Turbomachinery Aerodynamics

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Lecture No. # 03

A Two Dimensional Analytical Model Cascade

Hello and welcome to lecture number 3 of this lecture series on turbomachinery aerodynamics. In the last lecture that was lecture number 2, we had discussed about some simplified aero thermo analysis of compression systems, and where we had discussed about significances of such an analysis method. We will also we had also discussed about how we can construct a velocity triangle and use that in our preliminary design analysis, and how we can use this simplified analysis to help our detailed design analysis which will be taken up after this simple aero thermo dynamic analysis.

In today's class, we will continue with discussion on some of these topics. To begin with we will be talking about how we can calculate the pressure the work required for certain compression to be carried out, and how we can estimate the pressure ratio based on the work requirements for the compression process. So, we will basically be deriving an expression for the pressure ratio expressed in terms of the temperature rise and the efficiency. So, that is one of the first things that we will be doing today.

Subsequently, we will discuss about some of the design parameters, which are used in the initial design of of a compression system. We will be talking about the various parameters like the flow coefficient and the stage loading coefficient, the degree of reaction and the diffusion factor. So, we will discuss about all of these parameters in detail and the significance of these parameters. And then we will take up discussion on, what is known as a cascade aerodynamics, we will be discussing about the aero dynamics of a cascade. We will first take off what we mean by cascade and what a cascade wind tunnel looks like, and what is the significance of a cascade in the design process. So, these are some of the topics that we will be discussing in today's lecture.

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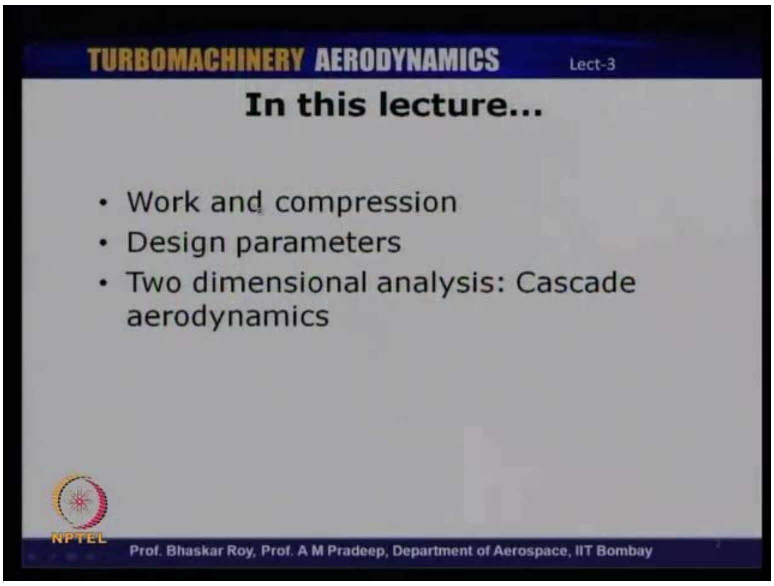

TURBOMACHINERY AERODYNAMICS Lect-3

In this lecture...

- Work and compression
- Design parameters
- Two dimensional analysis: Cascade aerodynamics

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So, we will start our discussion with the calculation of work for a compression process, we will then take up the design parameters, and then will do we will carry out a two-dimensional analysis of what is meant by cascade and the aerodynamics of a cascade flows. So, in the previous lecture, if you remember, we had drawn velocity triangle across the stage of a compressor, where we also expressed all the velocity components, the absolute velocity, the rate of velocity, the blade speed, then the tangential components as well as the axial component of the velocities. So, using what we using the discussion that we had in the last class. Let us now, carry out some analysis of the flow across a stage and how we can estimate the pressure ratio as the flow passes through a stage.

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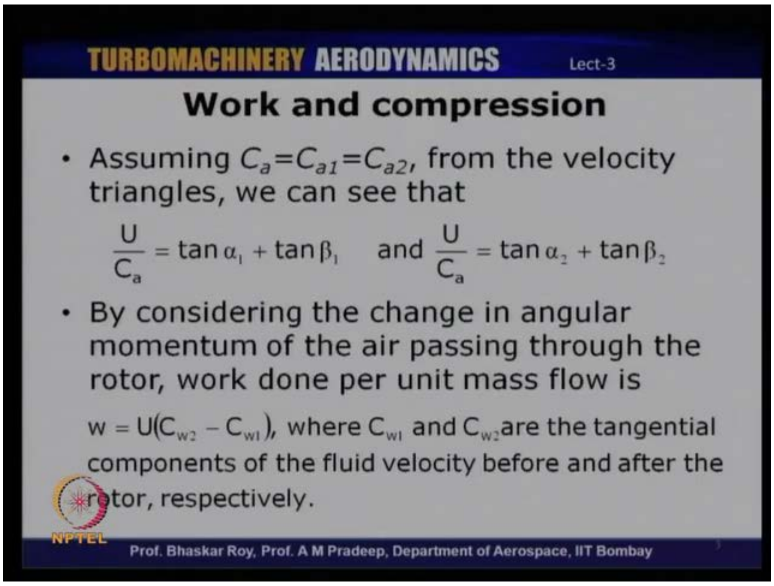

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Work and compression

- Assuming $C_a = C_{a1} = C_{a2}$, from the velocity triangles, we can see that

$$\frac{U}{C_a} = \tan\alpha_1 + \tan\beta_1 \quad \text{and} \quad \frac{U}{C_a} = \tan\alpha_2 + \tan\beta_2$$

- By considering the change in angular momentum of the air passing through the rotor, work done per unit mass flow is

$w = U(C_{w2} - C_{w1})$, where C_{w1} and C_{w2} are the tangential components of the fluid velocity before and after the rotor, respectively.

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So, we will begin this analysis with an assumption that the axial velocity remains the same. So, if we re-call the velocity triangle which we had seen in the last class. We will assume, if we assume the axial velocity to remain same that is C a should be equal to C a1, it is equal to C a2. What we see is that from the velocity triangle, we get the ratio U by C a that where U is the blade speed, C a is the axial velocity. This should be equal to tan alpha 1 plus tan beta 1. Similarly, U by C a, this is at the what you see here is at the leading edge of the rotor or the inlet of the rotor. Similarly, at the exit of the rotor, you have U by C a is equal to tan alpha 2 plus tan beta 2. And so, of course, if you take a relook at the velocity triangle, this is very simple, you you will see that this directly follows from the velocity triangle, if this assumption of axial velocity being the same ways is true.

Now, if we consider the change in angular momentum of the air as it passes through the rotor, you have seen that at the inlet of the rotor, the the whole component or tangential component of velocity of the absolute velocity is C w1 and the exit of the rotor it is C w2. So, what is the net change in tangential component of velocity across the rotor? It is basically, the difference between C w2 and C w1, which we had expressed as delta C w, if you remember in the last class, we had expressed difference between C w2 and C w1 as delta C w. So, this multiplied by the net speed, will tell us what is the net change in angular momentum as the flow passes through the rotor.

So, the net work done for driving this rotor should be equal to this net changing momentum. Which is why, we have now written w which is the work required for this compression process, as equal to the product of U, and the net change in the angular velocity. So, U multiplied by delta C w is that is U multiplied by C w2 minus C w1 gives us what is the net change in the angular momentum, as it passes through the rotor. And this is basically equal to the work required or work done on the flow per unit mass. Here, C w1 represents the tangential component of the velocity - before the rotor or the inlet of the rotor and the C w2 is the tangential component of velocity at the exit of the rotor.

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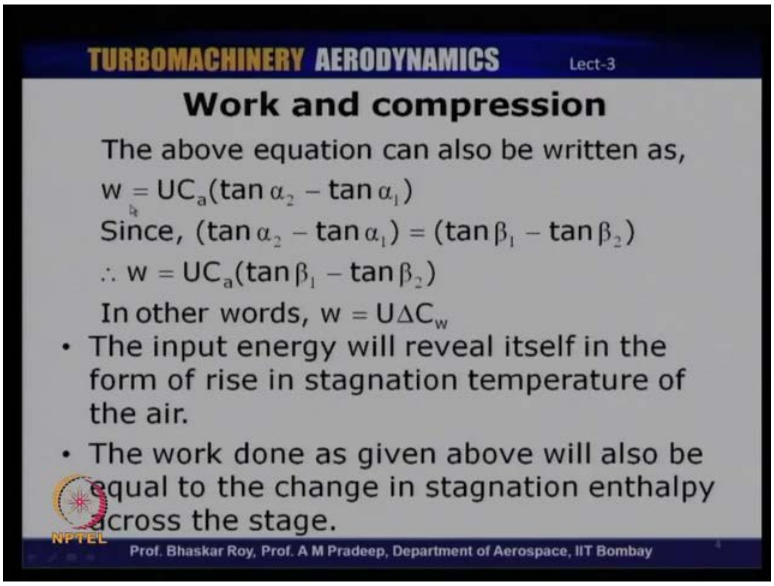

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Work and compression

The above equation can also be written as,

$w = UC_a(\tan\alpha_2 - \tan\alpha_1)$

Since, $(\tan\alpha_2 - \tan\alpha_1) = (\tan\beta_1 - \tan\beta_2)$

$\therefore w = UC_a(\tan\beta_1 - \tan\beta_2)$

In other words, $w = U\Delta C_w$

- The input energy will reveal itself in the form of rise in stagnation temperature of the air.
- The work done as given above will also be equal to the change in stagnation enthalpy across the stage.

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Now, so, from the velocity triangle, we can also write or express these velocity components that is C w2 and C w1 in terms of the axial velocities. So, C w2 should also be equal to C a times tan alpha 2 and C w1 is C a tan alpha 1. So, U times C a multiplied by tan alpha 2 minus tan alpha 1. So, this is also equal to the net work done.

Now, if you remember from the velocity triangle, we can also write the difference between tan alpha 1 and tan alpha 2; that is tan alpha 2 minus tan alpha 1 is also equal to tan beta 1 minus tan beta 2. Therefore, we have the work done has U into axial velocity C a tan beta 1 minus tan beta 2. And which is also basically equal to U times delta C w. So, what we have here is that we can express the net work done, which is a function of the blade speed as well as the change in the time mutual velocity in delta C w. So, the product of the blade speed and delta C w will give us, what is the or we will tell us what is the amount of work required or that is amount of work that is done on the flow by the rotor blades.

So, and this energy that is added on the flow will basically reveal itself in the form of an increase in the stagnation temperature of the air. Therefore, the work done has indicated here as equal to U times delta C w will also be equal to the net change in stagnation enthalpy in the stage. Because, you are hiding work on the flow through the rotor, which obviously leads to increase in stagnation enthalpy; that is something we have seen the last class that from the inlet of the rotor to the exit of the rotor, there is an increase in stagnation enthalpy, across the stator there is no change in stagnation enthalpy. So, the net change in stagnation enthalpy, taking place in the stage is basically as a result of the work done by the rotor on the flow.

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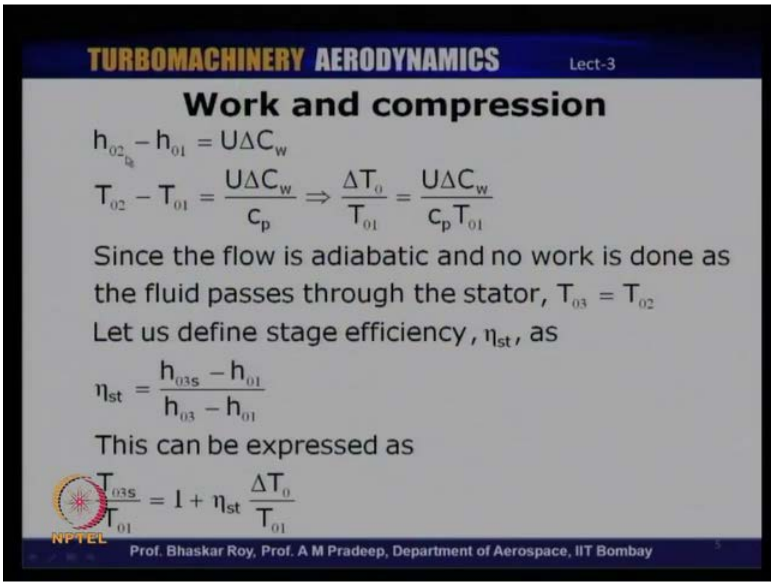

TURBOMACHINERY AERODYNAMICS Lect-3

Work and compression

$$h_{02} - h_{01} = U\Delta C_w$$

$$T_{02} - T_{01} = \frac{U\Delta C_w}{c_p} \Rightarrow \frac{\Delta T_0}{T_{01}} = \frac{U\Delta C_w}{c_p T_{01}}$$

Since the flow is adiabatic and no work is done as the fluid passes through the stator, $T_{03} = T_{02}$

Let us define stage efficiency, η_{st}, as

$$\eta_{st} = \frac{h_{03s} - h_{01}}{h_{03} - h_{01}}$$

This can be expressed as

$$\frac{T_{03s}}{T_{01}} = 1 + \eta_{st}\frac{\Delta T_0}{T_{01}}$$

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So, what we can do is that we can equate delta h across the rotor that is h 02 minus h 01 should be equal to the work done on the flow. That is U times delta C w. Now, here we will express now, the enthalpy in terms of the temperature, the stagnation enthalpy difference is also equal to stagnation temperature difference multiplied by C p. So, that is T 02 minus T 01 is equal to del U times delta C w by C p, which is also equal to delta T 0 by T 01, which is U delta C w by C p into T 01. Now, since the flow is adiabatic as that is one of the basic thing that we have seen in the last class. Since, it is adiabatic, there is no work done as the flow passes through the stator. And therefore, T 03 should be equal to T 02. So, what we will do now is to define what is known as stage efficiency. If you remember in the last class, we had expressed the compression in a stage in terms of temperature and entropy diagram, and I specifically mentioned that deviation of the process from an isentropic behavior is expressed in terms of what is known as isentropic efficiency.

Now, for a stage, what we will do is that we will express this as what is known as the stage efficiency. Which is basically the difference between the enthalpy at the exit of the stage for an isentropic process minus the inlet enthalpy, this divided by the actual enthalpy at the stage exit minus the inlet enthalpy. So, we will define stage efficiency, as equal to h 03s or h 03 prime. That is the stagnation enthalpy at the exit of the stage minus h 01 which is the stagnation enthalpy at the inlet divided by h 03 minus h 01. We can express this further as T 03s or T 03 prime, which is the isentropic stagnation temperature at the exit of the stage divided by T 01. This is equal to 1 plus this stage efficiency multiplied by delta T 0 divided by T 01. So, here we have expressed this stagnation temperature ratio in terms of the stage efficiency and delta T 0, and the inlet stagnation temperature.

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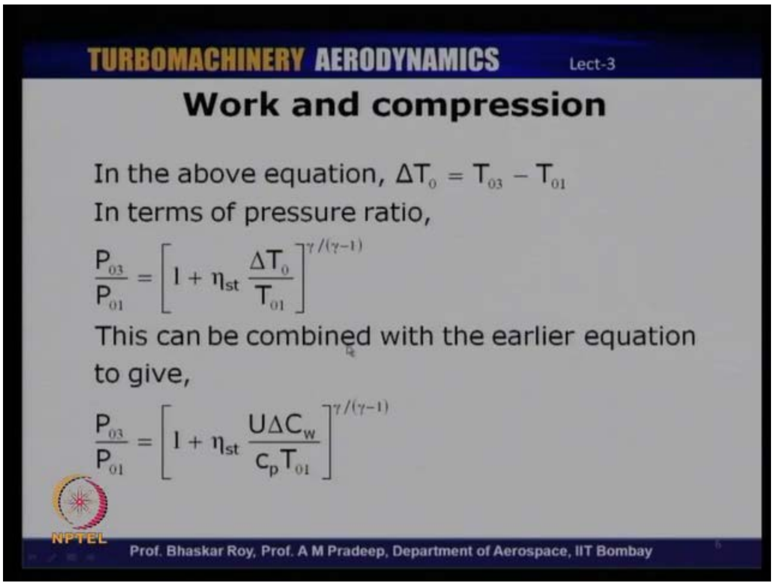

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Work and compression

In the above equation, $\Delta T_0 = T_{03} - T_{01}$

In terms of pressure ratio,

$$\frac{P_{03}}{P_{01}} = \left[1 + \eta_{st}\frac{\Delta T_0}{T_{01}}\right]^{\gamma/(\gamma-1)}$$

This can be combined with the earlier equation to give,

$$\frac{P_{03}}{P_{01}} = \left[1 + \eta_{st}\frac{U\Delta C_w}{c_p T_{01}}\right]^{\gamma/(\gamma-1)}$$

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So, here in this expression that we have written just now, delta T 0 represents the net change in the stagnation temperature across the stage, which is basically the actual stagnation temperature rise T 03 minus T 01. Now, we will express this expression that is T 03s by T 01 is the stagnation temperature ratio for an isentropic process. This is related to the pressure ratio through the isentropic relation. So, P 03s by P 01 is equal to P 03s by P 01 rise to gamma minus 1 gamma. Therefore, we have pressure ratio P 03 by P 01 is

1 plus the stage efficiency into delta T 0 divided by T 01, the whole rise to gamma by gamma minus 1.

So, this we will combine with the previous expression, which we have written here. For delta T 0 by T 1, in terms of the components from the velocity triangle; that is U times delta C w. What we get here is P 03 by P 01 is 1 plus this stage efficiency multiplied by U times the delta C w by C p T 01 rise to gamma by gamma minus 1. So, we have now here, an expression for the pressure ratio across a compressor, expressed in terms of a certain parameters which for example, U times delta C w is something you can find from the velocity triangle itself. The inlet stagnation temperature is known the stage efficiency if known, then the pressure ratio across this stage of a compressor can be quite easily determined.

So, we have now, derived an expression which can basically tell us, the amount of pressure ratio that you can get from a stage, if one can estimate the… if of course, the blade speed is known and the change in the tangential velocity across the rotor is known one can estimate the pressure ratio per stage. So, this is one way of estimating, the amount of pressure ratio that one can express that one can expect from a stage of a compression process. Or the other way round to look at is, if this there is certain pressure ratio requirement, what is the kind of work that will be required for generating this amount of pressure ratio per stage.

So, we have expressed the pressure ratio that can be developed per stage in terms of some of the parameters which can be obtained, quite easily from the velocity triangle. And therefore, the pressure ratio that one can achieve per stage of compression can be directly expressed in terms of some of these parameters.

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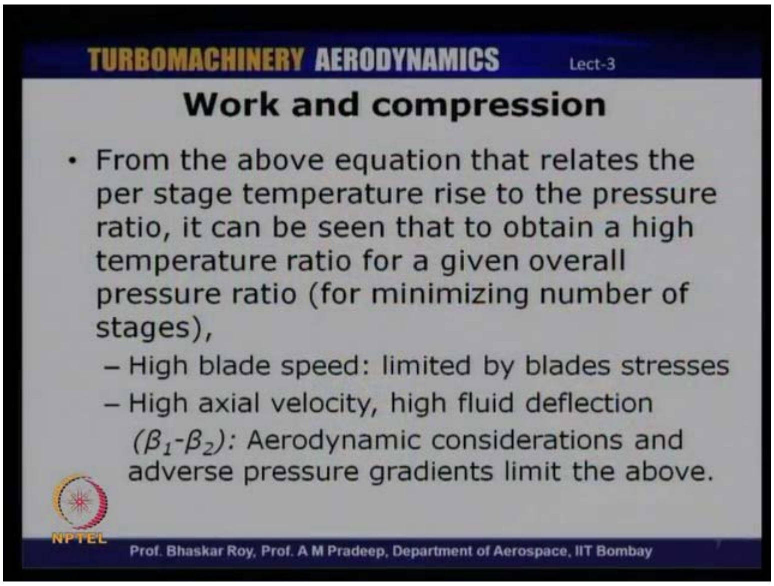

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Work and compression

- From the above equation that relates the per stage temperature rise to the pressure ratio, it can be seen that to obtain a high temperature ratio for a given overall pressure ratio (for minimizing number of stages),
  - High blade speed: limited by blades stresses
  - High axial velocity, high fluid deflection (β_1-β_2): Aerodynamic considerations and adverse pressure gradients limit the above.

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Now, if you take a closer look at the expression that you have just derived. We can see that the per stage temperature rise of, that is which basically the equation which relates the temperature rise to the pressure ratio. We see that to obtain a high temperature ratio, for an overall pressure ratio, there are two possibilities here, which can lead to minimum number of stages; one is that you can increase the blade speed that is if you increase the blade speed here, as shown here, because the pressure ratio is directly a function of the blade speed, and the other is of course, is to increase the axial velocity, because that is something, which would come up here.

Now, or change the fluid deflection that is you have a higher fluid deflection, which means that the difference between the inlet and exit blade angles that is beta 1 minus beta 2 would be quite high. Now, all the 3 obviously have certain limitations. That is high blade speed of (( )) is limited by the blade stresses, and high axial velocity or fluid deflection is limited in terms of the aerodynamics performance penalty is that one may have to pay, and also the adverse pressure gradients, which might lead to that is if fluid deflection is very high; the fluid is forced to take an increased turn, which means that it leads to increased pressure gradients - adverse pressure gradients, and that the performance of the blade of the compressor would be drastically effected. But of course, one can see that from this pressure ratio relation with the temperature rise or blade speed. One can at least estimate the parameters on which the increase in pressure ratio can be achieved.

So, what we have discussed now is, from simple thermo dynamics how we can estimate the pressure ratio that one can achieve per stage. You seen, some of the parameters like the temperature and the inlet conditions as well as some of the parameters coming in from the velocity triangles. You can estimate, what is the kind of pressure ratio that one can expect from a certain stage of an axial compressor. Now, I would understood this, what will do next is to try an understand the different design parameters, which have usually used in the initial design stages. So, let us try to understand, what are those parameters which are used in the design exercise, and what is the significance of each of these design parameters? So, we will see that there are primarily four types of design parameters that we need to look at, and these are parameters which are usually used in a parametric analysis of axial compressor.

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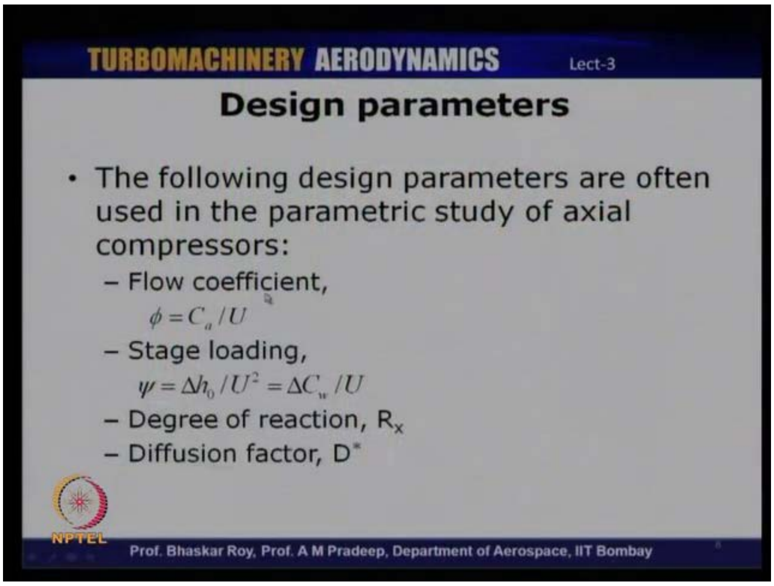

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Design parameters

- The following design parameters are often used in the parametric study of axial compressors:
  - Flow coefficient, $\phi = C_a / U$
  - Stage loading, $\psi = \Delta h_0 / U^2 = \Delta C_w / U$
  - Degree of reaction, R_x
  - Diffusion factor, D^*

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Now, one of the important parameters is what is known as the flow coefficient. Flow coefficient is usually expressed in terms of phi. And flow coefficient is the ratio of axial velocity to the blade speed U. So, C a by U tells us what is meant by the flow coefficient. And you can see here, that the numerator of this flow coefficient is the axial velocity. Significance of this parameter is in the fact, that flow coefficient is in some sense, non-dimensional form of a mass flow rate. That is primarily, because for a given blade speed as flow coefficient changes, it also tells us that there is a change in mass flow rate, because mass flow rate is directly a function of the axial velocity. And therefore, we will see little later in some of the later lectures; that the performance characteristics of it typically a single stage of axial compressor is expressed in terms of one of the parameters, you using which one would express the performance parameters characteristics is the flow coefficient, because it is an indication of the mass flow rate.

The other parameter that we shall be interested in is known as the stage loading coefficient, usually denoted by psi, and this is equal to delta h 0 by u square; that is delta h 0 represent the change in stagnation enthalpy across the stage divided by U square. This is also equal to delta C w by U, because delta h 0 is U times delta C w. Therefore, you have delta C w by U which is usually denoted as the stage loading coefficient. And so, these are two parameters to begin with which we shall be interested in, and we will be using these parameters in in great detail, when we take up the performance characteristics of single stage, and subsequently multi stage axial compressors.

Now, there are two other parameters which are of interest; one is known as the degree of reaction, we will denote by symbol R subscript x, and the diffusion factor which we will denote by D star. So, we will discuss these two parameters in little more detail as to what is mean by degree of reaction, and what is meant by a diffusion factor, and how we can calculate these parameters.

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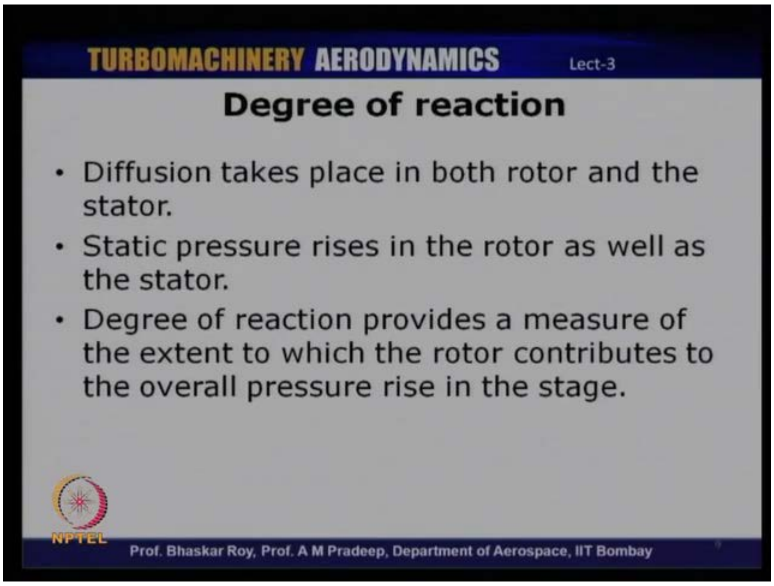

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Degree of reaction

- Diffusion takes place in both rotor and the stator.
- Static pressure rises in the rotor as well as the stator.
- Degree of reaction provides a measure of the extent to which the rotor contributes to the overall pressure rise in the stage.

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Now, let us take a look at what is meant by degree of reaction. Now, in a axial compressor stage, as we have seen diffusion is or the compression is shared by both the rotor as well as the rotor as well as the stator. That is part of the diffusion takes place in the rotor and part of the diffusion takes place in the stator. So, the degree of reaction is basically telling us, what is the extent to which the rotor takes part in the entire diffusion process, as compared to the entire stage. So, degree of reaction tells us, what is the fraction of the diffusion that has taking place in the rotor as compare to the diffusion across the whole stage itself.

So, we will be expressing diffusion factor in terms of pressure - static pressure as well as the enthalpy shortly. Now, static pressure rise take place as we have seen, it takes place both in the rotor as well as the stator, and degree of reaction give us some measure of the extent to which the rotor contributes to the overall pressure rise in the stage.

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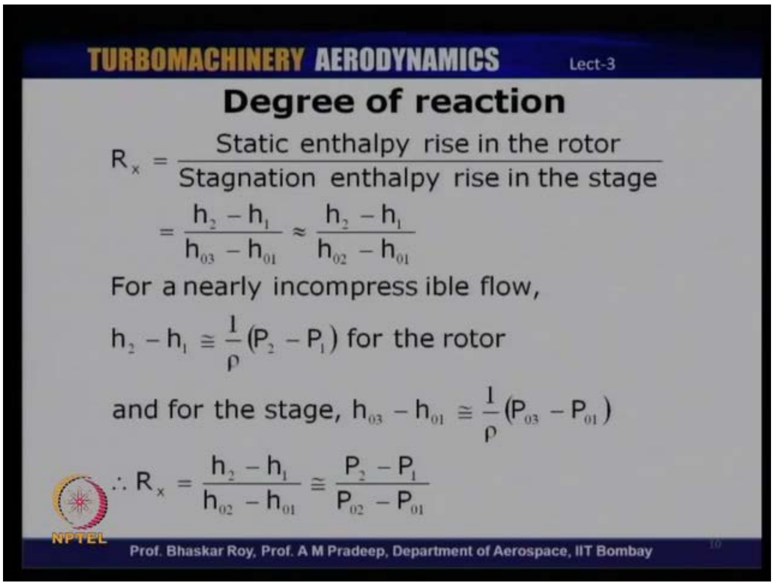

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Degree of reaction

$$R_x = \frac{\text{Static enthalpy rise in the rotor}}{\text{Stagnation enthalpy rise in the stage}}$$

$$= \frac{h_2 - h_1}{h_{03} - h_{01}} \approx \frac{h_2 - h_1}{h_{02} - h_{01}}$$

For a nearly incompress ible flow,

$h_2 - h_1 \cong \frac{1}{\rho}(P_2 - P_1)$ for the rotor

and for the stage, $h_{03} - h_{01} \cong \frac{1}{\rho}(P_{03} - P_{01})$

$$\therefore R_x = \frac{h_2 - h_1}{h_{02} - h_{01}} \cong \frac{P_2 - P_1}{P_{02} - P_{01}}$$

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Now, to express this in terms of the enthalpy or the rotor or the static pressure; degree of reaction by definition is the ratio of the static enthalpy rise in the stage in the rotor divided by the stagnation enthalpy rise in the stage. That is h 2 minus h 1 which is the static enthalpy rise in the rotor divided by h 03 minus h 01 which is the stagnation enthalpy rise taking place in the stage. This is equivalent to h 2 minus h 1 divided by h 0 2 minus h 01, because h 02 is equal to h 03, there is no stagnation enthalpy change in the stator.

Now, for a incompressible flow, we can express enthalpy can be equated to the change in static pressures divided by density. That is h 2 minus h 1 can be expressed in terms of p 2 minus p 1 divided by delta by rho. So, for a rotor, we can see that h 2 minus h 1 can be expressed in terms of p 2 minus p 1 divided by rho. Similarly, for the stage h 03 minus h 01 is equal to p 03 minus p 01 divided by rho. Therefore, this degree of reaction is which was expressed in terms of enthalpy that is h 2 minus h 1 divided by h 2 h 02 minus h 01 is equivalent to p 2 minus p 1 divided by p 02 minus p 01. That is following from this previous expression that I just mentioned. So, we can express the degree of reaction in terms of these static pressure rise in the rotor divided by the stagnation pressure rise in the rotor.

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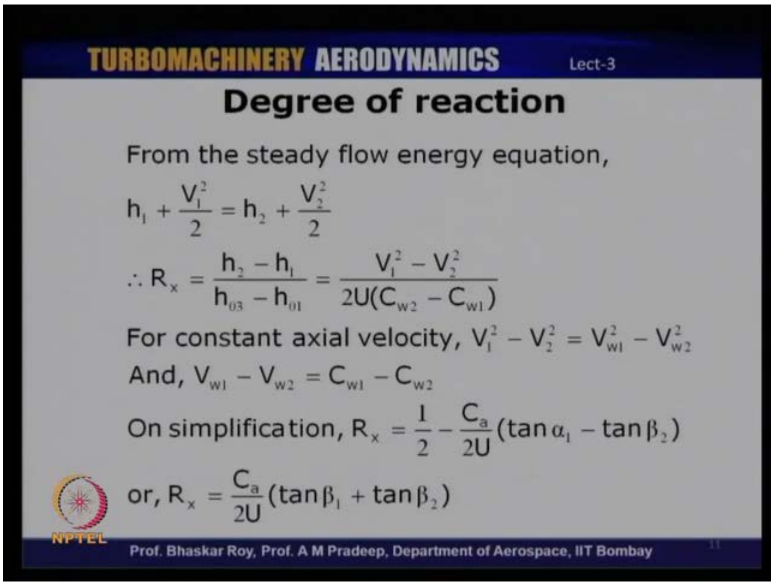

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Degree of reaction

From the steady flow energy equation,

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

$$\therefore R_x = \frac{h_2 - h_1}{h_{03} - h_{01}} = \frac{V_1^2 - V_2^2}{2U(C_{w2} - C_{w1})}$$

For constant axial velocity, $V_1^2 - V_2^2 = V_{w1}^2 - V_{w2}^2$

And, $V_{w1} - V_{w2} = C_{w1} - C_{w2}$

On simplification, $R_x = \frac{1}{2} - \frac{C_a}{2U}(\tan\alpha_1 - \tan\beta_2)$

or, $R_x = \frac{C_a}{2U}(\tan\beta_1 + \tan\beta_2)$

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Now, we will further simplify this expression, and express that in terms of velocities and angles, which one can determine easily from the velocity triangle. The intent here is that we can express the degree of reaction in terms of parameters that can be derived from a velocity triangle. From the velocity triangle, one can one should be in a position to estimate the degree of reaction, for a particular rotors or stator combination. Now, from the from a classical thermodynamics that I assume you would have undergone, the study flow energy equation which you might have studied in your thermodynamic course, tells us that the sum of enthalpy plus the kinetic energy is conserved. So, h 1 plus V 1 square by 2 should be equal to h 2 plus V 2 square by 2 for a flow through a compressor or a turbine.

Therefore, we have degree of reaction which was expressed in terms of h 2 minus h 1 divided by h 03 minus h 01. We will have now now express that in terms of the velocities, so, on the numerator we have this difference h 2 minus h 1 has V 1 square minus V 2 square divided by on the denominator, it is stagnation enthalpy change in the row in the stage. This is equal to U times delta C w. So, we have in the denominator 2 U into C w2 minus C w1. So, if you assume that axial velocity is a constant in a stage, then V 1 square minus V 2 square can be equated to V w1 square minus V w2 square. And you also know that V w1 minus V w2 is also equal to C w1 minus C w2. Assuming that, axial velocity remains the same in the stage. So, if we substitute for all this in the degree of reaction expression, we get degree of reaction as 1 by 2 that 0.5 minus C a by 2 U into tan alpha 1 minus tan beta 2 or C a by 2 U into tan beta 1 plus tan beta 2. So, this of course comes from the velocity triangle directly.

So, what we have done now is to express the degree of reaction in terms of parameters, which one can easily determine from the velocity triangle. So, here we have axial velocity degree of reaction is now a function of axial velocity and the blade angles tan beta 1 plus tan beta 2, and denominator we have the blade speed. All these parameters can be quite easily estimated from the velocity triangle. And therefore, if one can construct a velocity triangle for a certain rotor, stator combination, one can estimate the degree of reaction, simply by developing a velocity triangle based on the flow conditions.

So, degree of reaction is something that you can estimate from the velocity triangle, and the significance of a degree of reaction is the fact that it basically tells us, how much does the rotor contribute to the overall pressure rise or the diffusion taking place in the compressor. So, this is one of the parameter that will be of significance when we carry out detailed design exercise, one would be looking at what is the degree of reaction in a typical compressor stage. So, what we will do is now, we will now take a look at some of the special cases of degree of reaction which are likely to which one might likely one might encounter while we carry out an design exercise, what happens to the different values of degree of reaction. For example, what if degree of reaction is 0 and what if degree of reaction is equal to 1 or 0.5.

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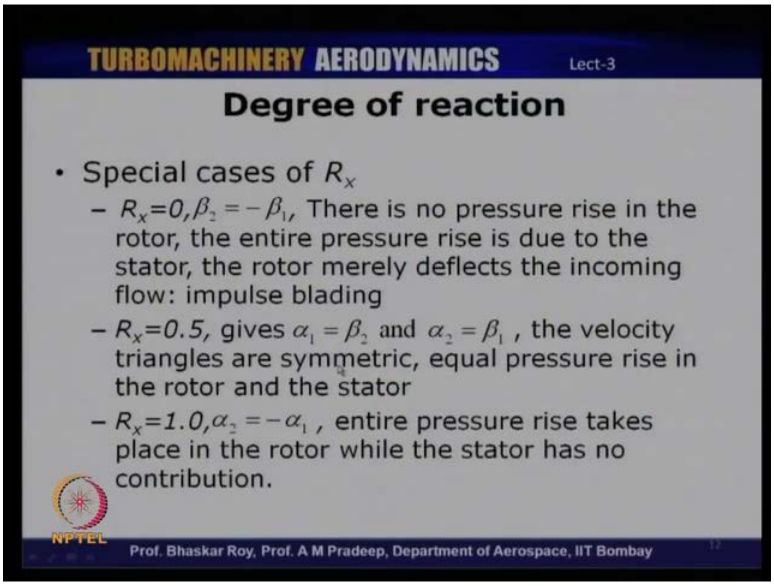

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Degree of reaction

- Special cases of R_x
  - $R_x=0, \beta_2=-\beta_1$, There is no pressure rise in the rotor, the entire pressure rise is due to the stator, the rotor merely deflects the incoming flow: impulse blading
  - $R_x=0.5$, gives $\alpha_1=\beta_2$ and $\alpha_2=\beta_1$, the velocity triangles are symmetric, equal pressure rise in the rotor and the stator
  - $R_x=1.0, \alpha_2=-\alpha_1$, entire pressure rise takes place in the rotor while the stator has no contribution.

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Now, let take a look at a few special cases of degree of reaction; when degree of reaction is 0 what happens. When degree of reaction is 0, it means that the blade angles beta 2 will be equal to minus beta 1. When degree of reaction is 0, it basically means that there is no change or there is no contribution of the rotor to the overall pressure rise, because degree of reaction is 0 it tells us what is contribution of rotor to the overall pressure rise.

So, when degree of reaction is 0, and if we substitute degree of reaction R x equal to 0 here, what you will see is that beta 1 beta 2 is equal to minus beta 1. There is no pressure rise in the rotor, the entire pressure rise is basically taking place in the stator, and the rotor merely deflects the incoming flow, and this basically represents an impulse blading. So, we will probably discuss little more details of these in later lectures. That is in this particular configuration, if degree of reaction is 0, the rotor primarily does not do any work on the flow, it is simply deflects the flow, the entire pressure rise takes place in the stator.

Now, the second possibility is, if degree of reaction is equal to 0.5, so, if you substitute R x is equal to 0.5 here, what you will observe is that you will get alpha 1 is equal to beta 2, and alpha 2 is equal to beta 1. And if you re-call the velocity triangles which also, I will show in the next line. In such a situation, what you will have to observe is the velocity triangles are symmetric, and since degree of reaction is equal to 0.5, it also means that there is an equal pressure rise taking place in the rotor and the stator.

And the third extreme case is, if degree of reaction is equal to 1. Degree of reaction is equal to 1; alpha 2 is equal to minus alpha 1. Entire specialized takes place in the rotor, the stator has no contribution to the pressure rise taking place in the rotor. So, these are three different special cases of degree of reaction, where one might encounter an actual design you are likely to encounter values in which are in between 0 and 1. So, if you now look at the velocity triangles corresponding to each of these three cases. One can see, what is the difference between the velocity triangle as we change the degree of reaction from 0 all the way to 1.

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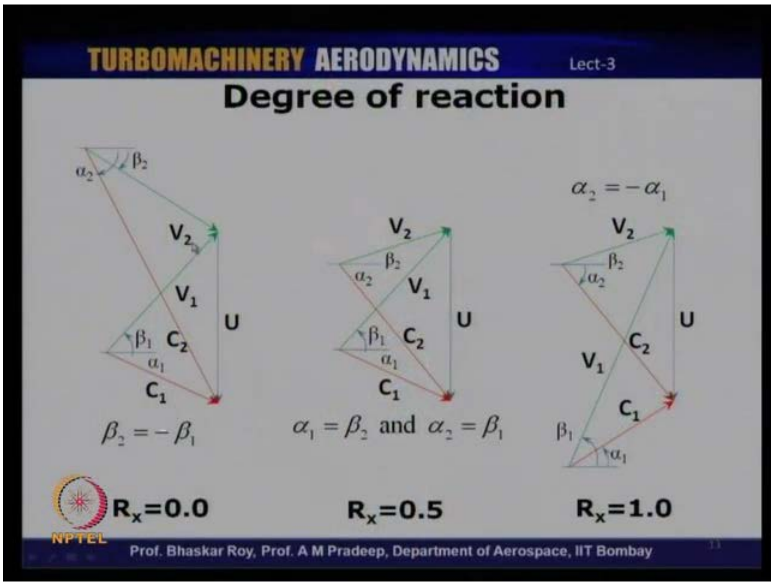

TURBOMACHINERY AERODYNAMICS Lect-3

Degree of reaction

$\beta_2 = -\beta_1$

$\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1$

$\alpha_2 = -\alpha_1$

R_x=0.0 R_x=0.5 R_x=1.0

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So, for a degree of reaction of 0, beta 1 was equal to minus beta 2. So, here we have degree velocity triangle for this particular case, we have beta 1 and this is beta 2. So, beta 1 is equal to minus beta 2. So, you can see the change in velocity triangle, you can see what happens to the velocity triangle in this case. So, this first one corresponds to the velocity triangle at the rotor, inlet and this corresponds to the velocity triangle at the rotor exit. And in such a configuration, the rotor does not basically contribute to any pressure rise, it simply deflects flow. Which is also true here, as you can see V 1 is in magnitude equal to V 2. So, that is the there is no change in relative velocity as it passes through the rotor, and since there is no change in relative velocity as it passes through the rotor, there is no diffusion taking place in the rotor, the entire diffusion takes place in the stator.

The absolute velocity on the other end of course, increases there is tremendous increase in absolute velocity from C 1 to C 2. When degree of reaction is 0.5, we have alpha 1 is equal to beta 2, and alpha 2 is equal to beta 1. We get a velocity set of velocity triangles which are symmetrical, velocity triangle at the inlet and exit of the rotor are mirror images of one another. So, one would have C 1 is equal to V 2, and C 2 is equal to V 1, because alpha 1 is equal to beta 2, and alpha 2 is equal to beta 1. And in this case, both the rotor and stator contribute equally to the pressure rise.

And the third case is when the degree of reaction is 1, we have alpha 2 is equal to minus alpha 1, and here the entire diffusion takes place in the rotor which is, because you can see that there is tremendous change in the relative velocity from V 1 to V 2. So, the entire diffusion is restricted to the rotor in this case. So, these are three different cases of degree of reaction possible values which the degree of reaction can take place can take during a particular design exercise.

Now, let us move on to the next parameter that is of interest to us which is known as diffusion factor. We will now, look at what you mean by diffusion factor, and how we can calculate diffusion factor for a certain design of a rotor combination.

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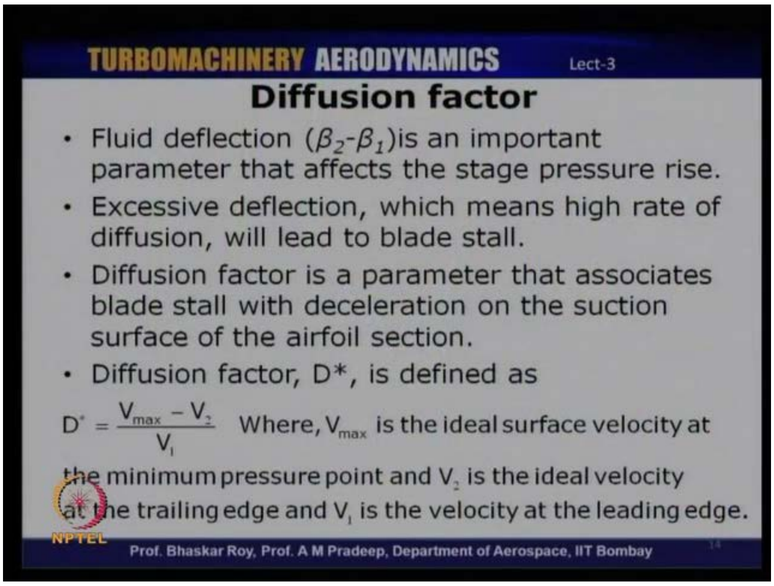

TURBOMACHINERY AERODYNAMICS Lect-3

Diffusion factor

- Fluid deflection (β_2-β_1)is an important parameter that affects the stage pressure rise.
- Excessive deflection, which means high rate of diffusion, will lead to blade stall.
- Diffusion factor is a parameter that associates blade stall with deceleration on the suction surface of the airfoil section.
- Diffusion factor, D*, is defined as

$D^* = \frac{V_{max} - V_2}{V_1}$ Where, V_{max} is the ideal surface velocity at the minimum pressure point and V_2 is the ideal velocity at the trailing edge and V_1 is the velocity at the leading edge.

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Now, diffusion factor is basically, looking at as the name suggests diffusion itself in the blade, and we have seen that the fluid deflection that is the difference between beta 2 and beta 1 is a very important parameter which effects the stage pressure rise. So, it means that as we increase this deflection, as we increase beta 2 minus beta 1, we can get a substantially high amount of diffusion. At the same time, we are also risking the fact that increasing the diffusion can also lead to increased pressure gradients, adverse pressure gradients eventually it might lead to the blade stall. Because, if you are operating a blade by the substantially high amount of diffusion, the fact that the blade might undergo stall is also true that. So, diffusion factor is a parameter which will now tell us, what is the possibility that the blade might stall, and what are kind of numbers that we need to look at to ensure that the blade stalling does not take place.

So, diffusion factor is a parameter which basically associates blade stall with deceleration primarily on the suction surface of the airfoil. So, that is where one would except that the blade to stall to takes place under normal incidence angles. So, diffusion factor will tell us, what is the what is the relation between the blade stall and the deceleration on the blade surface. So, it is basically defined as the difference between V max which is the ideal suction surface velocity at the minimum pressure point, and V 2 which is the ideal velocity at the trailing edge divided by V 1 which is the velocity at the leading edge. So, velocity difference between the maximum and the trailing edge velocity to the leading edge velocity.

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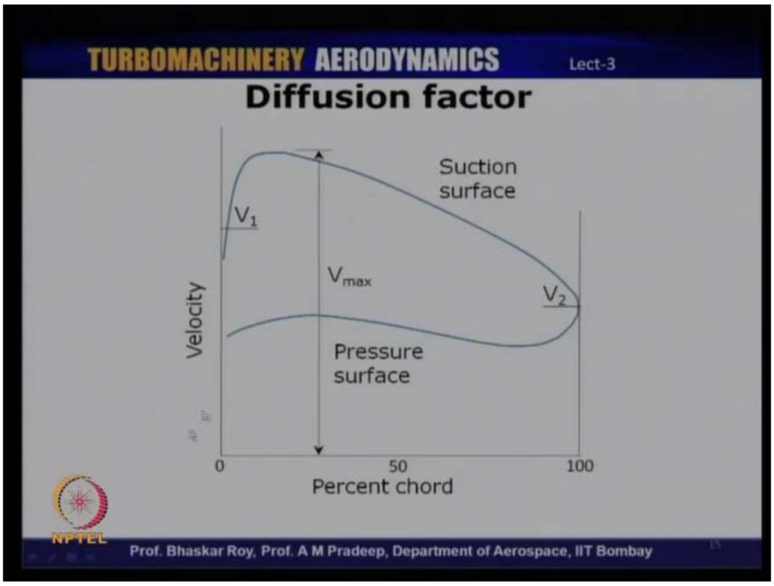

TURBOMACHINERY AERODYNAMICS Lect-3

Diffusion factor

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So, let us look at what diffusion factor definition basically means. So, what I have shown here is the distribution of velocity on the suction and pressure surface, as the percentage chord. So, on y axis we have velocity - the relative velocity, and on the x axis we have the percentage chord. So, this represents the leading edge of the blade, and this represents the trailing edge of the blade. So, as the flow proceeds from the leading edge, you can see there is an acceleration from the leading edge all the way to the minimum pressure point, which is where the velocity remain attains its maximum on the suction surface, subsequent to that the flow decelerates. So, you can see that, after the minimum pressure point the flow decelerates, and then you have a decrease in velocity, all the way up to the trailing edge. So, this here represents V 2, which is the trailing edge velocity.

So, diffusion factor is basically the difference between this velocity here that is V max minus V 2 as a function of the ended velocity. So, greater this difference between V max and V 2, the greater is the diffusion, but on the other hand it also indicates the fact that higher the diffusion means greater is the pressure gradient - the adverse pressure gradient. That is the static pressure at the trailing edge would be much higher than the minimum pressure point here. So, as you increase this difference between V max and V 2 in relation to the in that velocity, the adverse pressure gradient also increases. So, diffusion factor tells us that beyond a certain level, certain parameter or a certain level. If you try to increase, the diffusion even further, there are chances that the blade might stall is quite high. So, diffusion factor would in some sense tell us that what are the chances that the blade might stall given certain value of diffusion factor.

So, how do we calculate these maximum and minimum velocity. So, we have in the diffusion in the fundamental definition of the diffusion factor in terms of max velocity and the trailing edge velocity to the inlet velocity. So, for a long time, it was it was in the earlier days, when the significance of this parameter was realized, calculating the maximum velocity from experimental data was not quite easy. And therefore, they had come up with the some of the empirical correlation to estimate the value of diffusion factor from the data that one would get from internal testing.

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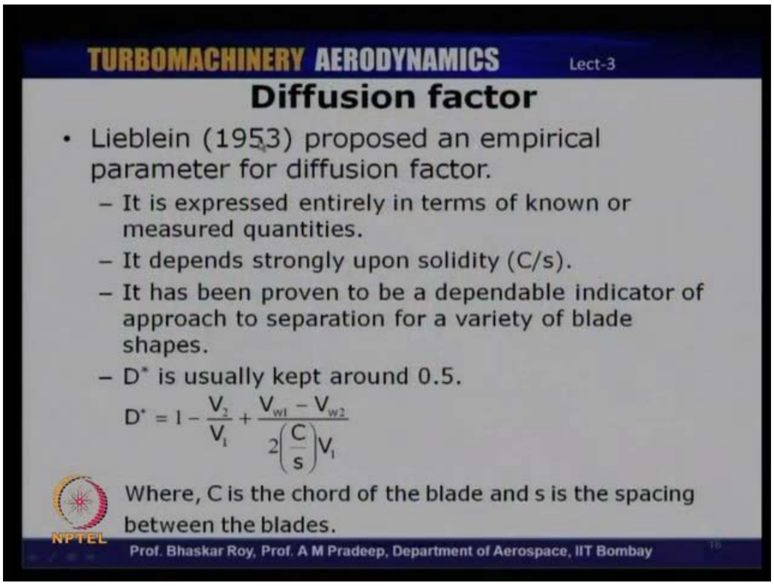

TURBOMACHINERY AERODYNAMICS Lect-3

Diffusion factor

- Lieblein (1953) proposed an empirical parameter for diffusion factor.
  - It is expressed entirely in terms of known or measured quantities.
  - It depends strongly upon solidity (C/s).
  - It has been proven to be a dependable indicator of approach to separation for a variety of blade shapes.
  - D^* is usually kept around 0.5.

$$D^* = 1 - \frac{V_2}{V_1} + \frac{V_{w1} - V_{w2}}{2\left(\frac{C}{s}\right)V_1}$$

Where, C is the chord of the blade and s is the spacing between the blades.

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So, in 1953, we went back in 1953, Leiblein proposed an empirical parameter for calculating diffusion factor. Now, the advantage of this is that you can express this diffusion factor entirely in terms of parameters which are measured, and also there is a strong dependent of the diffusion factor on the solidity, which is the chord to the spacing ratio. And the definition for what the Lieblein diffusion factor definition is 1 minus V 2 by V 1 plus V w1 minus V w2 divided by 2 into C by s into V 1. So, here you can see that the sides certain geometric parameters, there are also velocity components and tangential velocity components which are involved here, in this definition of the diffusion factor as stated by Lieblein.

And over the years from experience, it has been found that diffusion factor of around 0.5 is what is considered as a safe diffusion factor. Diffusion factors exceeding 0.5 might lead to the possibility increased threat of blade stall occurs at diffusion factor, which are much higher than 0.5. So, 0.5 is kind of considered as a safe diffusion factor in most of the preliminary design analysis.

So, we are now, looked at four difference performance parameters, starting with the flow coefficient which was the ratio of axial axial velocity to the blade speed, then the loading coefficient, and subsequently we have discussed about degree of reaction and the diffusion factors. So, these are four fundamental parameters which form part of the design optimization cycle. That of course, few more parameters which we will discuss little later. So, but to begin with these are the four parameters that we need to familiar with, and something that we will be taking up in discussion in detail in subsequent lecture as well. So, now that we understood the working of rotor stator combination of a compressor stage, and also the difference performance parameters.

Let us now, proceed towards discussion on a concept of what is known as cascade. Cascade is basically forms the fundamental basis for carrying out a two-dimensional analysis of compressor blades. And so we will discuss about what is meant by a cascade, and what constitutes a cascade wind tunnel, and what are the different parameters that one can expect from cascade testing.

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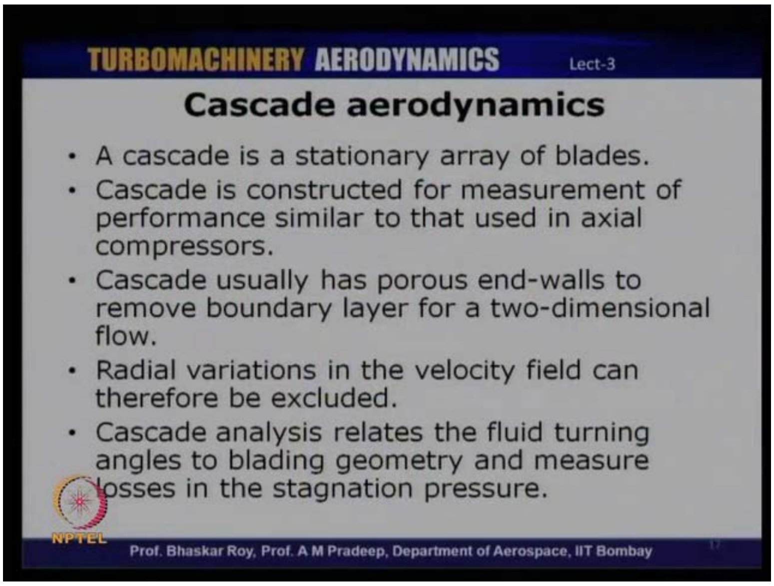

TURBOMACHINERY AERODYNAMICS Lect-3

Cascade aerodynamics

- A cascade is a stationary array of blades.
- Cascade is constructed for measurement of performance similar to that used in axial compressors.
- Cascade usually has porous end-walls to remove boundary layer for a two-dimensional flow.
- Radial variations in the velocity field can therefore be excluded.
- Cascade analysis relates the fluid turning angles to blading geometry and measure losses in the stagnation pressure.

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So, let us take a look at what is meant by cascade, and what are the constituents of a cascade. So, a cascade is basically a stationary array of blades, and the basic function of a cascade is to measure certain performance parameters which can be used in axial compressors designs. So, that is subsequent to preliminary design of an axial compressor blade, one may like to test these blades for its performance, and cascade is the simplest of experiment or performance test that can be done on a certain type of compressor blades. The compressor blades are simplified, and since the blades are stationery, detailed measurements are relatively easy on a cascade as compared to a rotational or on an actual compressor set up. So, cascade is a simplified form of an experimental test facility where one can do wind tunnel test to estimate the performance of a particular compressor blade - compressor or turbine blade designs.

Now, cascade wind tunnels usually have porous end walls; so, as to remove boundary layer, so, that you get a pure two - dimensional flow. One would not like to have a three-dimensional flow in a cascade in a typical conventional cascade tunnel. One would like to remove any three-dimensional effects, and this is also ensured by removing boundary layer from the casing using boundary layer. So that envolves of cascade or porous. Something, I will explain in the next slide. And since the flow is now two-dimensional radial variation in the velocity field can be neglected, because there the flow is purely two-dimensional. So, cascade analysis basically relates the fluid turning angles to the geometry, and also the losses in stagnation pressure that are likely to be incurred as the flow passes through a cascade blade.

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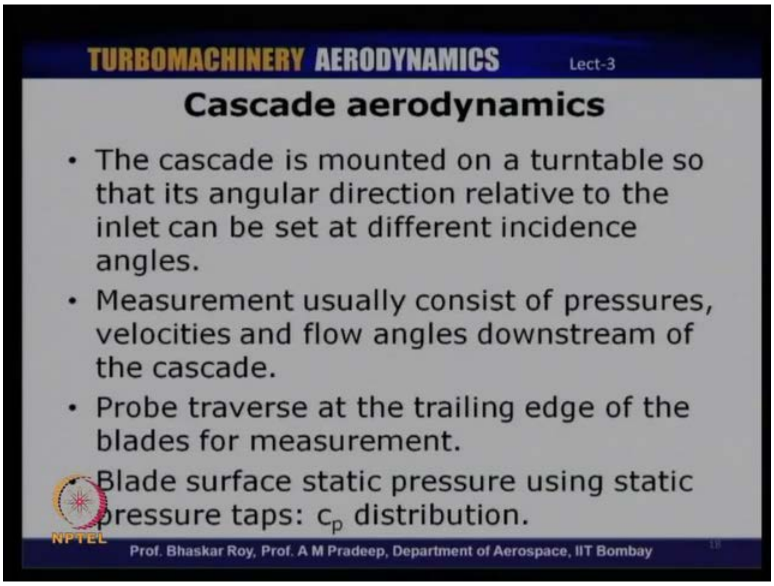

TURBOMACHINERY AERODYNAMICS Lect-3

Cascade aerodynamics

- The cascade is mounted on a turntable so that its angular direction relative to the inlet can be set at different incidence angles.
- Measurement usually consist of pressures, velocities and flow angles downstream of the cascade.
- Probe traverse at the trailing edge of the blades for measurement.
- Blade surface static pressure using static pressure taps: c_p distribution.

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Now, typical conventional cascade will consists of a turntable, on which all the blades are mounted, and so, a turned table will enable the enable us to rotate the blade, so as to change the angle of incidence of the flow. And in in a cascade, the conventional cascade one would like to measure the pressure distribution of the blades, and also the stagnation pressure lose taking place across the blades.

So, measurements in cascade basically consist of pressures, velocity and flow angles downstream and upstream of the cascade, and the cascade wind tunnels usually will have provision for traversing, these different types of probes at the trailing edge for measurement. And on the blade surface, one would measure the static pressure distribution. So that, one can estimate the C p distribution or the blade loading from the C p distribution that we get from the cascade.

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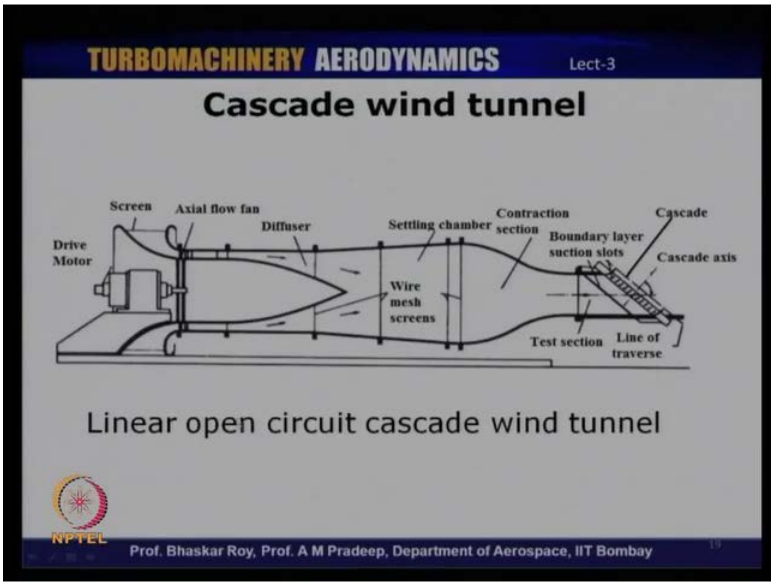

TURBOMACHINERY AERODYNAMICS Lect-3

Cascade wind tunnel

Linear open circuit cascade wind tunnel

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So, what you see here is typical open circuit, so called open circuit cascade wind tunnel. It is called open circuit, because it draws air from the ambian and releases air at the other end of the cascade back to the ambian. There are certain wind tunnels, which are also known as closed circuit wind tunnels where it is a same air which is circulated again and again within the cascade. So that in a closed circuit wind tunnel. And this is called a linear cascade, because all the blades that you can see here are arranged in a linear fashion. They are also cascade tunnels, which are annular in nature where the blades are arranged in annulus. So, those are annular wind tunnels cascade tunnels.

So, cascade tunnel, as you can see consists of various components, and if you have seen a wind tunnel, cascade tunnel is very similar to that of a conventional wind tunnel. It is all those components, there is a driving drive section here, where a fan drives the flow through a series of screens, and wire messed screens, to ensure that the flow that reaches the test section is uniform, and free of high levels of turbulence. Then there is a contraction just before the test section, to accelerate the flow and further reduce the turbulence levels. Now, upstream of the cascade, you can see these slots, these slots are basically meant for removing the boundary layer, to ensure two-dimensionality of the flow. As it reaches the blades and this is the cascade that you can see. You can notice a series of blades which are arranged in a linear fashion. So, all these blades together constitute a cascade. And at the trailing edge, you can see an axis shown here, which is basically a line of traverse that is where one would traverse the probes, and take measurements at the trailing edge of the blades.

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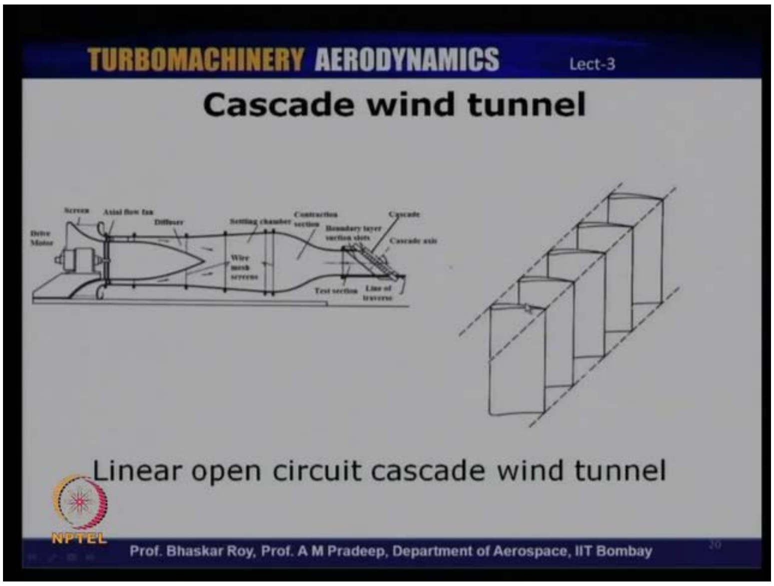

TURBOMACHINERY AERODYNAMICS Lect-3

Cascade wind tunnel

Linear open circuit cascade wind tunnel

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If you take a closer look at the same the test section, the cascade section would look like this. These blades either compressor or turbine blades are arranged in a linear fashion, and that is what is mounted on a turntable that is the entire section can be actually rotated. So that, one can change the incidence angle of the flow entering the cascade. So, as you rotate the cascade axis, one can change is the incidence angle.

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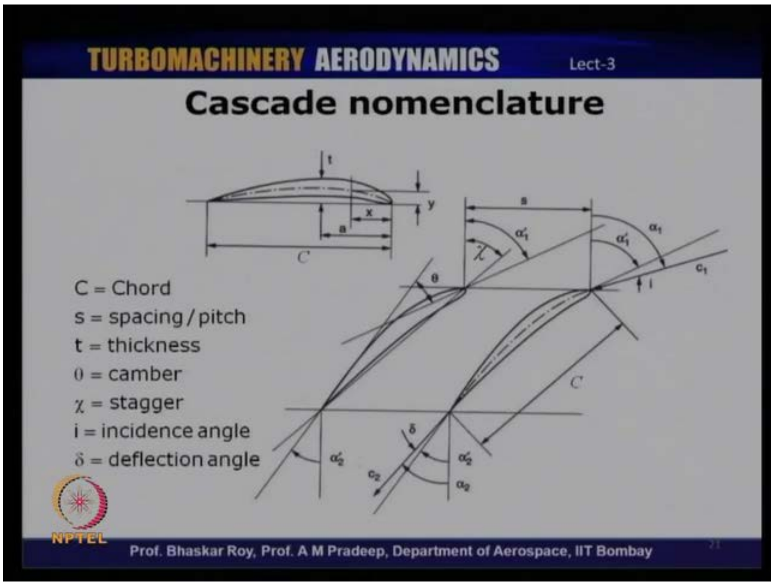

TURBOMACHINERY AERODYNAMICS Lect-3

Cascade nomenclature

C = Chord
s = spacing / pitch
t = thickness
θ = camber
χ = stagger
i = incidence angle
δ = deflection angle

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Now, that I mentioned about incidence angle, let us also look at cascade nomenclature. So, there are certain angles and geometric parameters that one needs to be familiar with when we are dealing with cascades. So, what you shown here, is the different parameters in terms of geometry In terms of the geometric parameters associated with the cascade. The most fundamental parameter being the chord of the blade which is indicated here by symbol C. So, C represents the chord of the blade and the distance between two blades is represented as by symbol s. It is known as the spacing or the pitch of the blades. And here t represents the thickness of the blades, and the angle subtended by tangent at the leading as well the trailing edge is known as the camber, it is indicated by symbol theta. So, theta tells us the camber of the blade. So, the higher the camber, the higher would be the turning of the blade. And theta is measured by tangent, the angle subtended between the tangent at the leading edge and the tangent at the trailing edge of the blade.

Now, psi here denotes the denotes what is known as the stagger of the setting angle. It is basically the angle between the chord of the blade and the cascade axis. This angle that you see here, represents the stagger of this angle, angular setting of a blade. Incidence angle i denotes the angle between the incoming velocity which is shown here as C i, and the blade angle at the leading edge. So, the angle between the blade and the velocity vector is known as the incidence angle. So, the difference between these two gives us the incidence angle. Similarly, at the trailing edge the difference between the velocity vector and the trailing edge tangent to the trailing edge gives us the deflection angle. So, incidence angle represents the angle between the velocity vector and the leading edge, and deflection angle gives us the angle between the velocity vector leaving the trailing edge and the tangent of the trailing edge.

In most of the designs, the one of the criteria would be to try it minimize the incidence and deflection angles. One would like to keep by design the incidence and deflection angles as low as possible, because as the angle incoming velocity vector deviates from the design angles of for higher and higher incidence angle, so, chances of deterioration in performance of the cascade is also higher. So, in for a compressor, one would like to minimize the deviation or at the trailing edge or incidence at the trailing edge to as low as value as possible. So, as to minimize the performance penalties associated with deviation of the incident angle of the deflection angle from what it is been designed for.

So, in a typical cascade, one can adjust all these parameters in terms of the for a given test section the chord is fixed, and the spacing for the given solidity is fixed, but one can change the incidence angles, one can change the stagger or setting angles, and see how all these parameters influence the performance of this particular cascade which has been or the blade which has been designed.

So, in a in a modern day cascade, one would like to take lot of measurements, because cascade facilitates, detail measurements on compressor blades primarily, because of the fact that the blades are stationary. And because the blades are stationary one can take very detailed measurements which probably acquire difficult to take in rotating machinery, and therefore, cascade testing forms is still part of modern day design exercise where one could carry out quite detailed measurements on compressor blades. In fact, there is substantial amount of literature available on series of different types of compressor blades, which are used and all these data has been generated primarily from cascade testing.

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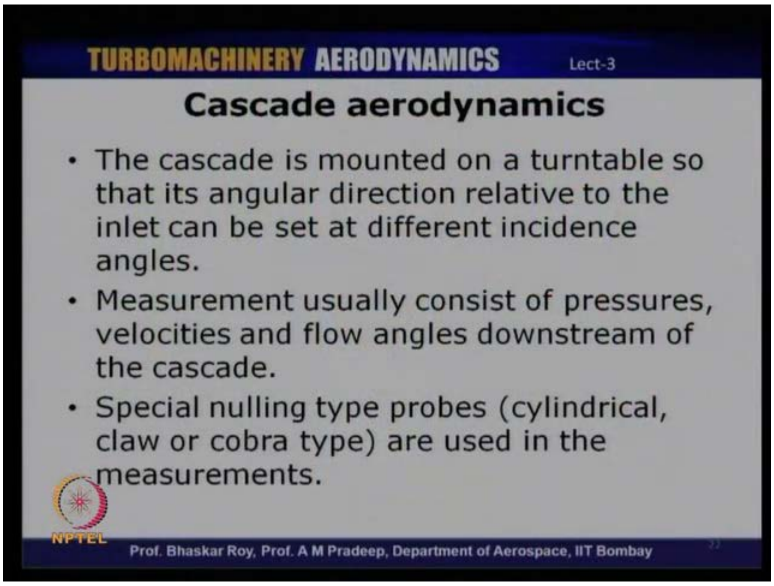

TURBOMACHINERY AERODYNAMICS Lect-3

Cascade aerodynamics

- The cascade is mounted on a turntable so that its angular direction relative to the inlet can be set at different incidence angles.
- Measurement usually consist of pressures, velocities and flow angles downstream of the cascade.
- Special nulling type probes (cylindrical, claw or cobra type) are used in the measurements.

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Now, in cascades, one would normally use different types of measurement devices, measuring probes for measuring total pressure losses and static pressure on the blade surface flow angles etcetera. So, one might use different types of probes which could be nulling probes or non nulling probes, either cylindrical type probe or cobra probes. These are different types of probes that one might like to use to carry out measurements in a cascade and in for data from the cascade testing.

So, what we have discussed in relation to cascade is a fact that cascade allows us cascade testing allows us to carry out detailed measurements in much more simplified geomentry without the complexities of rotation, which are present in an actual compressor. So, let me now quickly re-cap our discussions in today’s lecture.

We started our discussion today with the compression process taking place in an axial compressor stage, how we can estimate the pressure ratio, and how we can relate pressure ratio to the temperature rise in stage of an axial compressor. And what is that pressure ratio depends upon, that is increase in pressure ratio would require that we either have increase in axial velocity or blade speed or a deflection each of them obviously as we discussed have certain limitations.

We then discussed about different performance parameters or design parameters which are used in design of axial compressors, to begin with we have discussed about flow coefficient, and then we have the loading coefficient, and subsequently we also discussed about two other parameters; the degree of reaction which tells us what is the amount of static pressure rise that is generated in a rotor as compared to that of a stator. And in subsequently, we also discussed about what are what is meant by the diffusion factor. Diffusion factor basically tells us, the amount of diffusion that one can achieve in a rotor without there is cough flow separation that is likely to take place or diffusion factor indicates, the chances of flow separation due to adverse pressure gradients, which might occur with increase in deflection across rotor blade.

We then spend a few minutes on discussion on cascades, what is meant by cascade, and what is the significance of a cascade. And subsequently, we also discussed what is meant by a cascade tunnel, and how we can carry out testing of two-dimensional blades in a cascade, and what are the kind of measurements that one can take in a cascade. We also had some quick discussion on the nomenclature, which is used in a cascade, what are the different geometric parameters like this the chord, the spacing, the camber, the stagger and incidence and deflection angles.

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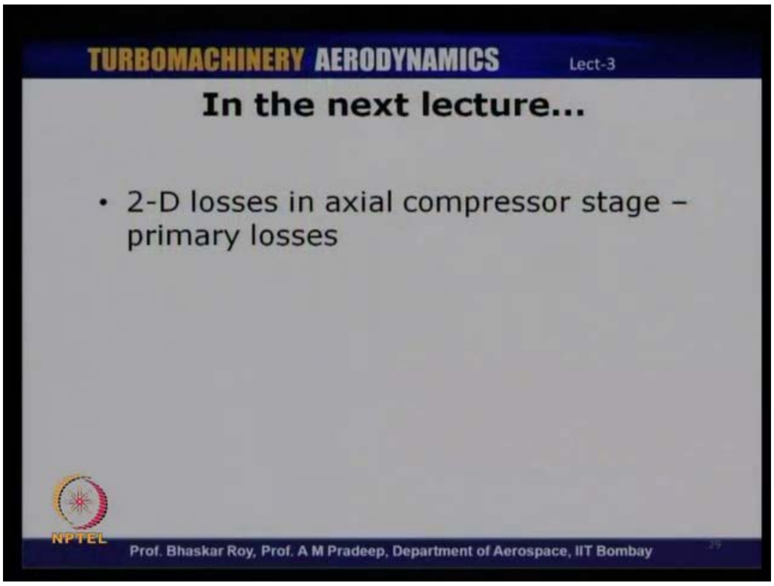

TURBOMACHINERY AERODYNAMICS Lect-3

In the next lecture…

- 2-D losses in axial compressor stage – primary losses

NPTEL

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In the next class, we are going to continue discussion on cascades and two-dimensional analysis. We will now extend the discussion to losses different types of two-dimensional losses that we can associate to a cascade, and how we can estimate some these two-dimensional losses in a typical axial compressor stage. So, we are going to discuss about the losses associated with 2-D flow which is basically a cascade flow, and how we can estimate these losses in an axial compressor. So, these are some of the topics that we will be discussed in our next lecture.