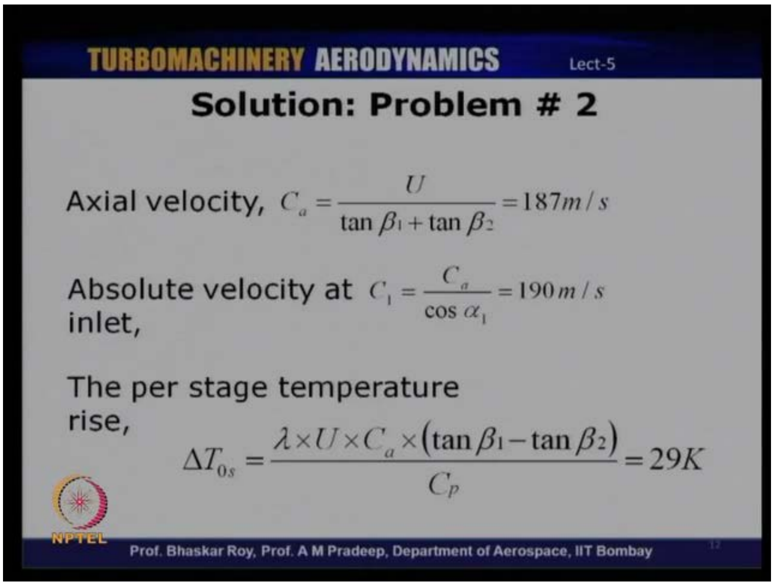

So, let us do that now. Now given the blade speed, we can calculate the axial velocity from the velocity triangle that is axial velocity divided by well axial velocity will be equal to this blade speed divided by tan of beta 1 plus tan of beta 2. Let me take a look at that once again. What is tan beta 1? Tan beta 1 is basically this component C a times tan beta 1 is this and C a times tan beta 2 is the remainder part of it. So, sum of these two will give us the blade speed U. Therefore, axial velocity is basically equal to U by tan beta 1 plus tan beta 2 and both this angles have been given to us, and therefore axial velocity can be calculated as U which is 150 divided by tan beta 1 plus tan beta 2 which is 187 meters per second. We can also calculate the absolute velocity at the inlet which is C 1 that is C a divided by Cos alpha 1. C 1 is this component, alpha 1 is Cos alpha 1 is basically the ratio of C a 2 C 1, C a divided by Cos alpha 1. Therefore C 1 is C a divided by Cos alpha 1, which is 190 meters per second.

Now like in the previous problem, we will first solve, we will first find the per stage temperature ratio, we will then use the isentropic efficiency to the find and the pressure ratio to find the total pressure, total temperature rise taking place over the entire compressor, ratio of these two should give us the number of stages. So, solving this part is rather straight forward identical to what we have solved in the previous question. Only difference here is the presence of a work done factor which was not mentioned in the previous problem and thus therefore, assume to be 0 or to be equal to 1. So, here there is a work done factor which has been specified, and therefore that has to be accounted for in our calculation.

So, the way that work done factor here works is that because of this blockage effect, there is a reduction in the net work done by the compressor, and so that factor gets multiplied to the delta T not part of it. So, instead of U times C a into the tan beta 1 minus tan beta 2 divided by C p which is delta T not for the stage, this gets multiplied by this work done factor which is usually denoted by a symbol lambda. So, lambda times U times C a into tan beta 1 minus tan beta 2 divided by C p is the stagnation temperature rise in one stage.

Now, let us calculate what the stagnation temperature rise is because all these parameters which are required for this calculation is already known to us. So, lambda times U, lambda is the work done factor multiplied by U into C a times tan beta 1 minus tan beta 2 this whole thing divided by C p gives us the stagnation temperature rise per stage and this comes out to be 29 Kelvin.

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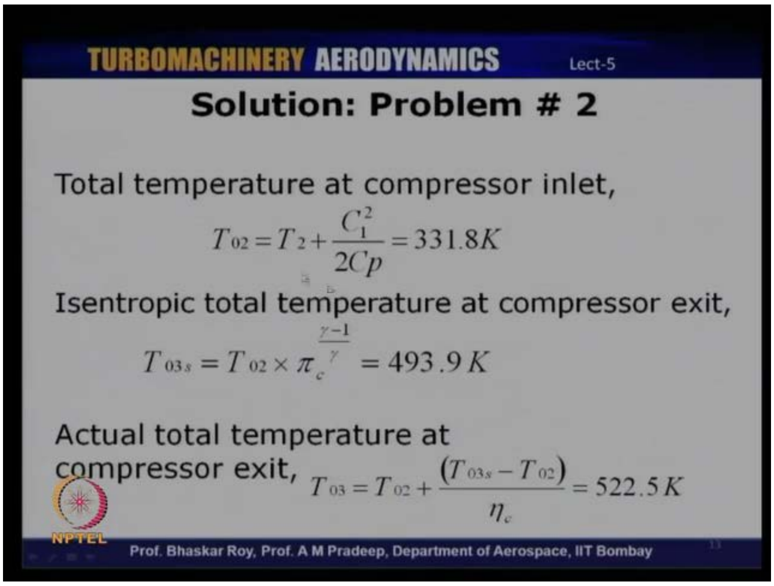

Total temperature rise we need to find the total temperature rise across the compressor and for which we will first find the total temperature rise at the compressor inlet, Which is this static temperature plus C 1 square by 2 C p, and that comes out to be 331.8 Kelvin and the isentropic total temperature at the compressor exit, which is T 03 s or the isentropic total temperature is equal to T 02 multiplied by phi c rise 2 gamma minus 1 by gamma this is 493.9 Kelvin.

Therefore the actual temperature at the compressor exit will be equal to T 02 plus the difference between this and T 02 divided by efficiency, T 03 s minus T 02 by eta c. Now if you carefully look at this equation here, which is basically equal to the efficiency definition, eta c should be equal to T 03 s minus T 02 divided by T 03 minus T 02. So, from that we simplified that for an expression for T 03. So, T 03 we can calculate and that comes out to be 522.5 Kelvin.

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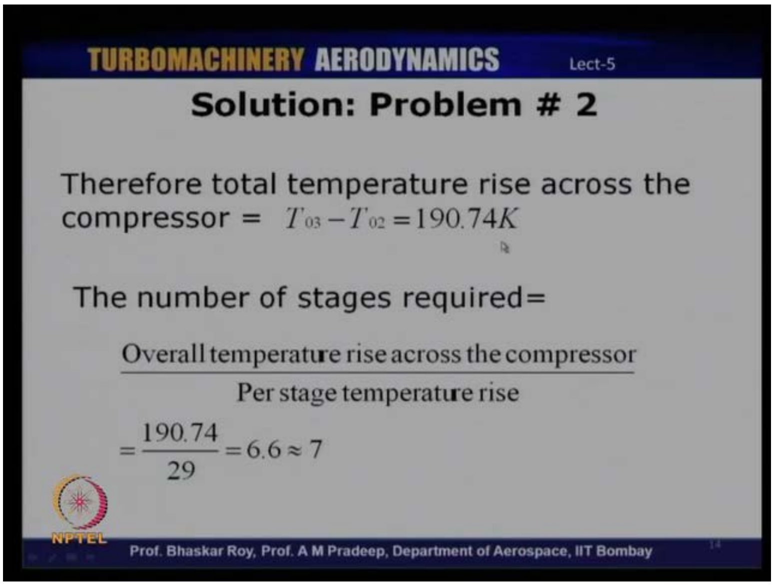

Therefore, the total temperature rise across the compressor is this temperature 522.5 minus 331.8. So, that is 190.74 Kelvin. This is the total temperature rise across the whole compressor. So, therefore, the number of stages required in this case will be the overall temperature rise across the entire compressor divided by per stage temperature rise that is 190.74 divided by 29, this is 6.6 and that has been rounded off to the next integer and that is 7.

So, in this case this compressor requires 7 stages for developing a pressure ratio of 6 and with an efficiency of 85 percent and other parameters as specified in the problem. So, this solves the first part of the problem which was basically to find the number of stages and this is the simpler part because that something we have already solved in the first question.

Now, second part of this question is to determine if this particular compressor is likely to suffer from shock losses as you probably are aware that shock losses are shocks are basically present in supersonic flows that is in the presence of supersonic flows, the likely hold of the presence of shocks are quiet high that is if the flow is in is supersonic then it is very likely that there may be shocks present in the flow. So, that is the basic principle which we are going to use in solving this part of the problem to estimate whether the flow is supersonic or not, that is if the Mach number exceeds 1 then there is chance that could be shocks present in the flow.

Now, here there are two ways of determining the Mach number. One is based on the absolute velocity and the other is based on the relative velocity. So, we basically going to make use of the relative Mach number and determine whether that is indeed greater than 1. If it is greater than 1 then there is a chance that could be shocks present in the flow. So, for this problem we will need to find out the relative Mach number at the tip of the compressor and then compare this and then find out whether this is greater than 1 or not. If it is exceeding 1 then there is a chance that the flow might undergo or might see shocks in the flow.

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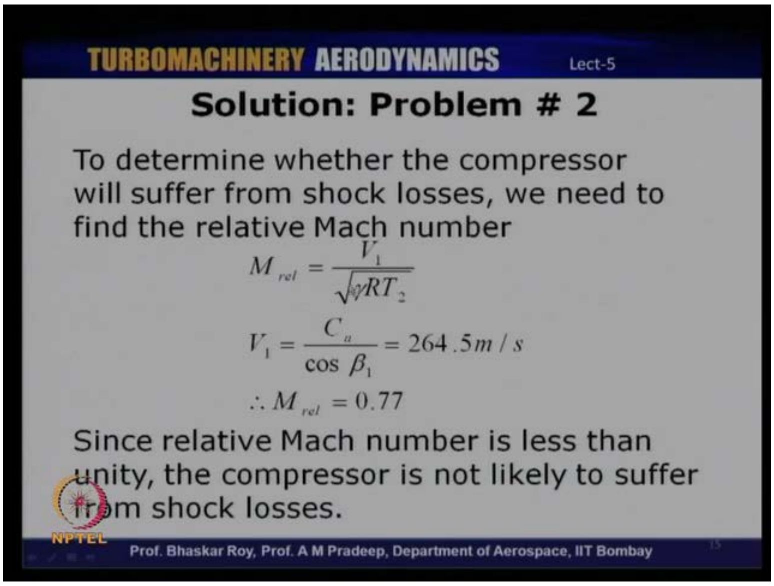

So, let us calculate the relative Mach number and then see if it is greater than 1. So, to determine whether the compressor will suffer from shock losses, we will need to find the relative Mach number. Relative Mach number is the ratio of the relative velocity to the blade speed of sound which is gamma R t, and V 1 from the velocity triangle is C a divided by Cos beta 1 and that is 264.5 meters per second. So, if we take the ratio of V 1 to square root of gamma R t, we get C a divided by Cos beta 1 which is V 1 that is 264.5. So, relative Mach number comes out to be 0.77.

Now, here we get the relative Mach number which is less than 1. So, what it means is that since the relative Mach number is less than 1, the chances that the flow might see shocks are not it is it is not likely to suffer from shock losses for this particular speed that is if we change the blade speed, rotational speed yes probably you might have shock loss, but given these conditions and data it is unlikely that this particular compressor is unlikely to suffer from any shock losses because the relative Mach number has been calculated and comes to be less than 1 - it is only.77.

So, in this particular problem, we have which we had we have two different aspects which we have solved. One is of course, the number of stages very similar to what we did in the first problem. Second part was to find out if there were any shocks present in the flow for which we have used the for which we have basically calculated the relative Mach number and determined whether it is exceeding one or not, in this case it so happens that it is less than one, and therefore the compressor is unlikely to suffer from shock losses.

Now, in some problems it might be specified that you might be given data both at the hub mean diameter and the tip. So, you might wonder under which case you should finding out this relative Mach number. Now if if the data is been given for both hub for hub mean diameter as well as the tip then it is obvious that we need to calculate this at the tip, because it is at the tip that you have the highest rotational speed of the blade speed is the maximum at the tip, and therefore relative mach number at the tip is what one needs to calculate and ensure that and and see if it is exceeding one or not.

In many of the modern, the compressors, the relative Mach numbers can the tip relative Mach numbers can be as high as 1.6 and that is true for many of the so called transonic compressors where the relative Mach numbers are supersonic - this is why they are called transonic. So, relative mach numbers can exceed one and often it reaches as high as 1.6 to 1.8 that is the kind of relative mach numbers that modern day fans, transonic fans and compressors are likely to operate, and we will of course, we discussing lot more details about transonic blades, and transonic compressors, and their properties and how they are different from the traditional low speed compressors, how the blades are different, and how the flow properties etcetera are different. Till we will probably discuss that little later during in some of the later later lectures.

So, we have now so far solved two problems on axial compressors discussing various aspects of the two-dimensional flow and how we can calculate different parameters associated with an axial compressor. So, let us now move onto the third problem which we have in hand and let us see how we can precede towards solving this problem.

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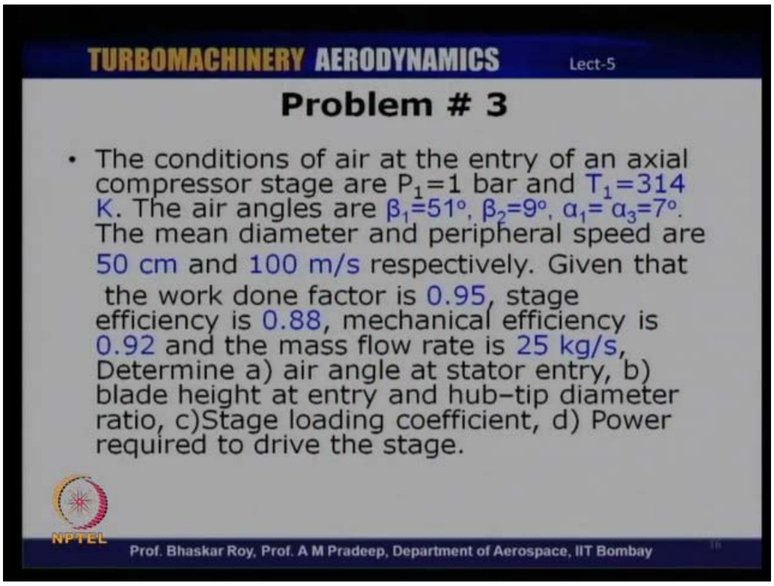

So, we have third problem statement here. The conditions of air at the entry of an axial compressor stage are P 1 is 1 bar and T 1 is 314 Kelvin. The air angles are beta 1 is equal to 51 degrees, beta 2 is 9 degrees, alpha 1 is equal to alpha 3 is equal to 7 degrees, the mean diameter and peripheral speed are 50 centimeters and 100 meters per second respectively.

Given that the work done factor is 0.95, the stage efficiency is 0.88, the mechanical efficiency is 0.92 and the mass flow rate is 25 kg per second. Determine part a - air angle at stator entry, part b - blade height at entry and hub to tip diameter ratio, part c - the stage loading coefficient and part d - power required to drive the stage. So, this is a problem where we have lot of data which is given to us, all the data required for determining the velocity triangle like all the angles are been given to us beta 1, beta 2, alpha 1, alpha 3 all the angles are been given to us, we have some of the velocity the peripheral velocity or U has been given to us, we have the mean blade diameter and also the temperature and pressure at the inlet. You also have the work done factor and efficiency. So, with those we need to find out the host of other parameters like the air angle at the stator entry and and work power required and so on stage loading coefficient etcetera.

Now, in this problem as before, we will have to draw the velocity triangle first. So, I am assuming that by now you have understood how to construct a velocity triangle, and so I am skipping that part I am leaving it to you to find the velocity triangle for this particular problem. I am sure based on what we have discussed for the previous two cases you will be able to calculate determine the velocity of triangle for this case. So, I am not solving the velocity of triangle right now, I am leaving that as an exercise for you. I will straight away go ahead and solve the problem for you assuming that you can construct the velocity of triangle on your own.

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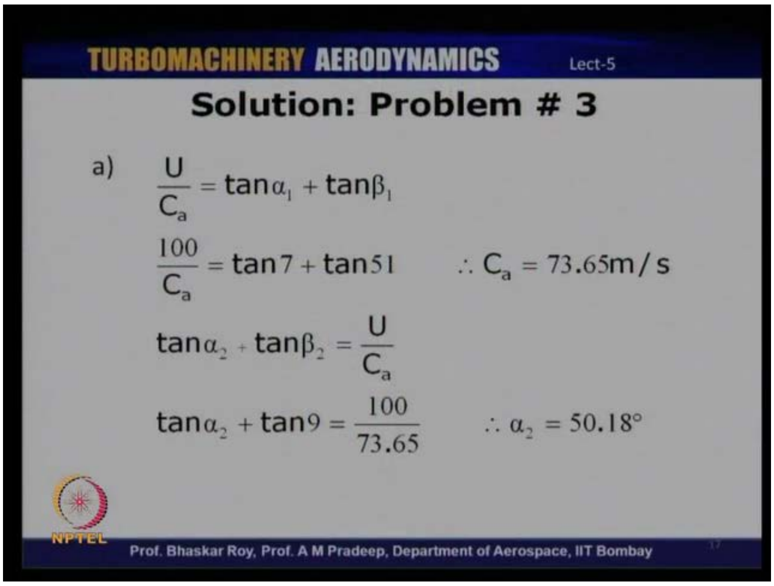

The first part of the problem is to find the air angle at the inlet which is alpha 2 at the stator entry basically for which, once you have constructed the velocity of triangle it makes it very simple to solve. From the velocity of the triangle you will see that the ratio U by C a should be equal to tan alpha 1 plus tan beta 1. This is basically because U is equal to C a times tan alpha 1 plus C a times tan beta 1, alpha one is known, beta 1 is also known and U is known. So, from this we can calculate C a, which comes out to be 73.65 meters per second.

Now, since C a known from the velocity of the triangle again we have tan alpha 2 plus tan beta 2 again is equal to U by C a. Since beta 2 is known and this ratio is also known, we can calculate alpha 2 which is the angle at the stator inlet, alpha 2 is 55.18 degrees. So, this is so, as I mentioned if you have the velocity triangle right this is very straightforward you can very easily find out alpha 2.

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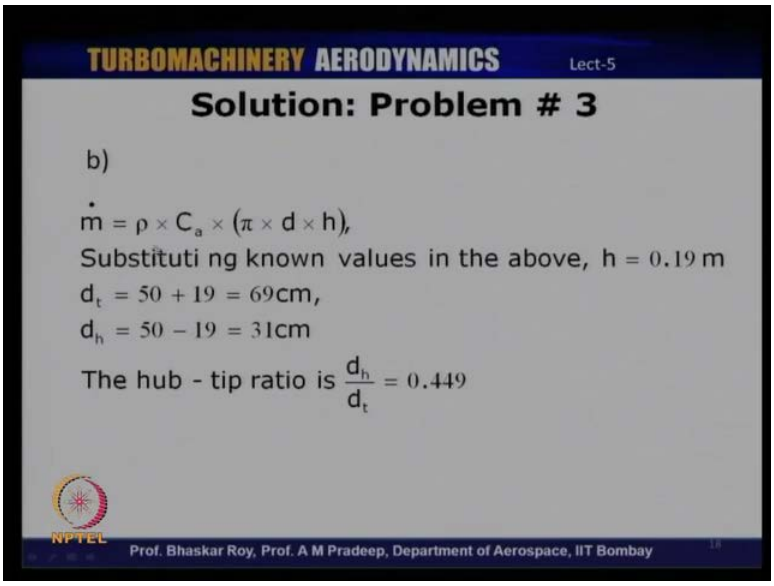

The second parameter to find is the hub to tip diameter ratio. Now we know the mass flow rate it is given as 25 kg per second mass flow rate is given as twenty five kg per second. This is basically equal to density times C a times phi times mean diameter into the blade height h. So, mass flow rate is known, density is known, because P 1 is known, T 1 is known. So, T 1 by R t 1 gives us density which is row 1. C a is known and diameter is known, mean diameter and the blade height from this can be calculated, because from mass flow rate density, hub diameter is mean diameter minus d blade height and that is 31 centimeters. So, the tip diameter is 69 centimeters, at the hub diameter is 31 centimeters, and therefore the hub to tip diameter ratio is 0.449.

The other way to find us probably is to use this directly in the mass flow rate formula by using the annular area where this ratio will appear as one of the fractions. So, that is another way of finding the hub to tip diameter ratio which is basically boils down to the same thing, mass flow rate is density times axial velocity into phi into annular areas that is dt square minus dx square by 4. From which we can take a ratio and find out the hub to tip diameter ratio. So, here it comes out to be 0.449.

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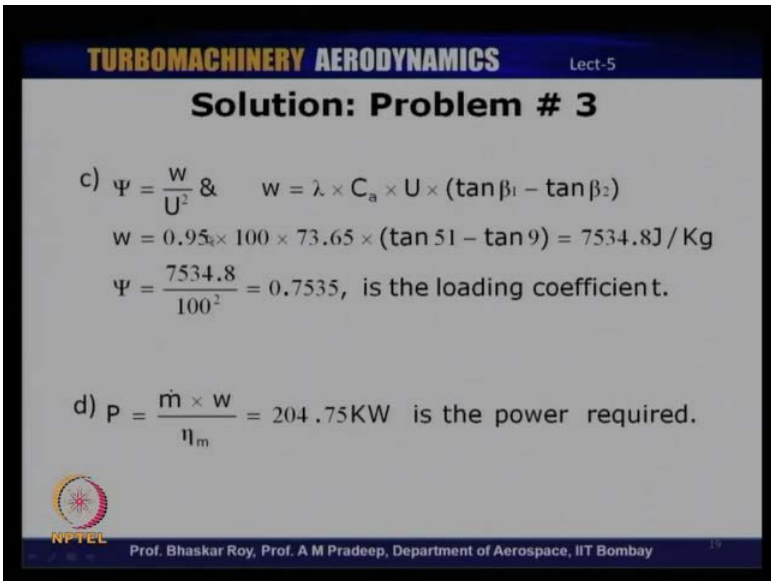

Now the third part of this problem is to find the loading coefficient. Blade loading coefficient is the work done by the blade divided by U square. So, work done by the blade as we have seen is lambda times C a into U into tan beta 1 minus tan beta 2 this is basically the work done which is the enthalpy rise across the blade delta h not multiplied by the work done factor will give us the net work done. All these parameters are known to us now. So, that the work done per unit mass is 0.95 into axial velocity 100 blade speed 73.65 into tan beta 51 minus tan beta 2 which is tan 9. So, this is 7534.8 joules per kilogram. Therefore the loading coefficient is the work done per unit mass divided by U square which is 100 square that comes out to be 0.7535 this is the loading coefficient of this particular geometry.

And the last part of course, is to find the power required which is very simple now because that the mass flow rate multiplied by the work done per unit mass divided by the mechanical efficiency. So, in this question it is also mentioned that there is a certain mechanical efficiency. So, mass flow rate times the work done per unit mass gives us the power divided by of course the mechanical efficiency, because the mechanical efficiency is basically an indicator of the amount of power loss taken place between the turbine and the compressor.

So, the turbine is the one which provides the work done work to the compressor and so, because of various losses taking place like frictional losses and so on, there is certain amount of power loss taking place and that is part is given by the mechanical efficiency. So, the actual power required by the compressor will be higher by this amount of mechanical efficiency. So, mass flow rate multiplied by work done per unit mass this is divided by mechanical efficiency gives us the power required.

So, we are now solved all the aspects required for solving this problem and I hope you will be able to calculate and determine the velocity triangle and use that in solving this problem. So, with this background of what we have discussed in today's lecture and also what we have been discussing in the last few lectures I have a few exercise problems for which you can solve based on our discussion.

So, let me take you straight to the first exercise problem.

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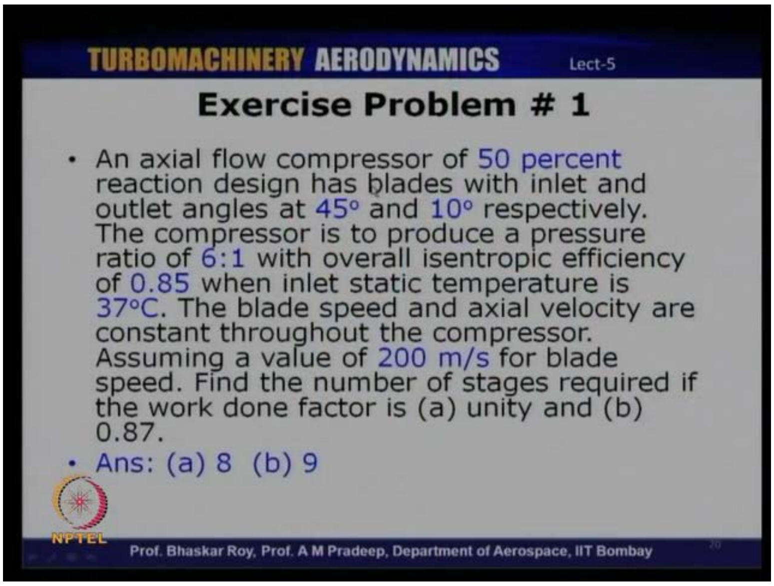

First exercise problem is an axial compressor of 50 percent reaction design that is the degree of reaction is.5 has blades with inlet and outlet angles of 45 and 10 degrees respectively. Compressor is to produce a pressure ratio of 6 is to 1 with an efficiency of 0.85 when the inlet static temperature is 37 degrees Celsius. The blade speed and axial velocity are constant throughout the compressor assuming the value of 200 meters per second for the blade speed. Find the number of stages required if the work done factor is part a unity, and part b - 0.87. Answer for this is part a is 8 stages if the work done factor is unity with the work done factor of 0.87. You need more stages to generate the same amount of pressure ratio that is 9 stages. So, this is the first exercise problem.

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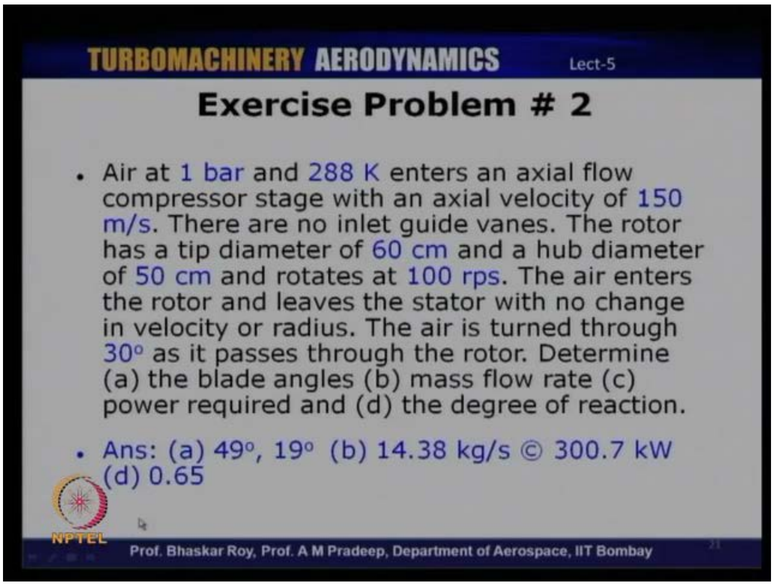

The second exercise problem is air at 1 bar and 288 Kelvin enters an axial flow compressor with an axial velocity of 150 meters per second. There are no inlet guide vanes. The rotor has a tip diameter of 60 centimeter and hub diameter of 50 centimeter and rotates at 100 rps. The air enters the rotor and leaves the stator with no change in velocity or radius. The air is turned through 30 degrees as it passes through the rotor. Determine the blade angles, mass flow rate, the power required and the degree of reaction. So, this is very similar to one of the problems which we have solved today. Answers for this are 49 degrees, 19 degrees, mass flow rate of 14.3 kg per second, power required of 300.7 kilo watts, degree of reaction 0.65.

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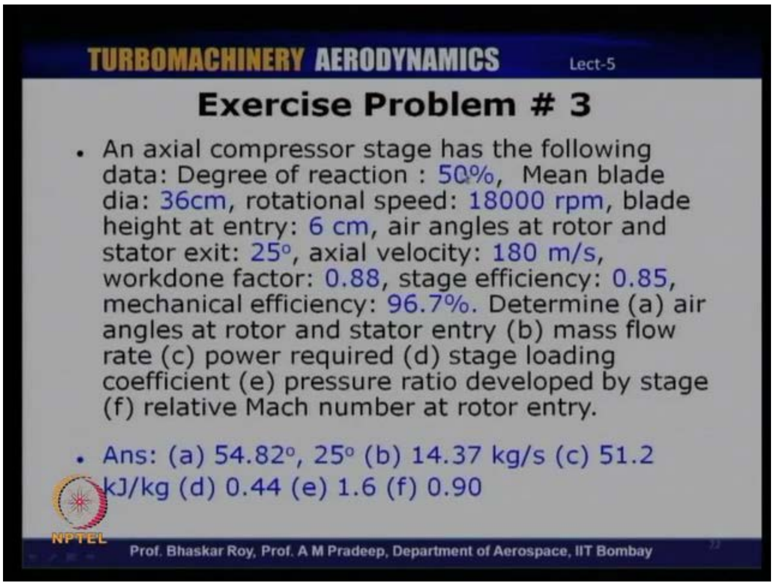

Third problem is axial compressor stage has the following data: Degree of reaction of 50 percent, mean blade diameter of 36 centimeter, rotational speed 18000 rpm, blade height at the entry of 6 centimeter, air angles at rotor and stator exit at 25 degrees, axial velocity 180 meters per second, work done factor 0.88, stage efficiency 0.85, mechanical efficiency of 96.7 percent. Determine the air angles at rotor and stator entry, mass flow rate, power required, the stage loading coefficient, pressure required pressure ratio developed by the stage and relative Mach number at rotor entry. Answers are the following: 54.82 and 25 degrees, part b is 14.37, part c is 51.2 kilo joules per kilogram, part d is 0.44, part e is 1.6 and part d is 0.9.

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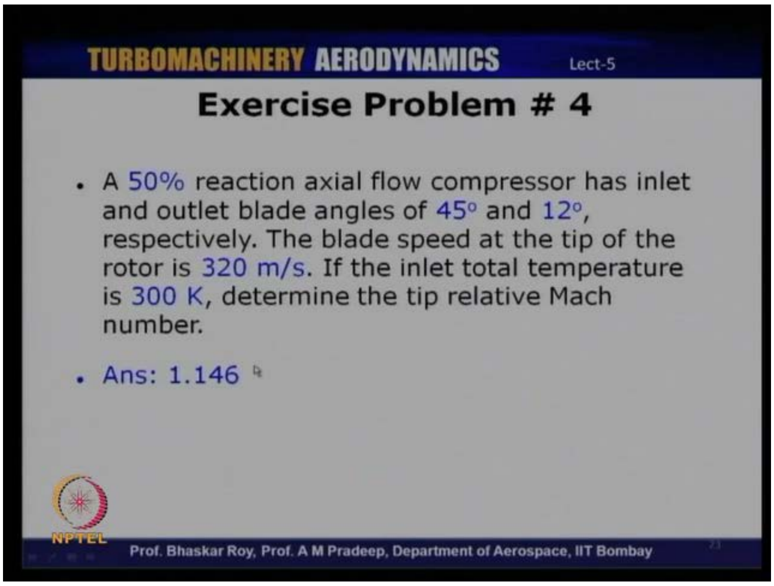

Fourth problem is a 50 percent reaction axial flow compressor has inlet and outlet blade angles of 45 degrees and 12 degrees respectively. The blade speed at the tip of the rotor is 320 meters per second. If the inlet total temperature is 300 Kelvin, determine the tip relative Mach number. In this case the Mach number is 1.146.

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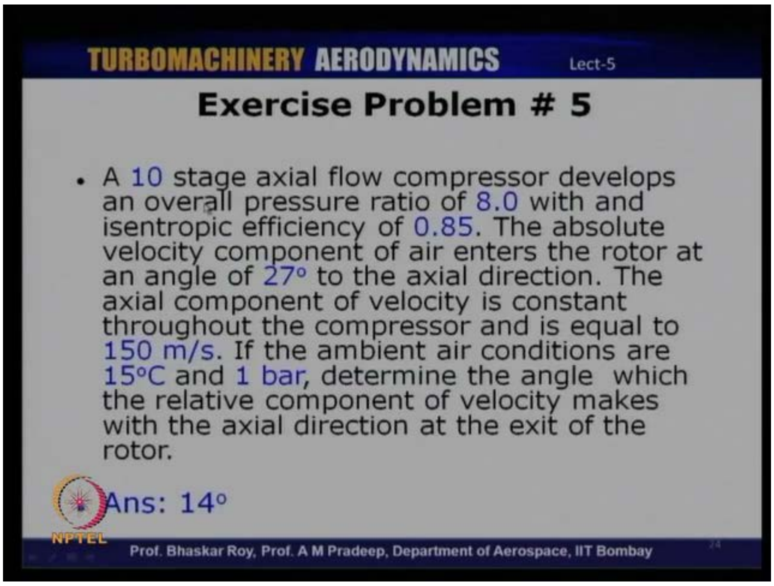

And the last exercise problem is a 10 stage axial flow compressor develops an overall pressure ratio of 8 with an isentropic efficiency of 0.85. The absolute velocity component of air enters the rotor at an angle of 27 degrees to the axial direction. The axial component of velocity is constant throughout the compressor and is equal to 150 meters per second. If the ambient air conditions are 15 degrees Celsius and 1 bar, determine the angle at which the angle which the relative component of the velocity makes with the axial direction at the exit of the rotor. So, in this case the angle is 14 degrees.

So, these are a set of 5 exercise problems which you can solve hopefully based on our discussion during today's lecture as well as the theory lectures which we had during the last 3 or 4 lectures. So, with this I would like to conclude the today's tutorial session and I hope you have had now some insight into how you can go about solving problems related to axial flow two-dimensional analysis of axial flow compressors and cascades and hopefully you will be able to use this knowledge based on the discussions in the last few lectures and today's lecture to solve problems and take up preliminary design of at least the two dimensional case of axial flow compressors and so, that brings us to the end of today's tutorial session, we will continue discussion on axial compressors and three-dimensional flow through axial compressors in future lectures.