Turbomachinery Aerodyanmics

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Lecture No. # 35

Radial Turbine: Thermodynamics and Aerodynamics

Hello and welcome to lecture number thirty-five of this lecture series on turbomachinery aerodynamics. We have been discussing about different types of turbomachines in the last 34 odd lectures, that we have had and in the last class I think I had mentioned, that we are going to take up a relatively new topic now from this lecture onwards, and we are going to discuss about a set of turbines, which kind of are contrary to the type of turbines, that we have discussed earlier on. We had some detailed discussion on axial turbines, which had the inflow as well as the exit flow as axial. In today's lecture, what we are going to discuss about are a set of turbines, which are in some sense very similar to a centrifugal compressor or these are the radial contra parts of turbines and these are known as the radial flow turbines.

We are going to have some detailed discussion on radial flow turbines. Today's lecture we are going to exclusively set aside for thermodynamics and the aerodynamics of radial flow turbines. We will also have some discussion in a later lecture on some other aspects of the blade's shapes and geometries of radial flow turbines, but today's lecture is going to be primarily on the thermodynamics and aerodynamics of radial flow turbines. We will first have some introduction to the different types of radial flow turbines, that are available or possibly followed by the thermodynamic working of the radial flow turbines and some elementary aerodynamics associated with radial flow turbines. We will also discuss in some detail about the losses in radial flow turbines and the methods of calculating or estimating these losses. So, we are going to discuss these topics in today's lecture.

Now, the history of radial flow turbines dates back to what, 150 to 200 years, now it is in the 18, mid 1800s or that the radial flow turbines were developed by a French engineer who was known as (( )) and this turbine was basically a radically outward flow turbine. And subsequently, a few years later, in 1850s, Francis and his colleague (( )) in developed the inward flow turbines, wherein the flow enters the turbine in radially and leaves the turbine in the axial direction and so these are known as inward flow turbines and some of the modern turbines are actually named after the inventors, are called Francis turbines. You must have heard of reaction turbines, which are also known as Francis turbines.

So, these are the earliest developments of turbines and were meant exclusively for use in hydraulic applications for power generation and so on. Some of these types are in use even to this date, even after about 200 years or more that these have been developed and so these are the basic types of turbines, which were developed in the 1800s and continue, some of the versions or modern versions of these continue to be used. And so, there are these two distinct types of turbines: the outward flow turbines, which were originally developed by the French engineer and subsequently, the inward flow turbine, which is the more popular version of the turbine, which is continued, which continues to be used even to this date.

And the inward flow turbine has a set of advantages, in the sense, that they can cover tremendous ranges of power, rates of mass flow and rotational speeds and therefore, they are used in a variety of applications, ranging from hydroelectric power plants, where these turbines generate hundreds of megawatts of power to micro or small gas turbines, where they generate probably a few kilowatts of power.

So, you can see, that there is a huge range of power spectra, which can be covered by these types of turbines, but in, in, if you look at aero-engine in specific, radial turbines, as I think I had mentioned few times earlier on, they are not really used in the sense, that the one of the main limitations of why radial turbines are not used is the fact, that there are certain limitations on them, how much temperature one can use in radial flow turbines when it is applied for a gas turbine application unlike an axial turbine, which is relatively easier to manage temperatures by applying artificial blade cooling. We have already discussed blade cooling in a lot of detail earlier on.

Now, blade cooling is not very easy to implement in a radial flow turbine, which means, that that puts a very significant limitation on the temperature, which these turbines can be operated at and that is the reason why, that is one of the reasons, why radial flow turbines are not very commonly used in gas turbine applications, but they are used in smaller aero-engines, where blade cooling is not really required because temperatures are not very high or of course, they are used in hydraulic applications for even huge capacity or plants, which generates hundreds of megawatts of power.

So, radial turbines have, as I said, a variety of applications, and the inward flow turbine is what we are basically looking at, because of the fact, that these inward flow turbines have inherent advantages, which is what I just mentioned. So, when we look at inward flow turbines, there are again multiple varieties of inward flow turbines.

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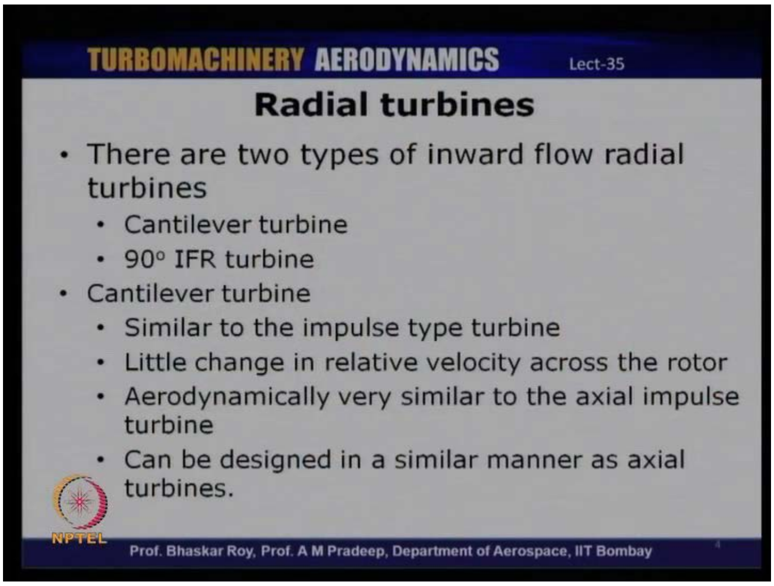

We will look at two distinct varieties, which have been used over the past many years and these are the cantilever type turbines and the 90 degree IFR or inward flow radial turbines. So, these are two distinct types of inward flow radial turbines, the cantilever type turbine and the 90 degree inward flow turbine. Out of this, of course, we will spend the rest of the lecture discussing primarily about the 90 degree inward flow radial turbine or IFR turbine.

Cantilever turbine is in some sense similar to the impulse type turbine, we discussed in detail when we were talking about axial compressors, which means, that there is hardly any change in relative velocity across the rotor and it is aerodynamically very similar to the axial impulse turbine. And it is in fact, designed in a manner very similar to how an axial impulse turbine is designed and so cantilever type turbines are in some sense similar to the impulse turbine.

We have discussed about impulse turbine in detail when we were talking about the axial turbines and these are very similar to that when it comes to the aerodynamics and design of these turbines. And it is called cantilever because I will show you now, that the blades of the turbines are actually suspended from one end and then, that resembles a cantilever beam because it supported only at only one end, which is also true for axial turbine blades because the blades are actually supported only at one end, the other end has to be free for mechanical reasons.

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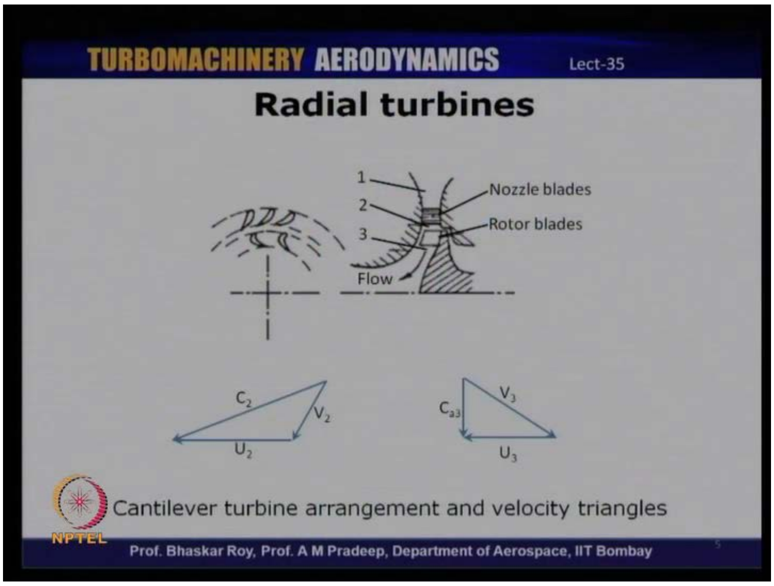

So, let us take a look at schematic of a cantilever turbine. This is how a typical cantilever turbine would look like and so there are two distinct view of, this is a front view of the turbine showing the nozzles and the rotor vanes or rotor blades. So, the nozzle blades actually accelerate the flow and then, discharge the flow in, through the, in a rotor blades and the flow leaves the turbine axially. So, you can see, that it enters the flow radically, it moves inward and that is why, these are all inward flow turbines and then it exhausts the flow in the axial direction.

Let us look at the velocity triangles. Velocity triangle at the inlet of the rotor or the velocity triangle leaving the stator, here station 1 is the nozzle entry, station 2 is nozzle exit and rotor entry station 3 is rotor exit. So, the flow leaves the, absolute flow leaves the angle at an angle of C 2 and because of the relative velocity, the blade speed U 2, the velocity which the rotor sees at its inlet is V 2, that is the relative velocity and as flow exits the rotor, we have the flow becoming axial, flow leaves the blades axially for nominal design condition. V 3 is the relative velocity with which the flow leaves the rotor.

So, you can see that at the exit of the rotor, the blade speed has actually changed. You have a different velocity blade speed because of the fact that these two are at different radial location, something we have also discussed. For centrifugal compressors, U 2 would actually be greater than U 3 because the whole thing is rotating at the same rotational speed because of the difference in radius. U 2 should be greater than U 3 and so the flow leaves the blades in the axial direction and V 3 is the relative velocity with which the flow leaves the rotor.

So, this is the typical arrangement of one type of inward flow turbine that is the cantilever type turbines and the corresponding velocity triangles, very similar to an impulse turbine that you have seen. So, in across the rotor V 3 and V 2 should really be the same, it should not change. As the flow enters and leaves the rotor, the relative velocity does not change, which means, that the entire static pressure drop as has actually occurred in the nozzle, the rotor does not really contribute towards the static pressure drop and that actually taken care of by the stator.

Now, the other class of turbines, that we are going to spend considerable time in of today’s lecture is the 90 degree IFR turbine, that is, 90 degree inward flow radial turbine; IFR stands for inward flow radial turbine. And we are saying 90 degree because flow actually takes a 90 degree turn as is, as it passes through the IFR turbine. Now, these are turbines, which are more commonly used for performance reasons, that these turbines have much better performance and the design complications are also much more simplified. This is also true for impulse and reaction turbines, which we discussed for axial turbines, impulse. The reaction turbines can actually have much better efficiencies than the impulse turbine and then we have seen when thermodynamically make sense.

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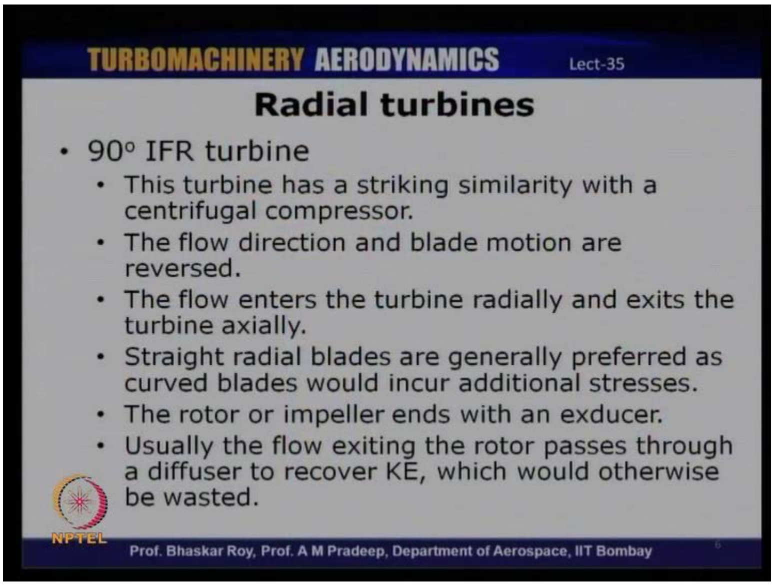

Now, in, if you look at a 90 degree IFR turbine, if you take cross-section of that, this turbine looks exactly similar to that of a centrifugal compressor and so an IFR 90 degree radial flow turbine, inward radial flow turbine has a very striking similarity to a centrifugal compressor. Just the fact, that the flow directions and the blade rotation is reversed in a centrifugal compressor, the flow enters the impeller axially, leaves the impeller radially and there is a certain direction of rotation.

Whereas, in an IFR turbine, the flow and direction flow enters the turbine radially and exits it axially and normally, the blades are straight radial. Whereas, in centrifugal compressor we have seen, that even it is possible to have backward leaning blades as well. In a turbine, well it is possible to have that theoretically, but then the curvature of the blade is going to induce even more stresses and therefore, it is very uncommon to see curved blades for IFR turbines. Normally, they are all straight radial blades and the rotor ends with the, with what is known as an exducer. In a compressor we have seen, that it as inducer, which turns the flow from axial into radial allowing smooth entry into the impeller.

Similarly, in a rotor of an IFR turbine, we have what is known as an exducer, it is like, it is exactly like an impeller inducer, just that the function is different. Here, it turns the flow from radial to axial direction and usually, the flow passing through the rotor also exhausts into a diffuser, which recovers part of the kinetic energy because the flow, still while exiting the rotor, will have a substantial amount of kinetic energy and if it is allowed to pass through a straight duct, that kinetic energy is going to get wasted, so that can be used to increase the overall work done by the turbine.

And so, diffuser helps us in recovering part of this kinetic energy. This is also very commonly used in hydraulic turbines, where they have what is known as a drafts tube; at the exit of the turbine they use a draft tube. So, the overall pressure ratio across the turbine is improved and that leads to an improvement in the overall work done. Let us now take a look at a schematic of a typical 90 degree IFR turbine and also the corresponding velocity triangles.

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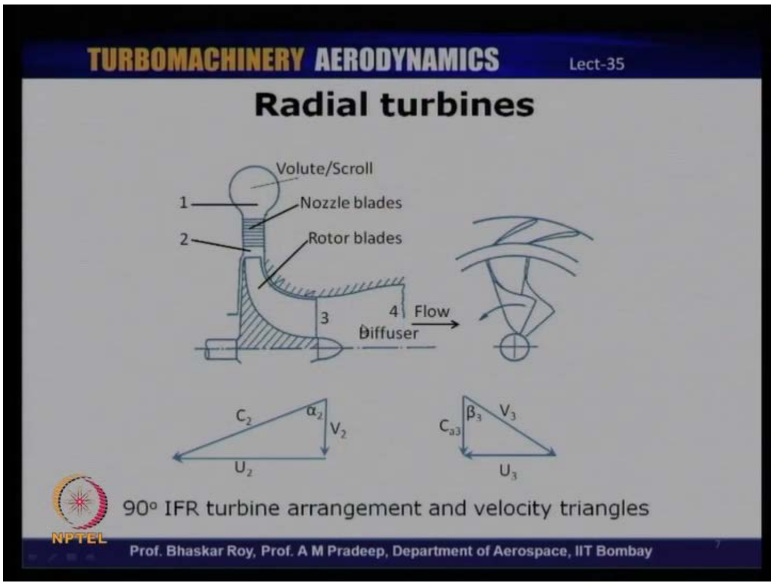

Now, as you can see the first on, if I had not written turbines here, one would be, one would possibly want to take consider this as a centrifugal compressor because it looks exactly like a centrifugal compressor. But of course, the rotor rotation is different in a centrifugal compressor; the rotor rotation would have been in the other way round. In this case, the rotation is in this way as well as the vanes, you can see, it is actually leading edge of the aerofoil, is towards the radial direction. It would have been other way round in a centrifugal compressor, but otherwise, they look very similar to a centrifugal compressor.

What are the different components of a radial turbine? Radial turbine has a volute or a scroll, as it is called in the turbine terminology. Scroll is like a volute or the, what we had seen in the case of centrifugal compressor, in turbines, it is usually called as a scroll. Flow from the volute or the scroll passes through the nozzle blades, which accelerate the flow and then allow the flow to pass through the rotor, wherein the work is done by the flow on the rotor blades. Flow exiting the rotor passes through a diffuser and therefore, the flow enters the turbine in the radial direction and leaves the turbine in the axial direction. It takes exactly 90 degree turn and that is why, it is called a 90 degree IFR turbine.

If you look at the corresponding velocity triangles, now these are the velocity triangles for the radial turbine, now this C 2 refers to the absolute velocity leaving the nozzle and then, it enters the rotor blades at a relative velocity of V 2 and it is radial because the blades are radial and U 2 is the blade speed at the tip of the rotor or the impeller. As the flow leaves the exducer, here the relative velocity is V 3 and the flow is axial and that is why it is called C a3. The flow leaves the turbine in axial direction, the blade speed of course lower here; U 3 is less than U 2 because of the difference in radii between stations 2 and 3. So, this is how a typical radial 90 degree inward flow radial turbine would look like and the corresponding velocity triangle.

What we are going to do next is to take up this particular turbine as an example. Consider the expansion of flow as it passes through this turbine and we will carry out an analysis of the flow as it passes through this turbine. We will also look at the governing equations with reference to a 90 degree IFR turbine. Of course, this is also applicable to other forms of turbine with corresponding variations in the blade speeds and velocities and so on. We will also then look at the losses incurred as the flow passes through such a turbine configuration.

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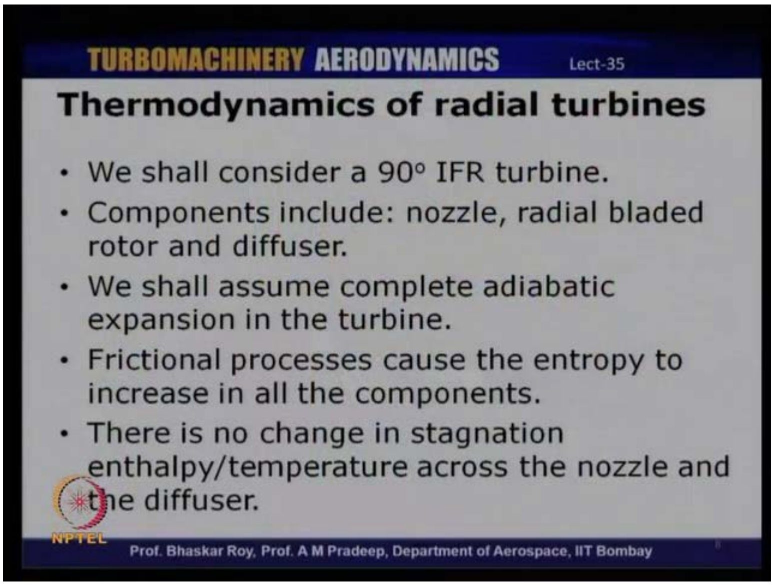

So, if you look at a 90 degree IFR turbine, the components, that constitute this turbine basically include a nozzle, radial bladed rotor and diffuser. For this analysis we are going to consider complete adiabatic expansion, which means, that there is no heat transfer across the walls of the turbine and the losses, that we are looking at would primarily, one of the components of the losses would be frictional losses and therefore, the entropy of course, changes, in fact, increases in all the components. The stagnation temperature or enthalpy does not change across the nozzle or the diffuser because we are firstly, considering an adiabatic flow and there is no work done on the system or by the system in the nozzle and the diffuser.

So, there is no change in stagnation parameters, stagnation temperature and enthalpy in the nozzle and diffuser. So, let us take a look at the temperature entropy plot, as we have been doing for all the other machines. We have seen compressor axial compressors and turbines and centrifugal compressor and we will look at thermodynamically, what is that is involved as the flow passes through these different components and what happens to the thermodynamic parameters and as the flow passes through these components.

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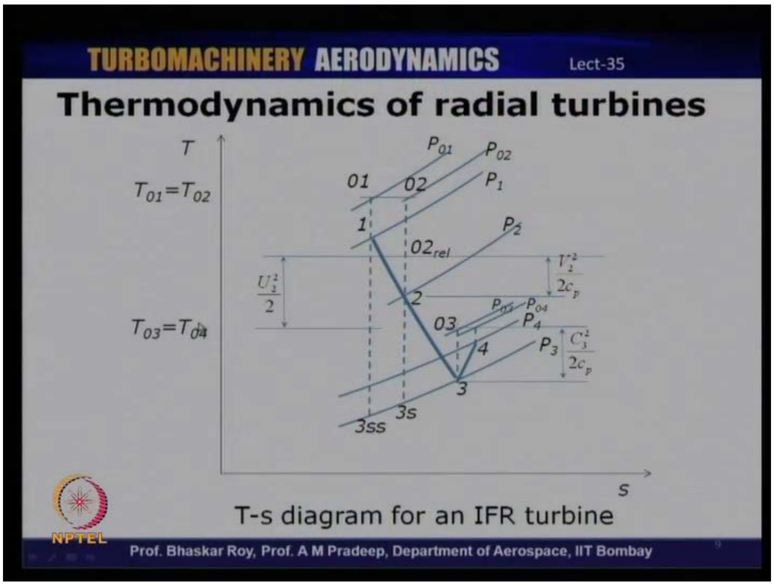

So, a T-s diagram for a typical inward flow radial turbine would look like this. Let me explain the different components associated with this particular T-s diagram. As we have seen before, station 1 is the nozzle entry, station 2 is nozzle exit, 3 correspond to rotor exit and 4 correspond to the diffuser exit.

So, if you look at the static pressure, there should be a continuous drop in static pressure between stations 1 to 2 and 2 to 3 because as the flow accelerates through the nozzle and then subsequently in the rotor, static pressure has to drop, whereas in the diffuser, there would be a rise in static pressure as the flow passes between stations 3 and 4. And in the nozzle, there is no change in stagnation, enthalpy and temperature, that is, T 01 should be equal to T 02; similarly, in the diffuser, T 03 should be equal to T 04.

So, let us take a look at the T-s diagram again. Here, T 01 is a stagnation pressure available at the nozzle entry, there is a certain amount of total pressure loss taking place in the nozzle due to frictional losses. So, there would be a small amount of pressure loss in the nozzle and then, the corresponding static pressures are p 1 and p 2, so there is a change, there is a drop in static pressure. The actual process, that is taking place in the turbine, is indicated by this bold line between stations 1 to 2 and 2 to 3 and between 3 to 4. So, between 1 to 2 is the nozzle and there is a drop in static pressure.

As you can see, there is also a drop in stagnation pressure because of the fact, that there could be some amount of frictional losses taking place in the nozzle, but the stagnation temperature does not change. You can see, T 01 should be equal to T 02 in the nozzle, this does not change irrespective of whether there are frictional losses or not. Between 2 and 3 we have the rotor and therefore, there is further drop in static pressure between stations 2 and 3, so, there is a drop in static pressure. As you can see, as the flow passes through the rotor, between 3 and 4 is the diffuser and in the diffuser, the flow actually decelerates and therefore, the kinetic energy is partly recovered in the form of rise in static pressure. So, there should be a change, there should be increase in static pressure between stations 3 and 4, which is why you can see, that there is an increase in static pressure between 3 and 4.

On the other hand, there could be a small drop in stagnation pressure in the diffuser and that is why, P 03 and P 04 do not really coincide, and there could be a small drop in the stagnation pressure, which is attributed to the frictional loss in the diffuser now. But the stagnation temperature, as you can see, does not change in the diffuser as well, T 03 and T 04 are the same. So, across the turbine, there is a drop in stagnation temperature, which is attributed to the rotor. So, the stagnation temperature, that, that you see is different, that between T 01 and T 03, because of the drop in stagnation temperature in the rotor, which is basically the work done by the rotor. So, that is the temperature drop, that is the potential, which is converted by the turbine to useful work outputs, that is, enthalpy drop, which the turbine is able to convert into useful work output and that is what we are basically interested, and we are trying to extract work from the flow through this drop in stagnation temperature.

So, what we will do now is to analyze the flow as it passes through the turbine and try to derive some equations, which we can use to analyze the flow as it passes through the radial flow turbine.

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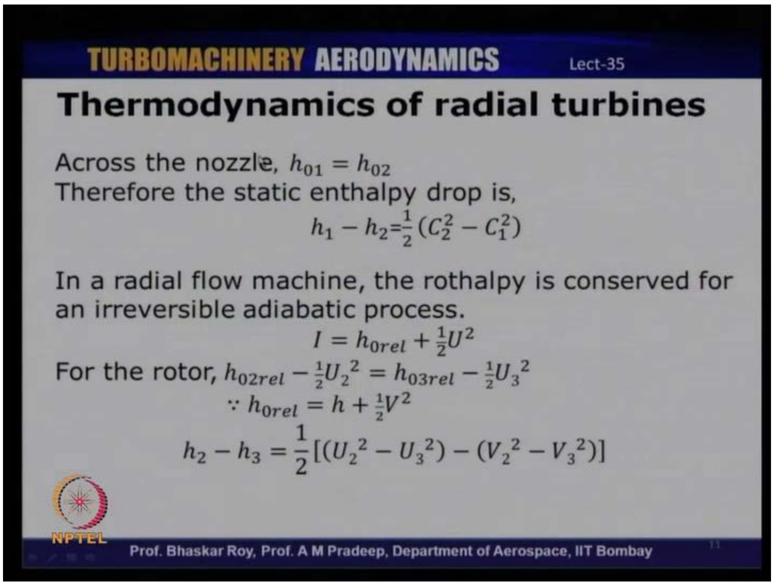

So, we will start with the nozzle now. Across the nozzle, we know that there is no change in stagnation enthalpy; h 01 should be equal to h 02. Now, therefore, the static enthalpy drop in the nozzle is h 1 minus h 2, there is a change in static enthalpy. As we have seen, the enthalpy between stations 1 and 2, there is a static enthalpy drop, but this stagnation enthalpy is the same and this static enthalpy drop is basically equal to the change in the absolute velocities, one-half of c 2 square minus c 1 square. And in a radial flow machine, as we have discussed a few times earlier, the rothalpy is conserved for an irreversible process. But an adiabatic process, the process could be irreversible, which means, that frictional losses are permitted, but it is adiabatic and in which case the rothalpy is conserved and therefore, rothalpy is denoted by I, this should be equal to h naught, that is, stagnation enthalpy in the relative frame plus half U square which is the blade speed.

So, for the rotor, if you look at the rotor, since rothalpy is concerned between the inlet and exit, h 02 relative minus half U2 square should be equal to h 03 relative minus half U3 square. So, this is at the inlet of the rotor, the left hand side; right hand side is the exit of the rotor. Now, you also know, that h naught relative is the static enthalpy plus half V square, where V is the relative velocity because this stagnation enthalpy we are defining in the relative frame of reference. Therefore, if you look at the static enthalpy drop in the rotor h 2 minus h 3, it, it consists of two components, one is because of the change in the blade speed between the inlet and exit and other is on account of the change in the relative velocity as it, as the flow passes through the rotor. So, h 2 minus h 3 is equal to half of U 2 square minus U3 square minus V2 square minus V3 square. So, this is the change in static enthalpy.

As the flow passes through the rotor, we will now look at the 3rd component, that is, the diffuser and then see, what happens to the flow as it passes through the diffuser. We will then derive an expression for the specific work done in terms of all these different parameters. Subsequently, we will define efficiency associated with such a turbine and then express efficiency in terms of parameters, which we can derive from the velocity triangle, that is, the velocity components and the blade angles.

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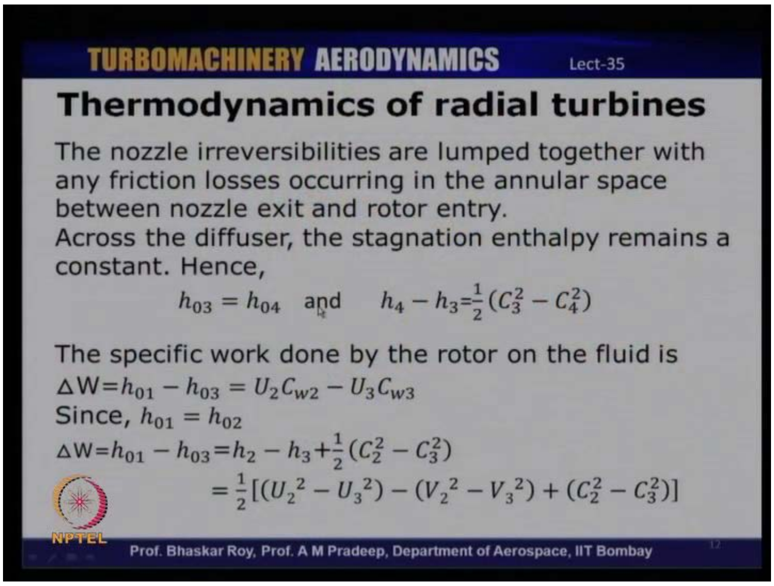

So, what is normally done is that the irreversibilities are basically, especially the nozzle irreversibilities are lumped together with frictional losses occurring between the space, between the nozzle exit and the rotor entry. So, the irreversibilities are basically lumped together in single parameter that we have just now seen. Across the diffuser, again stagnation enthalpy remains constant, therefore h 03 should be equal to h 04 and h 4 minus h 3, you can notice, that here it is h 4 minus h 3 unlike in the nozzle, where it was h 1 minus h 2. Here it is h 4 minus h 3 because there is a static enthalpy rise.

This is equal to half of C 3 square minus C 4 square. So, the specific work done by the rotor on the fluid is between stations 1 and 3, that is, delta W is h 01 minus h 03 and since h 01 is equal to h 02, we have h 01 minus h 03 as U 2 CW 2 minus U 3 CW 3.

So, delta W, that is, specific work done is equal to, we will express this in terms of static enthalpy. So, we have h 2 minus h 3 plus half into C2 square minus C3 square. This again can be further written down in terms of U and relative velocities. So, this delta W is half U2 square minus U3 square minus V2 square minus V3 square plus C2 square minus C3 square. So, you can see, that there are three distinct terms associated with specific work done and each of these different terms contribute towards the specific work done, and this is true for a specific or specifically for an inward flow radial turbine. And you can also see, that the axial turbine, in fact, happens to be a special case of this, where U 2 is equal U 3 and that component becomes 0 and the specific work done is basically because of the other two components.

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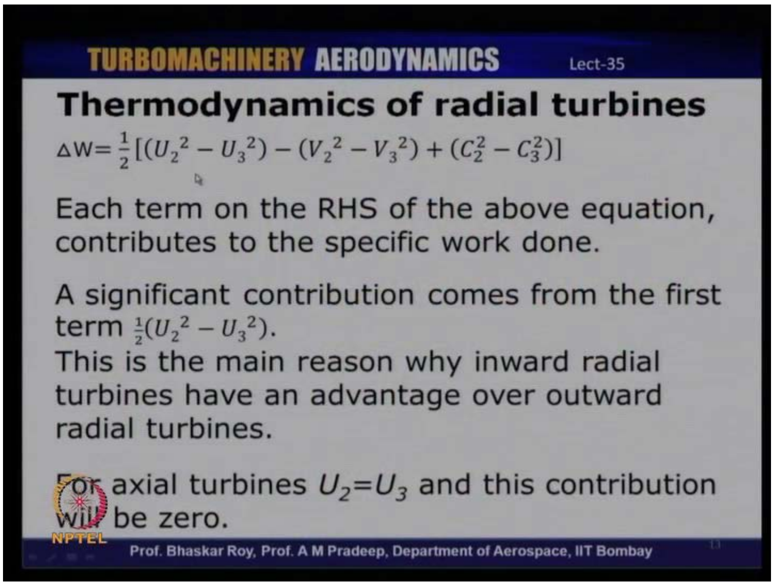

So, delta W, as we have seen in the specific work done by the turbine, has 3 components here, half U 2 square minus U 3 square, then the relative velocity change and the absolute velocity change. Now, for an inward flow turbine, a significant contribution comes from this term, that is, the change in the blade speed and this is basically one of the reasons why inward flow turbines have an advantage over outflow turbines or outward radial flow turbines because here, this contribution becomes positive in an outflow turbine; this actually becomes negative and that is the big disadvantage for an outflow, outflow radial flow turbines and that is why, they are not being used anymore in practice.

In an axial turbine, this component becomes 0 and basically work done is on account of the other terms. So, here, what we have done is that we have basically derived an expression for the work done in terms of velocity components that one can estimate from the velocity triangle. So, once you know the velocity triangle, the one, which I had shown earlier, could be used as a starting point to estimate these different velocity components and of course, the flow angles, which would also be known for the design condition. So, what we are going to do next is to discuss about what is known as the nominal design condition and then subsequently, the nominal design efficiency associated with a radial flow turbine.

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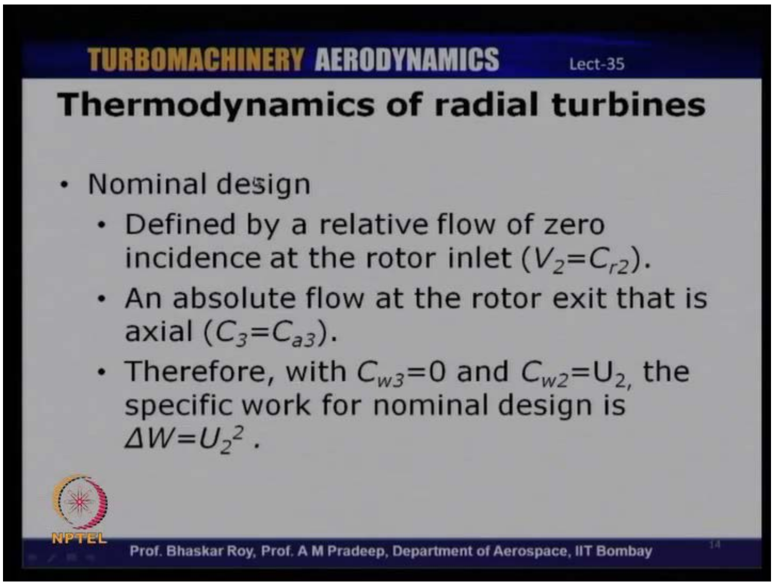

Now, nominal design is usually defined by relative flow of 0 incidence at the rotor inlet, that is, if the flow enters the rotor with no incidence, which basically happens when the relative velocity is actually equal to the absolute velocity in the radial reaction, V2 is equal to C r2. And the other condition, that is also satisfied is, that the flow exiting at the rotor is axial C 3 is equal to C a3. So, when C 3 is equal to C a3, then C W3 becomes 0, that is, the tangential component of absolute velocity at the rotor exit become 0. Therefore, C W2 becomes U 2 and so the specific work done for nominal design would now become delta W is equal to U 2 square. So, it basically becomes a function of purely the blades speed at the rotor entry.

So, we, in an ideal scenario, when the turbine is actually operating in design condition with 0 incidence at the inlet and the flow leaving the rotor without any deviation, then one can consider that the nominal design and work done can actually be estimated nearly based upon the blade speed at the rotor exit. We will also now look at specific terminology, that is sometimes used in the analysis of radial flow turbines and that is basically, which also will come up in our efficiency definition a little later on. And I am bringing up this particular aspect because in later analysis, the, this particular form of velocity is sometimes used in efficiency calculations as well.

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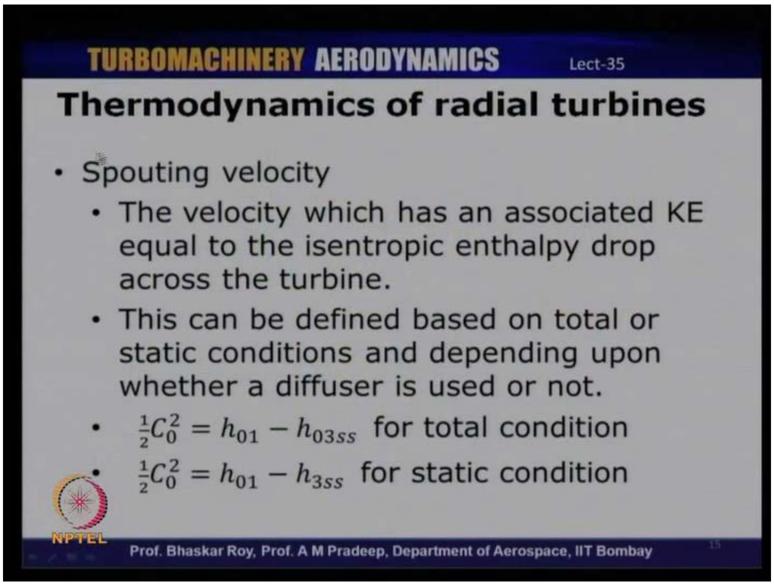

So, this is basically known as the spouting velocity and it has primarily been used in the hydraulic turbine context, but sometimes it is also referred to for normal gas turbine applications as well.

So, spouting velocity basically refers to the velocity, which has an associated kinetic energy equal to the isentropic enthalpy drop across the turbine. Now, they, depending upon how this is, what kind of an application it is, one can define spouting velocity in different ways, either for a total condition or for a static condition.

Spouting velocity, we will denote this by C subscript 0, C naught and so this is basically half C naught square, is either defined as h naught minus h naught 3 ss. We will come back to this what this means for total condition and if it for static condition, then half C naught square is equal to h naught 1 minus h 3 ss.

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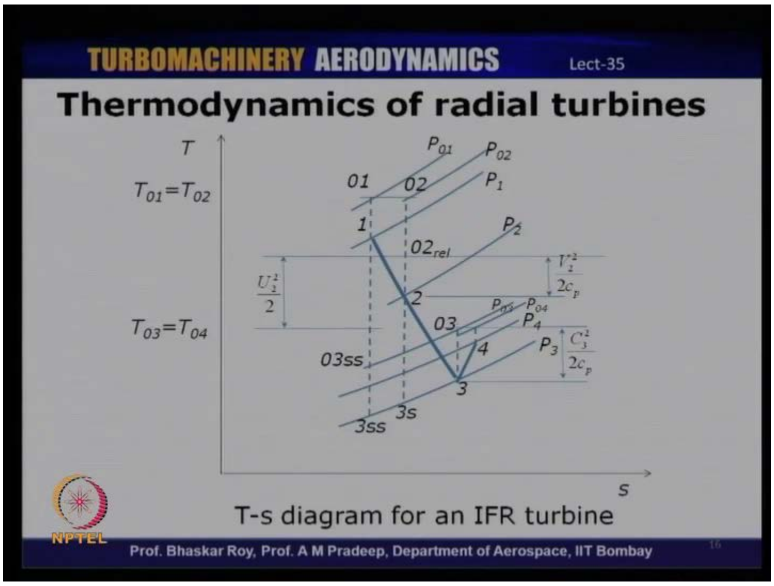

So, let us take a look at the T-s diagram once again, that I had plotted earlier. Now, there are two distinct points you can see here, which correspond to be isentropic enthalpy drop. So, the kinetic energy associated with this isentropic enthalpy drop is basically denoted by this spouting velocity. So, h naught 1 minus h naught 3 ss in one case and h naught 1 minus h 3s in the other case; so, these are the two definitions that one can have.

So, the amount of kinetic energy, that is involved in this much enthalpy drop between h 01 and h 03 for the isentropic case; similarly, h 01 and h 3 ss, that is for the static condition. So, this is basically defined depending upon applications, whether the diffuser is used or not, that is, if kinetic energy is recovered, then the flow basically reaches a static condition at the exit. In which case, one would define spouting velocity based on the 2nd definition, that is, enthalpy drop from h 01, isentropic enthalpy drop taking place between h 01 to h 3 s and if it is, if, if you do not use a diffuser, which means, there is stagnation, enough stagnation enthalpy is still available at the exit of the rotor, then one would prefer to use spouting velocity or define that in terms of h 01 minus h 03 s. So, this is one of the ways of defining kinetic energy and a term, which is sometimes used in analysis of these radial flow turbines.

Now, that we have discussed about nominal design point and, and conditions associated with nominal operation, remember, that the velocity triangles, that I had shown for a typical radial flow turbine was corresponding to a nominal operation because there was no incidence at the inlet and no deflection at the exit.

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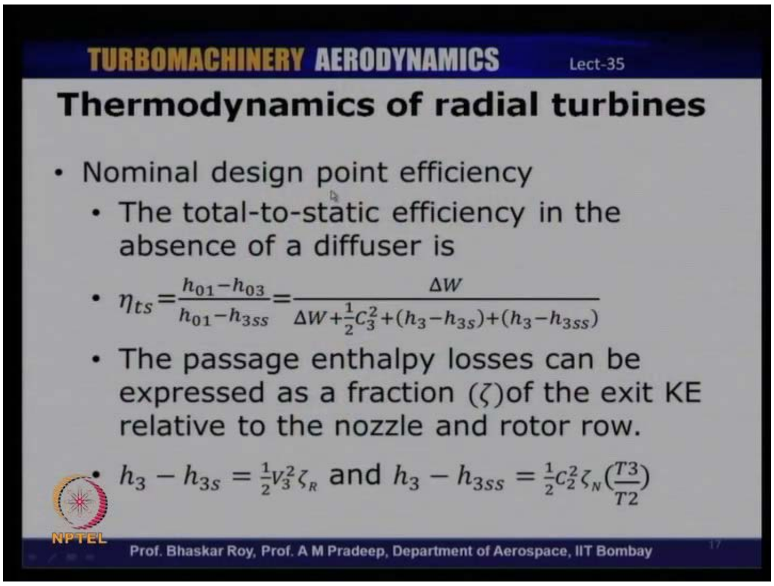

So, if you look at nominal design and try to define an efficiency for this particular design, we can define efficiency, we will define it first in terms of total to static efficiency, that eta T-s is total static efficiency and so we have h 01 minus h 03 divided by h 01 minus h 3 s, this equal to the numerator is basically the specific work done delta W divided by the denominator. We have specific work done plus a few additional components, that is, half C3 square plus h 3 minus h 3s plus h 3 minus h 3 s double, double s.

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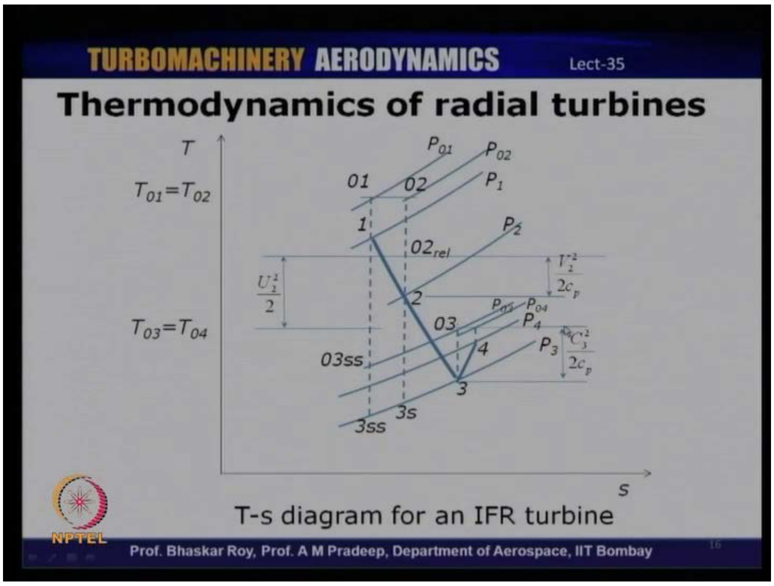

So, this look at the velocity the T-s diagram, we will once again be able to appreciate, that further, the numerator is h 01 minus the stagnation temperature corresponding to that h 03; denominator is h 01 minus the corresponding static enthalpy condition and the other components are what have been added up here. So, delta W plus what you see here has been added, is basically the difference between h 03, the .03 here to the 0.3 s. So, this difference is what constitutes these three other components, including half C3 square. The enthalpy difference h 3 minus h 3 s and h 3 minus h 3 double s.

We will now define a few loss parameters; let us define the passage enthalpy loss. We will define that as a fraction of the exit kinetic energy relative to the nozzle and rotor row. This fraction will denote by zeta, zeta subscript r for the passage losses in enthalpy, losses in the rotor zeta subscript n for the passage losses in the nozzle. So, we will define these enthalpy losses separately for the nozzle as well for the rotors. So, for the rotor we have h 3 minus h 3 s is equal to half V3 square into the passage enthalpy loss that is zeta subscript r. Similarly, for the nozzle we have h 3 minus h 3 double s is equal to half C 2 square zeta n into the temperature difference, that is, T 3 by T 2 ,where T 2 and T 3 are the static temperature at the rotor exit and rotor inlet respectively.

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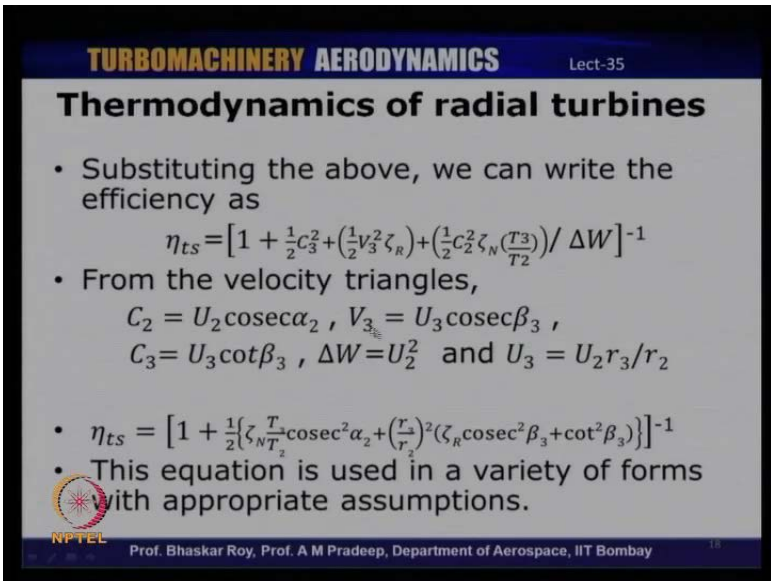

So, with this definition of the passage enthalpy loss, we will substitute these in the previous expression, that we have written down, that is, for the total to static efficiency and then, if we substitute these values here, we will get the revised formula definition for the total to static efficiency as 1 plus half C 3 square plus half V 3 square into zeta r plus half C 2 square zeta n into T 3 by T 2 divided by delta W raise to minus 1, so, or delta W divided by this.

Now, from the velocity triangles, that we have seen for typical 90 degree IFR turbine, we can also see, that the velocities can be related based on two angles, one is the nozzle angle, that is, alpha 2 and the other angle is the flow exiting the rotor, that is beta 3. So, based on these two different angles, we can actually calculate the velocity components based on these different angles. So, what we will do is that in the efficiency definition, we can substitute for these flow angles in the definition there and arrive at an expression, which is primarily in terms of some of these parameters, which are associated with the velocity triangle and the temperature ratio T 3 by T 2.

So, from the velocity triangles we can actually see, that nozzle exit flow absolute flow, that is, C 2 is equal to U 2 cosecant alpha 2 and V 3 is equal to U 3 cosecant beta 3. And similarly, C 3 is equal to U 3 cot beta 3 and assuming, that delta W is equal to U 2 square because for a nominal design, the flow delta C W is basically equal to CW 2 and so we have delta W is equal to U 2 square and the fact, that U 3 is equal to U 2 into r 3 by r 2, where r 3 is the radius at the rotor exit, r 2 is the radius at the rotor inlet. So, if you substitute all these different values in the efficiency definition here, we have this total to static efficiency in a, in a generic form, 1 plus half into zeta n, the enthalpy loss coefficient for a nozzle T 3 by T 2 into cosecant square alpha 2 plus r 3 by r 2 the whole square multiplied by zeta r cosecant square beta 3 plus cot square beta 3 inverse of this, looks like a very complicated formula here, but this is a very generic version of the total to static efficiency definition, that one can use and lot of approximations and simplifications to this formula is what have been used by the designers at a preliminary level to estimate the total to static efficiency. So, basically this involves a temperature difference or between the nozzle, between the rotor exit and inlet and the nozzle inlet angle and the rotor exit angle and as well as the enthalpy loss co-efficiency for the nozzle as well as the rotor. So, this can actually help us in estimating the total to static efficiency for a typical inward flow turbine configuration.

So, the generic formula, that I had derived, is actually used in variety of simplified versions by lot of assumptions and one can actually derive, simplify this further by calculating the temperature in terms of the velocity components and the blade speed. So, there is, there is one more step, that is involved. If, if you wish to do that, you can actually estimate the static temperature ratios T 3 by T 2 and express that in terms of the flow angles and the blade speed, that would complicate the formula even more. So, I am not going in to that as of now. So, this particular efficiency definition, what we will also do is to try and relate that to the total, to total efficiency.

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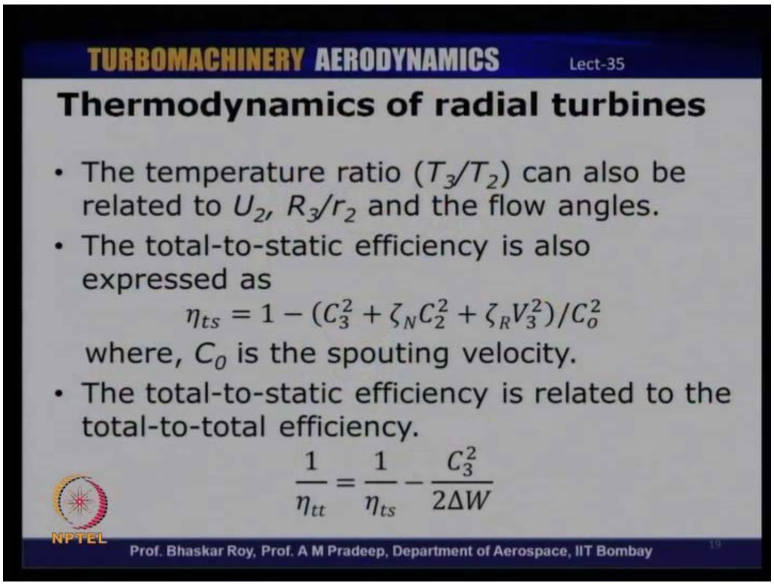

If one wants to use total to total efficiency as a performance parameter, which is sometimes used especially in aero-engines applications, now the temperature ratio, as I was mentioning, can be related to the blade speed at station 2 and the radius ratio and as well as the flow angles. Now, in, if you were to relate total to static efficiency to the spouting velocity, we can also relate it in terms of the velocity components, that you see here as 1 minus C 3 square plus zeta n C 2 square plus zeta r V 3 square divided by C naught square. So, if one can estimate the spouting velocity, then the efficiency definition can be simplified further and now you can see, there are 3 components of this efficiency definition, one corresponding to the nozzle zeta n and the velocity zeta r for the rotor and corresponding absolute velocity C 3 is the velocity leaving the rotor absolute velocity divided by the spouting velocity square. So, this is yet another way of expressing the total to static efficiency.

Now, the, the relation between total to static and total to total, we have already seen for an axial turbine. So, the very same relation also holds for radial flow turbine, whereas 1 by total to total efficiency is equal to 1 by total to static efficiency minus C 3 square by 2 delta W, where delta W is the work done by the rotor. So, this is basically relating total to static and total to total efficiencies.

So, what we have discussed now is about the definition of total to total and total to static efficiency in a variety of different ways and based on this understanding, let us also now take a better look at, closer look at the different losses and the sources of losses associated with the radial flow turbine. Now, there are different ways of representing losses, which have been used by different researchers over the years. We will look at two distinct groups, one set of loss parameters associated with the nozzle, another set of loss parameters associated with the rotor and then, we will also look at the sources of these different losses in a general format.

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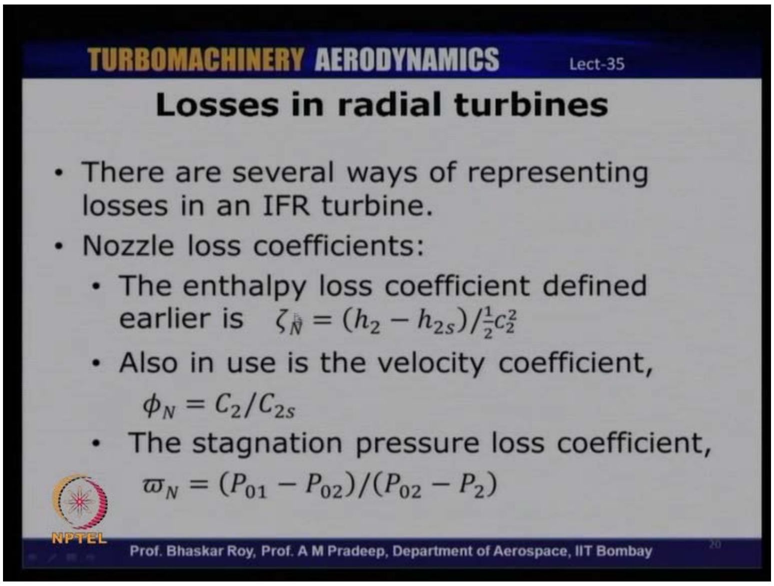

So, look at the nozzle loss coefficients; there are different ways of expressing the loss coefficients. We have seen one of them, that is, the enthalpy loss coefficient, that is, zeta N, which is basically defined as the h 2 minus h 2s divided by half C 2 square. In, in some of the literature, you might also come across what are known, what is known as velocity coefficient phi subscript N. This is defined in terms of absolute velocities C 2 divided by C 2s and of course, the standard stagnation pressure loss coefficient, which is denoted by omega N P 01 minus P 01 by P 1 minus P2. So, these are three distinct coefficients or loss parameters, which one would define for a particular nozzle configuration.

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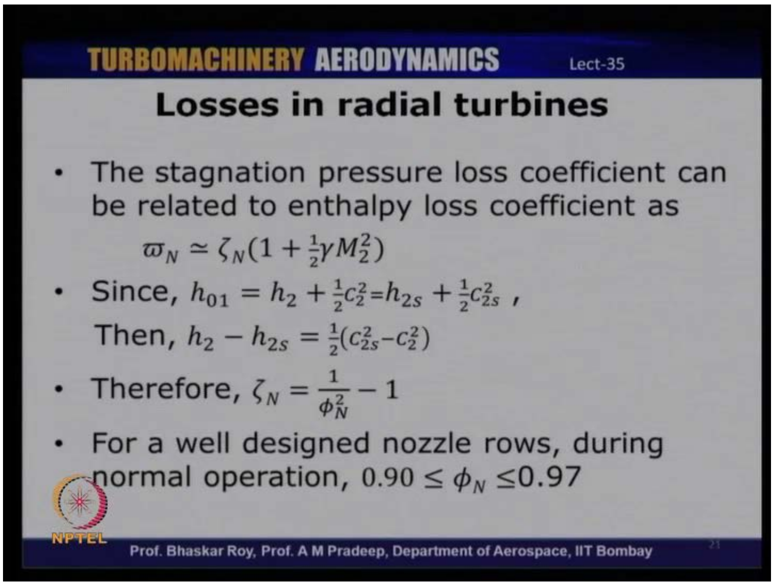

And you can actually relate some of these different loss parameters; the stagnation pressure loss coefficient is approximately related to the enthalpy loss coefficient through the Mach number. So, omega N is approximately equal to zeta N into 1 plus half gamma by 2 into M 2 square, where M2 is the absolute Mach number at the rotor entry, which is basically related to C 2 and the temperature at the rotor entry.

Now, we know that the h 01 is basically h 2 plus half C2 square or this is also equal to 02 and therefore, it is h 2 plus half C2 square. This is in turn equal to h 2 s plus half C 2 square s. Therefore, h 2 minus h 2 s is half C 2 s square minus C2 square. So, we can actually relate these two different components, that is, velocity coefficient and the enthalpy loss coefficient through zeta N is equal to 1 by phi N square minus 1. So, the velocity coefficient can be related to the enthalpy loss coefficient associated with the nozzle and it has been observed, that for a well designed set of nozzle blades during nominal operation, the typical value ranges between 0.9 to 0.97 for phi N. So, the velocity coefficient ranges between 0.9 to 0.97 for a well designed nozzle row, which is operating under a nominal operation condition.

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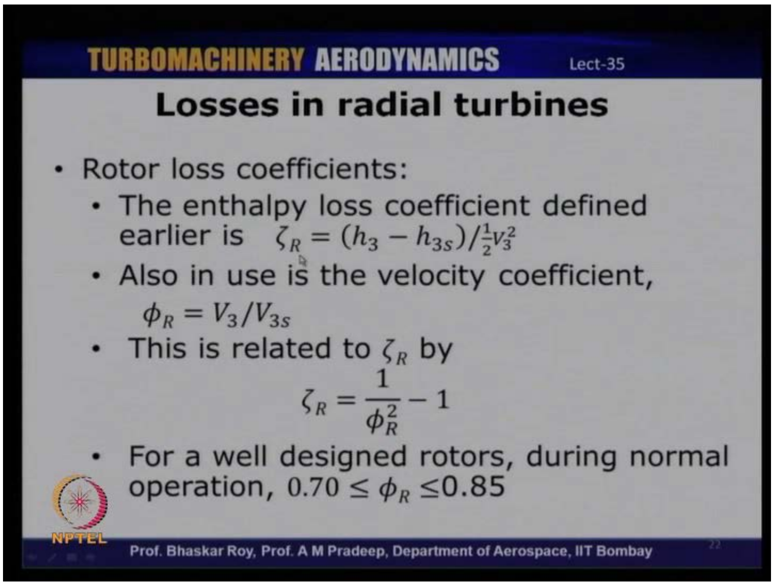

We now look at the rotor loss coefficient; we will define them exactly the same way. Enthalpy loss coefficient is zeta r h 3 minus h 3 s by half V 3 square velocity coefficient V 3 by V 3 s and this is related to zeta r the same way as we have done for nozzle zeta r is equal to half phi r square minus one. In the case of rotor we have seen, that it is noticed, that phi r ranges between 0.7 to .85. So, this is the typical range for the velocity coefficient for a nozzle and for a rotor. So, for a rotor you can see, it is much lower in the range of 0.7 to 0.85. On the other hand, for a, for a nozzle it can be quite high, between 0.9 to 0.95.

So, these are some of the methods of estimating some simplistic loss parameters for nozzle as well for rotor. So, there are other complex loss parameters, which look at the sources of these losses, which is what we will discuss a later now, which also involves the 3D loss sources, like the leakage flows and secondary flows and so on. These are the parameters we just discussed are overall loss parameters, which kind of puts all the other individual components of, or sources of losses into a single parameter.

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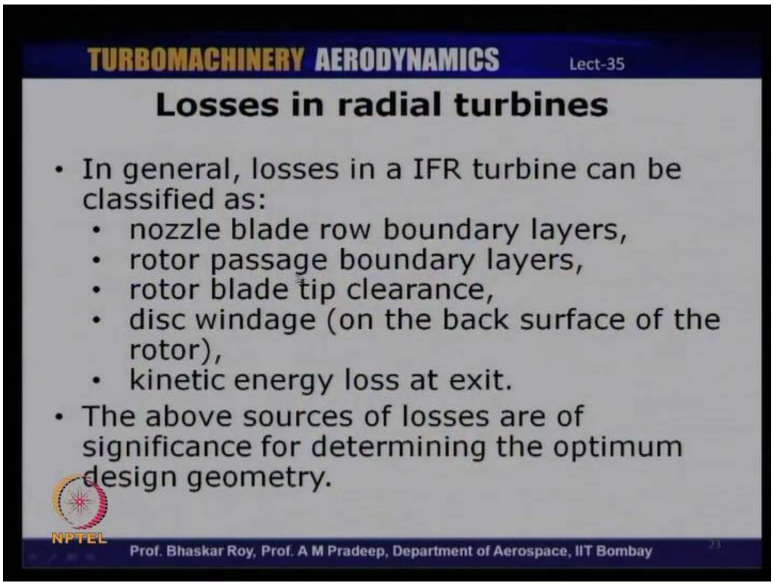

So, in general, if you look at the different sources of losses, let us be specific for an inward flow radial turbine, then the various sources of losses can basically be the nozzle blade row boundary layers, the rotor passage boundary layers, then you could have tip leakage or tip clearance effects at the rotor exit, the disc windage, that is, the rotor surface itself can lead to certain amount of windage losses and the kinetic energy loss at the exit.

So, these are the different sources of losses and the loss coefficient or parameters we just defined, do not really look at these individual sources of losses and they are combining the various losses into a single parameter for easiness of analysis. And there are of course, very complicated loss models, which can estimate these different sources of losses and of course, that is out of scope of this particular syllabus, that we are trying to cover here.

So, we are not going to details of these individual loss models, we are just trying to take overview of the different sources of losses. So, the knowledge of these different sources of losses, obviously, very significant in obtaining an optimum configuration for a particular design that is being attempted. One, there is one more source of loss, that I would like to highlight upon before I close this section here, that is to do with the incidence effects.

And when the turbine is operating at under off-design conditions, whether it is at a different mass flow rate or it is at a different rotational speed than what it has been designed for, all of these constitute the off-design conditions. So, under off-design condition, there is an additional source of loss that comes into picture, which is associated with a higher level of incidence than the nominal incidence itself.

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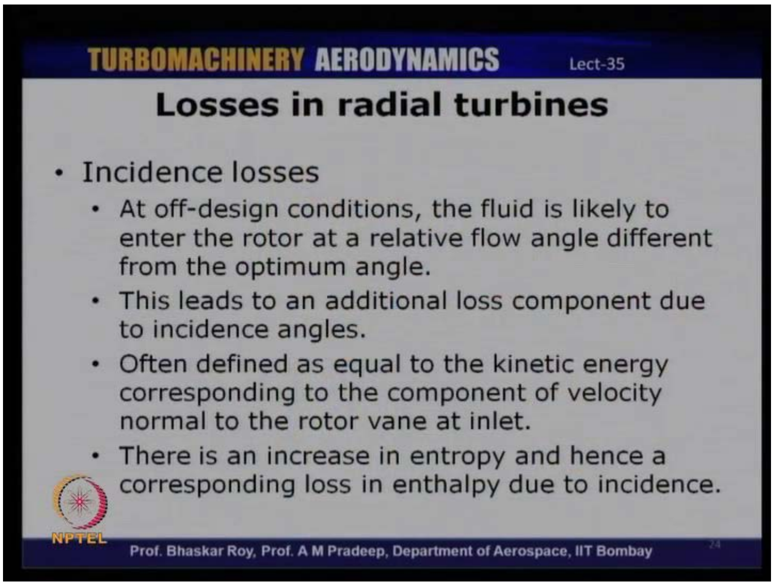

So, at off-design condition, the flow is likely to enter the rotor at a relative flow angle, which is different from the optimum angle and this leads to an additional loss component due to incidence and this is often defined as equal to the kinetic energy corresponding to the component of velocity normal to the rotor vane at the inlet.

So, what does basically does is that it leads to a corresponding increase in entropy and a corresponding drop in enthalpy due to incidence. So, enthalpy drop will directly correspond to drop in work done by the turbine and increase in entropy leads to a drop in efficiency for the turbine. So, there are multiple effects here, one is, there it leads to a drop in the work output of the turbine, at the same time it also effects the efficiency, which in thermodynamics sense is because of an increase in entropy.

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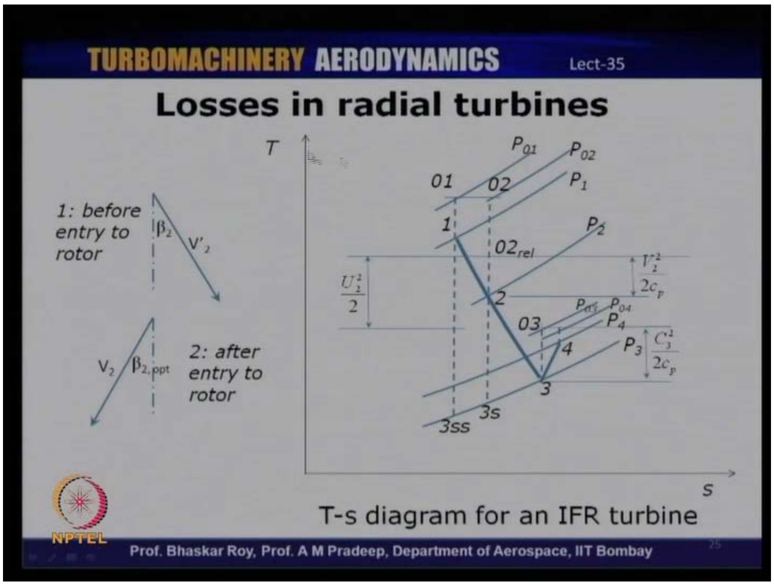

So, let us take a typical example of why this happens in, in a normal turbine. So, let us look at the T-s diagram that we had discussed earlier. Now, at the entry to the rotor, let us say the, the rotor has been designed for an angle of beta 2, which is the optimized beta 2 for a velocity, which is V 2 relative velocity and as the flow approaches the rotor because it is operating under off-design condition, the approach angle itself could be slightly different from what it has been designed and optimized for.

So, the velocity, approach velocity is at V 2 prime, which is slightly different from V 2 and this leads to a certain amount of a mismatch between the flows as it approaches the rotor and as it enters the rotors. So, there is a slight different between the flow as it enters the rotor leading to a certain amount of incidence and therefore, the flow enters the rotor at a slightly higher angle than what it was intended for, leading to multiple effects in terms of loss in enthalpy and an increase in entropy and therefore, a drop in efficiency. So, this is usually termed as an incidence loss; loss associated with incidence angle, which is greater than what it has been primarily designed for.

So, let me quickly now wind up this lecture with a recap of what we had discussed in today's class. So, today's lecture was an introductory lecture on axial, on radial flow turbines. We discussed the fact, that radial flow turbines were of course, developed in the early 1800s, primarily for hydraulic applications for power generation from hydraulic applications and of course, subsequently, they were, they have evolved and they have been used also for other applications, like gas turbine engines and so on. And so, we have seen that there are two different classes of radial flow turbines, either the outward flow turbines or the inward flow turbines. Inward flow turbines are, have inherent advantages, which is why they are used over a wide spectrum of power output ranges, like it they are used in hydraulic power plants, where they developed something like a few 100 megawatts of power, all the way to very small gas turbine engines, where (( )) turbines are used, which generate a few kilowatts of power. So, they have a very wide spectrum of applications. In the inward flow turbine we have seen, there are again, two classes of turbines, one is like the impulse flow turbine, these are also called cantilever flow turbines and the other is the 90 degree inward flow turbine, which is what we had spent lot of time discussing about, that basically a reaction turbine and where there, there is contribution from the nozzle and the rotor in terms of static pressure drop and work output of the turbine.

We then discussed in detail the, the governing equations of flow as it passes through these different components from the nozzle, through the rotor and diffuser and how we can estimate the work output based on the flow, these, this analysis. We also discussed about the efficiency definition in quite some detail and expressed it in different forms including the spouting velocity definition.

We also discussed in little bit detail about the various loss mechanisms and how one can estimate the loss in a very general sense, losses associated with the nozzle and loss parameters associated with the rotor. So, we covered these different aspects of radial flow turbines in today’s lecture, starting from an introductory part towards the thermodynamic analysis and also the different sources of loss and the components of losses in radial flow turbines.

So, we will continue with some more aspects of radial flow turbines in subsequent lectures as well.