

Turbomachinery Aerodynamics
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Lecture No. # 30
Turbine Blade Design: 3d Blade Shapes

We are talking about axial flow turbine design. In the last lecture, we had some look at their profiles basically airfoil kind of profiles which are used in turbine design and development.

We had a look at the profiles which as you might have understood are quite different from standard airfoil profile that you may be familiar with. The kind of airfoil profile that you used in aircraft wings or for that matter, even those are used in axial flow compressor which we had done in axial flow compressor chapter.

And now you can see that the profiles used in axial flow turbines are substantially different. You can make out from the look that their turbine profiles. We also had a look at some of the typical profile that may be used in supersonic axial flow turbines which are rather rare, but, still they are in using very special circumstances.

In today's lecture, we will take a look at how to build up a three dimensional turbine blade using some of the simple theories that we have been discussing about and how to put together those theories. In the process of typically creating a design steps, a set of design steps that help the design process in a step by step manner. We may use one or two new definitions, one or two new abstract definitions to help us in the design process. Typically a design process is indeed aided a lot by the databank that is normally available with the designer or available with the design bureau. Every big company or even small company who actually develop axial flow turbines would invariably have a lot of design data available in the form of databanks.

Now, those kinds of databanks are invaluable in case of turbine design or for that matter in compressor design as we have seen before. You need to kind of call upon that kind of

databank, which supplies initial data. For example, things like losses, what should be the losses of various profiles or the three dimensional loss parameters or the turbine tip flow losses, secondary losses. Many of those things are indeed extremely difficult to predict or predict through analysis early on when the blade is still being designed.

So, normally designers do take help from that kind of available databank. They also need some kind of a databank for choice of the profiles which are normally chosen on the basis of cascade data. So, again some kind of a cascade databank is available. The early ones were created by various agencies like NASA or NACA and they had created the early databank. Similar databanks are available in many other countries like England or Germany or Russia and they use their own databanks to supply data to the designers even today.

So, we will go through the various steps of the design at and at various stages. I will try to indicate to you what kind of databank would probably need to be looked into. Some of the very early data graphs, plots etcetera may be available in some of the references that we have given, the references of NASA or the books by Howelock. In many of those, some of the early plots, graphs which are created of two dimensional cascade formation may be available for you to make use of and you can put together what can be called a reasonable preliminary design of an axial flow turbine.

You need to realize that once this preliminary design is available, it needs to be submitted, nowadays, for very rigorous computational CFD analysis. Only after this CFD analysis is done, sometimes, as I mentioned it goes into a CFD design loop and the design may have to be done all over again. Once that is finalized, it goes into final prototype making and rig testing. So, the steps that we are looking at are meant for creating an axial flow turbine where nothing exists.

It is not meant for, strictly not meant for you know if you are just making a few changes in an available turbine design for which you do not need to go through all the steps. But, it is essentially meant when you need to design an axial flow turbine from scratch.

Let us go through the various steps in which some simple way of designing axial flow turbines have been enumerated. Now, we are talking about three dimensional blade. The reason is that, I mentioned that turbine blades specially the rotor blades do acquire a

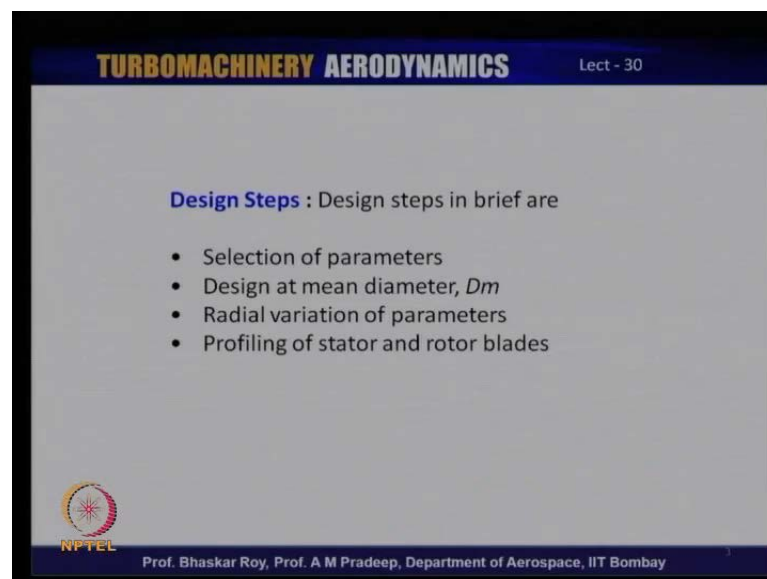
certain amount of twist. Maybe, not as much as the compressor blades, but, they do acquire quite a lot of twist.

On the other hand, the tendency of many of the modern designers, especially in the hp turbines is to avoid giving a twist to the stator nozzles. One of the reasons, as I mentioned earlier is to make sure that you have sufficient cooling technology embedded inside the blades. To accommodate the cooling technology, which is indeed you know, more important issue, the twist is often avoided in stator nozzles.

On the other hand, in the [row\rotor] rotor blades a certain amount of twist is inevitable. The modern hp rotor blades are indeed cool and hence twist is indeed kept to the minimum to accommodate the cooling technology which again is indeed the more important issue.

However, later on in the LP turbines, both the stator nozzle and the rotor blades do acquire somewhat more twist to accommodate better and better aerodynamic efficiency of these compressor stages. In the hp stages, the efficiency or more work done is accomplished through very high temperature operations, but, in LP turbine that high temperature is not available any more. So, one tries to accomplish higher turbine efficiencies. So, that the work extraction is as high as possible for the given shape and size that is available within the engine. So, those blades are quite often somewhat twisted.


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Design Steps : Design steps in brief are

- Selection of parameters
- Design at mean diameter, D_m
- Radial variation of parameters
- Profiling of stator and rotor blades

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Let us get into the fundamental issues of blade design. Now the first thing that you would see is that you need to select the parameters of design. As we had seen in the last class, the selection involves number of parameters. Some of the parameters that we had looked at in the last lecture were indeed thermodynamic parameters. They come out of the thermodynamic cycle design of a particular engine gas turbine engine. Then those thermodynamic parameters become the starting parameters for the axial flow turbine design. Now those parameters need to be factored in. We will look at them again in the next slide.

The second issue is that design is normally initiated at the mean diameter which is the mean length of the turbine. This means, if you have the annular space defined, the mean of the annular space is where the design is started or initiated and then you design the rest of the annular space.

So, the design is normally done at mean diameter. To begin with, if it is a multistage turbine, it is done for all the stages at the mean diameter first. It is only after the entire mean diameter design of all the stages are completed satisfactorily and they have been well matched with the compressors which they are powering, only after that the complete three dimensional design of the blades is normally undertaken.

So, design at mean diameter is an important step. Often, it is not a very quick method. Even though we will go through a very quick round of steps, but, quite often it does take a long time before the mean diameter design of all the stages of a multistage turbine is completed and appropriately matched with axial flow turbines axial flow compressors which they are supposed to power.

And then of course, you take the radial variation of all the parameters, the flow parameters and the geometrical parameters of the blade. So, the radial variation is normally guided by one of the laws of three dimensional blade. The primary one which everybody remembers is a free vortex law, but, as we have discussed before in one of the lectures earlier that in axial turbine design, free vortex law is not the most popular law.

There are other laws which are more popular for axial turbine design and we will have a look at some of those again in today's lecture. The design finally, would need to end with profiling of all the sections from root to tip of stator and rotor blades of all the stages.

We will go through the steps of designing a particular rotor and in the process look at how the stator is put together. Similar things need to be done for all the stages of a multistage axial flow turbine. For all the stages, then, the profiling of the stator and rotor blade; that means, essentially choice or selection or creation of the profiles of the individual blade sections would need to be undertaken. You may need to make something like hundred different profiles for all the stators and rotors to create a final multistage axial flow turbine.

So, that is the enormity of the job that we have at hand similar to what we had done in axial flow compressors. It is a time consuming job. Often, it is done in a **a** group of designers, very well qualified people and then only the turbine design is taken to be complete before it is handed over for computational fluid dynamic analysis.

So, let us look at the various steps starting at the beginning; that means the basic parameters.

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1. Design at Dm

a) Parameters to be selected from the preliminary cycle (thermodynamic) calculations are: (@ flt velocity & altitude), T_g, \dot{m}_g, P_a, T_a at design point P_g^*, π_T^*, P_T^* and the engine parameters

$$\eta_T^* = 0.88 - 0.90 \text{ for one stage}$$

$$= 0.91 - 0.94 \text{ for each stage for multi-stage}$$

b) Parameters selected from compressor-turbine matching are:
Exit $U_{mean}, D_{mean}, D_{tip}, \bar{d}, \lambda_2$ or M_2

[λ is a total temperature based critical speed ratio, $\lambda = v/a_{cr}$]
and, $a_{cr} = \sqrt{\gamma_g R T_{0i}}$

All these should lead to a work distribution, $H_T = \sum_1^Z H_i$
and number of stages Z.

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The parameters to be selected from the preliminary cycle, which includes factors like flight velocity and flight altitude, at which the design point has been selected. As we have discussed, the design point may be ground static condition for an aircraft engine or it may be a flying condition for an aircraft engine. Especially, if it is a supersonic aircraft and engine meant for supersonic flights.

Now, these parameters are typically the entry parameters placed on flight velocity and altitude temperature T_A and P_A . Then of course, the combustion chamber delivery temperature T_G or T_{03} or whatever we have talked about in terms of cycle. In terms of axial flow turbine, it is T_G which is t gas at the entry to the multistage turbine and that is coming from combustion chamber. Then of course, we need to know what is the mass flow inclusive of their fuel that has been burnt and that \dot{M}_g is the mass flow that is coming from combustion chamber. Then of course, the pressure that is being supplied from combustion chamber pressurization that is occurred in an axial flow compressor or any other kind of compressor and that pressured gas is also carrying a very high pressure that is to be supplied to the turbine.

And then from the cycle analysis, we have some idea what should be the turbine pressure ratio *prima facie* what should be the turbine pressure ratio to match with the compressor pressure ratio to supply the necessary amount of power that runs the compressor or any other load like propeller or fan.

The other thing of course, quite often important, is the turbine exit temperature which sometimes is prescribed. This also needs to be factored in, going into the exhaust or in case of aero engines, the nozzle of the engine.

Depending on the state of art of the engine design, some of the efficiency parameters can be aimed for. To begin with, some efficiency number or value needs to be used for design purposes. This would indeed have to be checked out later on thorough detailed analysis.

So, the efficiency values given here are kind of ballpark figures that one can use to start the design process. If it is a single stage turbine, quite often efficiency of the order of 88 to 90 percent is often possible. It is likely to be slightly high pressure and high pressure ratio axial flow turbine.

Whereas, if you have a multistage turbine, quite often, the individual stage efficiencies are somewhat on the higher side. The overall multistage efficiency would again be of the order of 90 percent or so. Whereas, the individual stage efficiencies could be higher than that. Individual stage efficiencies, nowadays, could be indeed as high as 94 percent.

The parameters that are selected for compressor turbine matching, these are normally the

mean blade velocity U , the mean diameter \bar{D} , the tip diameter, the hub to tip ratio of the diameters d_{tip} and two parameters namely, the Mach number at the exit of the stator of the particular stage being designed or an equivalent λ_2 , which is actually a critical speed ratio.

λ_2 is equal to V by 'a' critical where 'a' critical is a total temperature based on critical speed similar to a sonic speed, but, based on total temperature. As a result of which, it is often useful to essentially gas turbine designers because you do not have to convert all the time temperature from total temperature to static temperature because sonic speed is based on static temperature as per the basic definition of sonic speed.

'a' critical has been essentially defined for turbine designers. So that it does not have to all the time convert a total temperature to static temperature at every location. So, it is a useful definition and so, λ_2 is something similar to M^2 or Mach number.

Now, all these things should lead to the total work distribution amongst the various stages. This is typically done by distributing the number of stages over the many stages over hp and LP. You remember, the number amount of work done in turbine stage is substantially more than that of an axial flow compressor stage. As a result, the number of axial flow turbine stages is normally far fewer than that of number of axial flow compressors. As a result, the number of stages is far lesser and hence a little bit of extra work more or less would have to be accommodated within the integer number of stages that are possible.

So, if a little more work is to be done, that is to be done by those stages. Every one of them doing a little more or a little less power can be redistributed among some of the stages. So that you do not need to reduce or increase the number of stages at the design, in the process of design.

So, number of stages is decided by that aggregate amount of work that needs to be done and then split between hp and LP depending on how much power needs to be supplied to the hp compressor and to the LP compressor hot fan. Then correspondingly design LPT and HPT and distribute the work in those stages within hp and LP turbines. So that is how the distribution of work is done and number of stages often arrived at.

We have seen that the number of stages in LPT often is more than that of the number of

stages in HPT. That is because each stage of LPT is actually doing far less work than that of any stage of HPT. Hence, typically aggregate amount of work done in LPT could be of the same order as aggregate work done in HPT. But, the number of stages of LPT would be more than that of number of stages in HPT. So, these are some of the standard features of axial flow turbine.

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2. Design at D_m

Matching : Peripheral velocity, u is selected from turbine-compressor matching criteria. The outlet velocity from the nozzle may be supersonic, but inlet relative velocity to rotor is generally brought down below sonic speeds. The respective sonic speeds are:

At nozzle exit $a_2 = \sqrt{\gamma R T_2}$

At rotor exit $a_3 = \sqrt{\gamma R T_3}$

Total Temp based critical speed at the nozzle exit $a_{cr2} = \sqrt{\gamma_g R \cdot T_{02}}$

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Now, let us look at what do we do to initiate the process of design at mean diameter. One of the first thing we mentioned in the last slide, is that, you need to do a matching of compressor and turbine which means the compressor and turbine much must be at the same speed and thereafter the U is selected. Now compressor design if it is going on parallely or if it is being sort of already undertaken, the speed of the compressor at which it is running rpm may be already selected or in the process of being selected in which case, the rpm of the particular turbine spoon would be also selected.

Now, once you do that, given the annular space that is available near the combustion chamber let us say for HPT, the median of that space is that $U D$ mean and correspondingly U mean would come out of that particular selection process.

Now, whether that value of U mean is sufficient for you to accomplish the amount of work to be done on the basis of Euler's formula of work done, needs to be decided very quickly. Otherwise the value of rotating speed rpm would have to be reselected or changed. Alternatively, the annular space definition of the particular turbine, a blade

would have to be redesigned.

We have seen that LP turbine is often set at a higher radial station essentially as we have discussed to achieve a higher value of U . This is precisely the point that you may have to select the annular space and the rpm together to arrive at a value of U that matches with the compressor and supplies the work to be supplied to the compressor. The other thing that needs to be selected is the outlet velocity from the nozzle which may actually be choked. In which case, the velocity could be held at sonic velocity or just a little below sonic or sub supersonic.

Normally, most commercial axial flow turbines used in aero engines, the relative velocity to a rotor is generally below sonic speed that is subsonic. Normally, they are not supersonic rotors as of even today because of the need for cooling that needs to be embedded in the turbine blade shape. However, it is possible that in future some of the blades would go supersonic were cooling technology would be appropriately used or eliminated.

A few definitions are in order here. The respective sonic speeds are defined at the nozzle exit as A_2 is equal to $\sqrt{\gamma R T_2}$ and at the rotor exit as A_3 is equal to $\sqrt{\gamma R T_3}$. They are indeed being defined at the nozzle exit and rotor exit. Because in turbines, the velocity is actually the speeds of the flow through the blade passage actually accelerate and they reach near sonic or mildly supersonic near the exit of the blades unlike in compressor where they reach supersonic or transonic near the entry of the blade rows of rotor and stator. So, in case of turbines, it is the exit flow condition which is indeed more important and quite often used for normalizing purpose.

The other definition which we just mentioned in the last slide, is the total temperature based critical speed at the nozzle exit and this is now defined as $A_{critical 2}$ at the nozzle exit and $\sqrt{\gamma g R T_0 2}$. So, where there is a difference between A_2 and $A_{critical 2}$ and this is typically used by the turbine designers as I mentioned, to avoid the process of converting from total temperature to static temperature.

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Blade passage wall temperature at Dm

$$T_{wt} = T_{0t} - \frac{C_2^2 - V_2^2}{2C_p} = T_{0t} - \frac{\gamma - 1}{2\gamma R} (C_2^2 - V_2^2)$$

$$V_2^2 = C_2^2 + U^2 - 2UC_2 \sin \alpha_2$$

$$\frac{T_{wt}}{T_{0t}} = 1 - \frac{\gamma - 1}{\gamma + 1} (2\lambda_u \lambda_2 \sin \alpha_2 - \lambda_u^2)$$

and

$$\lambda_{2-abs} = \frac{\cos \beta_2}{\cos \alpha_2} \sqrt{\frac{T_{wt}}{T_{0t}}}$$

$\lambda_{2-rel} = \frac{\cos \beta_2}{\cos \alpha_2} \sqrt{\frac{T_{wt}}{T_{0t}}}$

where

 $\lambda_{2-abs} = C_2 / a_{cr}$
 $\lambda_{2-rel} = V_2 / a_{cr}$
 $\lambda_u = U / a_{cr}$

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As we have done in case of a turbine cooling, the blade passage overall temperature at mean diameter is also calculated. This gives us whole temperature of the turbine blade at a mean diameter on the basis of the velocity triangles that can be created at that stage which gives us the values of C_2 and V_2 .

Now, C_2 V_2 are the exit velocities from the stator nozzle which had gone in with the velocity C_1 and V_2 is the relative velocity to the rotor and C_2 is the absolute velocity from the stator nozzle coming out of the stator. The ratio of the overall temperature to the total temperature of the turbine at the throat of the stator nozzle is written down here as one minus gamma minus one by gamma plus one to multiplied by twice into lamda U into lamda 2 sine alpha 2 minus lamda U square from which we can get the ratio of lamda 2 absolute to lamda 2 relative. These are the ratios which are then proportional to the temperature ratio of the wall temperature and the total temperature at the throat of the stator nozzle.

Now, we are using that old temperature at the throat of the stator nozzle because that is where the flow essentially goes sonic. Now this again requires a little redefinition of the lamda where lamda 2 absolute is a C_2 divided by 'a' critical at to 'a' critical two and lamda 2 relative is V_2 divided by a critical 2 and lamda U which is a Mach number type of normalized blade velocity one can say that is U by 'a' critical.

So, these are normalized velocities with reference to a critical. These lamdas is

essentially, as I mentioned something similar to Mach number based on total temperature based sonic speed a kind of sonic speed. So, this allows us to relate the lambda 2 ratio of absolute and relative to the wall temperatures and of course, the relative and absolute flow angles at the stator nozzle exit.

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Choice of velocity triangle for the rotor inlet is obtained by the relation,

$$M_{2-rel} = J_1 (M_2 - M_{2u}), \quad \text{where}$$

$$J_1 = \sqrt{1 + \frac{1 - \sin \alpha_1}{\frac{M_1 M_{1u} + M_{1u} M_1 - 1}{2}}}$$

= 1.2 to 1.4, is a flow coefficient

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And now the velocity triangle can be completed by writing the rotor inlet condition at which M_{2-rel} can be written down in terms M_2 and M_{2u} . This comes out of the velocity triangle and here we use of flow coefficient J_1 which essentially derived flow coefficient and it is based on a number of parameters that we were just looking at M_1 . M_{1u} is again the blade velocity converted into a Mach number. Then of course, the α_1 with which the flow has gone in and this is value is normally of the order of 1.2 to 1.4. Now this is a kind of flow coefficient which is used to find out the relative Mach number at the rotor inlet.

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At the exit of the stage, Mach number can be expressed as,

$$M_3 = J_2 (M_{3-rel} - M_{3u})$$

Where, another flow coefficient J_2 is used,

$$J_2 = \sqrt{1 + \frac{1 - \sin\beta_3}{\frac{M_{3-rel}}{M_{3u}} + \frac{M_{3u}}{M_{3-rel}} - 1}} = 2.5 \text{ to } 4.5$$

$$M_{3u} = M_{2u} \cdot \sqrt{\frac{T_2}{T_3}}$$

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Now, similar flow coefficient is used for finding out the Mach number at that rotor outlet. That uses flow coefficient J_2 based on Mach number M_3 relative and blade velocity based Mach number M_{3u} . So, this gives us a J_2 and this again is normally else somewhere between 2.5 to 4.5. As you can see it is derived flow coefficient and the parameters here can be actually computed or assigned and a flow coefficient can be indeed actually calculated for the specific case.

In which case, we can find out then the Mach number based blade velocity related at the exit to that at the entry with relation to the static temperatures t_2 and t_3 . So, these are the steps that allow us to relate some of the thermodynamic and aerodynamic parameters across the rotor starting with the stator nozzle exit. This allows us to complete the aero thermodynamic flow features of the turbine blade that has been designed primarily at the moment at the mean diameter.

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For moderate pressure drops ($\pi < 2.0$) the flow in the rotor may be entirely subsonic. However for high pressure drops ($\pi > 2.5$), the flow becomes transonic at the stator trailing edge.

Stator Exit flow conditions:
 From continuity, for an unit length of the blade, at throat

$$\rho_t \cdot V_t \cdot s_t \cdot l = s \cdot l \cdot \rho_3 \cdot V_3 \cdot \sin \beta_{3\text{eff}}$$

s = blade spacing or pitch
 $s_t = t = \text{Throat width}$, Subscript t for throat
 β_t = blade exit angle
 β_3 = flow exit angle
 $\beta_{3\text{-eff}}$ = Mass - averaged effective flow exit angle



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Now, let us see that if you have moderate pressure drop across the turbine of the order of two or less, the flow in the rotor is most likely to be subsonic. That means you can do with the subsonic rotor and still be quite contain getting a pressure ratio of the order of two.

On the other hand, if the pressure drop required is higher than 2.5 or more, then the flow may need to be assigned a transonic flow which means it will go transonic near the stator nozzle exit. In which case, the flow would have certain transonic flow features and those would need to be then computed accordingly.

Now, the stator exit flow condition tells us that for an unit length of the blade, let us say at the throat of the blade, if we use the continuity condition, we can write down that the throat conditions at the stator $\rho_t v_t s_t$ being the spacing; that gives us the mass flow must be equal to the mass flow. That is going out of the rotor which is in terms of $\rho_3 v_3 \sin \beta_{3\text{eff}}$ that is going out of the exit of the turbine.

Now, this allows us to start talking about the blade spacing or the pitch or the packing of the blade which also of course, would tell us how many blades we would need to have. As we know, the spacing to chord ratio is an important parameter in blade design of turbines and indeed that of compressors as we have seen before. The additional important parameter here is a subscript t that is $t = 0$ and that is the throat width.

Because, throat is the place where the flow actually goes sonic and hence the throat needs to be decided very accurately because, the flow will indeed go sonic. Thereafter one has to decide whether it allows the flow to go mildly supersonic or to go back again to subsonic flow.

So, location of the throat, the throat dimension is an important parameter. Then of course, we get the various flow angles, the blade angle, the flow angle. The difference between the two of course, is the blade deviation. Others are small flow turbines and then of course, the effective mass average effective flow angle because the flow angle actually varies from the suction surface of one blade to the pressure surface of the other blade.

Now, this variation needs to be captured to begin with. Later on, this capture is normally done with the help of computational fluid dynamics to get a more accurate estimation of beta 3 effective. At the moment, one we use certain standard rules to get the beta 3 effective, but, these are the places where cfd would become more and more useful to give you a mass averaged beta 3 effective that needs to be used here in refinement of the design.

The other place where cfd would indeed be useful is finding out the throat more accurately because the throat is indeed where the flow gets actually sonic. You remember going sonic means the flow is getting choked. If the flow is getting choked in turbine, you remember the flow is getting choked over the entire engine of a gas turbine engine or the aircraft engine.

So, the choking in turbine essentially has to be matched with either the choking in the nozzle or choking in the other turbines. They have to be done in a matched manner and that choking is a very important issue because that maximizes the mass flow operation.

So, when you create a throat of a turbine blade typically near the nozzle exit, sometimes similar throat may appear at the rotor exit where the flow in also would get choked. That means, there is a matched choking between stator exit and rotor exit or even more continuous matching between various stages. Then, matching with the nozzle of the aero engine all of that can be done accurately only if the computations here are very accurate. Then of course, the dimensioning and the geometrical features are very accurately selected and geometrical modeling is done very accurately.

So, these are the places where when you get into cfd. More and more cfd would have to be used to get these features. They are refined and super refined if necessary to get those things done correctly.

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Generally, because of diffusion

$$\rho_t \cdot V_t > \rho_3 \cdot V_3$$

Subscript t for throat

$$\sin \beta_{3\text{-eff}} = \frac{s_t}{s} \quad \text{Throat area ratio}$$

The exact relationship between β_2 and s_t / s can be found experimentally by accurate cascade analysis

$$\beta_3 = \sin^{-1} \left(k_2 \frac{s_t}{s} \right) \quad \text{Initially assume } K_2 = 1$$

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Now, let us take a look at some of the other things that we need to get into. There is a possibility that some amount of small diffusion may occur from the throat, either of the rotor or of the stator. In which case, some amount of diffusion does occur and hence there is a change of a density and velocity which may happen during the process of the smile diffusion zone $\sin \beta_3$ effective. The effective flow angle can also be found out from the throat area or either of the rotor or of the stator. Definitely, you have to find it at the stator in which case it will be called $\sin \beta_2$ effective and that is the ratio of the throat dimension to the pitch or the spacing of the rotor or stator blades.

Now, as I mentioned this may have to be done definitely at the stator exit. Sometimes in modern turbines, you may have to do it again at the rotor exit. Now the exact relationship between β_2 and the $R \beta_3$ and the β_3 for a rotor β_2 for rotor inlet and the throat dimension to the pitch can be found out experimentally from accurate cascade analysis. This means, the cascade data becomes an important issue here. In that case, you can find the cascade databank to be of helpful for getting some of these dimensions and some of these flow features accurately decided at least prima facie decided before you embark on more involved cfd analysis.

So, this is done by using beta 3 is equal to sin inverse K 2 into S t by s, where K 2 is a value which is initially assumed as 1 to get the value of beta 3 or beta 2 as the case may be.

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TURBOMACHINERY AERODYNAMICS Lect - 30

a) Actual R_x ,

$$R_x = \frac{h_{rotor}}{h_{rotor} + h_{stator}} = \frac{h_{rotor}}{h_T} = \frac{V_3^2 - V_2^2}{2h_T}$$

Degree of Reaction - ideal to actual change

DR	0	0.1	0.25	0.35	0.45	0.5
DR_{act}	0.03	0.073	0.226	0.33	0.433	0.485

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Then of course, you need to find the value of degree of reaction. Now, degree of reaction as we have defined before is the static enthalpy change in the rotor to that in the rotor and stator put together. This typically can be denoted in terms of the change in relative velocity across the rotor and that gives us some idea about what the degree of reaction should be. If you relate that, we get a value of degree of reaction.

Now, what happens is, if you take the real value or a more actual realistic value of degree of reaction and the prima facie degree of reaction that may have been calculated earlier from some of the cycle analysis, one finds that when you get down to turbine design, the actual value of degree of reaction slightly varies from that of the original or earlier degree of reaction definition or values that may have come out of the earlier detail cycle analysis.

So, where the degree of reaction could be 0 which means an impulse turbine, it could actually have a very small degree of reaction. On the other hand, when the degree of reaction is 0.5 which actually gives you symmetrical blading, the actual value of degree of reaction is 0.485. This means that, the velocity triangles would not be exactly symmetrical in nature and when it is supposed to impulse it is not exactly symmetrical

blading.

So, this is something which the turbine designer would have to be careful about; that means, when you think he requires a symmetrical blading, he may not require a symmetrical blading. He may have to design which is not symmetrical blading or when he thinks he is can do with the impulse turbine rotor, he may actually have to do a slightly small reaction turbine, a turbine which has a small bit of reaction. So, these are the accuracies that the turbine designer would have to be careful about.

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b) Distribution of work in a multi-stage turbine

$$H_T = \sum_1^n H_i$$

c) Selection of flow track i.e. λ angle (local)

The diagram illustrates a conical flow track for a multi-stage turbine. The flow track is shown as a series of stages, each consisting of a nozzle (N) and a rotor (R). The flow track is defined by two dashed lines that converge to a point, forming a cone with a half-angle of γ , and a total angle of 2γ . The inlet height is h_1 and the outlet height is h_2 . The total axial length of the flow track is S . The axial distance between the centers of two adjacent stages is ΔS . The diagram also shows the radial positions of the nozzle and rotor blades.

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And then of course, as the distribution of work in multistage as we discussed earlier that distribution can be finally leading to an integer number of turbines. Now, if we look at the earlier flow track simple flow track design configuration that we had looked at, we see that finding out the flow track cone angle involves so many number of stages.

This means you have to decide the number of stages during which the work distribution must have been completed. If you have done that, you have an idea of how many stages are in hp and how many stages are in LP. Having decided that, you need to decide the annular space that need to be allotted to these turbines and choice of these annular space. Of course, this is based on whether you are going to have a constant mean diameter design or a variable mean diameter design as we have seen in some of the early lectures. So, this would allow the flow track to be designed at least a prima facie first cut design can be created through the hp and the LP turbine set.

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d) Velocity triangle and other parameters.

i) $H_T = \frac{H_{T-actual}}{W_{rotor}}$, where $\bar{w}_{rotor} = 0.97 - 0.98$
(Loss coefficient)

ii) $C_{2w} + C_{3w} = \frac{H_T}{U}$
 $C_{2w} - C_{3w} = 2U(1 - DR)$

From which C_w are selected at mean diameter

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And then you get down to the velocity triangles where you need to find out some of the work that actually needs to be accomplished where the losses now come into picture. The losses some value of losses need to be assigned to begin with.

So, the rotor losses for example, one can assign a 2 to 3 percent rotor loss in turbines depending on whether it is subsonic or supersonic. In supersonic, this will have to be probably twice the amount of losses inclusive of the shock losses. This gives you an idea how much is the actual work that the turbine would produce compared to the ideal work that are ideally ideal calculation tells us. This is the actual work that would indeed be communicated or transferred to the shaft which powers the compressor and it is less by the amount of losses. So, the amount of ideal work that needs to be attempted would have to be then found out.

So, one may first find the actual losses that need to be actually produced factoring the losses that would inevitably happen. Then find the ideal work that needs to be done by the turbines and attempt to get that ideal work done and in that attempt create the velocity triangles.

So, the whirl component C_{2w} plus C_{3w} as we know, gives us from the Euler's equation gives us the work done in the turbine in the particular turbine rotor. Then of course, we have the degree of reaction definition which is also in terms of the whirl component C_{2w} and C_{3w} .

If we solve these two equations, we get the values of c_w at station 2 and 3 at mean diameter. So, this sort of allows us to figure out all the aero thermodynamic parameters at the mean diameter.

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iii) Select three probable values of α_2 and for each one of them calculate the velocity triangle parameters at various radial stations

$$C_{2a} = \frac{C_{2w}}{\tan \alpha_2} \quad \text{where, } C_2 = \frac{C_{2a}}{\cos \alpha_2}, \quad \lambda_{2C} = \frac{C_2}{a_{cr2}}$$

And, $\lambda_{cr2} = 0.85 \text{ to } 0.9$

iv) Assume velocity coefficient ϕ , and calculate pressure loss coefficient δ_{noz}

$$\delta_{noz} = \frac{\pi \left(\frac{\lambda_2}{\phi} \right)}{\pi (\lambda_2)}; \quad P_{02} = P_{01} \delta_{noz}$$

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Now, what needs to be done is you need to start creating a three dimensional blade from the mean diameter which requires that the parameters that we were talking about. All these parameters would need to be now transported or transposed to the various stations radially from root to the tip of the blade.

Now, if you want to do that, you have to adopt a technique by which this can indeed be done. One of the ways of doing it is to select 3 probable values of α_2 at mean diameter for each one of them to calculate various velocity triangle parameters at various radial stations.

So, at from here onwards, from step three onwards, you are calculating for at least 3 probable values of α_2 at mean diameter **ok**. If you have capability, you can probably go for more values of probable values of α_2 . We have seen the probable values of α_2 quite often between 60 and 70 degree. However, for particular design you may like to choose some different values also and hold them within the probability and then select so many probable values of α_2 and then compute the velocity triangles at various stations.

So, first is the calculation of the axial velocity which you can get from the alpha 2. Then of course, you get certain restrictions may be imposed on the value of c 2 or on the value of lamda 2 lamda two absolute which may have a restriction of the order of 0.85 to 9 which would indeed probably give us a Mach number restriction of the order of 1 or there about which is the sonic speed at the stator nozzle exit. Then of course, with the help of that you try to find the velocity coefficient phi which is the flow coefficient or velocity coefficient and the pressure loss coefficient del nozzle.

So, the nozzle pressure loss coefficient can be found by using certain correlations. These correlations normally need to be made available from various cascade data and that cascade data has to made available to the designer. These are as you can see related to the values of lamda or Mach number M 2 and flow coefficient phi. When you have these parameters related to the nozzle loss coefficient del nozzle, you get a idea about the pressure loss. Then this pressure loss is factored into. Get the pressure loss at the exit of the nozzle related to the pressure that has been supplied from the earlier turbine or from the combustion chamber.

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v)
$$A_2 = \frac{\dot{m}_g}{\rho_2 \cdot C_a}$$

From which
Blade height
$$h_{2bl} = \frac{A_2}{\pi D_{m2}}$$

$$h_{2passage} = h_{2bl} (1 + \Delta \bar{r})$$

the rotor tip
radial gap
$$\Delta \bar{r} = \frac{\Delta r}{h_{1bl}} = 0.010 - 0.015$$

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And then with the help of that, you find out using the continuity condition, you find out the area at the nozzle exit from the continuity condition. From the area, you can find out what the blade height should be at the station 2 at the nozzle exit and then this blade height is the passage height. Now in case of a rotor, if you do this when you calculate the

rotor exit passage height, the rotor tip gap needs to be factored in. Normally, the tip gap is of the order of 1 to 1.5 percent. This allows us to calculate the rotor tip radial gap Δr by h_1 blade of the rotor and that allows us to calculate the h_2 passage. So, the passage annulus space that is available would not be fully available to the rotor blade. A little bit of gap would have to be left for the rotor to operate safely. Now with the help of this, we got the annulus space and the rotor and stator blade heights assigned.

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The slide contains the following content:

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vi)
$$\bar{d} = \frac{\left(\frac{D_{m2}}{h_{2bl}}\right) - 1}{\left(\frac{D_{m2}}{h_{2bl}}\right) + 1}$$

vii)
$$\beta_2 = \tan^{-1} \left(\frac{C_{2w} - U}{C_{2a}} \right)$$

Find V_2 and λ_{2-rel}

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Then we get the diameter ratio that is hub to tip diameter ratio. To begin with you get with respect to the mean diameter and then you can get with respect to the hub diameter. So, this is with respect to the hub. This is the hub diameter and this is the tip diameter and that gives you the hub to tip diameter ratio. Then of course, the relative velocity β_2 which you get from the velocity triangle that is being constructed at that particular station. You can find V_2 the relative velocity and then of course, the λ_2 , the critical speed ratio.

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viii) C_{2a} , C_2 , and V_2 , leads to λ_3 , α_3 and β_3 . Calculate for blade geometry and check for λ_2, α_2 within limits mentioned.

also $\lambda_{2-rel} \leq 1.0 - 1.1$

ix) Assume ψ for rotor, calculate P_{02-rel} and calculate P_{03-rel}
 From empirical rotor loss correlations, e.g. those given here:

$$P_3^* = P_{3-rel}^* \frac{\pi(\lambda_{3-rel})}{\pi(\lambda_3)}; \quad P_3 = P_3^* \cdot \pi(\lambda_3); \quad \pi_T^* = \frac{P_{01}^*}{P_{03}^*}$$


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Then these values lead to λ_3 , α_3 and β_3 and calculate these values and check for λ_2 and α_2 within the limits which are prescribed. λ_2 relative may be prescribed within 1 or 1.1, α_2 may be prescribed within certain values as we have seen somewhere between 60 and 70 and that needs to be checked at all the radial stations from hub to tip. Then assume now loss coefficient, for the loading coefficient, for the rotor ψ and this gives us certain idea about what should be the pressure values across the rotor P_{02} relative and P_{03} relative.

So, from the empirical rotor loss correlation to begin with, you have to use the empirical loss correlations, which are available in some of the old textbooks or NASA handbooks or if you have access to a databank of a particular design bureau.

So, that gives us the P_3 design value with reference to the P_3 relative. P_3 of course, is the absolute value. With the help of the various correlations that we need to inevitably make use of, we get the turbine pressure ratio P_{01}^* by P_{03}^* which the star represents the design value. Now, this gives us the pressure ratio across the turbine at various locations and together it gives us the pressure ratio of the entire turbine.

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The stage efficiency is calculated from :

$$\eta_{T_i} = \frac{H_{T_i}}{\frac{k_g}{k_g + 1} R_g T_{g_i} \left(1 - \frac{1}{\pi_T^{k_g}} \right)}$$

The best efficiency consideration often determines the selection of α_2 from the three initial considered. In some cases e.g. military a/c engine, best pressure ratio π_{0T} may be used for making the final decision on α_2

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Now next we can find the stage efficiency which is indeed calculated from the work that is being done with reference to large number of parameters that we have already used. The best efficiency consideration is often used to determine which value of alpha 2 of the 3 or more that we had started off with. At step three, to be finally decided for your design selection. In some cases, that is, if you were military aircraft engine, it is possible that best pressure ratio which we had done in the last slide here maybe the deciding factor. A slide sacrificing the efficiency parameter may be made especially if it is a military application engine.


Now, these are the parameters based on which you now finally go back to the step three and select the value of alpha 2 which meets at the selection criteria. Then all the values that were computed from step three onwards till the efficiency then become the selected values for the particular design.

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Exit area A_3 may now be found from various aerothermodynamic parameters and using the continuity condition.

$$h_{3_{bl}} = \frac{D}{2} - \sqrt{\frac{D^2}{4} - \frac{A_3}{\pi}} \quad \text{For } D_{tip} = \text{const} \quad d = D_m - h_{bl}$$
$$h_{3_{bl}} = \sqrt{\frac{d^2}{4} - \frac{A_3}{\pi}} - \frac{d}{2} \quad \text{For } d_{hub} = \text{const} \quad D = d + 2h_{bl}$$

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And then you can now calculate value of area at the rotor exit which gives you the rotor height or the blade height from which you can indeed find the D tip and the D hub so that you can find the hub to tip ratio of the particular rotor. We have seen in case of turbines across the rotor also, the height may change from leading edge to the trailing edge of the blade.


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After these calculations divergence angle is checked and if $\gamma > 15^\circ$, the blades angles are modified to allow for more expansion.

Radial variation: either use α_2 as constant from hub to tip or use some vortex law e.g. constant reaction law or the free vortex law

Profiling: same as in last lect use turbine specific airfoils e.g. T6 (HPT) or T106 (LPT) airfoils

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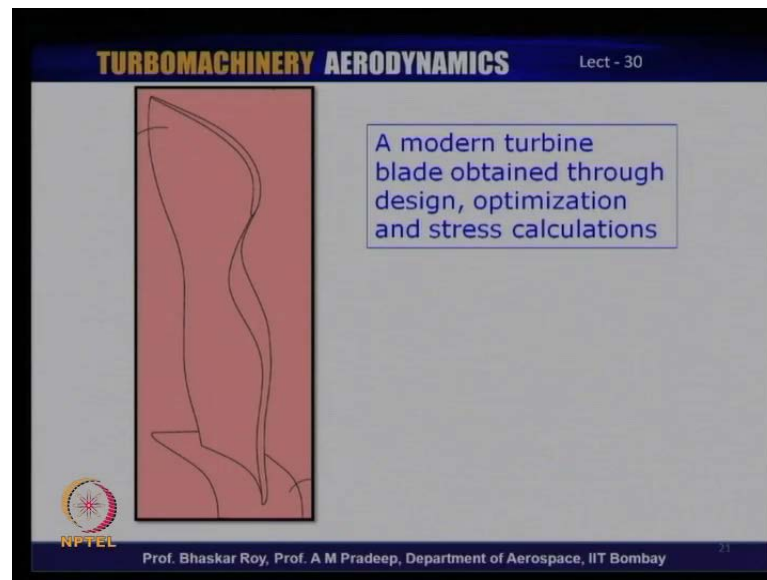
After all these calculations are done and if you go back to that of a conical flow track, you find the divergence angle. At no point of time, the divergence angle should be more

than 15 degree. If it is more, you have to redesign one or more of the stages all over again to meet the divergence angle limit.

So, these are the steps through which you can accomplish an overall turbine design, even a multistage turbine design. One has to go through the steps over the number of stages. We have only shown steps for one particular stage saying would need to be done over all the stages one after another. Having done that, you have the variation of all the parameters across the length of the blade and then across the all the stages. In variation across the length of the blade, you may use either a vortex law like free vortex law or more popularly these days, one uses α^2 constant as the variability as the law that carries the variation from root to the tip of the rotor and stator. That allows us to calculate all the steps from step three onwards all over again to get all the velocity triangles at all the stations from root to the tip of the blade.

If you have all those velocity triangles the alphas and the betas, then you can sit down and start profiling of each of those stations, each of those blades of stators and rotors using maybe the standard profiles the t 6 profile or the T 1 0 6 profile. These are the profile that you put together to create the turbine blades. Once you have created that, you have prima facie a turbine blade shape, three dimensional shape that is available to you. Now use a bit of this blade to intense refinement through the use of cfd and stress calculations in turbines. Stress calculation in turbine is extremely important because turbine is subject to multiple traces or compound traces due to gas loading as well as due to thermal loading. Once you have done that, you have the refine blade.

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I will try to show you a particular refined blade that has been used in turbine design after optimization through the stress calculation and the gas load calculation. As you can see, the load here is indeed actually factored into the design and it tells us that the final blade shape may not be a very straight blade. It may have curvilinear shapes similar to what is being shown here and these are the shapes that you finally get of a rotor or maybe sometime that of a stator. It may come out of this kind of shape typically of LP turbines where you do not have the cooling technology embedded inside it.

So, some of these refinements need to be done through CFD through optimization between gas dynamics and thermal load and stress calculations. So, you can get final design through very intense analysis process after the first cut design is done through the steps that we have gone through in today's lecture.

So, this is how you finally create an axial flow turbine blade. What we have seen now is that the turbine blade creation requires a lot of empirical correlations, a lot of use of computational fluid dynamics also optimization with the stress calculation. What I have not mentioned is, in case of low pressure turbines it is not only stress calculation, but, aero elastic calculation needs to be done; that means, that allows you to calculate the blade vibrations. All of it together finally, gives you the axial flow turbine blade. That brings us to the end of axial flow turbine chapter.

We will move on to the other kinds of a turbo machines, the centrifugal compressors and the radial turbines before we complete this lecture series with a few lectures on computational fluid dynamics. So, that brings us to the end of axial flow turbine chapter. We will go over to the various radial flow machines from next lecture onwards.