Turbomachinery Aerodynamics

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Lecture No. #25

3D Flow in Turbine: 3D Flow Theories, Free Vortex Theories etc

We are talking about turbine aerodynamics, in the course, lecture series of turbomachinery aerodynamics, and we are now on to the area of turbine aerodynamics, in which we can now, look at certain three-dimensional aspects of flow through axial flow turbines. We have done some of the area that is related to the two-dimensional flow, which normally is also called a cascade flow, as we have done in case of axial flow compressors. In case of axial flow turbines, some of the features, as we have seen, are somewhat similar to the axial flow compressors, but there are many areas that are quite different from that of axial flow compressors.

When we come to the three-dimensional flow, it is somewhat similar, that there are certain areas that have some overlap or similarity with axial flow compressors. On the other hand, there are quite a few areas and considerations that are different in case of axial flow turbines. So, today's lecture, we will be looking at three-dimensional flow in axial flow turbines.

Now, in three-dimensional flow, the axial flow turbine behaves in a manner that is somewhat different from the two-dimensional theory, which you have done in somewhat great detail in the earlier lectures, in this lecture series. The three-dimensional flow, as the name suggests, that the flow has acquired a third dimension in its motion and this third dimension is the radial flow. Normally, we assume that the flow is two-dimensional, which means, it has axial motion as well as tangential motion, which is imparted by the rotating blades. And hence, it is a 2-component flow, which is what was assumed in two-dimensional flow analysis. The moment we get into a three-dimensional flow, the third dimension, that is the radial component of the flow, becomes an important issue.

So, many of the theories that have been developed earlier on, were developed with the assumption, that the flow is essentially two-dimensional nature. So, when the three-dimensionality of the flow had to be factored in, the assumptions made were the three-dimensionality could be factored in a pseudo-three-dimensional manner; that means, when the blades or the airfoils are indeed stacked from root to the tip of a blade, the root to the tip stacking is done with the help of firstly getting the two-dimensional airfoil sections at each section, at each radial section, and then stack them up one on top of another from the root to the tip.

Now, this two-dimensional airfoil, fundamentally the airfoils are two-dimensional entities aerodynamic entities. So, when you stack them up in a three-dimensional manner, you get a 3D blade shape, which to begin with, under the assumption that it follows two-dimensional flow aerodynamics. However, the modern designs and the modern analysis have shown that some of the flows do acquire certain amount of three-dimensionality; when the flow acquires three-dimensionality, the problem essentially is that, predicting the turbine performance through analysis, various kinds of computational analysis, becomes quite a lot difficult proposition in the sense, that the prediction may not match with the, you know, actual tests.

On the other hand, when it was essentially two-dimensional flow, it was known that there would be some different between prediction and test results. However, prediction was much easier and much faster. So, many of the laws governing the turbine, you know, basic turbine analysis and indeed basic turbine design, were based on two-dimensional understanding; however, they quickly figured out a way to create a blade stacking methodology, by which three-dimensional turbine blades could be created. And hence, a certain three-dimensionality of the turbine blade design was brought into the picture or brought into the methodology.

In today's lecture, we will be we talking about some of these methodologies, where three-dimensionality of the flow has been attempted to be built into the turbine design methodology. And, we will say, that in what respect, certain amount of three-dimensionality of the flow could be avoided by design; you know, a large amount of three-dimensionality can be indeed avoided by design, and if it is a little bit is there, it would have to be found out through a numerical analysis, mostly simulation in terms of computational flow dynamics.

So, most of the turbine design laws, indeed are essentially pseudo-three-dimensional, in the sense, they give you a method by which two-dimensional blades or airfoils could be stacked from a root to the tip or quite often from mid-section design, which is normally done first, and then stacking downwards to the root and upwards to the tip.

So, in today's class we will be looking at some of these aspects which are essentially factoring in a certain amount of three-dimensionality of the flow and then, of course, telling us, how to create a three-dimensional axial flow turbine blades, for applications in modern gas turbines. So, some of the first assumptions that we would probably like to look at are related to the three-dimensionality.

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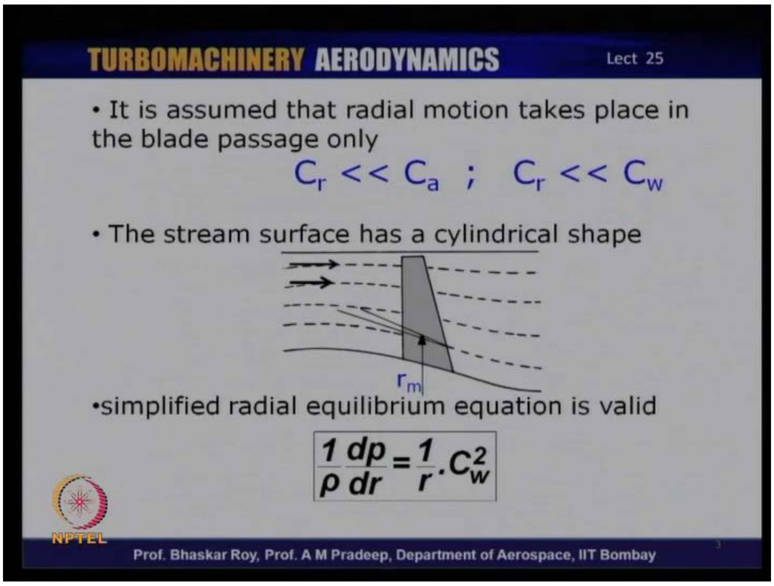

One of the first assumptions that we would like to take a look at, is the fact that the radial component of the flow is prima facie, very small, in relation to the axial component and the whirl component C a and C w. We would like to keep it that way by design. So, the designers attempt to create blades which would promote flow over the blade surfaces and through the blades, which have very small radial component C r; if it is in a rotor, we may call it v r; however, the radial component would be rather small compared to the whirl component.

Now, if you look at the picture just below, it shows that the flow coming into the turbine could have a situation where the flow actually acquires three-dimensionality because of the shape of the annular passage. The annular passage that goes through the turbine, depending on the turbine pressure drop a pressure ratio, would have to open up, and in the process of opening it up, it creates a divergence passage, as opposed to a converging passage in axial flow compressor. And then this diverging passage, you know, automatically promotes a certain amount of flow which is known axial; that means, it acquires a certain amount of three-dimensionality.

For example, flow coming in here, into the turbine, is most likely to be actual in nature; let us say, in the first stage of the turbine. And then, as it goes to the first stage which promotes very high pressure ratio, it has to open up. So, it has a large area of flow which has to be given as the density has fallen, and then it goes through a meridianal path, which, let us say, has a radius of curvature of r m. So, this meridianal path through which it goes, then automatically brings in radial component of the flow.

So, if you take the meridianal path of the flow at any point on the meridianal path, the flow would automatically would have an axial component; if it is passing through a rotor blade or edges parts of the rotor blade, it will have acquired a whirl component as shown here; and then, because of the nature of this meridianal path, it will acquire a certain amount of radial component. So, because of the geometry of the flow track, sometimes the flow acquires a radial component and that is inevitable because of the aero thermodynamics of the flow, as it is going through the axial flow turbine.

What people would like to do, is to ensure that this radial component is as small as possible, as is given out in the first assumption; and to do that, one may probably invoke this simple radial equilibrium equation that we have done in case of axial flow compressors.

So, this radial equilibrium equation, then simply equates the pressure, static pressure gradient along the radial direction with the centrifugal action that the flow is experiencing due to its whirl component or rotational component. And then, this balance of static forces on the left hand side and the dynamic forces on the right hand side, which is the centrifugal force, gives us equilibrium of forces. And, this equilibrium of forces, if it is adhered to or invoked in the design of the axial flow turbines, then each of those passages would have a radial balance of forces, and then it will be only due to this opening up of the passage; which means, near the tips, for example, in this particular shape, it would be quite flat. However, near the hub, it is entirely possible that it would be acquiring certain amount of radial components.

So, the designers, by design, quite often try to balance the forces the statics and the dynamics, and as a result of which, try to minimize radial flow component to the minimum value, so that, one can say that it is very small compared to axial and the lateral of the whirl component of the flow.

Now, this is something which the designers often would like to do by design, at the time of designing of the axial flow turbines, if you can do that properly. When the turbine is actually operating, it would be more or less, stay very close to this design, assumption or what is being invoked by design. Under certain of design operating condition, it is possible, that the turbine would indeed acquire a certain unbalance of forces, and a little more of the radial flow may come out or come up in the actual operation of the flow dynamics or the aerodynamics, and in which case, those are the things that would need to be found out through more intense analysis which is essentially a computational simulation.

So, some of those things would have to be found out through intense computational analysis before the turbine is, turbine design is finalized. So, some of those things we will be talking about when we talk about computational flow dynamics towards the end of this lecture series.

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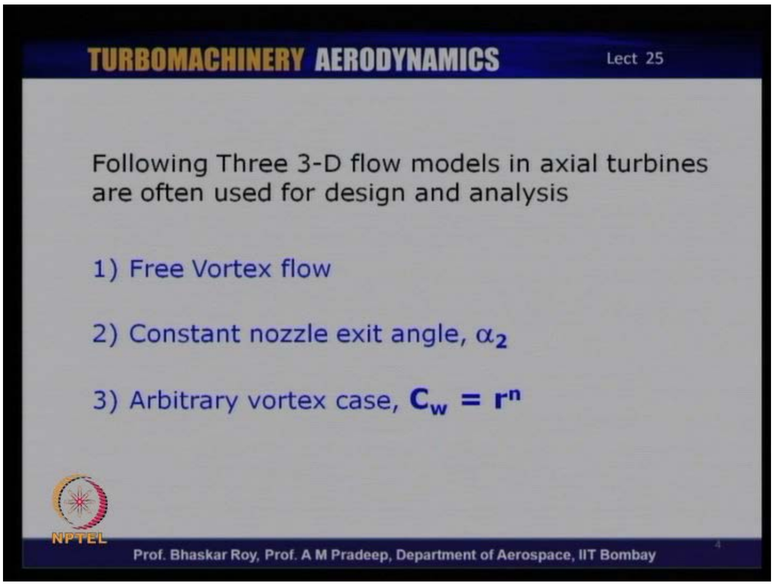

So, having invoked a simple radial equilibrium equation, the next thing to do is, you know, try to find out what are the 3D models or as I mentioned, they are more of pseudo 3D models, flow models, which you need to be invoked for design, and then immediate post design analysis.

So, the design and the post design analysis would create the turbine or predict the turbine performance and its fundamental characteristics, which would, as I mentioned, indeed you on would have to be checked or validated first with intense computational fluid dynamics, three-dimensional analysis and later on, much later, when the design is more or less final, through rig testing. So, those are time consuming and indeed rather costly business. We will look at the design features of modern axial flow turbines and the same features are used in immediate post design analysis.

So, some of the features that we would be invoking, our designers do invoke these days; first, of course, is the free vortex law. Now, free vortex law is something we have done in some detail with reference to axial flow compressors. So, it is essentially as you know, gives us a very simple, handy relation, which tells us how the whirl component should vary from root to the tip of a blade, especially of a rotor blade, where the flow otherwise is an unknown quantity.

If we do not invoke a free vortex or any such law, what is happening through the rotor, essentially would be unknown or untraceable. So, a vortex law is absolutely essential in tracking the three-dimensionality of the flow through the rotor. So, free vortex is the simplest thing that can be done and we have done in case of axial flow compressor in some detail.

The one that is used most in modern axial flow turbine is indeed not really the free vortex law; in modern axial flow turbine, there are different laws that govern the design and design related immediate analysis. And, one of them is our simply known as the constant nozzle, exit angle; alpha 2 as we have used in annotations earlier, coming out of the stator and the flow coming out of the stator would then be coming out at constant alpha 2 from root to the tip of a blade. So, it is going into the rotor with a constant flow angle from along the length of the blade, from root to the tip.

Now, this is to be invoked; this is not going to happen naturally, it does not happen naturally; it needs to be invoked or imposed on the design, which means, the blade shaped with created accordingly. So, you would have a completely different blade shape if you create it with free vortex law and if you create it with constant exit angle feature. So, they are two different design loss and indeed they would create completely two different looking turbine blade shapes; both stator as well as rotor.

The third case that we will be talking about today is a relaxation of the free vortex, which is called arbitrary vortex case or arbitrary vortex law, which is simply C w into r to the power n. Now, n is a variable and we will be talking about that also in today's lecture. So, we will talking about these three possibilities as design laws for axial flow turbine, and we shall see that there are different from each other, and they would indeed give completely different kind of turbine blade shapes. So, depending on what kind of blade you are designing, you need to invoke accordingly the design law.

The early turbine design, very early turbine designs where indeed made with the help of free vortex design, which was known to everybody more than fifty years back; and as a result in booking those design laws for turbines, as was being done for compressors, was a very easy thing to do. And, as in case of compressors, the blades that you get in turbine with using free vortex law indeed would tend to be a rather more twisted.

Now, those twisted blade of turbine that came about, was a bit of a problem later on, because a lot of cooling technology needs to be embedded in the turbine blades, which is not there in compressor blades. And, this creating, this cooling technology inside the turbine blades which are twisted or heavily twisted is a technological problem, it is a huge big technological problem; sometimes, it is quite impossible to actually do that and of course, it increases the cost of making the turbine blades hugely.

So, as it is, the cool blade cost is hugely more than a un-cool blade; however, the cooling is something we will be talking about very shortly in this lecture series, in some detail. So, because of that one reason and the fact that the blades are made of high temperature material; the nickel alloys, the mnemonic and the Inconel, and those are high temperature, very costly material. So, twisted blade was something that also produces high stress levels. So, turbine is a very heated area, the entire blade is highly, you know, heated up through high temperature gases and then a twisted blade creates lot of stresses due to the temperature gradient along the length of the blade, from root to tip, as well as from leading edge to trailing edge.

So, that kind of an environment for turbines, tells us that, somewhat simple blade shape is probably a better choice rather than somewhat twisted, complicated blade shape that we have seen in case of compressors. So, in compressors, they are okay, those complicated blade shapes; but if you try to use them in turbine, you get into all kinds of problems, which are other than aerodynamic problems. And in fact, in case of turbine blade design, those are the people - the mechanical designer, the heat transfer people, indeed, often have the veto power. They can overrule the pure aerodynamic design on the basis of other considerations.

So, the aero dynamic designer would have to modify his design to accommodate or keep room for a blade cooling, as well as to ensure that the blades are not unduly stressed during its operation. Because, turbine blades suffer from huge temperature gradient, that gives rise to creep and fatigue failure.

So, those are the very strong issues, based on which turbine design is carried out. And, that is one of the reasons why, after the early era of turbine, which were un-cool, the blades are not made of, normally not made of free vortex law; they are made of the other laws. And, we will talking about all these laws into today’s lecture, one after another.

So, let us look at some of these laws.

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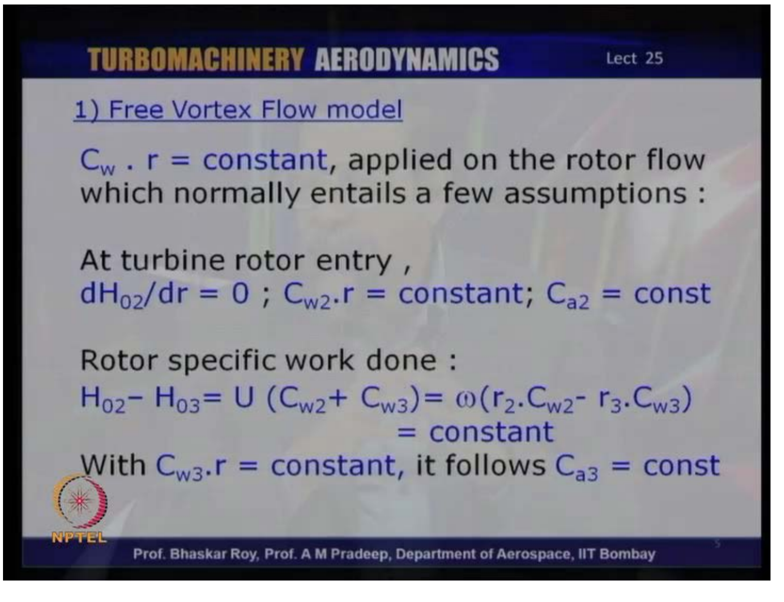

First, we take up the free vortex law, which is of course, you know, known to all of you, and it simply gives us that C w r equal to constant, and it is applied to the rotor flow which, you know, it has a few assumptions behind it. It comes with a radial equilibrium and then it comes with the assumption that the enthalpy gradient along the radius is 0; that means, it is a, it is a constant enthalpy or iso-enthalpic flow through the turbine. And then, of course, one assumes that the flow is going into the rotor; that is, C w 2 into r equal to constant. The rotor entry flow is indeed following the free vortex law; that is the important consideration, because the rotor flow is the more complex flow. And then, of course, a C w is constant from root to tip, C a 2 is constant from root to tip; that is, the axial velocity. So, these three are the regular normal assumptions, if you remember, for free vortex law.

So, if you invoke them in the turbine design, you end up getting a rotor specific work done; which is also normally part of free vortex assumption, that rotor specific work done; that is, h 02 minus h 03. Normally, we write it as U into C w 2 plus C w 3 at any station; and, that is into omega r 2 C w 2 minus r 3 C w 3 and this is constant from root to tip.

So, in the third dimension, that is, in the radial direction, now we see that enthalpy at the entry is constant, C w 2 into r is constant, C a 2 axial velocity is constant, and now we find that specific work done is also constant; that is, work done per unit mass flow is also constant from root to tip. That automatically tells us that C w 3 into r is also going to be constant from root to tip and it follows that axial velocity at the exit C a 3 is also constant from root to tip.

So, it actually, aerodynamically gives us a very simple flow situation, where a lot of things have very nicely constant from root to tip; however, it does give us a very complex blade shape, highly twisted blade shape; also in a blade shape, that in which the degree of reaction would vary substantially from root to the tip of a blade. Now, that variation of degree of reaction may have certain issues. Degree of reaction in a turbine can vary again from near 0 to very high degree of reaction, which could be 0.5 0.6; not as high as you get normally in axial flow compressors at the tip, but quite high. And, the mean, or the, or the mid radius degree of reaction for turbines is somewhat less than that of an axial flow compressors.

So, typically, that symmetrical blading that we have done in case of axial flow compressors is normally not done at the mean radius of turbine design. However, at the root, the design could go to degree of reaction of 0. Now, degree of reaction 0, as you well know, actually produces impulse turbine, which is acceptable; there is no problem with thing impulse turbine. But, once the degree of reaction close to 0, you know that the reaction component from the gas turbine is now very small.

Now, in a highly twisted free vortex design, this is a possibility that, that is very strong that some part of the blade would have very low reaction component, whereas the upper part may have reasonably good reaction component. So, the reaction blade that we have talked about in the earlier lectures, would then be actually valid for some part of the, outer part of the blade and may not really be valid or available for the inboard part of the turbine rotor blade. In which case, you know that the amount of work done by the turbine would be somewhat lesser, because you create reaction blades essentially to get more work done out of a single rotor. So, if the reaction is not available or somewhat none existent in a particular design model, you would know that the work done would be somewhat limited by the reaction availability only to the outboard portion of the blade. So, that is one of the limitations among the other limitations that we have talked about with reference to the degree of reaction, which indeed varies from root to the tip of the blade.

Of course, that brings us to the point, that you could have a constant reaction, axial turbine blading; that is entirely possible. Normally, it is not a done thing; people have designed axial flow turbines with a constant reaction from root to tip, there is nothing fundamentally wrong with that kind of design. But normally, in modern axial flow turbines, especially with the aero engines, it is not a done thing; you normally have a variable degree of reaction, constantly degree of reaction.

We are going to not do the details here, but just to mention that, it is a possibility; that does exist. And, in the past, long past, long back, people have designed axial flow turbines with constant reaction, and it works; there is no reason why it should not work. And, the free vortex design, as I just mentioned, have been used earlier, pretty widely, but it is not a use thing in many of the modern designs anymore.

So, if you get degree of free vortex design, this is what you would normally get; very similar to what we have received in axial flow compressors.

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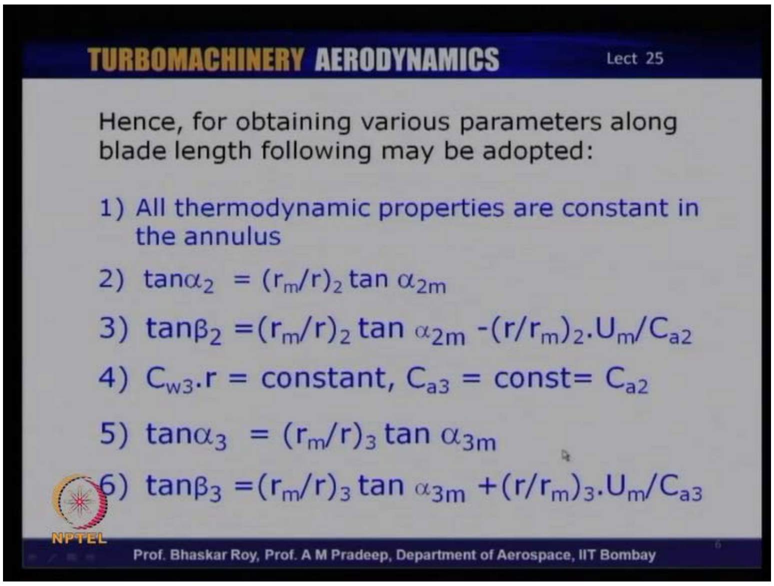

So, the free vortex design essentially gives us the thermodynamic properties are constant from root to tip, which means they are constant in the annulus, through which the flow is passing in the axial flow turbine. Remember, it is an annular flow and hence its annular flow passage over which we are now, kind of invoking, a law which promotes constancy over the entire annulus. Then, over the help of all those things, it is comparatively easy now to find out that the entry flow angle from the to the rotor; tan alpha 2 can be given at any radius can now be related to the one at the mean radius or the reference radius, simply by invoking the radius ratio.

So, everything kind of gets into the radius ratio in the free vortex designs. The tan beta 2, which is a relative flow angle going into the rotor, which is the flow angle which the rotor indeed feels, is also relatable to the radius ratio. And then, of course, the inverse of the flow coefficient to the turbine, that is U m by C a 2; C a 2 by U m, of course, would be the average flow coefficient through the particular turbine rotor.

In which case, as we have seen, C w 3 into r is constant from root to tip. C a 3 is constant, and we can also assume that we have normally used it in free vortex law; it is held constant across the rotor; that is, C a 3 is equal to C a 2. So, free vortex law actually makes the flow variables in a very simple manner, and this simplicity, of course, is the first attraction of this particular law. At the exit, a tan alpha 3, again is a, at any radial station, is relatable to the mean value tan alpha 3, which is where normally the design is done, as I mentioned earlier; and, we have done that in case of axial flow compressors also. So, that is relatable again through simple radius ratio.

And then, of course, the relative flow angle tan beta 3, which is again relatable to the radius ratio and the inverse of the flow coefficient at the exit. The rigorous designers may find a these two values different, but a simple design would tell us that C a 2 is equal to C a 3. So, the flow coefficient at the entry and the flow coefficient at the exit are indeed equal to one another. However, as we have seen that the length of the blade at the leading edge and at the trailing edge could be different. So, the values of the variation of this could be different at the exit of the rotor, compared to that at the entry to the rotor

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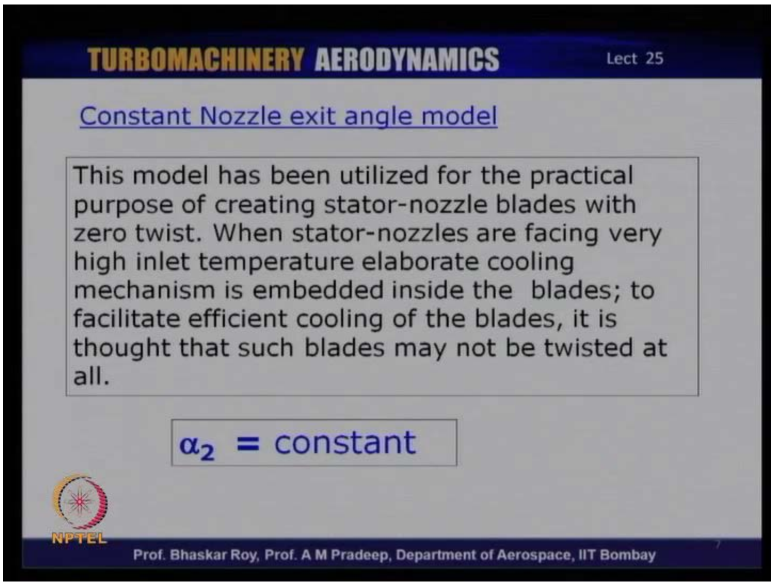

If you look at the drawing here, it tells you very clearly that at the exit side, the radial variation is far more than at the leading edge or the entry of the axial flow turbine rotor. So, those are the simple things that we can get out of a free vortex design. And as I mentioned, it does give us a very simple flow feature; however, it does create a complex, somewhat twisted and more complex a blade shape.

The other blade shapes which people then resorted to, is simply called the constant nozzle exit angle model. Now, this model has been essentially created for the more practical purpose of accommodating cooling. It creates a nozzle or stator blades with 0 twists, so that, the flow comes in at alpha 1, goes out at alpha 2, saying from root to the tip of the blade.

So, this untwisted stator nozzle is the first thing that we wanted and one of the reasons is, the stator nozzles face very high inlet temperature coming from the combustion chamber; and, it normally, for a long, long time now, at least, last 40, 50 years. The first stator nozzle is embedded with very elaborate cooling mechanism. To have all that cooling mechanism inside the blade requires that the blades blade itself does not have complex shape or a twist or a large twist.

So, the turbine blade designers, very quickly, decided to adopt this particular design philosophy to create untwisted stator blade, so that the cooling can be efficient. Because, for the last 50 years, the advancement of turbine design has been more through the cooling technique and by increasing the turbine entry temperature to get more work out of turbine, rather than creating more and more complex aerodynamic shapes. So, in some sense, the aerodynamics of the flow may have been slightly compromised to create an elaborate cooling mechanism.

So, what we get is, alpha 2 is constant from root to the tip of the blade; if alpha 1 is also constant from root to the tip of the blade, which is sometimes true, especially in the first stage of the HP turbine, then you indeed have a untwisted blade. If alpha 1 is some variable in the later stages, you have a mildly twisted blade, which can stay, accommodate or have embedded cooling mechanism.

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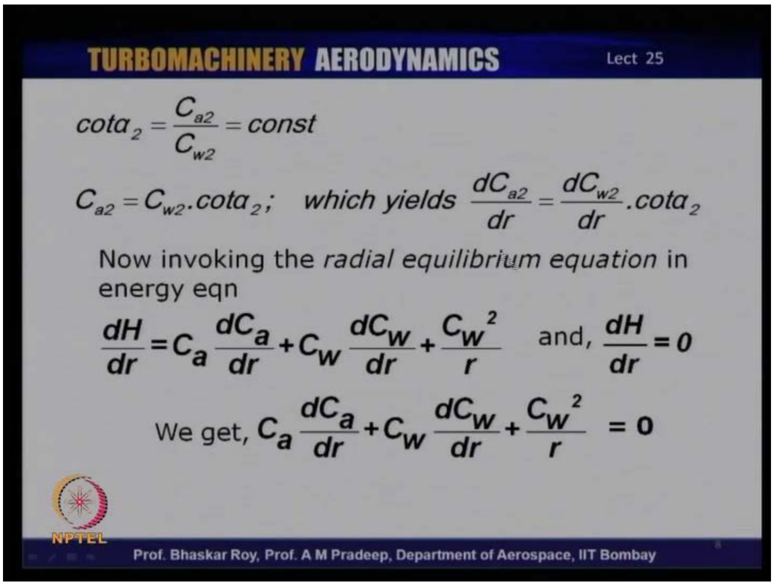

So, with this simple invocation of the constant exit angle, if we move forward, what we get is a simple mathematical model in which now cot alpha 2 which is equal to C a 2 by C w 2, the ratio of axial and whirl component; now, that is a constant from root to the tip of the blade.

Now, as we can see, C a 2 would be equal to C w 2 into cot alpha 2; if you take a simple differentiation of that, that yields that d C a 2 d r will be equal to d C w 2 d r cot alpha 2. And, if we use this in the energy equation, if we import the energy equation that we have used in creating the radial equilibrium; in case of compressors, if you remember, it was and can be invoke it here or bring it back here again, d h d r that is the enthalpy variation in the radial direction a C a into d C a d r plus C w into d C w d r plus a C w square by r, and the last term, of course, comes from the radial equilibrium. And then, if we invoke d H d r equal to 0; that means, enthalpy variation in the radial direction is 0, that is enthalpy is constant, it is iso-enthalpic flow over the entire annulus; if that is so, then the right hand side goes to 0 and on the left hand side, we have C a to C a d r plus C w d C w d r plus C w square by r, and that would then be 0.

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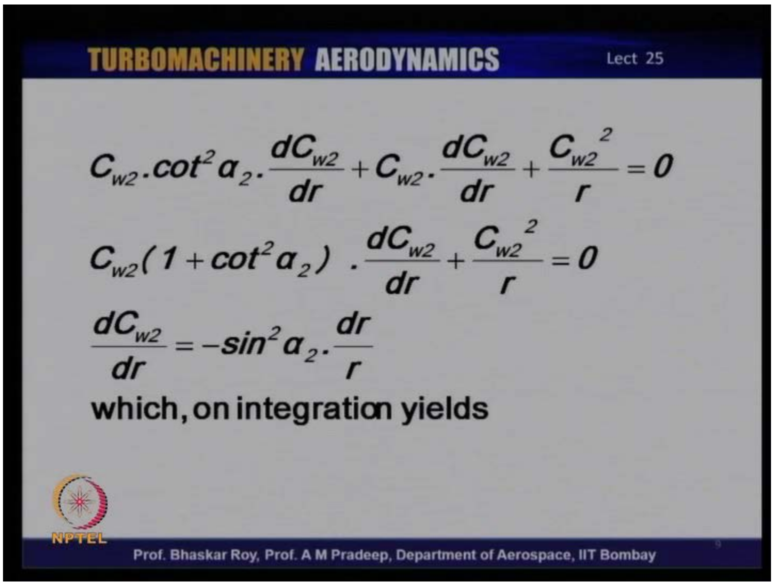

So, energy equation is brought in here. If we now, substitute this in this energy equation, we get C w 2 into cot square alpha 2 into d C w 2 d r; invoke it at the entry to the rotor. We are invoking the energy law at the entry to the rotor, as we have done in case of compressors, and then we get with this design law of a turbines; we get a situation that C w 2 to d C w d r and C w 2 square by r, and that is equal to 0.

So, rewriting that, we get C w 2 into 1 plus cot square alpha 2 into d C w 2 d r plus C w 2 square by r equal to 0, and this yields 1 plus cot alpha square is sin square alpha 2 and then, you will get d C w d r equal to minus sin square alpha 2 d r by r.

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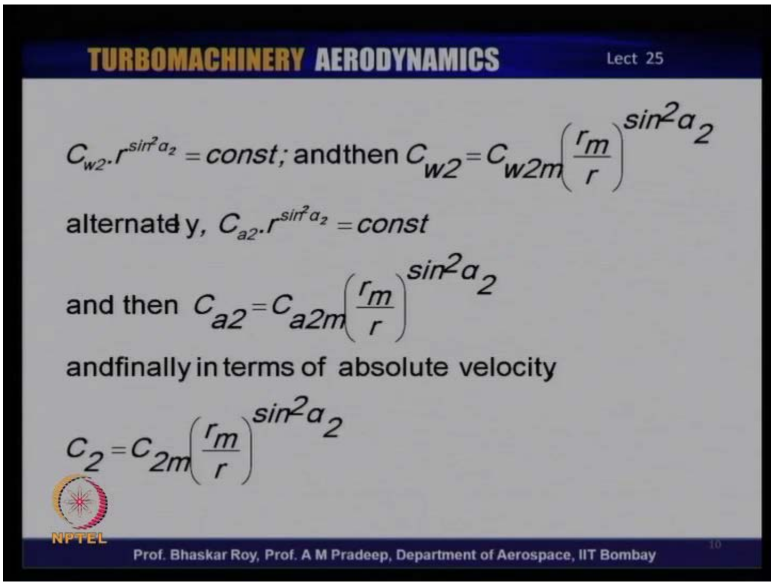

Now, this on integration, this is derived equation from the energy equation by invoking the constant exit angle law, turbine design law. And, having arrived at here, if we simply carry out the integration, it gives us C w 2 into r to the power sin square alpha 2 equal to constant. Now, this you can see is a different equation, very different from the one we had for free vortex law. And then, of course, we get C w 2 into C w 2 m; that is related to the mean radius or the mid radius of a turbine, with the radius ratio into sin square alpha 2. So, the variation of the parameters now is beginning to look quite different from that we had got in case of free vortex design. And then, of course, the axial velocity variation also falls in line in the same manner and hence we can write, C a 2 into r to the power sine square alpha 2, and that would be equal to constant from root to tip.

So, the variation in the radial direction is now governed by sine square alpha 2 that is the index. And then, we can get C a 2 variation along the radius, similar to C w 2 as C a 2 m into r m by r to the power sine square alpha 2. And, in terms of the absolute velocity C 2, we can then write down C 2 would be C 2 into C 2 m to the into r m by r to the power sin square alpha 2. So, all of them now use this alpha 2 as a parameter, which is invoked in case of this present design law.

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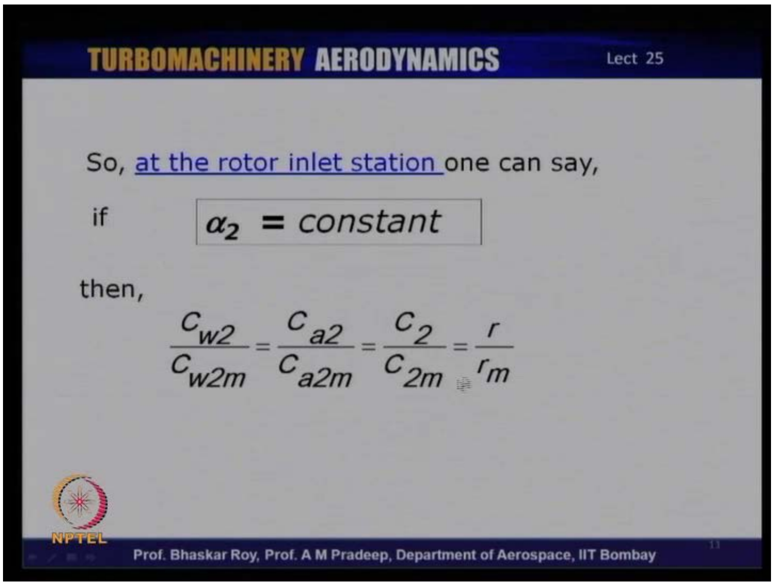

So, at the rotor inlet station, we have put alpha 2 equal to constant and hence C w 2 by C w 2 m is equal to C a 2 by C a 2 m is equal to C 2 by C 2 m and all of them are equal to r by r m. So, they are all related to the radius ratio; all velocity components are directly related to the radius ratio.

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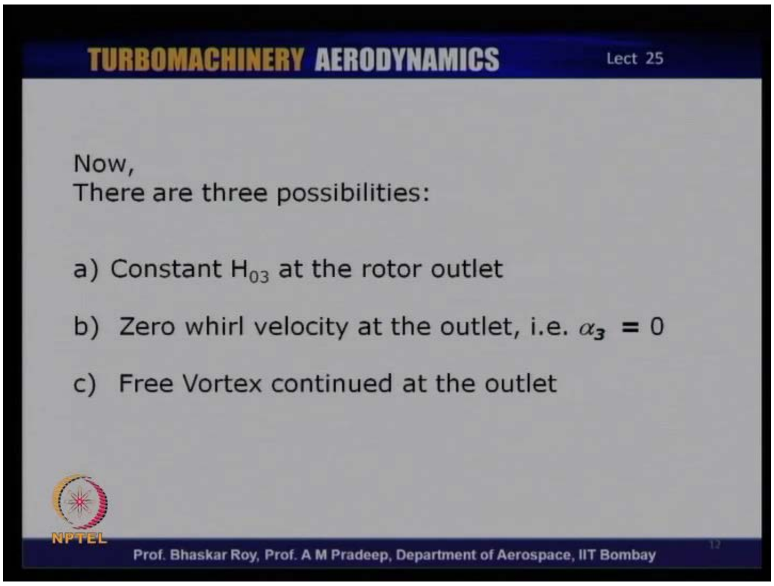

Now, this constant exit angle from the stator nozzle has still three possibilities, additional three possibilities. The additional possibilities are, you can have constant H 03 at the rotor outlet; that is, enthalpy is constant at the rotor outlet, as we had indeed got in case of free vortex law or you can have 0 whirl component at the rotor outlet; that means, alpha 3 equal to 0, indeed C w 3 would be equal to 0. And then, we could have free vortex again brought back, continued at the rotor outlet. So, it is possible to look at now, what would be the flow at the rotor outlet from three different possibilities.

So, let us look at the three possibilities.

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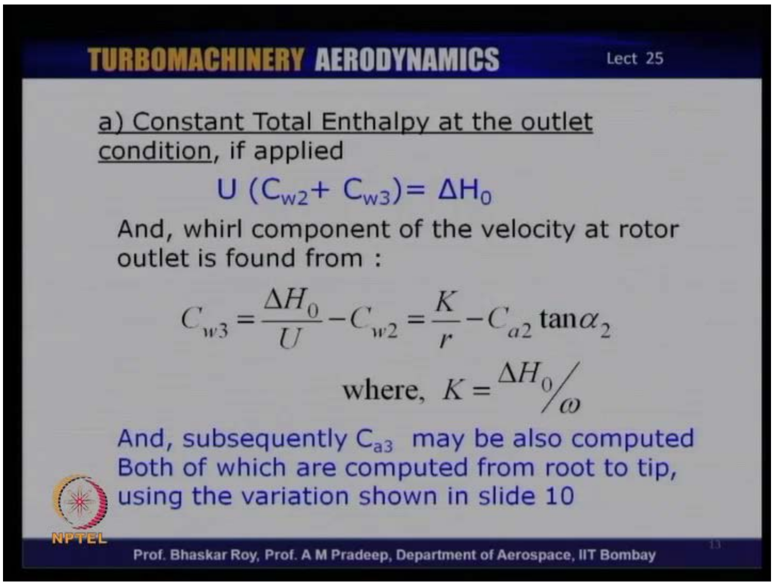

If you have constant total enthalpy at the outlet, U is equal to C w 2 plus C w 3, that is equal to delta H 0, that is the work done through the turbines or work being created by the turbine through the hot gas passing and the whirl component of the velocity at the rotor outlet is indeed found form from C w 3 is equal to delta H 0 by U minus C w 2. Now, the first time here, can be written in terms of K by r, K being delta H 0 by omega; omega is the angular velocity of the rotation of the turbine rotor.

Now, rotor is the only one which is doing work; stator, if you remember, it does not do any work. So, the angular velocity of the work done of the rotor and then that ratio can be taken to be some kind of a constant value for design purposes. Now, you can also find C a 3, which can be computed; once you get C w 3, you can sit down and do a little bit of a vector analysis, very quick velocity diagram, with the velocity diagrams, and you can find what would be the value of axial velocity at the rotor outlet, which could be again take a form out of this. Now, both of which can be then computed from root to the tip of the blade.

Now, using the kind of variation that we have seen in the earlier slides, so, it is now possible to create C w 3 and C a 3 at the rotor outlet, with the help of this particular assumption.

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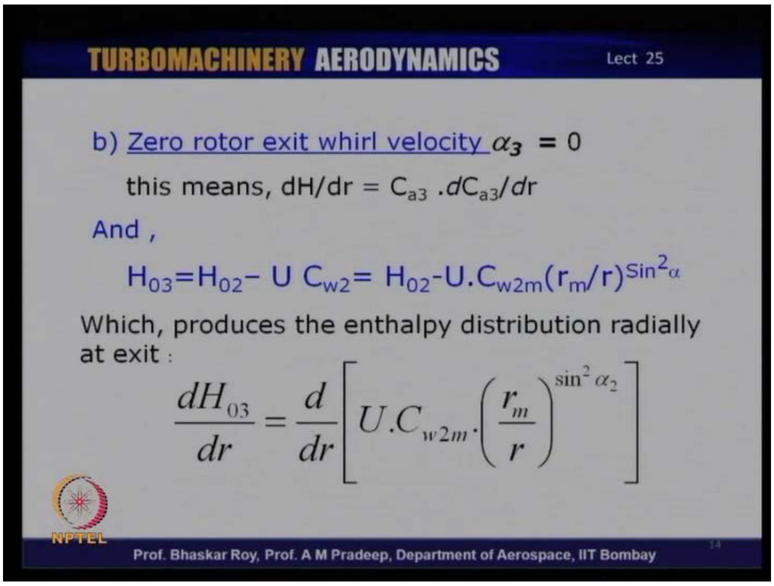

If we go for the next assumption, that is the whirl component is 0 at the rotor outlet, that is alpha 3 is 0; this means, that d H d r would be equal to C a 3 into d C a 3 d r. And then, one can write down C 0 3 is equal to H 0 2 minus U into C w 2; and, a C w 2 can be now written down in terms of C w 2, at any station, any radial station, can be written down in terms of the mean radial station as we have done before; I substituted with C w 2 m into r m by r to the power sin square alpha 2. Now, this produces the enthalpy distribution radially at the exit as d H d r equal to d d r into U into C w 2 m into r m by r to the power sin square alpha 2.

So, this is how you get the enthalpy variation radially; the earlier one which we did just did, was that the enthalpy was constant; that means, the radiation would be 0. Now, we find that there is enthalpy variation along the length of the blade.

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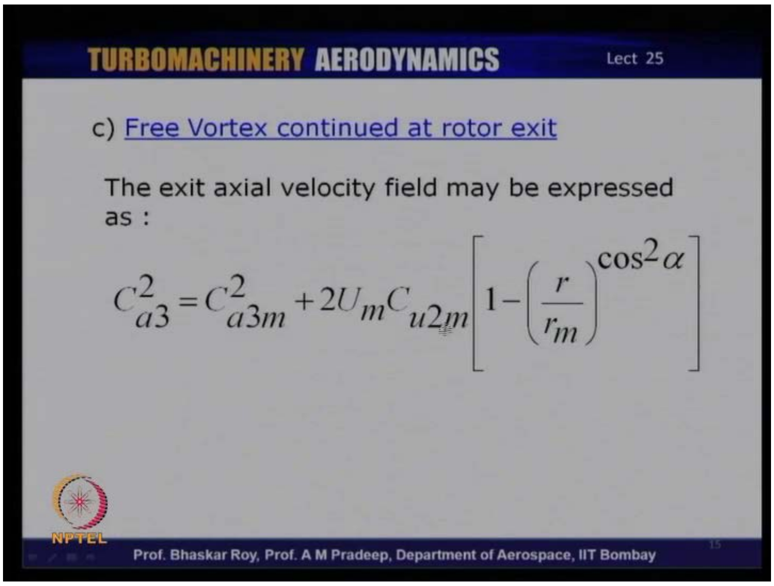

The third condition that we can invoke is the free vortex law, which is to be applied at the rotor exit. Now, free vortex law is somewhat we are familiar with. And, if you do that, invoking the free vortex laws that we have done in the earlier slides, if you bring them over here and simply apply them at the rotor exit, you would get a C a variation; C a which can be written down or expressed in terms of C a 3 square equal to C a 3 medial square plus twice u m into a C w 2 m into 1 plus r by r m to the power cos square alpha 2.

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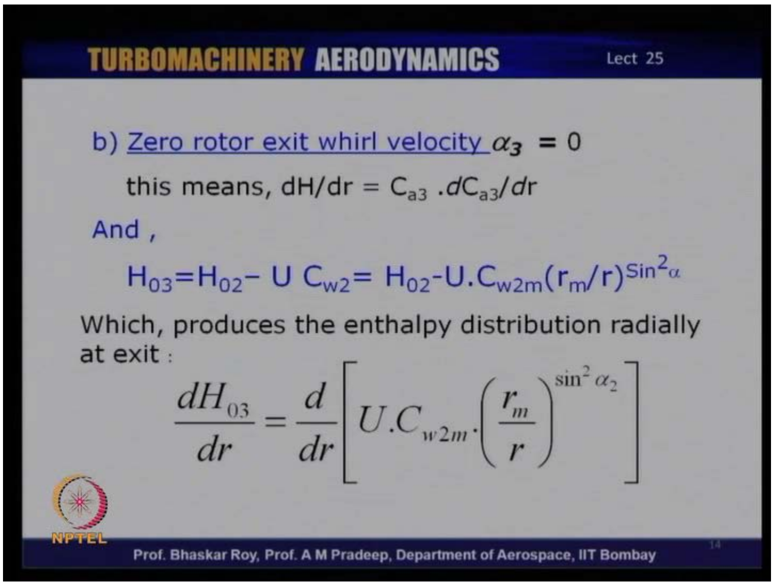

Now, this expression would also be valid for this particular case; which means, the C a variation that you get in case b, would be very similar to that you get in case 3. So, axial velocity variation can also be found by using the laws that we have prescribed.

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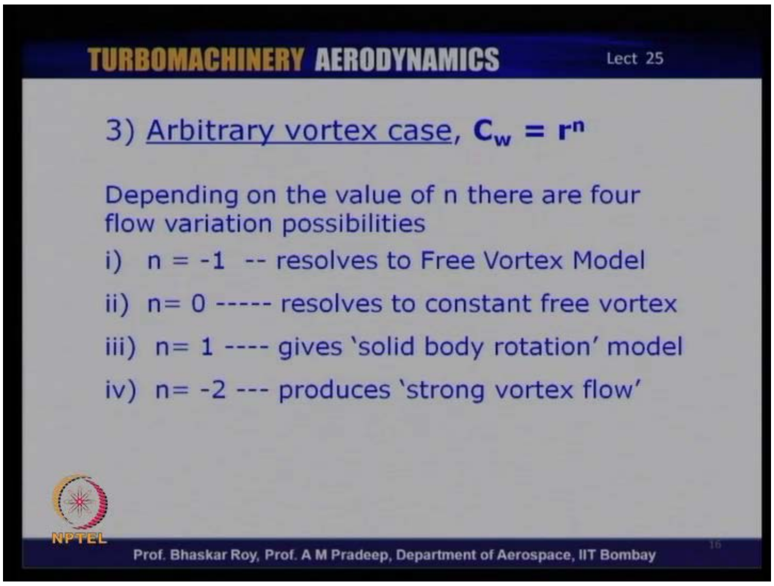

The third possibility that we have talked about is the arbitrary free vortex case or the relaxed free vortex case, in which C w is equal to r to the power n; instead of writing C w 2 into r equal to constant, we are saying that C w has an equality to r to the power n. Now, this value of n is, can be now varied. Now, if you put n equal to minus 1, it actually, if you remember, resolves to free vortex model. So, that something is which we have already done before. The other one which, if you put n equal to 0, it resolves to constant vortex model or constant; it is not necessarily constant reaction, it is a constant vortex law; not constant free vortex, but constant vortex from a root to the tip of the blade.

On the other hand, if you put n equal to 1, it gives what is simply known as solid body rotation; that means, the flow would be simply directly proportional to r; like, for example, u is omega r, so all other parameters would also be directly proportional to r, rather than being inversely proportional, as in case of free vortex laws. So, in some sense, this is the inverse of free vortex model.

The third possibility is where you have n equal to minus 2; in case of compressors, we had seen it produces somewhat different kind of a blade loading. It is same in case of turbine, it produces a different kind of blade loading and indeed, it produces vortex strength that is different, most likely higher vortex strength at the various sections of the blade, if you invoke this law.

Now, what the modern designers are doing, turbine designers, as in case of axial flow compressor designers, they invoke these laws in various parts of the blade. So, you do not have the whole blade; actually, a design as per any one of these laws, various length of the blade is designed as per one of these laws. So, you can have one kind of law in one part of the blade and other kind of law in other part of the blade, and then, of course, the blade would have to blend into a smooth shape through geometrical modeling and aerodynamic analysis. So, modern designers often invoke more than one law in creation of one single turbine blade rotor. So, this is how the design indeed proceeds in the modern axial flow turbine blade creation.

So, we have gone through various design models, the design models essentially are, what I mentioned, pseudo three-dimensional flow models in the sense, they tried very assiduously, very consciously and deliberately to avoid creating the radial flow model, a radial flow complaint. So, the radial component is very deliberately tried to be avoided in the design, so that, radial flow is not created through aerodynamic pressure gradient or aerodynamic laws; however, if the geometry of the flow, somehow brings in certain amount of a radial flow component and there is nothing much, you know, you can do about that. And we have seen in the earlier lecture, that some of the blades indeed do have strong radial component due to the variation of the annular flow track, which is curvilinear and diverging flow track.

So, there is nothing much you can do there, but the blade designer tries to ensure that radial flow is not created by the operation of the turbine blade rotors. So, this is how the design is normally proceeded with in the modern axial flow turbine. And so, in today's lecture, we have talked about the design loss. In a lecture later on, we may have a look at some of the design features, design steps and indeed some of the aerofoil sections or the blade sections; I do not know whether you can really call the aerofoils, but the blade sections that are used in axial flow turbine design.

In the next lecture, we will be actually looking at some of these 3D or pseudo-3D or quasi-3D flow theories that we have done today and try to use them in solving a few very simple standard problems. So, we shall have a quick understanding of the numericals, of how this design laws actually come up with numbers in case of axial flow turbines.

So, next class will be a problem solving class. So, I will bring a couple of problems for you, for you, to have a look at solved problems and then I will leave you with a few problems for you to solve by yourselves. So, in the next class, we will be doing problems on axial flow turbines using three dimensional flows, laws that we have done in today's lecture.