

Turbomachinery Aerodynamics
Prof. Bhaskar Roy
Prof. A M Pradeep
Department of Aerospace Engineering
Indian Institute of Technology, Bombay

Lecture No. # 26

Tutorial – 4: 3D Flows in Axial Flow Turbines

We are talking about axial flow turbines, and in last class, last lecture, we talked about how to design and bring into the design steps and design features, three dimensionality of flow through axial flow turbines. Now, we know that the flow through the axial flow turbines, as in axial flow compressors, is normally annulus in nature and that flow through the annulus space that is available to the turbines is often not exactly uniform.

So, some aspect of the non uniformity or some aspect of the variation from the lower radius to the outer radius or inner radius to the outer radius, and more specifically from the root to the tip of the blade of a rotor needs to be factored into to begin within the design, and later on, in the computations, and of course, later on in the analysis. We have seen how some of these things can be brought into simple analysis. That we had done in the last class without getting into more complex computational analysis.

As I mentioned, we shall do computational, an introduction to computational analysis through turbo machines towards the end of this lecture series, but right now, we are looking at turbines specific certain theories which factor in the three dimensionality of the flow and built it into the design. So, most of it is indeed used for design, and then, immediate pose design analysis to find out how the turbine is actually going to behave.

So, in today's lecture, we will look at some problems that actually use these theories that we have done in the last class, and then, try to actually solve some problems which are prescribed problems, and from the problem statement, we tried to figure out what kind of solutions can be arrived at using the theories that we have done in the last lecture.

And towards the end, I will leave you with a few problems to solve by yourselves so that you can get the feel of the application of these theories to actual problems, realistic

problems and you also get a feel of the numbers. It is essential for an engineer to get the feel of the numbers and that is why it is essential that we look at a few problems which are probably a little textbook problems, simplicity problems where, you know, typically you have all the data you require. In a real life problem, you often may not actually have all the data that is require to solve, but we are dealing with problems where the problem statement gives you all the data that you require.

And now, you need to use the theories that we have done to arrive at solutions, and in a process, one learn how the solutions would look like and indeed as I said you get a feel of the numbers. What the numbers would look like? So, that is very important for all practical engineers. So, let us look at some of the problems related to three-dimensional flow in axial flow turbines. Now, in axial flow turbines, that we are going to do some solved problems, and then, I leave you with some exercise problems to solve for yourselves.

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Example .1.

Following data apply to a constant nozzle exit angle (α_2) axial turbine design :
Temp. drop, $\Delta T = 150$ K; at hub $U_{2h} = 300$ m/s ; at tip $U_{2t} = 400$ m/s ; $\alpha_2 = 60$; $\alpha_3 = 0$; and Radius ratio given is, $r_h / r_t = 0.75$

(a) Complete the design velocity diagrams at hub, mean and tip of the stage
(b) Calculate the velocity components if the design is free vortex for the turbine and compare the values with (a)

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Now, in the solved problems, we will first look at the problem statement. In the first problem, the problem statement shows that a constant nozzle exit angle which we have designated earlier is alpha 2 is being use for axial flow turbine design. In which, it is prescribed that the temperature drop should be 150 k, and at the hub, the flare velocity a blade speed u is 300 meters per second, and at the tip, it is 400 meters per second.

At it is prescribed that alpha 2 is 60 which is constant as prescribed from a root to the tip hub to the tip and alpha 3 is 0, that is, 0 whirl at the exit of the rotor. The radius ratio given for this particular problem statement is 0.75, that is, hub radius to tip radius ratio is 0.75 that is prescribed here. The problem asks a solution in which he should complete the design velocity diagrams at hub mean and tip of the stage, and thereafter, calculate the velocity components if the design is a free vortex design for the turbine and compare the results of that with that of constant nozzle exit angle. So, one can look at the whole thing from a free vortex point of view and compare the results. So, this is a problem statement which we can now try to find a solution to.

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Solution 1 :
At the rotor inlet station we know,

$$\frac{C_{w2}}{C_{w2m}} = \frac{C_{a2}}{C_{a2m}} = \frac{C_2}{C_{2m}} = \left(\frac{r}{r_m}\right)^{\sin^2 \alpha_2}$$

And, at the rotor exit

$$C_{a3}^2 = C_{a3m}^2 + 2U_m C_{w2m} \left[1 - \left(\frac{r}{r_m}\right)^{\cos^2 \alpha_2} \right]$$

and

 $r_m / r_t = 0.875$, and $r_m / r_h = 1.166$

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Now, at the rotor illustration, we have done in the last class - the equation for variation of whirl component is defined by the variation of α_2 and that is given by C_{w2} by C_{w2m} which is the mean and that is also equal to C_{a2} by C_{a2m} , which is also equal to C_2 by C_{2m} which is mean and all these velocity ratios are equal to radius ratio r by r_m being at any radius to the power sine square α_2 . Now, this is what we had done in the last lecture following which, what we get is, at the rotor exit, the actual velocity C_{a3} square is equal to C_{a3m} square plus twice U_m into C_{w2m} whole thing multiplied by 1 minus radius ratio r by r_m to the power cosine square α_2 .

Now, this is also the theory that we had the expression; we had shown in the last lecture. Now, from the prescribed radius ratio that is given, we can see that the mean to tip radius ratio would be 0.875 and mean to hub radius ratio would be 1.166 , that is calculated from the hub to tip radius ratio that is prescribed in the problem statement.

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Work done by the rotor is given by (for $\alpha_3 = 0$)
$$U (C_{w2} + C_{w3}) = \Delta H_0 = c_p \cdot \Delta T = U_m \cdot C_{w2m}$$

From which we can write $C_{w2m} = 492$ m/s
 $C_{a2m} = C_{w2m} \cot \alpha_2 = 284$ m/s = C_{a3m}

At the rotor hub inlet

$$C_{a2h} = C_{a2m} \left(\frac{r_m}{r} \right)^{\sin^2 \alpha_2} = 318.8$$
 m/s
$$C_{w2h} = C_{w2m} \left(\frac{r_m}{r} \right)^{\sin^2 \alpha_2} = 552.2$$
 m/s

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Now, the work done by the rotor is given by the Euler's equation which you are all aware of. Now, in this particular problem statement, α_3 would be equal to 0; exit whirl component is 0, which means C_{w3} indeed would be 0, and in which case, the specific enthalpy rise or, you know, specific work that we normally use for aerodynamic relations, that is, $c_p \Delta T$ would be equal to $U_m C_{w2m}$ as C_{w3} being 0, and from which, we can write that C_{w2m} would indeed be 492 meters per second and corresponding C_{a2} would be from the velocity triangles that you can draw at station 2 that is before the rotor and that would come out to be 284 meters per second. As per the prescription, that also would be equal to a actual velocity at the exit of the rotors C_{a3m} .


Now, at the rotor hub, at the inlet of the rotor, one can write a axial velocity C_{a2h} is equal to C_{a2m} multiplied by a radius ratio r_m by r to the power $\sin^2 \alpha_2$ and this would yield a axial velocity of 318.8 meters per second. On the other hand, the whirl component at hub, at the inlet C_{w2h} would be a C_{w2m} into r_m by r to the power $\sin^2 \alpha_2$ and that would be 552.2 meters per second. So, you now start getting the components of the velocities at station 2, that is, before the rotor at the rotor inlet.

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At the rotor tip inlet

$$C_{a2t} = C_{a2m} \left(\frac{r_m}{r} \right)^{\sin^2 \alpha_2} = 257 \text{ m/s}$$

$$C_{w2t} = C_{w2m} \left(\frac{r_m}{r} \right)^{\sin^2 \alpha_2} = 447 \text{ m/s}$$


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Now, at the rotor tip of the inlet, one can now find the actual velocity using the same relation, that is, C_{a2m} into r_m by r to the power $\sin^2 \alpha_2$, and in which case, the actual velocity would now be 257 meters per second, and at the whirl component, C_{w2t} would be 447 meters per second. So, as one can see that the whirl components has actually decreased from hub to the tip, whereas, the actual velocity is also decreased a little from hub to the tip and this is a consequence of the fact that α_2 has been held constant from root to the tip of this prescribed problem. Soon as a result of which, we now have all the velocity components that are required at the inlet to the rotor.

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At the rotor tip outlet

$$C_{a3} = \sqrt{C_{a3m}^2 + 2U_m C_{w2m} \left[1 - \left(\frac{r}{r_m} \right)^{\cos^2 \alpha_2} \right]}$$

From which we can calculate the axial velocities,

$C_{a3t} = 262 \text{ m/s}$

$C_{a3h} = 306 \text{ m/s}; \quad C_{w3} = \text{is constant radially}$



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Now, at the rotor tip outlet, we can find what the actual velocity would be and this can be found from the axial velocity relation that we had done earlier and that is given by $C_{a3} = \frac{C_{a2}}{\sqrt{1 - \frac{2U_m}{C_{w2}} \frac{r}{r_m} \cos^2 \alpha_2}}$, and if you do that, we can calculate the actual velocities at tip and hub as at the outlet as 262 and 306 meters per second.

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(b) Free vortex stage design and comparison
 For Free vortex design we have established in the last lecture
 $C_{w3} \cdot r = \text{constant}, C_{a3} = \text{const} = C_{a2}$

	Constant Nozzle	Free Vortex
C_{a2h}	318.8	284
C_{a2m}	284	284
C_{a2t}	257	284
C_{w2h}	552	574
C_{w2m}	492	492
C_{w2t}	447	430
C_{a3h}	306	284
C_{a3m}	284	284
C_{a3t}	262.6	284

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So, once you apply the relations that we had done in the last class for three-dimensional flow estimation, we now have the axial velocities at tip and hub at the outlet to the rotor. On the other hand, if we calculate all the values with reference to the free vortex design, we can see that the velocities that we get in comparison to the nozzle angle can be now written down in a tabular form, and one can see that these values are the ones in constant nozzle angle are given in red and the free vortex values are given in black, and the actual velocities as you can see in, **in**, front of the rotor and behind the rotor are constant for free vortex, only the whirl component is changing as per free vortex variation from tip to hub. On the other hand, for constant nozzle angle as one can see all the velocities are varying, and essentially only at the mean, the values of free vortex and the constant nozzle angle are same at the tip as well as at the mean, the axial velocity the whirl

component velocity and the exit axial velocity they are all same at the mean, but at the hub and the tip, the constant nozzle angle gives completely different values compared to that of free vortex.

Now, this is something which you may like to take a look at and you may like to sit down and draw all the velocity triangles that are born out of this two calculations, two sets of design calculations and you will probably find that you get completely different blade shapes. The blade shapes that come out of this two design exercises would indeed be quite different from each other and that would tell you that, if you use different kind of design law or design philosophy, you would in end up getting quite different blade shapes, the three-dimensional blade shapes.

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Example 2
It is proposed that for design of an axial flow turbine two design methods are to be explored :

A) $C_{w2m} = C_{w2h} = C_{w2t}$

B) $C_{a2t} = C_{a2h} \left(\frac{r_h}{r_t} \right)^{\sin^2 \alpha_2}$ and,

C) $C_{w2t} / C_{w2h} = r_h / r_t$

Common design data prescribed are: $C_{am} = 200$ m/s
; $\alpha_2 = 60$; $\alpha_3 = 0$; $R_x = 0.5$; and $r_h / r_t = 0.8$

Complete the velocity diagrams for all the cases.

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Now, if we go onto the second example in which, the problem statement reads that it is propose that for design of an actual flow turbine two design methods are to be explored. Now, first design method that is prescribed is C_{w2m} is equal to C_{w2h} that is equal to C_{w2t} ; that means the whirl components at hub mean and tip are equal to each other. Now, this type of design is what we have called earlier solid body essentially design; that means the fluid behaves more like a solid body and correspondingly the whirl component everywhere are same.

The second design that is prescribed is where C_{a2} is equal to C_{a2h} into the radius ratio hub to tip radius ratio to the power $\sin^2 \alpha_2$, which somehow tries to

make use of the constant α_2 prescription and that is your Case B. The second design that is suggested here in Case B, and in case C, we have $C W$ ratio $C W_2 t$ to $C W_2 h$ has equal to the radius ratio r_h by r_t .

Now that, if you remember is nothing but you free vortex design. So, we have three design that is suggested for design of an actual flow turbine - one in which the flow behaves like a solid body; the second one in which one may use the α_2 equal to constant that is constant nozzle angle prescription and Case C is which resembles of that of a free vortex design.

What is prescribed are some common data and these are the actual velocity at the mean is prescribed as 200 meters per second; entry α_2 is 60 degrees; exit α_3 is 0. The degree of reaction is 0.5 at mean and radius ratio prescribed is 0.8. What is asked for is complete the velocity diagrams for all the cases. The velocity diagrams are asked for because those are the velocity diagrams based on which final blade shapes would be created. So, the velocity diagrams would give a fairly good idea about the blade shapes that are being created by three cases - a b and c.

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Solution 2 :

From the prescribed data :
One can calculate that: $r_m / r_t = 0.889$; $r_t / r_m = 1.11$

$C_{w2m} = C_{a2m} \times \tan \alpha_2 = 346.5 \text{ m/s}$; and $C_{w3m} = 0$

For all the cases, $R_x = 0.5$ is prescribed at mean
Hence, from symmetrical blading concept
 $\alpha_{2m} = \beta_{3m} = 60^\circ$; $\alpha_{3m} = \beta_{2m} = 0^\circ$

Also, $U_m = C_{w2m} = 346.5 \text{ m/s}$ and hence at any
radius, $U_h = 308 \text{ m/s}$; $U_t = 385 \text{ m/s}$

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Let us look at how to go about finding a solution to this particular problem statement. From the prescribed data of the example 2, one can calculate that radius ratio r_m by r_t would be 0.89 and r_t by r_m would be 1.11. This is calculated from the hub to tip radius ratio that is prescribed in the problem, and then, the whirl component at mean C_{w2m} can be found from the mean velocity diagram that is a C_{a2m} into $\tan \alpha_2$ and that gives C_{w2m} equal to 346.5 meters per second and it is prescribed that α_3 is 0. So, C_{w3m} would be 0.

Now, C_{w3m} equal to 0 is what we had done in the first problem also, where it is prescribed that C_{w3m} could be given as 0 or α_3 is given as 0. This is a fairly standard prescription for many of the designs, because what happens is, when the flow is going out of the turbine quite often, the prescription often encourages that the flow going out of the turbine does not have any whirl component, because the whirl component going to going out of a turbine typically of a single turbine or a multistage turbine, last stage would be quite useless.

So, typically of a turbine it is quite often unless you know it is one of the earlier turbines are one of the middle stage turbines. The prescription quite often comes with α_3 equal to 0 which leaves 0 whirl component, and if the flow is going in to a nozzle or going into exhaust, any whirl component present is quite useless, only component that is useful for nozzle effect for thrust making is the axial component. So, giving a

prescription of whirl component 0 is pretty much a practical and standard prescription for turbine design as I mentioned, unless you are designing a turbine, which is one of the middle stages of a multistage turbine.

So, as we have seen in both the problems C W 3 and alpha 3 of the prescribed to be a 0, and as a result of which, the problems does become a little simple, but the prescription is realistic it is not really idealistic. Let us get back at the problem. For this particular problem statement, it is given the degree of reaction r_x is 0.5. Now, that means, it brings us back to the symmetrical balding concept that you may have done earlier and certainly done in some detail in axial flow compressors.

So, the moment you put a degree of reaction 0.5, the symmetrical bleeding concept comes in, and then, you have at the mean, α_{2m} equal to β_{3m} that would be equal to 60 degrees as prescribed and α_{3m} equal to β_{2m} equal to 0 degrees as prescribed. This of course, makes the problem a little simple to handle. However, many turbines in the past and the early days of turbine design have been used, using these kind of a somewhat simpler design prescriptions and those turbines were a operating quite well.

It makes the designers job definitely a simple and a probably analysis also become simple in those days and long back thirty forty years back if you may remember, the aid from a computational fluid dynamics was not available, and as a result, the refinement that is possible in today's turbine design was not really available in those days, and as a result, somewhat simpler design prescriptions were often used for design and those were functional, they worked fine cell. So, a similar somewhat simpler design prescriptions has been also prescribed here.

Now, the blade velocity U_m would come out to be same as a C W 2 m because $u_r \beta_{2m}$ is 0, and as a result of which, 346.5 meters per second as calculated just a little above, and at any radius, we can now calculate the blade velocities from the radius ratios that we have just written down. So, the hub blade velocity would be 308 meters per second and the tip blade velocity U_t would be 385 meters per second. So, the blade velocity is vary all the way from root to tip as a per ω into r concept.

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For Case (A)

This is a fluid behaving like a 'solid body' case for which $n = 0$ in the equation $C_w = r^n$

The axial speed is calculated from the axial velocity expression derived from the energy equation for the case $n=0$

$$C_{a2} = C_{a2m} \sqrt{1 - 2 \tan^2 \alpha_{2m} \ln \left(\frac{r}{r_m} \right)}$$

All the angles across the rotor may be also calculated from above

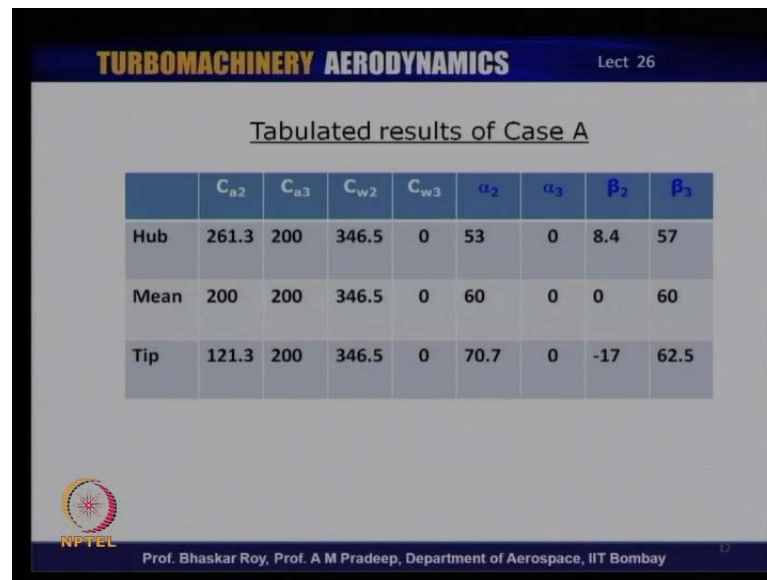


Now, for case A, we have three cases to be actually looked at, and as I indicated, this is a fluid which is prescribed to be behaving like a solid body for a cases which we have done in the last lecture n equal to 0 for the equation $C_w = r^n$. Now, when you put n equal to 0, it is a Cases which fluid behaves like a solid body.

Now, the actual speed is calculated from the actual velocity pressure derived from the energy equation for the cases n equal to 0 and this comes out to be C_{a2} is equal to C_{a2m} whole thing root over $1 - 2 \tan^2 \alpha_{2m} \ln \left(\frac{r}{r_m} \right)$. Now, this can be derived; you can said an and derived in the same way we were done earlier for the cases n equal to 0 from the energy equation that we are written down involving the various velocity components.

And you have to put the value of n equal to 0 there and he would arrive at this a solution which we are looking at for a axial velocity at any station with relation to axial velocity at a mean radius along the blade length.


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Tabulated results of Case A

	C_{n2}	C_{n3}	C_{w2}	C_{w3}	α_2	α_3	β_2	β_3
Hub	261.3	200	346.5	0	53	0	8.4	57
Mean	200	200	346.5	0	60	0	0	60
Tip	121.3	200	346.5	0	70.7	0	-17	62.5

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Now, using this, you can also calculate the angles across the rotor from the above considerations. If you do that, the solutions that you get for the case A tells you that the actual velocity prescribed at the mean was a 200 and C_{w2} at the mean was found to be 346.5. On the other hand, we found the axial velocity is varying from tip to hub and both at the rotor entry as well as at the rotor exit. At the rotor exit, the C_{w3} is prescribed to be actually 0, and correspondingly the value of α_3 also has been prescribed to be 0. The α_2 variation is shown here as part of our results. It varies all the way from hub to the tip. Correspondingly the β_3 also varies exactly in the same manner from hub to the tip, and the β_2 values are shown here. It is 0 at the mean as we have calculated, as off we prescribed the degree of reaction being 0. However, there is a small value β_2 at the hub and a small value of β_2 at the mean, at the tip that is been shown here.

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Case (B)

Prescribed condition is $C_{a2t} = C_{a2h} \left(\frac{r_h}{r_t} \right)^{\sin^2 \alpha_2}$

Which essentially means : $\frac{C_{a2t}}{C_{a2h}} = \frac{C_{a2t}}{C_{a2m}} = \frac{C_{a2m}}{C_{2h}}$

For constant nozzle angle: $C_{a2} = C_{a2m} \left(\frac{r_m}{r} \right)^{\sin^2 \alpha_2}$

$$C_{w2} = C_{w2m} \left(\frac{r_m}{r} \right)^{\sin^2 \alpha_2} ; C_2 = C_{2m} \left(\frac{r_m}{r} \right)^{\sin^2 \alpha_2}$$



Now, if we move to the results of a case B, the prescribed condition is C_{a2t} is equal to C_{a2h} into radius ratio r_h by r_t prescribed here in the problem to the power sine square alpha 2. Now, this of course, brings us to the fact that all the velocities at the tip and hub and mean can be the ratios of them can be put down as equal to each other. All of them would be equal to some relation to alpha 2.

Now, this allows us to write down that far constant nozzle angle, which is the prescribed case B that we were looking at. C_{a2} can be now written down as C_{a2m} into radius ratio r_m by r to the power sine square alpha 2, and in case of C_{w2} , that would be C_{w2m} to the into radius ratio r_m by r to the power sine square alpha 2 and C_2 is equal to C_{2m} to the multiplied by radius ratio r_m by r to the power sine square alpha 2.

So, far constant nozzle angle case, all the velocity C_{h2} , C_{w2} and C_2 which is the absolute velocity can be found from the mean values, that is, at the mean radius to hub to tip at any radius by using this radius ratio concept.

If we do that at the station 3, at the exit of the rotor, it is prescribed that alpha 3 is equal to 0 and C_{w3} is also equal to 0. The expression for axial velocity comes out as we have done in the last lecture a C_{a3}^2 is equal to $C_{a3}^2 - 3 C_{a3m}^2 \cos^2 \alpha_2 + C_{w2m}^2$ multiplied by 1 minus radius ratio to the power cos square alpha 2. Now, this allows us to calculate C_{a3} at exit of the rotor. If we now use the relation that we have done which is essentially for the case B, which is a exit angle from the rotor is held constant from root to the tip of the blade.

Now, this is something which we have discussed in last lecture that, holding the exit angle from the rotor constant from root to the tip of the blade makes the rotor untwisted or very likely twisted. Now, this is important for a stator nozzle blade cooling purpose. We shall be doing a cooling technology from next lecture onwards, but this particular design philosophy of holding α_2 equal to constant from hub to tip or root to tip of the blade of the stator essentially caters to cooling technology, and if we apply this, in this present problem what we see is the result that we get for case B.

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The slide displays a table of aerodynamic parameters for Case B. The parameters include axial velocity (C_{a2} , C_{a3}), tangential velocity (C_{w2} , C_{w3}), flow angles (α_2 , α_3), and flow angles relative to the axial direction (β_2 , β_3). The values are constant for α_2 and α_3 across the span, while C_{a2} , C_{a3} , C_{w2} , β_2 , and β_3 vary from hub to tip.

	C_{a2}	C_{a3}	C_{w2}	C_{w3}	α_2	α_3	β_2	β_3
Hub	218.5	216.7	378.5	0	60	0	17.9	54
Mean	200	200	346.5	0	60	0	0	60
Tip	185	183.3	320	0	60	0	-19.4	69

Let us look at the results of Case B. What we see here is the axial velocity is now vary all the way from hub to the tip. At the hub, it is 218.5. At the mean, it is prescribed as 200. So, the variation of axial velocity at the entry as well as at the exit is quite pronounced from hub to tip. The values of C_{w2} are also variables from hub to tip point. Substantially C_{w3} is being held constant; α_2 by prescription is held constant; α_3 by prescription is 0.

We get a variation of β_2 from 17.9 to minus 19.4 and we get a variation of β_3 from 54 to 69 from hub to tip. So, these are the results of the Case B which is of a constant α_2 from hub to the tip of the stator nozzle. Now, we can move to the Case C. Now, Case C is what we had seen was actually the free vortex design. Now, the free vortex design as we have mentioned before is not the most popular design for actual flow turbine even though it is a very popular design for axial flow compressors.

For turbine, it is not the most popular design, but it is the simpler design it of course works. If you make actual flow turbine with a free vortex design, it will surely work; there is no reason why it should not work, but it is not the most popular design today, and ever since the cooling technology came into the market, the free vortex design has been essentially replaced by the constant α^2 design which is the more popular design philosophy for turbines essentially as I mentioned to cater to a cooling technology. But let us look at the results that we get for this problem a statement problem to Case C for a free vortex design philosophy.

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For Case (C)

Since $C_{w2t} / C_{w2h} = r_h / r_t$ – this is Free Vortex law

Same may be applied at rotor outlet also :

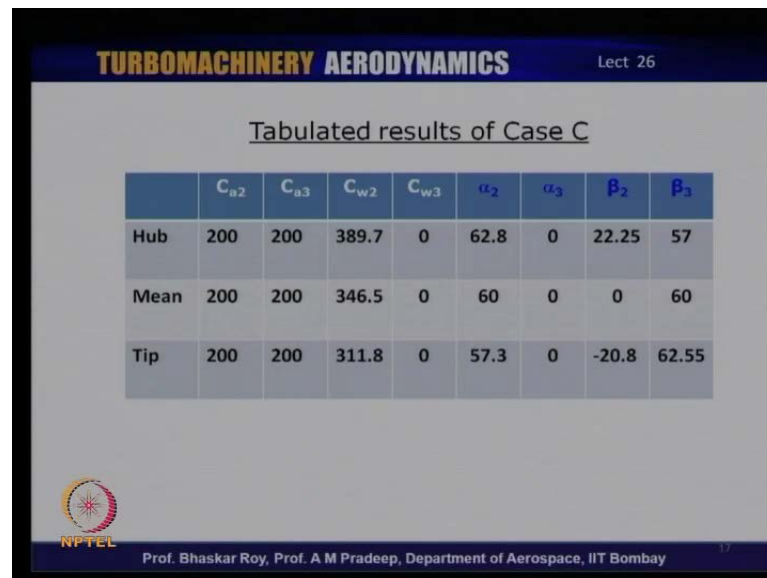
$C_{a2} = \text{const} = C_{a3}$ at mean radius

The results are summarized in the table :

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If you apply free vortex design philosophy, the whirl components at tip and hub the ratios are directly related to the hub to tip radius ratio. Now, if we apply the same at the outlet also, C_w also is held constant as per free vortex principle across the blade at mean various. So, C_w , C_{a2} is equal to C_{a3} , and if you apply all these, in the, as per the free vortex law, we get a set of results directly as per very well known free vortex law prescriptions.


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Tabulated results of Case C

	C_{D2}	C_{D3}	C_{W2}	C_{W3}	α_2	α_3	β_2	β_3
Hub	200	200	389.7	0	62.8	0	22.25	57
Mean	200	200	346.5	0	60	0	0	60
Tip	200	200	311.8	0	57.3	0	-20.8	62.55

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The results that we get that the axial velocities are held constant from hub to tip as well as across the rotor. C_{W2} varies substantially from hub to the tip as per free vortex law and which would tell you that you would end up getting a substantially twisted blade. C_{W3} is being held constant. So, the trailing edge would be rather leaner. On the other hand, the value of α_3 is 0 corresponding to the prescription. α_2 varies from hub to tip and the C_{W2} variation shows that, and then, of course, you have the variation of β_2 which goes minus at the tip and this is something that comes out of the free vortex design, and as a result of which, you get a variation of β_3 which also varies from hub to the tip of the blade.

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All Three cases compared : Design velocity diagrams

Strn	Case	C_{n2}	C_{n3}	C_{w2}	C_{w3}	α_2	α_3	β_2	β_3
Hub	A	261.3	200	346.5	0	53	0	8.4	57
Hub	B	218.5	216.7	378.5	0	60	0	17.9	54
Hub	C	200	200	389.7	0	62.8	0	22.25	57
Mean	A	200	200	346.5	0	60	0	0	60
Mean	B	200	200	346.5	0	60	0	0	60
Mean	C	200	200	346.5	0	60	0	0	60
Tip	A	121.3	200	346.5	0	70.7	0	-17	62.5
Tip	B	185	183.3	320	0	60	0	-19.4	69
Tip	C	200	200	311.8	0	57.3	0	-20.8	62.55


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So, you have the results of the Case C Tabulated here, and then, we have three Cases – a, b and c these variations can now be all put together into one table which are essentially tries to compare the 3 Cases – a, b and C, and as you can see the Case A is given in red, the Case B that is a constant nozzle angle is given in b and the free vortex design is given in Case C and all three of them are brought together. If you sit down, use the velocity is that I given the angles that are given, and if you draw the velocity triangles of all the cases, you will probably get a very clear picture of what kind of blades actually come out of the three-dimensional blade shapes that should come out of the three Cases that are prescribed here. The three cases that are prescribed here had certain commonalities, that is, the mean actual velocity prescribed by 200. The exit the angles were prescribed to be 0. The whirl components at the exit were prescribed to be 0.

So, with those common prescriptions, we try to put together; the blades that would come out even with those commonalities, and we see that three completely different blade shapes are likely to result from three cases that are prescribed here for axial flow turbine design. So, as I mentioned, you can probably sit down and actually draw the velocity triangles and you would find that three different cases or three different blade shapes and he would indeed need to choose different aerofoil shapes, different blade sections from hub to tip for the each of these three cases. So, you end up having three completely different blade shapes for three different design philosophies even though we started out with a common data prescription for all three cases. So, this is an example which tells

you with certain simplifications or simplified problem. It still tells you that, if you have three different design philosophies, you end up with three completely different blade shapes. I will now leave you with a few problems which you can sit down and solve for yourself and get a feel of the numbers that come out of solving of examples problems that are prescribed with numerical values. So, let us look at some of the problems you can solve for yourselves.

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Exercise problems

1. (a) An axial turbine rotor is prescribed with rotor inlet and outlet flow in radial equilibrium. The whirl component of the flow is designed to vary radially as : $C_{w1} = a.r - b/r$; and $C_{w2} = a.r + b/r$ Where, a and b are constants. Find the inlet and outlet axial velocities (C_{a1} and C_{a2}) from above.
 $[C_a^2 = K - 2.a^2 [(r^2-1) - 2(b/a)\ln r] \text{ \& } C_{a1} = C_{a2} \text{ axially}]$

(b) It is prescribed that at mean radius = 0.3 m , axial velocity=150 m/s, degree of reaction $R_x = 0.5$, blade loading coefficient, $\psi_{rotor} = H_0/U_{tip}^2$. Rpm=7640 Hub/tip ratio of the rotor =0.5. At 80% rotor radius, find the rotor relative flow inlet and outlet angles.
 $[43.2^\circ \text{ and } 10.4^\circ]$

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The first exercise, **exercise**, problem, the problem statement reads that an axial turbine rotor that is prescribed with rotor inlet and outlet flow in radial equilibrium, which means the static pressure and the dynamic pressure are balanced at the inlet and outlet of the rotor. The whirl component of the flow is designed to vary radically as per this prescription as C_w at the inlet as $a r - b/r$ and at the outlet as $a r + b/r$. Now, a and b are the constants, and what is required for you to find is find the inlet and outlet exit velocities and the expressions for those velocities.

In this case, you can see the answers given here and the actual velocity is of course would remain constant across the rotor. So, we are dealing with rotor only. So, the problem statement is essentially for rotor. Part b of the problem is - it is prescribed that at mean various given value is point 3 meters. The actual velocity is 10 meters per second and the degree of reaction is 0.5. The blade loading coefficient is prescribed as, **as**, per the definition, ψ_{rotor} is equal to work done, specific work done H_0 by U_{tip}^2 and

the r p m in 7640 rpm.

The hub to tip radius ratio is given as 0.5, and at 80 percent of the rotor radius, it is required for you to find the rotor relative flow inlet and outlet angles. So, what you are required to find the beta values of a beta 2 and beta 3 for the particular problem statement that is given here. Now, the answers given are 43.3 degree and 10.4 degree for beta 2 and beta 3. So, you can try to sit down and see whether you can arrive at those answers.


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2. Hot gas exits from a turbine stator-nozzle at a radially constant angle, α_2 . The gas is also prescribed to be in radial equilibrium. Axial velocity variation at that station is given as :

$$C_{a2} \cdot r^{\sin^2 \alpha_2} = \text{const}$$

For a turbine in which the axial velocity at radius 0.3m is 100 m/s. If the turbine, as stated above, is designed with constant $\alpha_2 = 45^\circ$, find the axial velocity at that station at 0.6 m radius. [70.7 m/s]

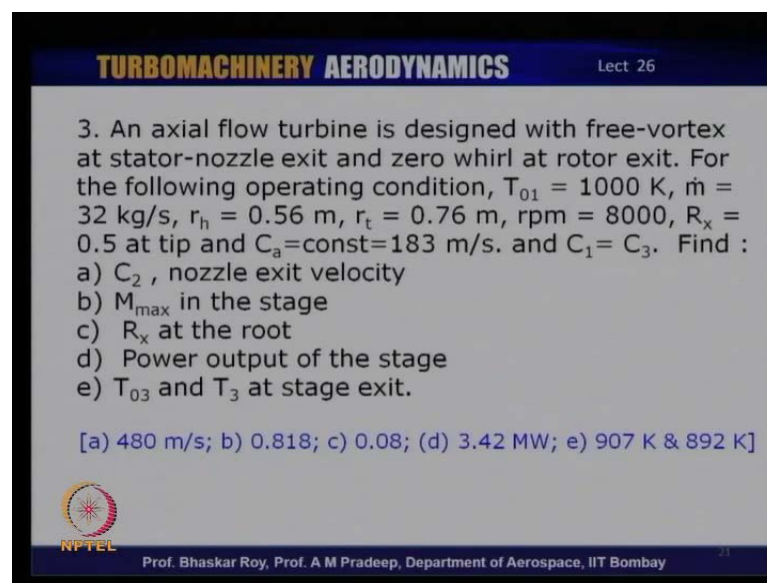
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The second exercise problem that is given is the gas exits from the turbine stator or nozzle at a radically constant angle alpha 2. So, it is a constant nozzle exit angle problem. The gas is prescribed to be in radial equilibrium. The axial velocity variation at that station is given as C a 2 in to r to the power into sine square alpha 2 equal to constant. This is what we have done in our lectures also, and for a turbine in which the axial velocity at the radius, 0.3 is a again prescribed 100 meters per second, and if the

turbine has stated above is designed with a constant α_2 equal to 45 degree, find the actual velocity at that station at 0.6 meters radius.

Now, the answers given here is very simple and that is 70.7 meters per second. So, it is a constant exit, stator exit angle problem and you have to apply the relations that we have done in the lecture or in the earlier problem that has been solved for you.

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3. An axial flow turbine is designed with free-vortex at stator-nozzle exit and zero whirl at rotor exit. For the following operating condition, $T_{01} = 1000$ K, $\dot{m} = 32$ kg/s, $r_h = 0.56$ m, $r_t = 0.76$ m, rpm = 8000, $R_x = 0.5$ at tip and $C_a = \text{const} = 183$ m/s. and $C_1 = C_3$. Find :

- C_2 , nozzle exit velocity
- M_{\max} in the stage
- R_x at the root
- Power output of the stage
- T_{03} and T_3 at stage exit.

[a] 480 m/s; b) 0.818; c) 0.08; (d) 3.42 MW; e) 907 K & 892 K]

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The third and the last problem that is prescribed for you to solve is an axial turbine is designed with free vortex at stator nozzle exit and 0 whirls at stator at rotor exit. So, it is a free vortex problem applied at stator and nozzle exit and a rotor inlet. So, the station between the stator and the rotor carries free vortex prescription. For the following operating condition, that is, at the entry T_{01} is equal to 1000 k mass flow is given as the thirty 2 kegs per second. The hub radius is 0.56; the tip radius is 0.76 meters; rpm prescribed is 8 thousand; the degree of reaction is point 5 and a actual velocity is constant that is 1 into 3 meters per second, and it is prescribed that the inlet and exit absolute velocities are equal to each other, that is, c_1 is equal to c_3 .

You are required to find C_2 that is the nozzle exit velocity. As you know, it expands ha hugely from C_1 to 2, and then, you are required to find the mach number, maximum mach number at the stage. In this particular stage, typically it is most likely to be a

somewhere in the nozzle exit, and then, the reaction at the root, the power output of this particular working turbine and t_0 and t_3 at that stage exit. So, these are the numerical values that you need to find out of this prescribed problem.

The answers solutions that are given here is that the nozzle exit velocity C_2 is would be for in 80 meter per second. The maximum mach number in this stage is so solved as point 0.818. The reaction at the root is 0.08, and you remember at the root, it is quite often specially free, **free**, vortex. It is likely to be very close to 0 and 0 of course would mean an impulse turbine, and we are looking at a problem in which, the solution actually comes pretty close to giving you an impulse station, impulse section at the root of this particular turbine, and then, the power output are work done for this particular rotor given the mass flow is 3.42 mega watts, and the temperatures at the exit t_3 is 907 and the static temperature t_3 is 8 892 k.

Now, you can sit down and try to solve this problem and see whether you can come. You can standard values of ah gas constant r , that is, 247 joules per kg k and value of C_p as thousand 147 joules per kg k. So, you can use those standard values to solve this problem in which, numerical are given and you are required to find the certain prescribed velocities, mach numbers, work done and the exit temperature. So, I leave you with these problems for you to solve yourself so that you can get a feel of the numbers the typically come out of axial flow turbine design. So, some of these problems would give you an idea how the turbine designs indeed proceeded with, and what kind of numbers you get? What kind of variations? You get a you get a feel of the numbers by solving these problems.

In the next class, we will be looking at turbine blade cooling, because in this class and in the earlier lecture, we had looked at the design philosophy of α_2 is constant from root to the tip and we have stated again and again that this particular design philosophy essentially caters to turbine blade cooling. In the next lecture onwards, we will devote ourselves to looking at this turbine blade cooling technology and how it impacts the turbine blade, the turbine blade shape, and essentially, the aerodynamics or the aero thermodynamics of the flow over the turbine blades is very strongly impacted, and essentially, the aerodynamics of the blade changes hugely by application of cooling technology.

We will look at various cooling technologies and how do they actually impact the turbine design of modern actual flow turbines specifically these cooling technologies are very widely used in aero engines, and we will look at some of the typical examples of these applied cooling technologies in turbine, actual turbine, rotor and stator. So, we shall be doing turbine cooling technologies from next lecture onwards.