Turbomachinery Aerodyanmics

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Lecture No. # 36

Turotial 6: Radial Turbines

Hello and welcome to lecture number 36, of this lecture series on Turbo-machinery Aerodynamics. In the last class we had introduced a very new topic, which is quite different from what we have discussed in some sense. That is, on radial flow turbines. Of course, we have already discussed its counterpart centrifugal compressor. But, in the context of turbines, radial flow turbines operate in a quite different manner as compared to the axial flow turbines.

So, last lecture was exclusively devoted towards understanding of radial flow turbines, the basic working of radial flow turbines and the thermodynamics of radial flow turbines. That is, as the flow passes through radial flow turbine, what in a Thermodynamics sense happens to the flow, as it passes through the different components of the turbine? We have discussed different types of radial flow turbines, which are possible. The two broad classes are the outward flow radial turbines and the inward flow radial turbines. We have seen that though the development, the original development of radial flow turbines was in the outflow mode, very soon it was realized that the inflow mode or the inflow radial flow turbine is a much more efficient way of managing the flow, and achieving work output than the outflow type of turbine.

So, majority of the turbines, which are being used currently are inward flow radial turbines. The radial turbines as a concept, was originally developed towards meeting the hydraulic power requirement, that is, to convert hydraulic power into work output. And then, that continued for a very long time. In fact, it even continues till date. And, the hydroelectric power plants that we are aware of use one or more types of radial flow turbines. Of course, some of them also have axial flow types, but apparently radial flow turbines are very commonly used. And, the use of radial flow turbines for gas turbines was much later; that it was also being considered as one of the options for use in gas turbine engine.

And, the modern day gas turbine technology has restricted the use of radial flow turbines. It is primarily used in the smaller engines, smaller thrust class engines. And, it is the main disadvantage, when it comes to use of radial flow turbines; for gas turbine applications, is the fact that in order to achieve very high efficiencies and power output, the turbine performance is a strong function of the inlet temperature. And in axial turbines, we have already discussed that modern day gas turbines use different cooling technologies, which can be used to, which can actually permit us to use turbine…temperatures, which are much higher than their material limits.

This is not. It is still possible in radial flow turbines, but it is it is much more complicated to introduce and use some sort of cooling mechanisms in radial flow turbines. There are complicated methods, which are being proposed and probably being used in some types of engines. And, this is one major disadvantage. While radial turbines are not really used in gas turbine technology, in gas turbine engines, for larger thrust class engines because the turbine inlet temperature is sort of restricted.

We saw that there are two different types of inward flow turbines: the cantilever type and the 90 degree inward flow turbines. And, we discussed in the previous class. We devoted most of the time towards discussion on the 90 degree inward flow turbine because that is more commonly used.

The cantilever type turbine is very similar to the impulse turbine that we have already discussed in the axial turbine context. And therefore, there are certain disadvantages associated with impulse turbine as we have already seen. And, so the inward flow, 90 degree inward flow turbine is like reaction turbine. And, in geometrical terms, a 90 degree inward flow turbine looks exactly similar to a centrifugal compressor. And, but of course, just at the direction of flow and the rotation of the impeller or the rotor is exactly opposite in each cases.

We also discussed about the governing equations, which are used when we analyze the flow in these different components. Starting from the outlet flow, the volute or the scroll, and then the nozzle blades, and then it goes into the rotor or impeller. And then, there is the exducer, which forms the later part of the impeller. Its counterpart and centrifugal compressors was the inducer. And then, in typical turbine, we may also have a diffuser, downstream of the rotor to recover part of the kinetic energy, which would otherwise be lost.

And, I think, we also discussed two other aspects of radial flow turbines. They are the efficiency and performance parameters. We defined different forms of efficiency, the total-to-static efficiency and how it is related to total efficiency and the work output and so on. We also discussed or spent quite some time discussing about the different loss parameters, which are used in radial turbines, like the nozzle flow coefficient on the rotor flow coefficient. We also look at the incidence losses, which is the flow entering the rotor, need not necessarily be at the 0 incidence. Under off-design conditions, the incidence angles could be quite high, which leads to a substantial increase in the losses; that is, an additional component of loss that comes into picture when a turbine is operating in an off-design condition.

So, these are the different loss mechanisms that we discussed in a little bit detail. We, then of course, spent lot of time discussing them because I think, it is fairly out of the scope of this course to discuss the loss mechanisms and design optimization techniques in detail; that it is a vast subject on it is own. So, I decided not to spend too much time on that. We will discuss some aspects of design and performance in the next class.

But, today's class I thought it is a good idea to have a tutorial session. We shall be discussing a few problems, which I will solve for you here. And then, I also have a couple of exercise problems, which I think, you should be able to solve based on what we had discussed today as well as in the previous class.

So, today's lecture is going to be a tutorial session. So, let us take a look at the first tutorial problem that we have. Let us take a look at what the problem statement is.

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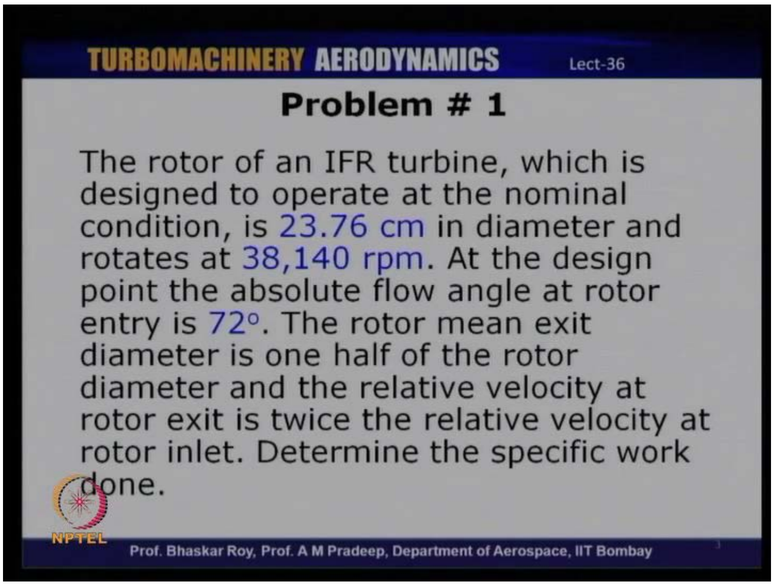

So, the problem number 1 that we have for today is the following. The rotor of an inward flow radial turbine, which is designed to operate at the normal condition is 23.76 centimeters in diameter and rotates at 38,140 revolutions per minute. At the design point, the absolute flow angle at the rotor entry is 72 degrees. The rotor mean exit diameter is one half of the rotor diameter and the relative velocity at the rotor exit is twice the relative velocity at the rotor inlet. Determine the specific work done.

So, in this particular problem, it is basically a very simple problem that, of course the first problem that I normally solve is a very simple problem. We have some dimensions of the rotor; we have rotor diameter and we have the rotational speed and the nozzle inlet angle. We also have been given that the rotor mean diameter is one half of the rotor exit diameter and the relative velocity at rotor exit is twice the relative velocity at the rotor inlet. So, based on this data that we have, we need to find the specific work done.

So, I would say this is a very simple problem but, again as I keep emphasizing every time I have a tutorial session is that, we start solving a problem with the velocity triangles. So, let us construct the velocity triangles in this particular case. And then, we shall see and proceed towards solving this problem because velocity triangle will help us to understand, what the known parameters are and what are those parameters, which we need to estimate and calculate?

So, for a 90 degree IFR turbine, we have the velocity triangles as shown.

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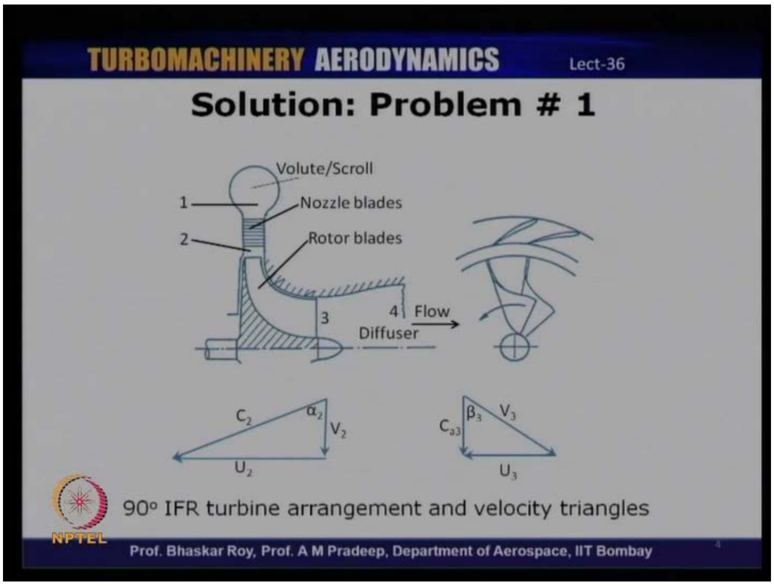

Here, we had already discussed this in the last class. Let me, again quickly explain the construction of a 90 degree IFR turbine.

We have volute or a scroll, which sort of acts like (( )) chamber, through which the flow enters stagnation chamber here, where the flow is stagnant and then that gets accelerated through the nozzle blades. So, these are the nozzle blades.

Nozzle exit station is denoted by station two. The flow then passes through the rotor blades or the impeller and exits at station three before going into the diffuser, which is the exit of the diffuser denoted by station four. You can see that, as I mention that is very similar to centrifugal turbine, a centrifugal compressor. In that case, of course, the velocity, the flow direction is the other way round. It is reverse the flow, actually proceeds in this direction. And, instead of nozzle blades, we have diffuser vanes. And therefore, this aerofoil orientation will also be the reverse. Let us look at the velocity triangles for this case. We normally have V 2 which is entering the rotor radically. So, the nozzle flow leaves relative velocity; entering nozzle is in the 90 degrees and C 2 is at an angle of alpha 2. In this case, it is given as 72 degrees. So, alpha 2 in this particular case is 72 degrees. U 2 is the blade speed at the station, that is, station two.

Now, as the flow leaves the rotor, the absolute velocity leaving the rotor is axial. Under design condition, relative velocity that an angle of beta 3 with the acceleration, that is, B 3 and U 3 is the rotor speed at station three. So, this is typical inward flow radial turbine and the corresponding velocity triangles at the rotor inlet and rotor exit. So, based on the data that we have, for this case we basically have the rotational speed, we have the exit diameters. So, I think we should able to find out U 2 and then subsequently, we also have been given some ratio of the blade speed at the rotor exit to the mean diameter and the relative velocity at rotor inlet and exit. So, with this data, we should be able to find the specific work done.

Now, let me recall what we had discussed in the last class, when we had derived a very general expression for a 90 degree IFR turbine, where the specific work was, if you recall a function of three distinct parameters. One is a difference between the blade speed at the inlet and outlet is U 2 square minus U 3 square. The second term was a function of the relative velocity and third term function of the absolute velocities. So, we are going to do exactly the same thing here to calculate the specific work done. Let us calculate these three individual components, and then add up all of these and that gives us the specific work done. So, specific work done was 1 by 2 into three different terms, U 2 squares minus U 3 square plus W 2 square V 2 square minus V 3 square. And, the third term was the absolute velocity. So, let get these individual terms first, add them up and then we get the specific work done.

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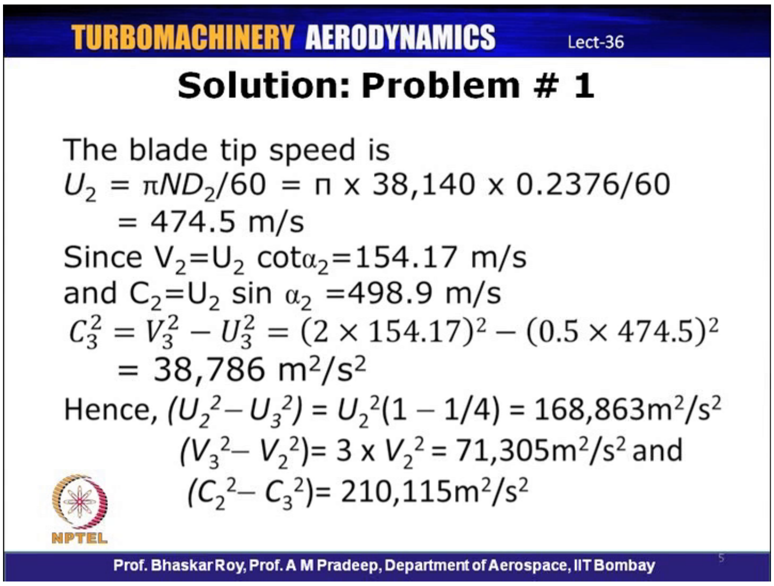

So, blade speed at the tip is U 2, which is basically pi D into N divided by 60. And, so n has been given as 38,140 revolutions per minute, the diameter is given as 23.76

centimeters. So, it is 0. 2376. This divided by 60. So, here U 2 comes out, may be if you substitute these values, we get U 2 as 474.50 meters per second.

Now, this is where the velocity triangles come into picture that, if we need to calculate V 2, if you have the velocity triangles in front of us it is very straight forward, it is ratio between V 2 and U 2 is related by tan alpha 2 or cotangent alpha 2 or cot alpha 2. So, let us look at the velocity triangle V 2 and U 2. So, tan alpha 2 is U 2 by V 2 and therefore, V 2 is U 2 into cot alpha 2. Alpha 2 is given as 7 2 degrees and therefore, V 2 the relative velocity at rotor inlet is 154.17 meters per second. Similarly, C 2 is U 2 sin alpha 2. C 2 is this, and this. So, sin alpha 2 is U 2 by C 2 and therefore, C 2 is U 2 sin alpha 2. And, since alpha 2 is 70 degrees, we get C 2 as 498.90 meters per second.

Now, C 3 square, that is, the absolute velocity at the exit, C 3 square is V 3 square minus U 3 square. V 3 is given as twice of V 2, and, that is, 2 into 154.1 7 square. And, U 3 is related to U 2 because the diameter at U 3, station U 3 is half that of U 2. Therefore, U 3 is 0.5 times U 2. So, 2 into V 2 and 0.5 times U 2.

So, what we get is that the square of this. We basically get C 3 square. C 3 square is 38, 786 meters squared per second squared.

Similarly, let us find out the three individual components, U 2 square minus U 3 square. This is U 2 square into 1 minus 1 by 4; this is 1 by 2 square. And, so it is 1 by 4. This should be 1, 68, 863 meter squared per second squared. The second term is V 3 square minus V 2 square, that is, 3 into V 2 square because V 3 is twice of V 2. So, this is 71, 305 meter squared per second squared. The third term is C 2 square minus C 3 square. We already know C 2 and we know C 3 square. So, that difference is 2, 10, 115 meter squared per second squared.

So, these are the three individual components, which contribute towards the specific work done. So, the next specific work done would be 1 by 2 times the sum of these three components. So, we add up all the three, divide that by two we get the specific work done.

Now, this is one way, while probably the direct way of calculating specific work done. we can also approximate specific work done without having to undergo any of this, but of course, this is an approximate estimate of the specific work done, that is, simply equal to the square of the blade speed at the exit. So, delta W is U 2 square and why is it U 2 square?

That is because flow enters the rotor and leaves the rotor in the axial direction. So, C w 3 is 0, C w 2 will basically be equal to U 2, specific work done is U 2 C w 2 minus U 3 C w 3. The second term would become 0; the first term is equal to U 2 square. So, delta C w should also be equal to simply U 2 square. So, if we do that if we calculate work done in both ways, let us see what happens.

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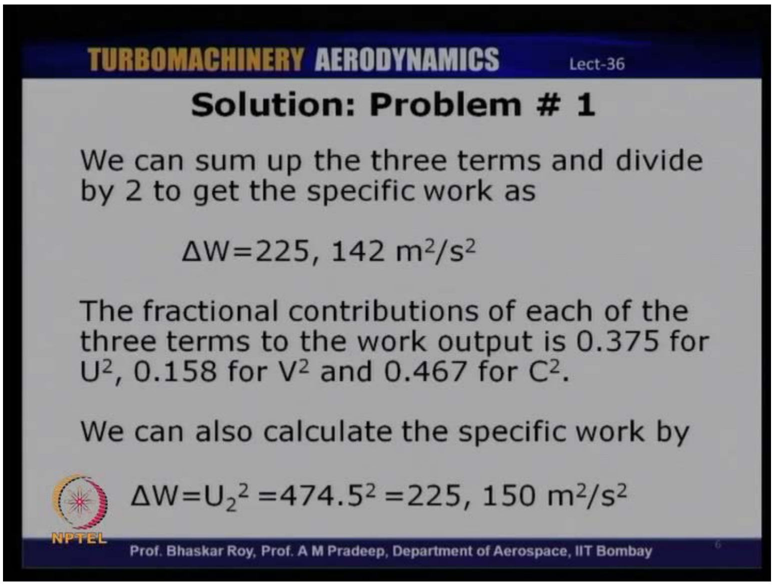

So, if we add up all the three different terms and divide by 2, we get the specific work done as 225, 142 meter squared per second squared. And, what we see is that, the fractional contribution of each of these three terms. The first term which is U square is .375. V square is .158 and C square is .467, that is, you can see that there is a fair share for all these three components in the specific work done.

So, we can calculate specific work done also by the second method, which is basically because in this particular case, beta 2 is 0 and alpha 3 is also 0. In which case, delta W simply becomes U 2 square. So, if you simply square 474.5 whole squares, you get 2 25,150 meter squared per second squared. You can see that, they are very close. Specific work done calculated in either of these ways, should give you the correct answer. Both these methods give you identical specific work done. So, one way is to calculate the individual components and add up all of them, which is a more general method because irrespective of how the velocity triangles are, you could, that is, still valid; whereas delta W is equal to U 2 square is valid, only if in this case. Like in this case, the incidence is 0, the deviation also 0, in which case you can directly calculate delta W. And, one would get the same delta W expecting the round of errors, which is seen in the both this calculation.

So, this first problem as you can, already as we have seen is a very simple problem, which involves simple application of mine to solving the velocity triangle to calculate the different components or constituents of the specific work done. One is blade speed; the other is relative velocity and the absolute velocity. We add up all three, divide them by 2; we should get the specific work done.

So, let us now, move on to the next problem that I have for you. And, it is a slightly more involved problem, but of course, again I normally keep these problems limited to very fundamental aspects of the particular aspect that we have designed working on. In this case it is the radial turbines. So, we are just looking at very fundamentally thermodynamics of radial turbine and how we can apply some of these principles to calculate some parameters associated with radial flow turbines.

Let us take a look at the second question we have for today.

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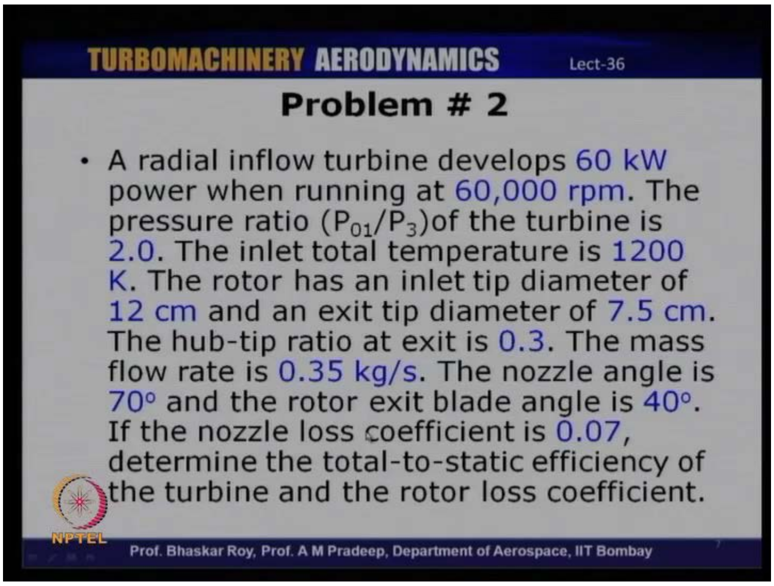

Problem statement number 2 is: a radial inflow turbine develops 60 kilowatts power when running at 60,000 revolutions per minute. The pressure ratio P 0 1 by P 3 of the turbine is 2.0. The inlet total temperature is 1,200 Kelvin. The rotor has an inlet tip diameter of 12 centimeters and an exit tip diameter of 7.5 centimeters. The hub to tip ratio at the exit is 0. 3. The mass flow rate is 0.35 kilograms per second. The nozzle angle is 70 degrees and the rotor exit blade angle is 40 degrees. If the nozzle loss coefficient is 0.07, determine the total-to-static efficiency of the turbine and the rotor loss coefficient.

So, here we can see, of course that the problem involves lot more data than we had for first case. We had the power input or power developed by the turbine is 16 kilowatts, the rotational speed is 60000 revolutions per minute. The pressure ratio is 2. Turbine inlet temperature is 1200. The dimensions of the rotor the tip diameter is 12 centimeter, exit tip diameter of 7.5 centimeter. Hub to tip ratio is 0.3. Mass flow rate and the angles, and additionally, the fact that the nozzle has a loss coefficient of 0.07.

We need to find total-to-static efficiency and the rotor loss coefficient. So, this is the problem statement for this second question that we have. As always, we will first start with the velocity triangle. It is exactly the same, as we have seen in the first problem. Nevertheless, let us just quickly look at the velocity triangles and understand, what are the data provided for using this question and what is that we need to find.

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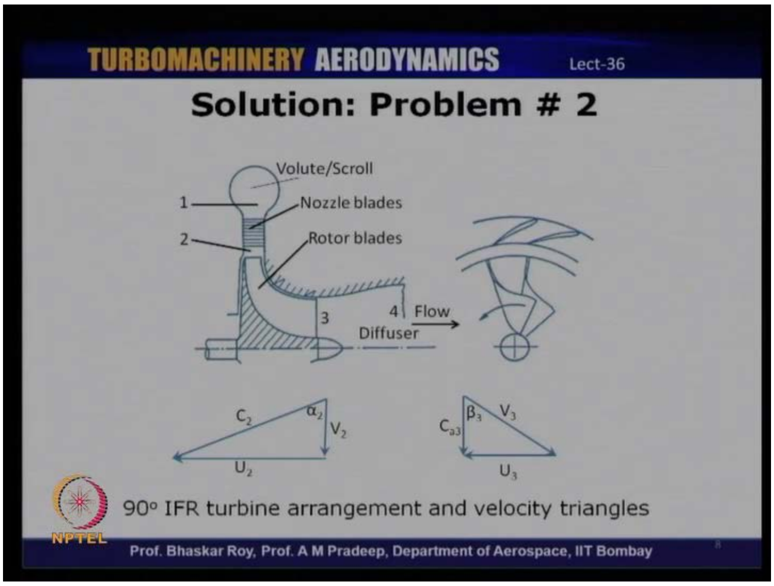

So, in this case, the velocity triangle again is the same as we have seen in the previous question. This angle is given to us as 70 degrees and the exit blade angle beta 3 is given as 40 degrees. We have the dimensions at station two and station three and hub to tip diameter ratio is also given to us. We have the power output and the rotational speed. So, quite a bit of data is given to us. Rotational speed is given; means U 2 minus U 3 already known, since angles are specified the other components can also be calculated.

So, let us start calculating some of these parameters. The rotor tip rotational speed is we can calculate from pi D N by 60, which is 377 meters per second. And here, D 2 is given in this question. It is given as 12 centimeters; rotational speed is 60,000 revolutions per minute. So, if we substitute pi and the rotational speed and the diameter, is divided that by 60, we get 377 meters per second.

Now, from the velocity triangle we know that, at the rotor inlet beta 2 is 0. And therefore, sin alpha 2 is simply U 2 by C 2, where C 2 is U 2 into cosecant alpha 2. Alpha 2 is given as 70 degrees. And, U 2 we have already calculated as 377 meters per second. And therefore, C 2 is equal to 377 into cosecant 70 degrees, that is, 401.185 meters per second.

T 0 2 is given to us; turbine inlet temperature is 1200 kelvin. Since C 2 is now calculated, we can calculate T 2 static temperature, that is, T 0 2 minus C 2 square by 2 C p. That should come out to be 1130 kelvin.

So, this is the preamble of a question, that is, we solve some simple parameters, which anyways is required for solving the rest of the problem. So, first part of the question is to find the total- to- static efficiency. And, that is where, we will make use of the pressure ratio that we have been given to, we have given as 2 .0.

From the pressure ratio, we should able to use that data to calculate the total-to-static efficiency. And, so that the next part of the question, we are going to solve is to find the stagnation temperature drop across the turbine. And, from the power output, we can actually find the isentropic or the actual power output, actual temperature drop across the turbine. We should take the ratio of the two; we get the total-to-static efficiency.

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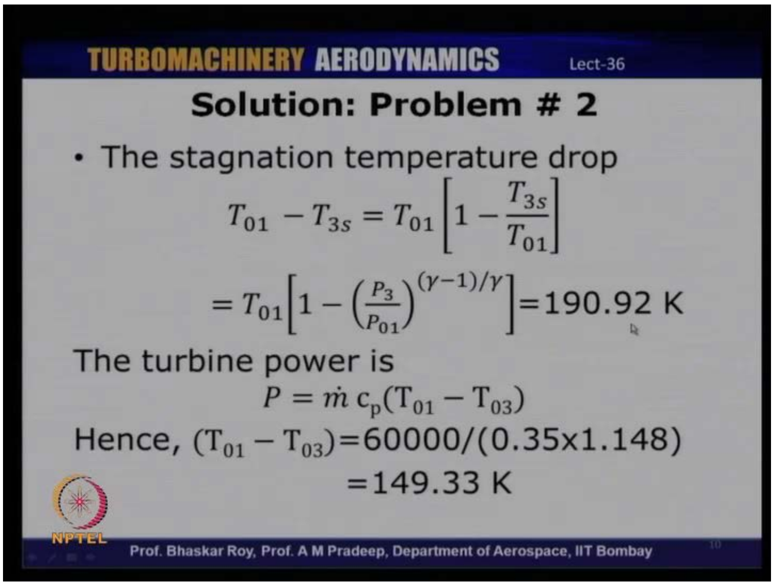

So, the stagnation temperature drop, which is given by T 0 1 minus T 3 s is equal T 0 1 into 1 minus T 3 s by T 0 1. We can convert this ratio into the pressure ratio because this ratio has been given as 1 by 2, that is, 0.5. T 0 1 is known to us. And therefore, this temperature drop is required for calculating the total-to-static efficiency because total-to-static efficiency is T 0 1 minus T 0 3, divided by T 0 1 minus T 3 s.

So, T 0 1 is 1200. This multiplied by 1 minus 0. 5, rise to gamma minus 1 by gamma. You can calculate this. And, this should come out to be 190.92 Kelvin. Turbine power output, as we know is m dot C p into T 0 1 minus T 0 3. Power output is given as 60 kilo watts; mass flow rate is 0.35 kilograms per second. C p, we are assuming for gasses as 1.148. Therefore, T 0 1 minus T 0 3 is 149.33 kelvin. So, if we simply take of ratio, this total-to-static efficiency is 149.33 kelvin divided by 190. 92. And, that is 0.782. So, total-to- static efficiency in this case is 0.782.

So, that solves the first part of the question where we are required to calculate the total-to-static efficiency. Now, the next part of the question involves or requires us to calculate the rotor loss coefficient. We have been given the nozzle loss coefficient as 0. 07. And, so we will need to make use of that, rather complex formula that we had derived in the last class. If you remember it was a very long formula, which was relating the last coefficient to the radius ratio and the static temperature ratios.

So, we will make use of that formula to calculate the nozzle, well the rotor loss coefficient and that formula involves both the nozzle coefficient, as well as rotor loss coefficient multiplied by the angles and so on. So, we know that in this case, the radius ratio because this will be required in that formula. Radius ratio r 3 by r 2 is basically the hub diameter at station plus the shroud diameter divided by 2, that is, the mean diameter divided by diameter station two. And, this we know the hub to tip ratio is given as 0.4. So, we substitute zeta here and that is basically 0.4 in this question. This is multiplied by the d 3 s plus d 3 s divided by 2 into d 2. So, if we substitute these diameters, which have been given, as well as the hub to tip ratio we can get the radius ratio as 0. 406.

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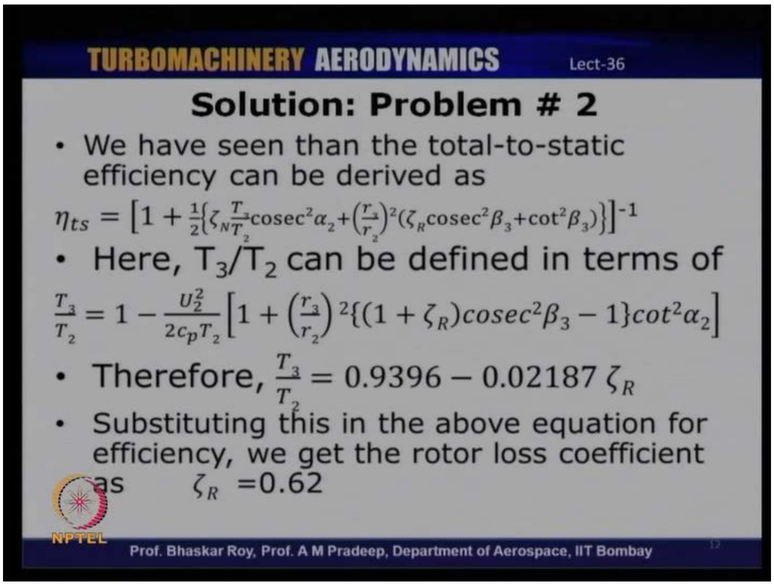

Now for efficiency this was the long expression, I was mentioning total-to-static efficiency is 1 plus 1 by 2 into zeta n, the nozzle coefficient, T 3 by T 2 into cosecant square alpha 2 plus r 3 by r 2 square into zeta r, which is the rotor loss coefficient to cosecant square beta 3 plus cot square beta 3, this whole raise to minus one.

Now, this, though it looks rather complex formulae, we have derived this expression right from the first principle. So, if we know, if we can actually relate the first principle formulae of total-to-static efficiency, which is T naught 1 minus T naught 3 by T naught 1 minus T 3 s, the denominator get expressed in terms of the nozzle and the stator or the rotor loss coefficients. Numerator is expressed in terms of static temperature ratios and so on.

So, from that, we can actually derive this without much difficulty. The numerator actually becomes C p times U into C p times C w delta C w. And then, delta C w we express in terms of the angles and so on. So, it is a very simple two to three step derivation from which we can derive this long, rather longest expression, which we see here for total-to-static efficiency.

Now, in this expression, we still have an unknown that is T 3 by T 2. T 2 we have already calculated, but we do not know the value of T 3, which also can be. In fact calculate, provided we know the exit stagnation temperature and the exit absolute velocity. So, if that is known, we can actually, we should be able to calculate because in this case, beta 3 is given and from the velocity triangle, since beta 3 is given, we can. And, U 3 is known, we can calculate C 3. What about T naught 3?

For T naught 3, the power output is given, inlet stagnation temperature is given. So, we can calculate T naught 3 from there. T naught 3 minus C 3 square by 2 C p will give us T 3. And then, we can take ratio T 3 by T 2. That would be deriving the whole thing from the first principle or the whole thing can be expressed in a single definition term, which also I think I had mentioned in the last class. Which is basically, in terms of some of these parameters, which we know and T 3 by T 2 is actually defined terms of the velocities and loss coefficients. 1 minus U 2 square by 2 C p, T 2 into 1 plus r 3 by r 2 whole square multiplied by 1 plus zeta r cosecant square beta 3 minus 1 into cot square alpha 2.

And, so if you substitute for these values here, the only unknown is zeta r. So, we should get 0.9396 minus 0.02187 zeta r. So, this, if you substitute in this expression where zeta n is also known, you get a quadratic equation and you can solve that zeta r, which is the rotor loss coefficient can be calculated as 0.62.

So, this is one way of calculating zeta r. The other way, of course is to calculate T 3 by T 2 using what I had mentioned. T 2, we have already calculated. We know what the value of T 2 is. For calculating T 3, it involves two to three steps. One is to calculate the stagnation temperature at exit T naught 3, which can be calculated from the power expression. Power is equal to mass flow rate into C p into delta T. T naught 1 minus T naught 3. So, all the three parameters are there. All the parameters are known except T 0 3 we can calculate stagnation temperature. Then, to calculate T 3, we also need to know the velocity at the exit, that is, C 3. And, to calculate velocity at the exit C 3, we know beta 3, we also know U 3. And, how do we know U 3? Because rotational speed is given to us and the diameter at the hub or at the exit of the rotor is also known.

So, from that we can calculate U 3. Since U 3 is known, beta 3 is known and we can calculate C 3 that is basically, U 3 cot alpha 3 should be equal to C3. So, once C 3 is also known, the static temperature as at the exit T 3 is equal to stagnation temperature T 0 3 minus C 3 square by 2 C p. And, so that, it is a rather easier way rather than this longest formula, I just shown.

And then, substitute for that in the expression and we should able to calculate zeta r, which is the rotor loss coefficient. Nozzle loss coefficient is already known to us. It is 0.07. So, what you can see is that the loss coefficient for the nozzle. This case has been given as 0.07. And, for the rotor it is 0.62. And, so I think, I have also given some range of these values in the last class. I had mentioned typically, the rotor loss coefficient are on the higher side because of the fact that there are rotational effects come into picture losses, associated with the rotor are much more than the rotor losses associated with the stationary component like a nozzle; especially when the flow is accelerating. So, in this case the rotor loss coefficient is, in fact close to one order magnitude higher than the nozzle loss coefficient.

So, this solves our second problem, which required us to calculate the total-to-static efficiency as well as the rotor loss coefficient. So, you can clearly see that this is slightly more involved question. In this, of course I have taken the easier route of directly substituting this in the formulae. What I would strongly urge you to do and probably leave that as an exercise for you to derive these equations from the first principles and not simply use the direct longest formulae. It is very easy to derive the equations from the first principles. In the total-to-static efficiency definition term, it is basically T 0 1 minus T 0 3 divided by T 0 1 minus T 3 s. Or, let us express that in terms of enthalpy, h 0 1 minus h 0 3 divided by h 0 1 minus h 3 s. The denominator gets expressed in two separate forms. One is to do with nozzle loss coefficient; second is the rotor loss coefficient. Numerator gets expressed in terms of mass flow rate C p and delta T and so on. So, from this, you can actually say denominator has a nozzle and rotor loss coefficient term. Numerator is already known to us.

So, this can be simplified and you can actually calculate the rotor loss coefficient, given the nozzle loss coefficient in a much simpler, less confusing manner than simply substituting them in the formulae. I picked up this method because if we take up any textbook, you would normally see this kind of method where they would refer to the derivation, which was discussed earlier on what I have done. And, in the problem we just simply substitute, plug in the values and calculate the corresponding efficiency and other terms required.

Now, that brings us to the third question. Let us, now proceed towards the third problem that we have for us to solve. And then, we will see how this problem is different from the previous problems.

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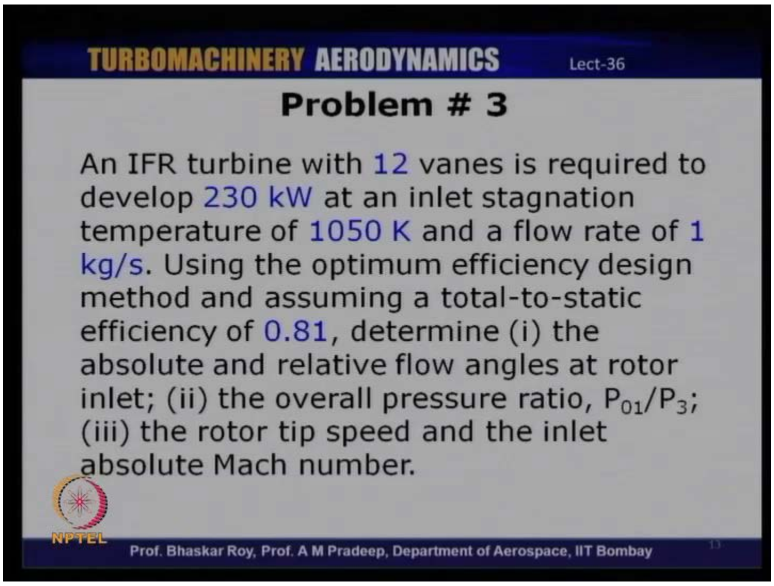

An inward flow turbine with 12 vanes is required to develop 230 Kilowatts at an inlet stagnation temperature of 1,050 kelvin and a flow rate of 1 kilogram per second. Using the optimum efficiency design method and assuming a total-to-static efficiency of 0.81, determine the absolute and relative flow angles at the rotor inlet; part b is the overall pressure ratio, P 0 1 by P 3; part c is the rotor tip speed and the inlet absolute Mach number.

So, this is a three part question where we are required to calculate three different part aspects. One is the angles, absolute and relative flow angles and the pressure ratio, and of course, the rotors tip speed and the Mach number. And of course, you can see what is mentioned here is that we can assume optimum efficiency design method. I mentioned some preliminary aspects of this in the last class. I would urge you to take up slightly more detail reading of this in any of the text books that we have mentioned. You can take up, pickup any book on the turbo machines and we will find a small section and optimum efficiency design in methodology, wherein a few formulae would basically be derived again from the first principles. And, you can see what it is basically trying to tell us.

So, in this question, it is question which involves optimum efficiency design methodology, which can be assumed. And then, we can, we are required to calculate the angles, the Mach number and so on. So, the first part of the question is to find the flow angles; the relative and absolute flow angles.

For optimum design, it is known that the absolute angle at the nozzle exit, that is, alpha 2 is simply related to the number of blades. And, this comes from what is known as the Whitfield's formula, which basically equates or which basically relates the alpha 2 to the number of blades. So, cos square alpha 2 is equal to 1 by N; so, that is, where n is the number of vanes.

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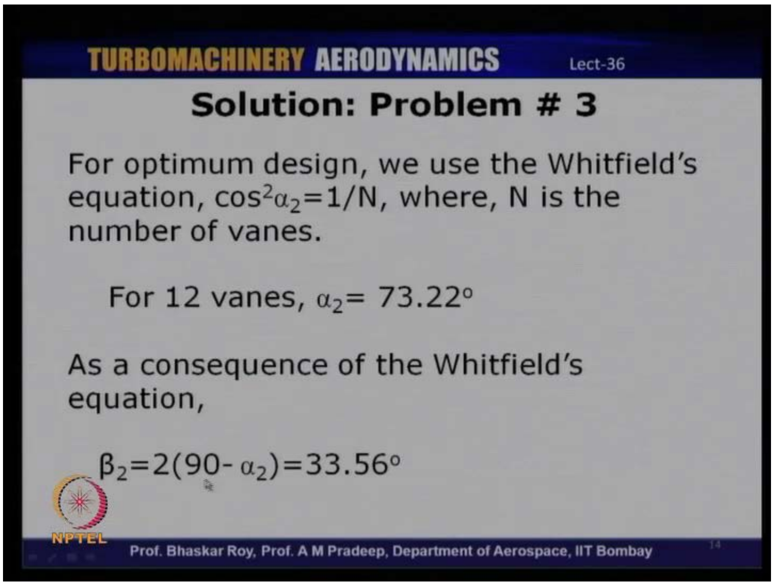

So, for optimum design, it actually has been shown by Whitfield that, cos square alpha 2 is 1 by n. So, that is, where n is the number of vanes. So, in this equation we have 12 vanes and since we have been told to assume optimum design, we can simply substitute the number of vanes here and calculate alpha 2. So, alpha 2 would be 73.22 degrees.

Also, another consequence of the Whitfield's equation is that, we have beta 2 is equal to 2 into 90 minus alpha 2. Beta 2 is the blade angle at the inlet of the rotor. And, that is equal to 2 into 90 minus alpha 2. And, so this comes to be 33.56 degrees.

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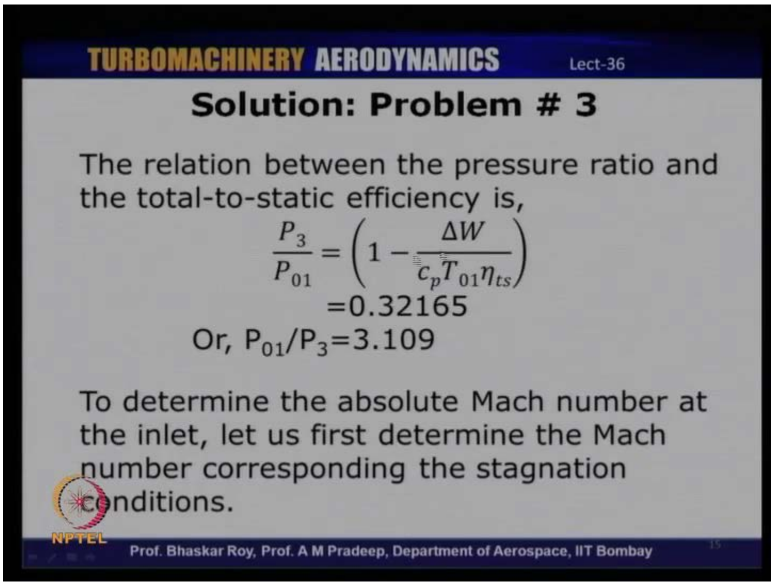

So, the next part of the question is to find the pressure ratio, well, basically the total-to-static efficiency. And, for which, of course we will use the total-to-static efficiency basically is T 0 1, 1 minus T 0 3 by 3, T 0 1 minus T 3 or T 3 s. And, from there we can express the denominator in terms, P 3 by P 0 1. And, in this question, we know that the power developed is given as 230 kelvin, inlet stagnation temperature is given, mass flow rate is given and the total-to-static efficiency is given. So, we need to find the pressure ratio in this case. So, all we have do is, we just substitute the power output here, which is m dot into C p into delta T. C p is known, assumed and the stagnation temperature is also can be calculated or it is given at the inlet.

Since the efficiency is known, we can calculate the pressure ratio, P 3 by P 0 1 as 0.32165. The inverse of this is the turbine pressure ratio, which is P 0 1 by P 3 and that is 3.109. So, this basically comes from the efficiency definition eta T s is T 0 1 minus T 0, T 0 3 divided by T 0 1 minus T 3 s. The denominator gets expressed in terms of the pressure ratio, from the isentropic relation numerated is simply C p times delta T. And therefore, that is the power output of the turbine.

Now, the third part of the question is to find the Mach number and then, we should also need to find the blade speed at the tip of the rotor. So, we will first find the absolute Mach number at the inlet, for which we will find first the Mach number corresponding to stagnation conditions.

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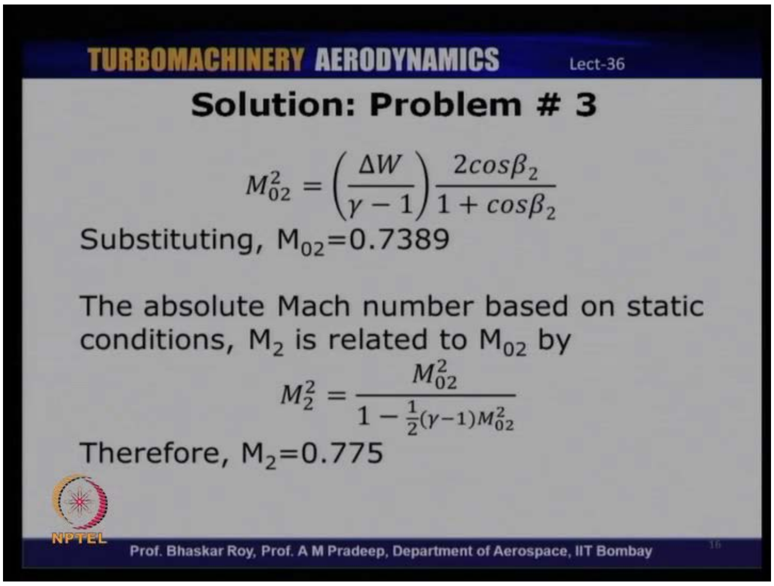

So, M 0 2, this again is coming from the very basic definition for delta M 0 2 square is equal to delta W by gamma minus 1 into 2 cos beta 2 by 1 plus cos beta 2. Now, where does this equation come from? Again I will urge you to try to derive this equation from the fundamentals. Delta W is h 0 1 minus h 0 3, which is m dot C p into T 0 1 minus T 0 3. And, there the temperatures can actually be expressed in terms of the corresponding velocities. And therefore, we can express the Mach number in terms of the, well, part of the temperature get expressed in terms of Mach number, wherein this flow angles alpha 2 and beta 2 will also come into picture.

So, from those fundamental equation we can actually derive the stagnation Mach number, which is M 0 2 Mach number at the inlet of the rotor, M 0 2 square is delta W by gamma minus 1 into 2 cos beta 2 by 1 plus cos beta 2. So, all the parameters on the right hand side are known to us. We just, substitute these different values and we get M 0 2 is 0.7389.

The absolute Mach number which is basically based on this static conditions, M 2 is related to M 0 2 as we know. So, M 2 square is M 0 2 square by 1 plus gamma minus 1 by 2 M 0 2 square. This again follows from the isentropic relations. We have already seen that, it is the relation between stagnation temperatures to static temperature, stagnation pressure to static pressure and so on. We can also relate the corresponding Mach numbers in this way.

So, since we have calculated M 0 2, we substitute that here and then, we get the static (( )) Mach number based on static conditions that the absolute Mach number as 0.775. So, this is again, this can be calculated in multiple ways. The other way, which I suggest you can try to calculate would be to take the ratio of T, absolute velocity; that is, C 2 divided by square root of gamma or T 2. Stagnation temperature is known at the inlet and to find static temperature, we need T naught minus C squared by 2 C p. So, is basically we need to calculate C square at the inlet. And, to calculate C square we will of course, need the blade speed because you need to calculate because blade angle is known. And so, if you know the blade speed at tip, that is U 2, you can solve C 2, and then, therefore, calculate Mach number.

In this case, of course we are doing at other way round. We are calculating the Mach number first and then, we are now going to calculate the blade speed at the tip. But, it can also be done the other way round.

So, let us now calculate the blade speed at the exit of the nozzle or, that is the entry rotor, entry tip of the rotor. Delta W by C p T 0 1 as we know is equal to gamma minus 1 into coos beta 2 into U 2 square by a 0 1 square.

So, here a 0 1 is speed of sound based on stagnation temperature, square root of gamma r T 0 1. This can be calculated as 633.8 meters per second. Assuming, T 0 1 is equal to T 0 2. Since in this case, we know the power output as stagnation temperature and beta 2, we can simply substitute for these values and then, calculate the blade speed at the tip U 2. And, that comes out as 538.1 meters per second. So, the other approach to calculate Mach number would be to actually calculate the blade speed first and then, since blade speed is known and alpha 2 is known, you can calculate C 2. And, from C 2 you can calculate the static temperature T 2. T 0 2 minus C 2 square by 2 C P. And therefore, Mach number would be C 2 by square root of gamma or T 2. So, this is the other way of calculating the Mach number. We have calculated that in the slightly different way.

So, that completes the third problem that we had set aside for today's tutorial. So, I have one more problem to solve, which is a very simple problem, basically not involving in the calculation. But, this is just to compare performance or operation of two different types of turbines that we have discussed about. One is the axial turbine, which we had rather detailed discussion, several lectures earlier on and of course, the radial turbine. So, let us compare under certain given operating conditions, how the work output of these two different turbines can be calculated and how do they compare with each other.

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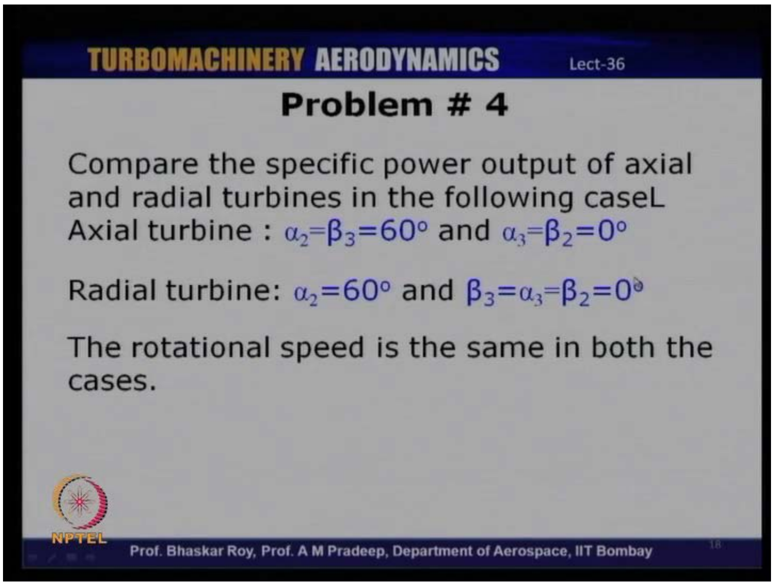

So, the fourth problem statement is the following: Compare the specific power output of axial and radial turbines in the following cases. Axial turbine, in this case is given as alpha 2 is equal to beta 3 is 60 degrees and alpha 3 is equal to beta 2 is 0 degrees.

For the radial turbine, alpha 2 is given as 60 degrees and beta 3, alpha 3, beta 2; all of them are 0. If the rotational speed is the same in both this cases, we are required to calculate this specific power output. So, what we can see is that immediately for the axial turbine, we have, we can see that alpha is equal to beta 3 and alpha 3 is equal to beta 2. Immediately, tells us that this is a 50 percent reaction turbine, which means that the velocity triangles would be symmetrical. And, for the radial turbine we have already seen the velocity triangle, which is for the nominal operation condition, where alpha 2 is given, beta 2 is 0, alpha 3 and beta 3 are respectively 0 at the exit.

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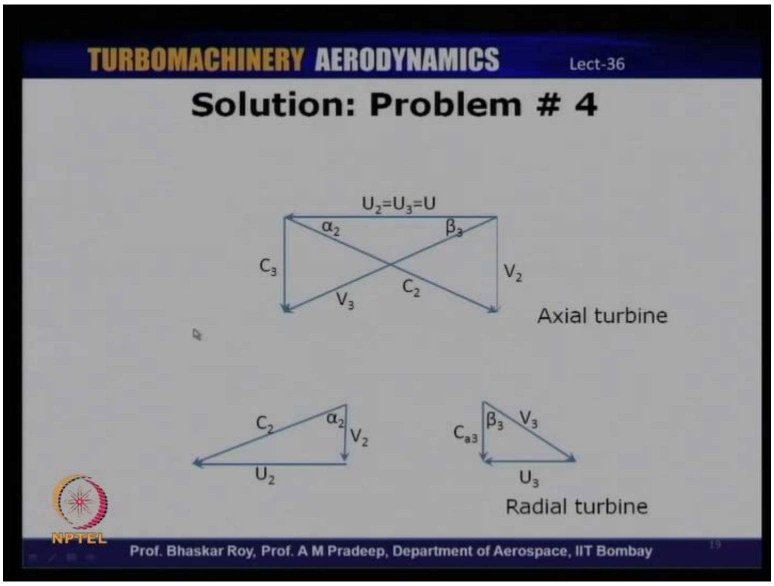

Let us take a look at the velocity triangles first. And, so here, we have the velocity triangle for both these cases, the axial as well as radial turbine. Let us look at the axial turbine first. Velocity triangle at the inlet is given by this. Alpha 2 is given as 60 degrees. And, so alpha 2 at 60 degrees and that is equal to beta 3, that is, also 60 degrees and beta 2 is 0, as you can see and alpha 2 is 0, that is, C 3 makes 0 degrees with the axial direction.

And, since it is an axial turbine, we will assume U 2 is equal to U 3 is equal to U, blade speed. It is for the same circumferential radial location. So, U 3 and U 2 are the same and that is equal to U.

And, these are the corresponding velocities, C 2. At the inlet, the absolute velocity V 2 is related and at the rotor exit, we have C 3 and correspondingly V 3, which is the relative velocity.

For the radial turbine, we have been given that alpha 2 is again 60 degrees and beta 3 is also 60 degrees. And, this is typical velocity triangle for an inward flow radial turbine. Since alpha 2 is 60 degrees, we have C 2, which is at 60 degrees to the radial location here. V 2 is the relative velocity which is entering in the radial direction. U 2 and U 3 they are not the same. They are different in this case. and, the exit of the rotor we have beta 3, which is again 60 degrees and axial velocity which is C 3 and C a 3, they are the same. V 3 is the relative velocity at the exit of the rotor.

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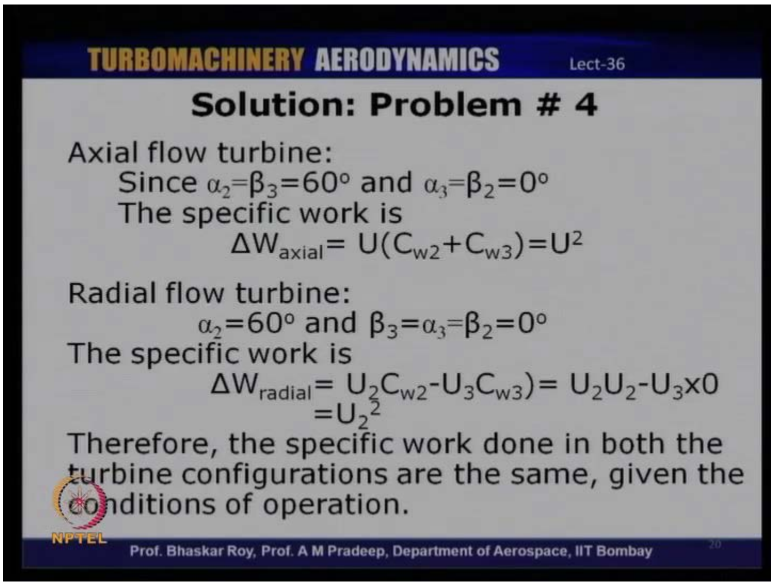

So, for the axial turbine, we have alpha 2 and beta 3 as 60 degrees; alpha 3 and beta 2 as 0 degrees; specific work for this case, in this fifty percentage reaction case, would be U times delta C w; U times C w 2 and C w 3. Here as you can see, C w 3 would be equal to 0 for the axial turbine because the flow is leaving in the axial direction. So, C w 3 is 0 and C w 2 is equal to U 2. C w 2 is the tangential component of C 2, which is equal to U 2. And therefore, the specific work done for the axial turbine would be simply U square.

For the radial turbine, alpha 2 and beta 3 are 60 degrees; beta 2 and alpha 3 are equal to 0. And, specific work done is U 2 C w 2 minus U 3 C w 3. In this case of course, C w 3 is again 0, like in the case of axial turbine. And, C w 2 is at the inlet, is equal to U 2. And, of course at the exit U 2 and U 3 are not the same. But, since C w 3 is 0, the radial turbine also develops a work which is equal to U square. In this case it is, U 2 square. So, specific work done in this particular case one of them is for an axial turbine and other is for radial turbine. For given these conditions for the same rotational speed, both of these turbines generate the same work output. They both are functions of square of the blade speed.

So, in this specific case that for example, that we have looked at because of the conditions that have been specified to us for the same blade angles and rotational speeds, both these turbines generate the same work output. So, just to give you an idea of how the work done can be calculated for different types of turbine configurations, of course, we have seen axial turbines in greater detail, including a tutorial session earlier on. And, this is for just to make a comparison between the work outputs of these two different types of turbine configurations.

So, that completes the fourth problem, as well that we have solved today. And, what I have for you are two exercise problems, which I would leave it for you to solve. And, of course I had also left few exercises in between the couple of problems where I had requested that you should try to solve it in a different way. The method I had used to solve is one of the ways of solving the problems. You can also attempt to solve the problem in a different way, for which I had given some hints. So, I suggest that you would also solve those problems using the other alternative approach that I had suggested.

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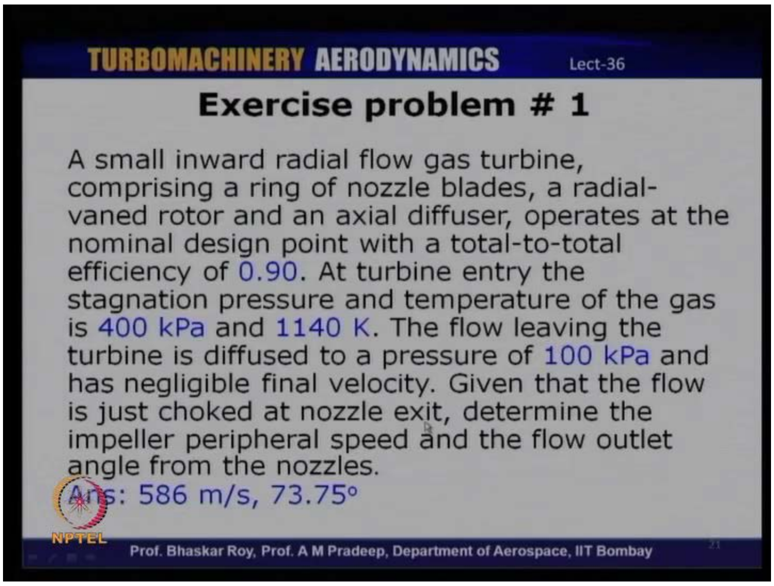

So, let us take a look at the first exercise problem that I have: a small inward radial flow gas turbine, comprising a ring of nozzle blades, radial-vaned rotor and an axial diffuser, operates at nominal design point with a total-to-total efficiency of 0.90. At the turbine entry the stagnation pressure and temperature of the gas is 400 kilopascals and 1140 kelvin respectively. The flow leaving the turbine is diffused to a pressure of 100 kilopascals and has negligible final velocity. Given that the flow is just choked at the nozzle exit, determine the impeller peripheral speed and the flow outlet angle from the nozzles.

So, this question has an additional component, that is, diffuser and it is given that at the exit of the diffuser the flow has negligible velocity. So, we can assume that the flow exits the diffuser with almost 0 velocity, and the flow at the nozzle exit is just choked, which means that Mach number at nozzle exit is 1, velocity there would be equal to square root of gamma r T. And, stagnation temperature is given to us. And, so that should help you in finding out the parameters at the rotor entry. And, since rotor exit conditions are lot of fix. You can also calculate the conditions at the rotor exit using the fact that the diffusion pressure is given and the fact that velocity at the exit is close to 0. So, in this case the tip speed comes out 586 meters per second and the angle alpha 2 is 73.75 degrees.

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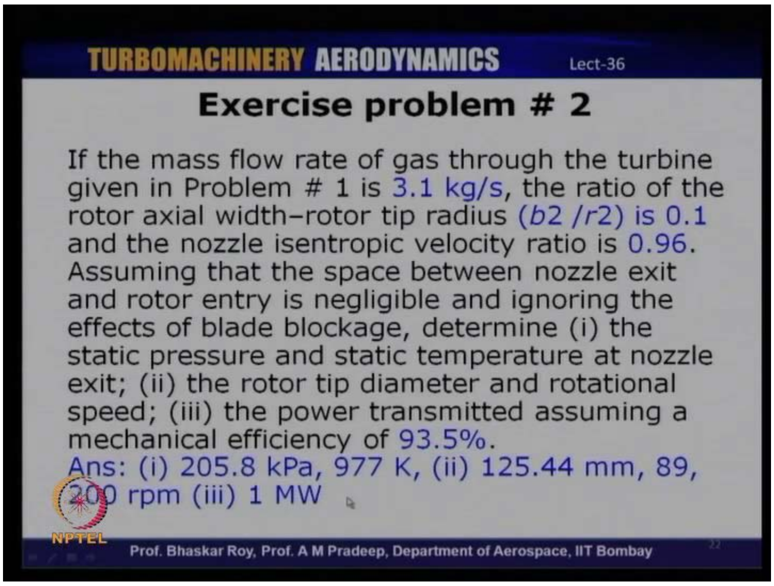

Now, the second question exercise problem is: if the mass flow rate of gas through the turbine given in this problem 1, the previews problem is 3.1 kilogram per second, the ratio of rotor axial width to rotor tip radius is 0. 1 and the nozzle isentropic velocity ratio is 0.96. Assuming that the space between the nozzle exit and nozzle rotor entry is negligible and ignoring the effects of blade blockage, determine the static pressure and static temperature at the nozzle exit; the rotor tip diameter and rotational speed; the power transmitted assuming a mechanical efficiency of 93.5 percentage.

So, we need to use part of data which is given in the first question, to be able to solve this question as well. So, answer to part 1 is the pressure static pressure is 205. 80 kilopascal; temperature is 977 kelvin; the rotor tip diameter is 125.44 millimeters and rotational speed is 89,200 revolutions per minute, power transmitter with this mechanical efficiency would come out to be 1 megawatts.

So, these are two exercise problems that I have for you. You can solve this based on what we have discussed in the last couple of lectures including today's. And, I would also suggest that you solve couple of those problems, which we solved in today's lecture that is problem number 2 and 3 in a different approach, from what we have solved in the tutorials. So, I would suggest that you also solve those problems using a slightly different approach.

So, we would have just one more lecture on radial turbines, where we would discuss some aspects of performance and some preliminary design aspects related to radial flow turbines.

So, these we will take up in the next class, which would be lecture number 37.