Turbomachinery Aerodynamics

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Lecture No. # 23

Tutorial - 3:

Axial Flow Turbines

Hello and welcome to lecture number 23 of this lecture series on turbo machinery aerodynamics. We have been talking about actual turbines in the last several lectures, and we had a chance to discuss about different aspects of axial turbines, starting from very fundamental thermodynamics of axial turbines, and moving towards two-dimensional cascade analysis, and also the 3D analysis, which we are going to do subsequently. So, based on what we have discussed so far, on the basic thermodynamics and the two-dimensional analysis; as I mentioned in the last class, it is time that we, have a tutorial session on understanding of axial turbines, and how to solve problems, which are related to axial flow turbines, but before I go into the tutorial, let me just quickly recap, what we have discussed in the last several lectures.

We had an introductory lecture on axial turbines, where we discussed about different types of axial turbines, like well different types of turbines in general, like the axial, the radial and the mixed flow turbines; of which the axial turbine happens to be the one which is the most commonly used in, especially in aero engine applications and also in marine as well as land based power plant applications; for a variety of reason, basically to do with the efficiency and and the convenience in arranging axial turbines, as compared to the other counter parts like the radial or the mixed flow turbines.

Now, when we talk about the axial turbines alone, we can further classify axial turbines based on the nature of the flow through the turbine itself. So, based on this classification, we could have either an impulse turbine or one could have reaction turbine; and so, the turbine axial turbine may operate in either of these modes, either an impulse turbine, where in the entire pressure drop takes place just in the nozzle; and there is no pressure drop taking place in the rotor, the rotor simply deflects the flow and does not contribute in any way in the pressure drop that is taking place in the turbine. So, this is true for an impulse turbine.

And the reaction turbine is one, where the pressure drop is actually shared between both the nozzle as well as the rotor. So, part of the pressure drop takes place in the nozzle and the remaining part of that takes place in the rotor. And so, we have defined what is known as degree of reaction and how we can calculate degree of reaction. We also had a session on, understanding the velocity triangles, which are applicable for an axial turbine and also these different types of axial turbine like the impulse or the reaction type of axial turbine. So, where in we discussed about, how we can construct a velocity triangle starting from the fundamental principle.

And then, we discussed about the various losses, which occur in axial turbine and how some of these losses can be quantified; one may have two-dimensional losses or one may have 3D losses like the secondary flows and deep leakage losses and so on. We had a rather detail discussion on these losses in our discussion on axial compressors. And I think, I mentioned that one could simply extend the discussion, which was applicable for axial compressors also for an axial turbine, so which is why we restricted our discussion on losses to the bare minimum, because it is already been covered.

We also defined different mechanisms or methods of calculating efficiency for axial turbines. There are two commonly used efficiency definitions; the total to static efficiency and total to total efficiency; each of them are used primarily, according to the applications, for which the turbine is being used; for example, if if it is a land based gas turbine engine, which is not expected to produce any nozzle thrust, one would like to expand the flow to the maximum extent possible without too much of kinetic energy leaving the turbine. In this case, we would define the total to static efficiency; we would want the flow to have or to reach the static conditions without much dynamic head as it leaves the turbine.

Now, in an aero engine kind of application, where the turbine is meant only to drive the compressor and some accessories; one would still want from kinetic energy available at the turbine exhaust, which can be further expanded in in a nozzle and therefore, in such applications, we would define the total to total efficiency. We have discussed some of these topics in the last several lectures, and we also of course, discussed about the performance characteristics of axial turbines very important. We have discussed how what it is significance is in relation to the engine as a whole and the significance of matching of turbine with compressors as well as matching of turbine with the nozzle.

So, these were some of the topics that we had discussed in the last few lectures; and in today's lecture, which is basically a tutorial session; what we going to do is, to try and solve a few problems. And use our understanding of the working of the axial turbines, and put that to practice in terms of solving problems. So, what I have for you today is a set of four problems on axial turbines, which I will solve for you; and after this, I would also give you a few exercise problems, which I would like you to go ahead and solve based on our discussion today, as well as our understanding of these concepts in the last few lectures. So, this is, what we are going to discuss in today's class. We basically be having tutorial session on axial turbines.

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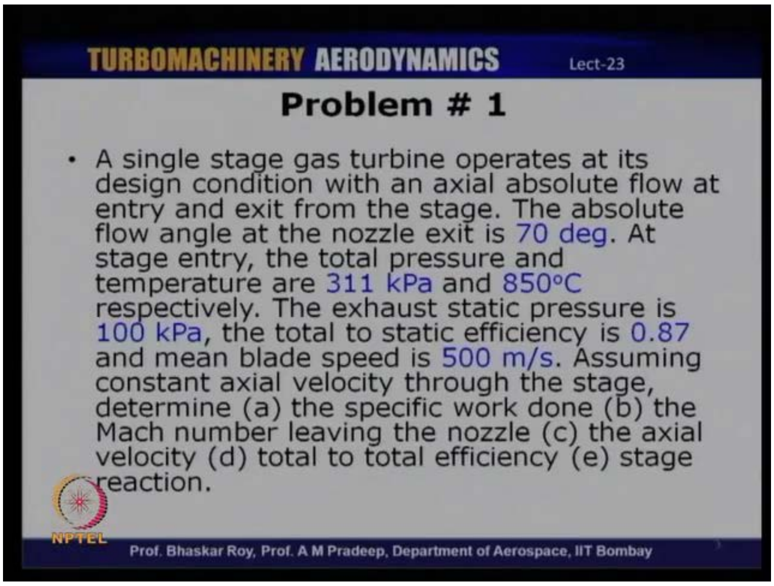

TURBOMACHINERY AERODYNAMICS Lect-23

Problem # 1

- A single stage gas turbine operates at its design condition with an axial absolute flow at entry and exit from the stage. The absolute flow angle at the nozzle exit is 70 deg. At stage entry, the total pressure and temperature are 311 kPa and 850°C respectively. The exhaust static pressure is 100 kPa, the total to static efficiency is 0.87 and mean blade speed is 500 m/s. Assuming constant axial velocity through the stage, determine (a) the specific work done (b) the Mach number leaving the nozzle (c) the axial velocity (d) total to total efficiency (e) stage reaction.

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So, let us go to the the first problem. So, the first problem statement is the following, its states that a single stage gas turbine operates at its design condition with an axial absolute flow at entry and exit from the stage. The absolute flow angle at the nozzle exit is 70 degrees, and at the stage entry, the total pressure and temperature are 311 kilo Pascal and 850 degree celsius respectively; the exhaust static pressure is 100 kilo pascal; the total to static efficiency is 0.87 and the mean blade speed is 500 meters per second.

Assuming constant axial velocity through the stage, determine part (a) the specific work done, part (b) the Mach number leaving the nozzle, and part (c) the axial velocity, part (d) the total to total efficiency, and last part is to find the stage reaction. So, this problem statement is to do with single stage gas turbine, which is operating under certain design conditions, the inlet stagnation pressure and temperature at given the exhaust static pressure is given, the total to static efficiency and the mean blade speed are given, based on this data we are required to calculate a variety of other parameters and solve this particular problem.

Now, as we have done in the past the with reference to turbo machines, the very starting point of solving a problem if you recall, when we had discussed about compressors, is to get the velocity triangles right. So, the first point is to draw the velocity triangles; and that is the starting point of solving any a such problems associated with turbo machines. So, let us construct a generic velocity triangle, here we are not given, whether it is a reaction turbine or an impulse turbine nothing is mentioned. So let us construct a generic velocity of triangle like what we had done a few lectures earlier on; and then from there we will see what are the parameters, which have been specified, and what is that we need to calculate to be able to solve this problem.

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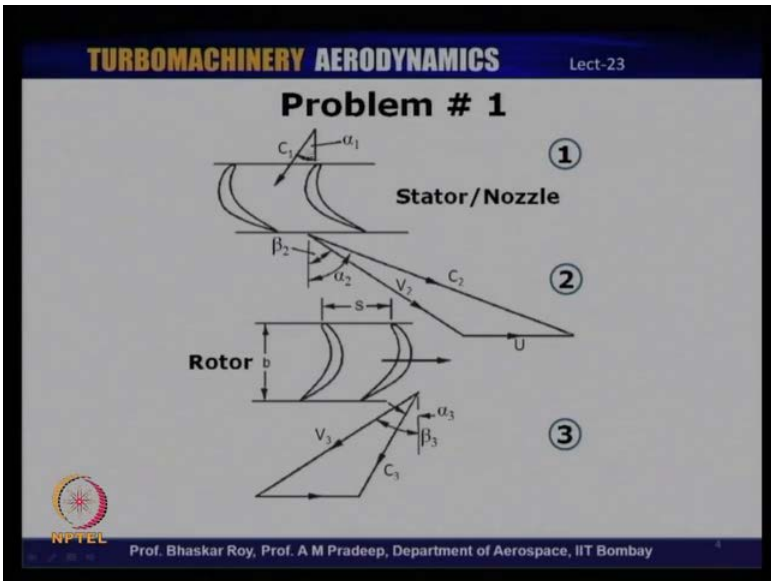

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Problem # 1

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So, let us construct a velocity triangle in general. So, this is a very generic velocity triangle, which is well, which is also true for a reaction turbine stage; it is not really an impulse turbine as you can see, it is an indeed a reaction stage. So, station 1 denotes the nozzle entry; station 2 denotes the nozzle exit or the rotor entry; station 3 denotes the rotor exit. So, these are the three parameters that have been the station numbers that we have defined.

And then the flow enters the nozzle with a velocity of C 1, and it exits the velocity absolute velocity of C 2; relative velocity at the nozzle exit is V 2, which is the velocity, which the flow actually enters the rotor, and the corresponding blade angles also been marked here; U is the mean blade speed the flow exits the rotor with the velocity of V 3, and an absolute velocity of C 3 with the corresponding angles of alpha 3 and beta 3. So, this is a typical velocity triangle of an axial turbine stage; and, what we will do is, we will take a look at what are the parameters that we know at this stage; so from this velocity triangle, we basically know the blade speed that something, which has been specified, and we also know the inlet conditions and exit conditions with static pressure, and we are required to calculate a variety of other parameters.

So, let us, begin with the part (a) that is to calculate the first parameter, which is to basically calculate the specific work done. Now, the first parameter that we need to calculate is the specific work done; and yes we also been given the flow angle at the nozzle exit that is alpha has been 2 given to us; so there are two parameters in this velocity triangle specified, we have alpha two and the blade speed U, so these are the parameters, which we know.

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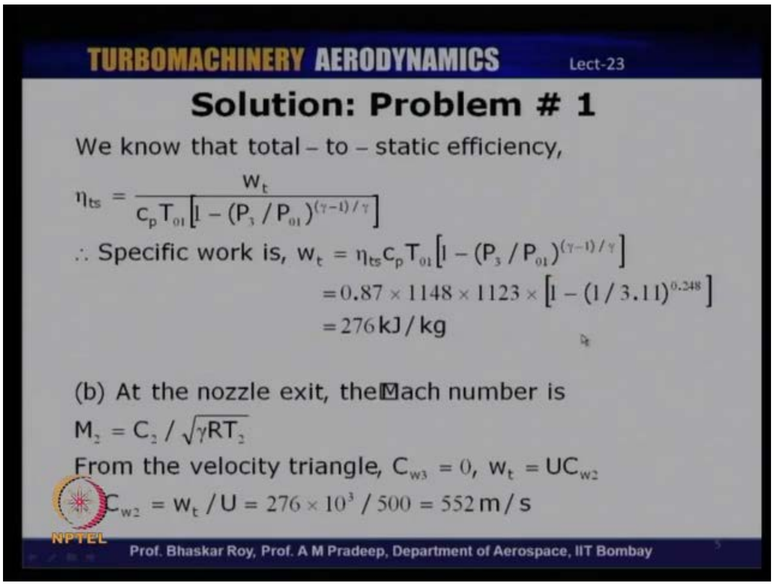

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Solution: Problem # 1

We know that total – to – static efficiency,

$$\eta_{ts} = \frac{w_t}{c_p T_{01}\left[1 - (P_3 / P_{01})^{(\gamma-1)/\gamma}\right]}$$

$\therefore$ Specific work is, $w_t = \eta_{ts} c_p T_{01}\left[1 - (P_3 / P_{01})^{(\gamma-1)/\gamma}\right]$

$$= 0.87 \times 1148 \times 1123 \times \left[1 - (1/3.11)^{0.248}\right]$$

$$= 276 \text{ kJ/kg}$$

(b) At the nozzle exit, the Mach number is

$M_2 = C_2 / \sqrt{\gamma R T_2}$

From the velocity triangle, $C_{w3} = 0$, $w_t = UC_{w2}$

$C_{w2} = w_t / U = 276 \times 10^3 / 500 = 552 \text{ m/s}$

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So, let us calculate the specific work done, we know that the total to static efficiency is basically, a function of the specific work done with reference to the static pressure at the exit and isotropic conditions; and therefore, the specific work done is basically total static efficiency into C P into T 0 1 1 minus P 3, where P 3 is a static pressure at the rotor exit divided by P 0 1 raise to gamma minus 1 by gamma. So, these are parameters, out of this, all these parameters are known to us; we know the total static efficiency it is given as 0.87 C P is known 1148, T 0 1 is 1123, P 3 by P 0 1 is 1 by 3.11, because the inlet condition is given as 311 kilo Pascal. So, that is 1 bar by 3.11 raise to 0.248, which is basically gamma minus 1 by gamma 0.33 minus 1.33 minus 1 by 1.33 so we get 0.248. So, the specific work done or the work done by the turbine can be calculated, and it comes out to be 276 kilo joules per kilogram.

Now, at the nozzle exit the Mach number, the second part of the question is to calculate the Mach number at the nozzle exit, now Mach number as we know is defined as ratio of absolute velocity to the speed of sound; so M 2 is C 2 divided by gamma R T 2. So, here we need to calculate two things; one is to calculate C 2 we also need to calculate the static temperature at the nozzle exit that is T 2. Now velocity triangle, because it is stated that the flow enters and leaves the stage axially, we have C W 3 is equal to 0; and since, we know that W t is a product of mu times delta C W that is mu into C W 2 minus C W 3; and since, C W 3 is 0; we can calculate C W 2, which is W t divided by U and that is 276 into 10 raise to 3 divided by 500 that comes out to be 550 meters per second.

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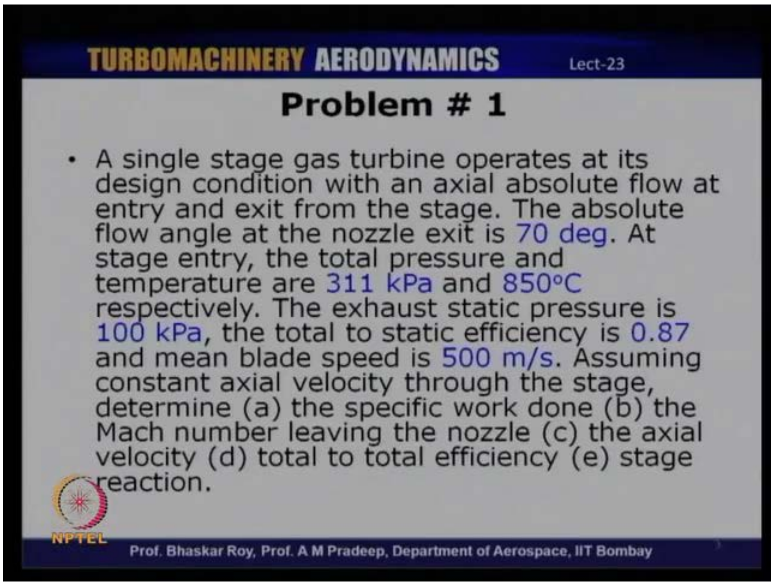

TURBOMACHINERY AERODYNAMICS Lect-23

Problem # 1

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So, let us understand this a little better, it is mentioned here that the flow enters the the well assuming a constant axial velocity through the stage, and that the flow leaves the stage in an axial direction. So, we have the turbine, which is operating in design conditions with an axial absolute flow entry and exit from the stage. So, this actually, gets needs to be modified. In the sense that C 1 is axial, C 3 is also axial, as per this question; so this general velocity triangle, which I had drawn should have actually have been modified in the sense, that alpha 1 should be 0 alpha 3 should also be 0; and which is why we get C W 3 as 0, because since, C 3 is axial, C W 3 is also 0.

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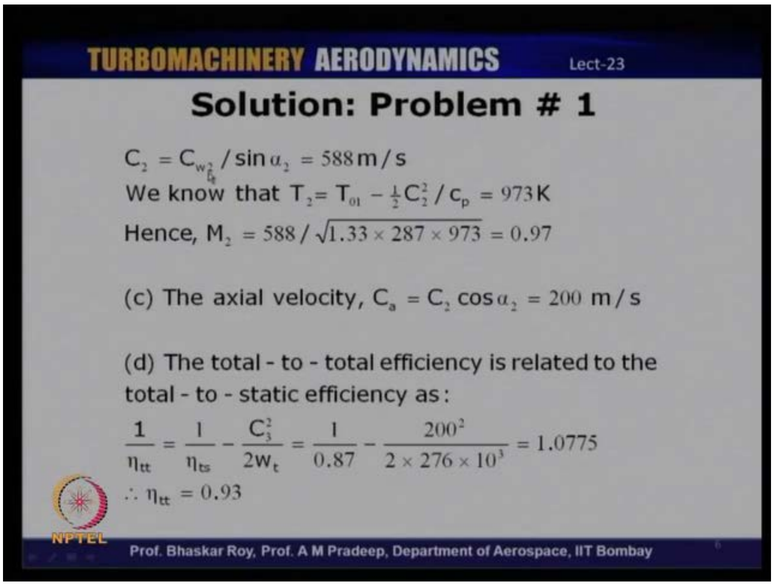

TURBOMACHINERY AERODYNAMICS Lect-23

Solution: Problem # 1

$C_2 = C_{w_2} / \sin\alpha_2 = 588\,\text{m/s}$

We know that $T_2 = T_{01} - \frac{1}{2}C_2^2 / c_p = 973\,\text{K}$

Hence, $M_2 = 588 / \sqrt{1.33 \times 287 \times 973} = 0.97$

(c) The axial velocity, $C_a = C_2 \cos\alpha_2 = 200\ \text{m/s}$

(d) The total - to - total efficiency is related to the total - to - static efficiency as:

$$\frac{1}{\eta_{tt}} = \frac{1}{\eta_{ts}} - \frac{C_3^2}{2w_t} = \frac{1}{0.87} - \frac{200^2}{2 \times 276 \times 10^3} = 1.0775$$

$\therefore \eta_{tt} = 0.93$

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So, C 2 we know from the exit velocity triangle is C w 2 divided by sin alpha 2. Let us, see that once again C 2 is C W 2, which is this component and divided by sin alpha 2, and that is 588; alpha 2 is given as 70 degree, so C 2 comes out to be 588 meters per second. We know that T 2 static temperature is T 0 2 minus C 2 square by 2 C p T 0 2 and T 0 1 are same, because there is no change in stagnation temperature in the rotor, in the stator. There is no work done in the stator, and so stagnation temperature in the stator has to be unchanged.

So, T 2 is static pressure at rotor entry, which is T 0 1 minus C 2 square by 2 C p C 2. We have just now calculated, so we can calculate static temperature at the nozzle exit; so once we calculate static temperature, we can now calculate the Mach number, because Mach number is simply the ratio of absolute velocity to the speed of sound. So, we have already calculated the absolute velocity that is 588 meters per second and the static temperature 973 kelvin; so M 2 is 588 divided by square root of 1.33 into 287, which is a gas constant, multiplied by 973, so the Mach number comes out to be 0.97. So, that is that solved the second part of the question.

Third part of the question is to find the axial velocity; axial velocity can be found from the inlet velocity triangle, because C 2 is known alpha is known; so, C a is 2 times cos alpha 2 and that is simply 200 meters per second. Now, the fourth part of the question is to find the total to total efficiency. I think, during our discussion on the efficiencies, I had derived an expression, which relates the total to total efficiency, and the total to static efficiency. And, we will simply make use of that equation, to calculate the total to total efficiency, total to static efficiency has been given as 0.87, and so, let us, calculate now the total to total efficiency.

So, based on that equation, if you recall and go back to that lecture, you will find that this derivation was shown. So, this is 1 by eta tt, which is the total to total efficiency; this is equal to 1 by eta ts minus C 3 square by 2 W t where W t is the specific work done. So, all these parameters we have already calculated, so we get 1 by 0.87, which is total static efficiency minus C 3 square; well C 3 is basically equal to C a, because the flow exits the stage axially, so C 3 and C a, are same and therefore, this is 200 square divided by twice of W t into 276 into 10 raise to 3.

So, this comes out to be 1 by eta tt is 1.0775, so the inverse of that eta tt is 0.93; total to total efficiency, we have calculated is 0.93. If you compare this with the total static efficiency, it is 0.87. So, you can see that total to total efficiency is indeed greater than the total to static efficiency; this is a comment, which I had made even during our discussion on efficiency is that in general, the total to total efficiency is comes out to be higher than the total to static efficiency, because of the very nature of the definition. I think, I had shown the T-S diagram to demonstrate, why this efficiency total to total efficiency has to be higher than total to static efficiency?

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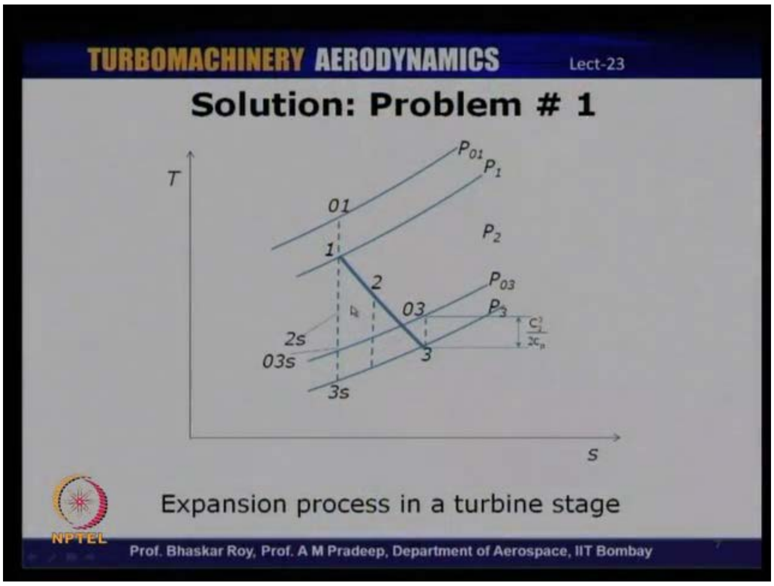
TURBOMACHINERY AERODYNAMICS Lect-23

Solution: Problem # 1

Expansion process in a turbine stage

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So, if you look at the expansion process here, which is something I had mentioned in the during our definition of the efficiency; total to total efficiency is defined, based on the temperature stagnation temperature here, with reference to the stagnation temperature at the exit isotropic; where as total to static efficiency is defined, based on the stagnation condition, here with this static condition here and the corresponding isotropic parameters, so we can see that this difference is always less than this other difference’ and therefore, it is inherent that total to total efficiency has to be higher than that of total to static efficiency. So, that is coming from the very basic definition and therefore, that should be indeed true; so there is something that which I have also seen in this particular problem that if we calculate that this actually, it should come out to be true.

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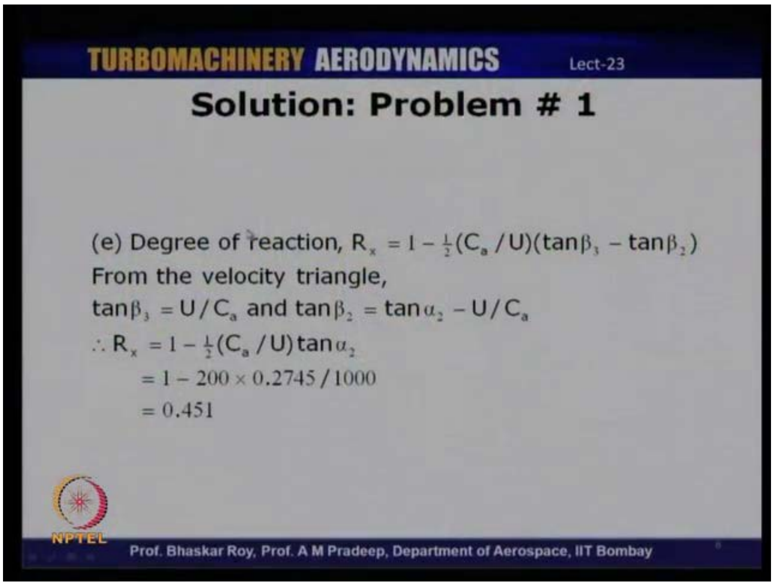

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Solution: Problem # 1

(e) Degree of reaction, $R_x = 1 - \frac{1}{2}(C_a / U)(\tan\beta_3 - \tan\beta_2)$

From the velocity triangle,

$\tan\beta_3 = U / C_a$ and $\tan\beta_2 = \tan\alpha_2 - U / C_a$

$\therefore R_x = 1 - \frac{1}{2}(C_a / U)\tan\alpha_2$

$= 1 - 200 \times 0.2745 / 1000$

$= 0.451$

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Let us, move on to the last part of the question, which is to calculate the degree of reaction. Now degree of reaction again; we have defined derived an expression for degree of reaction in the one of the earlier lectures. So, this is basically, equal to 1 minus C a by 2 U into tan beta 3 minus tan beta 2. So, from the velocity triangle let us see, what these parameters are; and how do we calculate tan beta 3. Let us, go back to the velocity triangle here; tan beta 3 is equal to this component divided by the axial velocity that is U divided by C a. And similarly, tan beta 2 is equal to the component given by this that is C W 2 minus u divided by C a.

So, from these two equations, we can calculate tan beta 2 as well as tan beta 3; and therefore, what you can see here is that tan beta 2, beta 3 is U by C a, tan beta 2 can be equated to tan alpha 2 minus U by C a, so if you substitute both these expressions here, we can simplify the expression for the degree of reaction, and that is 1 minus 1 by 2 C a by U into tan alpha 2. So, all these parameters, we have already calculated, and so let us, just substitute that here, and then we get the degree of reaction as 0.451

So, we have now calculated all the five parameters, which were required to be calculated for this question, we have calculated specific work done and axial velocity, the total to total efficiency and now the degree of the reaction as well; so that completes the first question, which was to do with very simple single stage axial turbine with certain parameters, which has been specified, and, how do you proceed towards calculating the other parameters and which obviously started from the velocity triangles, trying to solve the velocity triangles to calculate the flow angles, and therefore, the other parameters which we were require to be calculated.

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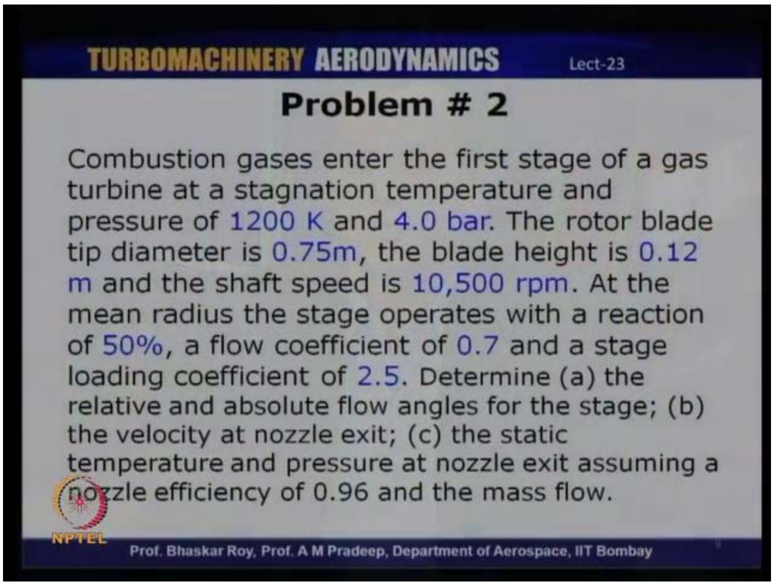

Problem # 2

Combustion gases enter the first stage of a gas turbine at a stagnation temperature and pressure of 1200 K and 4.0 bar. The rotor blade tip diameter is 0.75m, the blade height is 0.12 m and the shaft speed is 10,500 rpm. At the mean radius the stage operates with a reaction of 50%, a flow coefficient of 0.7 and a stage loading coefficient of 2.5. Determine (a) the relative and absolute flow angles for the stage; (b) the velocity at nozzle exit; (c) the static temperature and pressure at nozzle exit assuming a nozzle efficiency of 0.96 and the mass flow.

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So, let us, now move on to the next question. So, this is the problem statement for this question; this is basically, 50 percent reaction stage question; so the problem statement is the following; combustion gases enter the first stage of a gas turbine at a stagnation temperature and pressure of 1200 kelvin and 4 bar; the rotor blade tip diameter is 0.75 meter; the blade height is 0.12 meter, and the shaft speed is 10,500 rpm. At the mean radius, the stage operates with a reaction of 50 percent; a flow coefficient of 0.7 and a stage loading coefficient of 2.5. Determine, part (a) ,the relative and absolute flow angles for the stage; part (b), the velocity at nozzle exit; part (c), the static temperature and pressure at nozzle exit assuming a nozzle efficiency of 0.96 and the mass flow.

So, here this is a question, which contains to 50 percent reaction stage, and we have the temperature and pressure at the inlet the degree of reaction is given to us, the rotational speed, and also the mean diameter is specified; and based on this, we are required to calculate basically, solve the velocity triangle and calculate the flow angles the relative and absolute flow angles; part (b) was to calculate velocity at nozzle exit that is ah the absolute velocity, and the static pressure and temperature at nozzle exit, with certain nozzle efficiency, which has been specified and also the mass flow rate; so these are the parameters that we will need to calculate for this particular question. So, as always we will start with the velocity triangle, let us construct the velocity triangle first, and then we will proceed towards solving the question.

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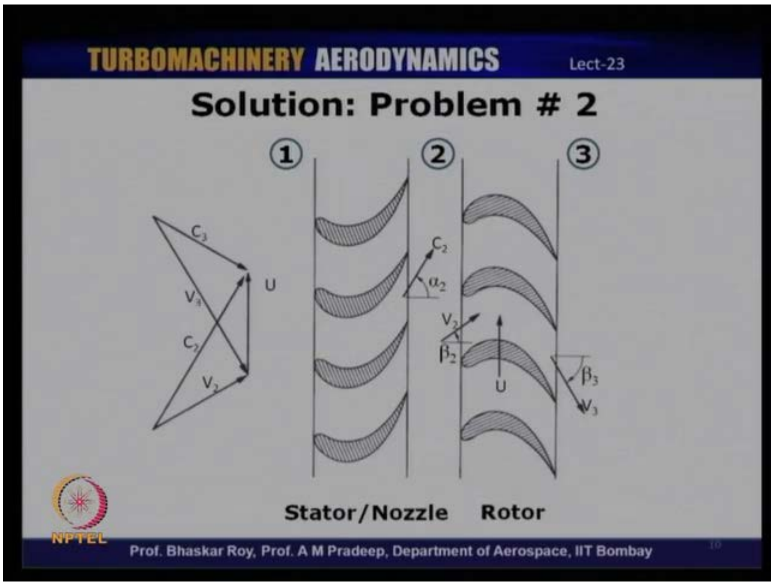

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Solution: Problem # 2

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So, this is what we had drawn for 50 percent reactions stage during our discussing on this couple of lectures .early, on so this is a velocity triangle for a typical 50 percent reaction stage, and the the mean feature that you can observe for 50 percent reactions stage is that the velocity triangles are symmetrical or mirror images; so what you have at the inlet of the rotor is, what happens at the exit as well it just a mirror image; and which means that the angles should also be equal alpha 2 will be equal to beta 3 and beta 2 will be equal to alpha alpha 3 and so on. Similarly, the velocity components if the axial velocity is constant, then we have C 2 is equal to v 3 and V 2 is equal to C 3.

So, in this question we have been given the blade height at the mean diameter and the speed, which means that we can calculate the blade speed that is U flow coefficient is given as 0.7, which means we can also calculate the axial velocity from their, because flow coefficient is the ratio of the axial velocity to the blade speed, so we have the blade speed as well as the axial velocity. We are now required to solve the velocity triangle and calculate few other parameters.

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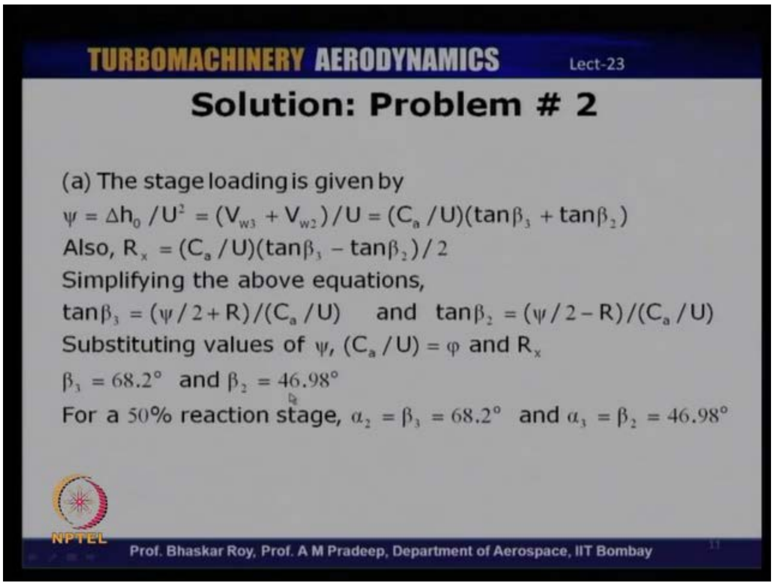

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Solution: Problem # 2

(a) The stage loading is given by

$\psi = \Delta h_0 / U^2 = (V_{w3} + V_{w2})/U = (C_a/U)(\tan\beta_3 + \tan\beta_2)$

Also, $R_x = (C_a/U)(\tan\beta_3 - \tan\beta_2)/2$

Simplifying the above equations,

$\tan\beta_3 = (\psi/2 + R)/(C_a/U)$ and $\tan\beta_2 = (\psi/2 - R)/(C_a/U)$

Substituting values of ψ, $(C_a/U) = \varphi$ and R_x

$\beta_3 = 68.2°$ and $\beta_2 = 46.98°$

For a 50% reaction stage, $\alpha_2 = \beta_3 = 68.2°$ and $\alpha_3 = \beta_2 = 46.98°$

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So, let us start with the first part of it, which is to calculate all the angles involved in the velocity triangle. Now for this case, we also have been given the loading coefficient psi which is delta h not by U square; and for a 50 percent reaction stage, delta h not equal to U times delta C W, which is also same as delta V w. And therefore, the stage loading coefficient reduces to delta h not by U square, which is V W 3 plus V W 2 divided by U; and so this if we express in terms of the angles, let us take a look at what is V w 2 and V w 3; V w 2 corresponds to this component here, which is in terms of beta 2 here and the axial velocity; similarly, V w 3 is for the exit in terms of the axial component and the blade angle that is beta 3.

So, this we have expressed in terms of axial components V w 3 is C a times tan beta 3; and V w 2 is c a times tan beta 2. So, from our discussion on degree of reaction, the specifically for 50 percent degree of reaction case, we can basically, equate degree of reaction as C a by U into tan beta 3 minus tan beta 2 divided by 2; and that is, because the angles are similar so if you if you replace alpha 3 by beta 2 and beta alpha 2 by beta 3, we can express degree of reaction in terms of this.

So, if we look at these two equations and we try to simplify them, what we basically get is an expression, in terms of the degree of reaction, the loading coefficient and the flow coefficient, C a by U is the coefficient, which we know is equal to 0.7 in this question; so what we get tan beta 3 is equal to psi by 2 plus degree of reaction divided by the flow coefficient phi similarly, tan beta 2 is equal to psi by 2 minus degree of reaction divided by phi; this basically, happens if we simply add and subtract these two equations, we can simplify them and get this equation.

So, since we have been we already know the loading coefficient psi, the flow coefficient C a by U and degree of reaction; we substitute for those values here and we get the angles beta 3, which is 68.2 degrees and beta 2, which comes out to be 46.98 degrees. Now so for 50 percent reaction stage, we do not need to actually calculate the other angles, because other angles are equal to what we already calculated; alpha 2 will be equal to beta 3, which is in term equal to 68.2 degrees; and alpha 3 is equal to beta 2, which is 46.98 degrees. So in this question that we have, because it happens to be 50 percent reaction stage, we could make lot of simplifications in calculation of degree of reaction or equating that to the loading coefficient and so on. And, also the fact that we do not need to really calculate the other angles, because beta 2 is equal to alpha 3 and beta 3 is equal to alpha 2.

So, we just calculated beta 2 and beta 3 and the absolute angles or indeed equal to these angles as well; so that completes the first part of the question. Now, let us look at what is the second part? Second part is to calculate the velocity at nozzle exit; subsequently we need to calculate static pressure and temperature at nozzle exit with a certain nozzle efficiency specified. So let us calculate the velocity at the nozzle exit, and for which we will make use of the fact that the dimensions have been specified, the mean, the radius has been given the blade speed is known. And, so we can basically, calculate the axial velocity and also the absolute velocity.

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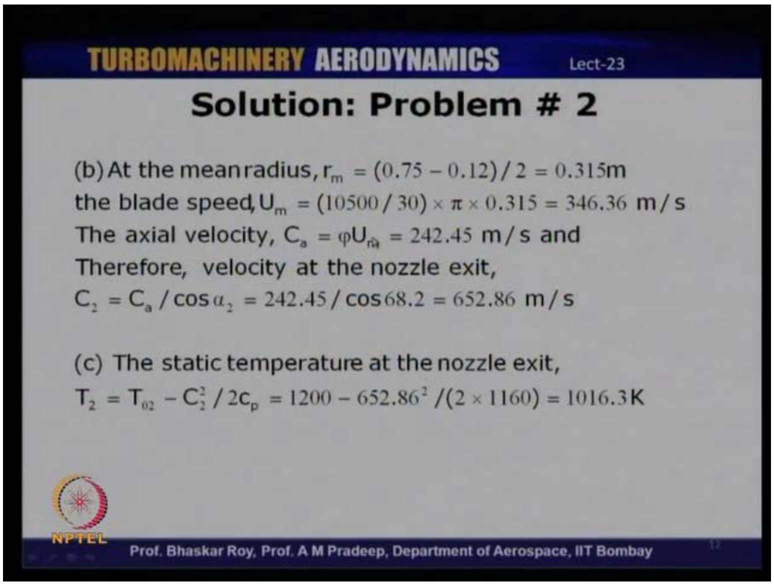

TURBOMACHINERY AERODYNAMICS Lect-23

Solution: Problem # 2

(b) At the mean radius, $r_m = (0.75 - 0.12)/2 = 0.315$m

the blade speed, $U_m = (10500/30) \times \pi \times 0.315 = 346.36$ m/s

The axial velocity, $C_a = \varphi U_m = 242.45$ m/s and

Therefore, velocity at the nozzle exit,

$C_2 = C_a / \cos\alpha_2 = 242.45/\cos 68.2 = 652.86$ m/s

(c) The static temperature at the nozzle exit,

$T_2 = T_{02} - C_2^2/2c_p = 1200 - 652.86^2/(2 \times 1160) = 1016.3$K

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Now, at the mean radius we can for, which the tip diameter and the blade height will be used. So, the tip diameter is given as 75 centimeters so 0.75 minus blade height 0.12 divided by 2, so this is the mean radius that is 0.315. So, at the mean radius, we can calculate the blade speed, the speed of rotation is given here, and the so phi D n by 60 will give us the blade speed mean, blade speed that is 346.36 meters per second. Since, the flow coefficient has already been specified, we can calculate the axial velocity as 5 times U mean that is 70 percent of this, so that is 242.45 meters per second.

So, the absolute velocity at the nozzle exit is C a by cos alpha; and alpha is given, we already calculated in the previous part of the question that was 68.2 degrees; so C 2 is equal to C a by cos alpha 2 and that is 652.86 meters per second. So, having calculated the absolute velocity, we can now calculate the static temperature based on the absolute velocity, because stagnation temperature is known. It is gives as 1200 kelvin. So, in this question, we have been given the stagnation temperature at the inlet of the turbine as 1200 kelvin, and we know that in the nozzle ,there is no change in the stagnation temperature, so T 0 1 should be equal to T 0 2. And so, T 2 is basically, equal to T 0 1 minus C 2 square by 2 C P and same that is, because T 0 1 and T 0 2 are same; so, from there you can calculate static temperature; static temperature at the nozzle exit will be equal to T 0 2, which is T 0 1 minus C 2 square by 2 C p. So, we get 1200 minus C 2 square 652.86 square divided by 2 into C p, so this can be calculated at 1016.3 kelvin.

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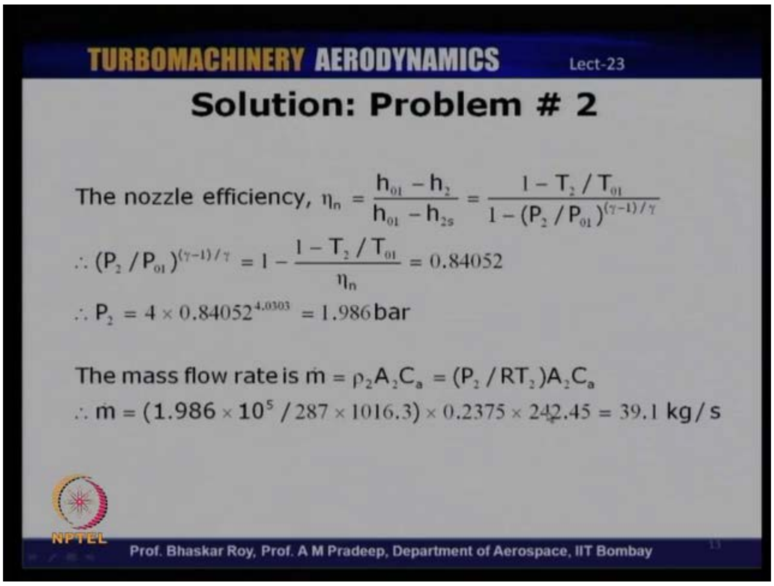

TURBOMACHINERY AERODYNAMICS Lect-23

Solution: Problem # 2

The nozzle efficiency, $\eta_n = \dfrac{h_{01} - h_2}{h_{01} - h_{2s}} = \dfrac{1 - T_2 / T_{01}}{1 - (P_2 / P_{01})^{(\gamma-1)/\gamma}}$

$\therefore (P_2 / P_{01})^{(\gamma-1)/\gamma} = 1 - \dfrac{1 - T_2 / T_{01}}{\eta_n} = 0.84052$

$\therefore P_2 = 4 \times 0.84052^{4.0303} = 1.986\,\text{bar}$

The mass flow rate is $\dot{m} = \rho_2 A_2 C_a = (P_2 / RT_2) A_2 C_a$

$\therefore \dot{m} = (1.986 \times 10^5 / 287 \times 1016.3) \times 0.2375 \times 242.45 = 39.1\ \text{kg/s}$

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Next part is, calculate the stagnation static pressure at the nozzle exit, for which we will make use of the nozzle efficiency definitions; so this is again from the fundamental cycle analysis that I assume you would be aware of for rate and cycles, where we could, where we have defined nozzle efficiency, in terms of enthalpies h 0 1 being the inlet enthalpy of the nozzle, h 2 is the inlet static exit static enthalpy divided by h 0 1 minus h 2 S. So, we can express this in terms of temperatures 1 minus T 2 by T 0 1 divided by 1 minus P 2 by P 0 1 raise to gamma minus 1 by gamma, this is because we can express the denominator in terms of 1 minus T 2 S by T 0 1, which is basically, in terms of pressure ratios for an isotropic condition.

So, from this we can simplify this and calculate the pressure ratios P 2 by P 0 1, in terms of the temperature ratio as well as the nozzle efficiency and so, the static pressure at the nozzle exit can be simply calculated in terms of inlet pressure, stagnation that is 4 bar multiplied by 0.84052 rise to 4.03; so this is 1.986 bar; and the last part of the question is to calculate the mass flow rate; mass flow rate is expressed, in terms of density, the area and the axial velocity. Now density is P 2 by R T 2, and how do you calculate the annulus area; so, for calculating this area we have the tip diameter as well as the blade height; so from there you can calculate the annulus area and multiply that with the axial velocity, one can calculate the mass flow rate. So, density is P 2 by R T 2, annulus area in terms of the blade tip diameter and the blade height and the axial velocity, which is already something calculated. So, if you multiplied all of them the mass flow rate as 39.1 kilo gram per second.

So, this completes the second question, which was very similar to what we have solved in the first question except the fact that here, we had also firstly, reaction turbine 50 percent reaction turbine stage, the first question was on an impulse turbine stage; and second difference being the fact that here we had nozzle efficiency and correspondingly, we had calculated the static temperature and pressure at the exit of the nozzle using the nozzle efficiency definitions. So, having understood two distinct problems, one to do with an impulse turbine and this second question was 50 percent reaction turbine, we will now proceed to solving two other problems, which are slightly different from what we have already solved.

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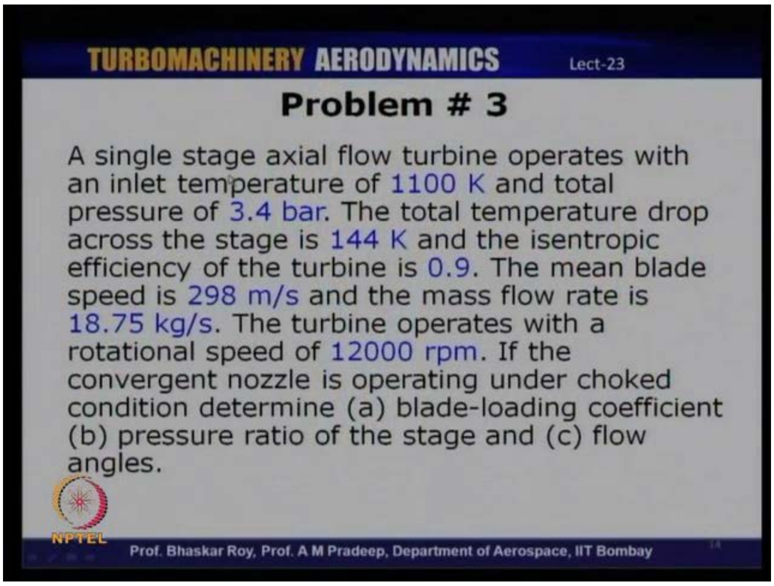
TURBOMACHINERY AERODYNAMICS Lect-23

Problem # 3

A single stage axial flow turbine operates with an inlet temperature of 1100 K and total pressure of 3.4 bar. The total temperature drop across the stage is 144 K and the isentropic efficiency of the turbine is 0.9. The mean blade speed is 298 m/s and the mass flow rate is 18.75 kg/s. The turbine operates with a rotational speed of 12000 rpm. If the convergent nozzle is operating under choked condition determine (a) blade-loading coefficient (b) pressure ratio of the stage and (c) flow angles.

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So, the third question, the problem statements is the following; the inlet a single stage axial flow turbine operates with an inlet temperature of 1100 kelvin, and total temperature of 3.4 bar, the total temperature drop across the stage is 144 kelvin, and the isotropic efficiency of the turbine is 0.9, the mean blade speed is 298 meters per second, and the mass flow rate is 18.75 kilo grams per second, the turbine operates with a rotational speed of 12,000 rpm. If the convergent nozzle is operating under choked condition, determine the part (a) blade loading coefficient part (b) the pressure ratio of the stage and part (c) flow angles.

So, in this question, we have been given that the nozzle is operating under a choked condition that is one of the things that we need to keep in mind, we also have been given isotropic efficiency of the turbine, besides few other parameters like the temperature and pressure; the temperature drop in the stage, blade speed and the mass flow rate. So, as we have been doing in the past being construct the velocity triangle for generic turbine, axial turbine; and so that we are familiar, with the parameters, which are specified and those we should be need to the calculate. And so in this question we have the blade speed, which is given 298 meters per second and the mass flow rate is given to us, and the pressure and temperature at the inlet have been specified.

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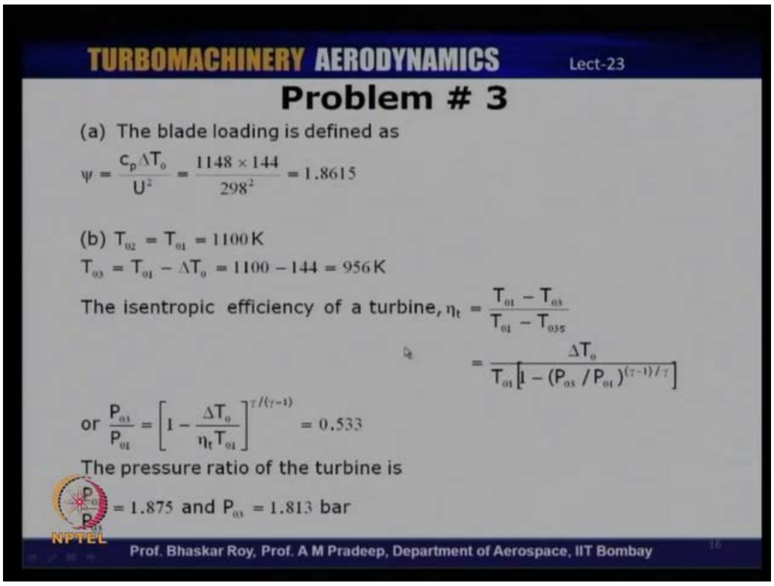

TURBOMACHINERY AERODYNAMICS Lect-23

Problem # 3

(a) The blade loading is defined as

$$\psi = \frac{c_p \Delta T_0}{U^2} = \frac{1148 \times 144}{298^2} = 1.8615$$

(b) $T_{02} = T_{01} = 1100\,K$

$T_{03} = T_{01} - \Delta T_0 = 1100 - 144 = 956\,K$

The isentropic efficiency of a turbine, $\eta_t = \dfrac{T_{01} - T_{03}}{T_{01} - T_{03s}}$

$$= \frac{\Delta T_0}{T_{01}\left[1 - (P_{03}/P_{01})^{(\gamma-1)/\gamma}\right]}$$

or $\dfrac{P_{03}}{P_{01}} = \left[1 - \dfrac{\Delta T_0}{\eta_t T_{01}}\right]^{\gamma/(\gamma-1)} = 0.533$

The pressure ratio of the turbine is

$\dfrac{P_{01}}{P_{03}} = 1.875$ and $P_{03} = 1.813$ bar

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So, let us solve the first part of the question, which was to calculate the loading coefficient, which is very straight forward, because we know the temperature drop and we know the blade speed. So, psi is simply C p times delta T not divided by U square, so that comes out to be 1.8615. Now, second part of the question is to find the pressure ratio of the stage, so we need to find P 0 1 by P 0 3, so to calculate that we know the temperature drop in this stage, it is given us 144 kelvin. The inlet temperature is given T 0 2 equal to T 0 1 that is 1100 kelvin, based on that we can calculate the exit stagnation temperature T 0 3, which will be T 0 1 minus delta T not and that is 1100 minus 144, so that is 956 kelvin.

So, having calculated the exit stagnation temperature, we can we need to also calculate the pressure ratio, because that is the objective of this second part of the question; for this we will make use of the nozzle efficiency definition; so isotropic efficiencies, I mean isotropic efficiency of a turbine, which is defined in terms of these the actual drop in stagnation temperature divided by isotropic drop, so isotropic efficiency is defined as one T 0 1 minus T 0 3 divided by T 0 1 minus T 0 3 as numerator is already known that is delta T not divided by T 0 1 into 1 minus P 0 3 by P 0 1 rise to gamma minus 1 by gamma .

So, if we simplify this, because the temper the efficiency is already given to us, we can substitute for the efficiency, the temperature drop is known, inlet stagnation temperature is known, and therefore, P 0 3 by P 0 1 comes out to be 0.53; and therefore, the pressure ratio of the turbine is an inverse of that is 1.875. So, that solves the second part of the question were we are required to calculate the pressure ratio of this particular turbine stage single stage axial turbine. The next part is to calculate the flow angles, we need to calculate all the angles that are involved in the velocity triangle; so we are going to use the fact that the nozzle is operating under a choked conditions, so that is the one of the information that we have; and that means that mach number at the nozzle exit is one and therefore, the absolute flow at the nozzle exit C 2 will be equal to square root of gamma R T 2.

And similarly, that is also fixes the temperature pressure at the nozzle axial, because the of the temperature and pressure would be the critical condition and therefore, we can actually, calculate static pressure and temperature from the isotropic relations, because operating under choked condition with Mach number equal to 1. So, once we calculate the absolute velocity, we can also calculate some of the flow angles that are involved, because blade speed is known and therefore, we can proceed towards solving the velocity triangle calculated all the angle.

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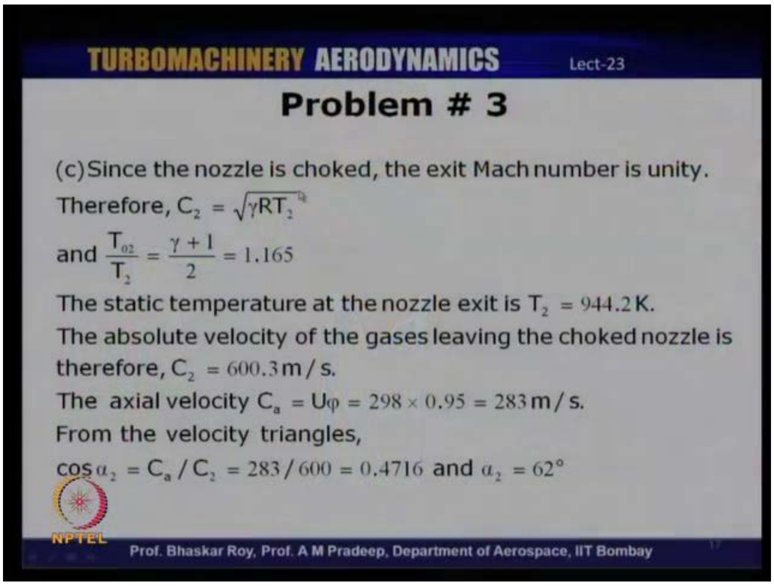

TURBOMACHINERY AERODYNAMICS Lect-23

Problem # 3

(c) Since the nozzle is choked, the exit Mach number is unity.

Therefore, $C_2 = \sqrt{\gamma R T_2}$

and $\frac{T_{02}}{T_2} = \frac{\gamma + 1}{2} = 1.165$

The static temperature at the nozzle exit is $T_2 = 944.2$ K.

The absolute velocity of the gases leaving the choked nozzle is therefore, $C_2 = 600.3$ m / s.

The axial velocity $C_a = U\varphi = 298 \times 0.95 = 283$ m / s.

From the velocity triangles,

$\cos\alpha_2 = C_a / C_2 = 283 / 600 = 0.4716$ and $\alpha_2 = 62°$

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So, the nozzle since, is it operating under choked condition mach number is exit mach number is unity, so C 2 is a equal to square root of gamma R T 2 and therefore, T 2 by T 0 2 by T 2 is equal to gamma plus 1 by 2 and that is 1.165. Since, T 0 2 is equal to T 0 1, that is known the static temperature at the nozzle exit is T 2, which is 944.2 kelvin; and therefore, the absolute velocity of the gases leaving the choked nozzle will be square root of gamma R T 2 that is 600.3 meters per second. And, now we can calculate the axial velocity, because the blade speed is known, the flow coefficient is known, so U times C a equal to U times that is 298 multiplied by 0.95; and at that is 283 meters per second; and from the velocity triangle, we can now calculate cos alpha 2, which is C a divided by C 2 that is 283 by 600, and therefore, alpha 2 is 62 degrees.

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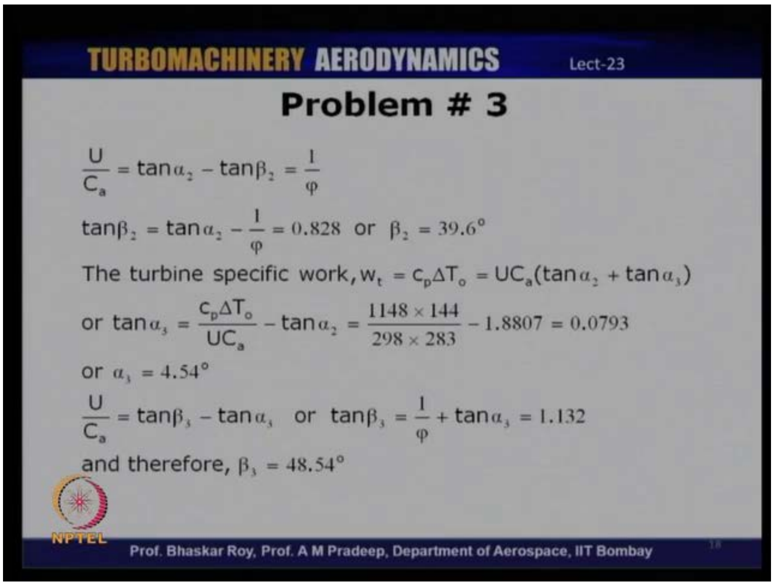

TURBOMACHINERY AERODYNAMICS Lect-23

Problem # 3

$$\frac{U}{C_a} = \tan\alpha_2 - \tan\beta_2 = \frac{1}{\varphi}$$

$$\tan\beta_2 = \tan\alpha_2 - \frac{1}{\varphi} = 0.828 \text{ or } \beta_2 = 39.6°$$

The turbine specific work, $w_t = c_p\Delta T_o = UC_a(\tan\alpha_2 + \tan\alpha_3)$

$$\text{or } \tan\alpha_3 = \frac{c_p\Delta T_o}{UC_a} - \tan\alpha_2 = \frac{1148 \times 144}{298 \times 283} - 1.8807 = 0.0793$$

or $\alpha_3 = 4.54°$

$$\frac{U}{C_a} = \tan\beta_3 - \tan\alpha_3 \quad \text{or} \quad \tan\beta_3 = \frac{1}{\varphi} + \tan\alpha_3 = 1.132$$

and therefore, $\beta_3 = 48.54°$

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We now need to calculate beta 2, which is the blade angle at the inlet of the rotor and for which we will make use of the velocity triangle again; and that is from the velocity triangle, you can see that this ratio U by C a is tan alpha 2 minus tan beta 2, which is inverse of the flow coefficient. Since, tan alpha 2 is known and phi is also known we can calculate tan beta 2, which is tan alpha 2 minus 1 by phi that is 0.828 or beta 2 is 39.6degrees.

Now, the third part of the question well, where in we are calculating all angle, we need to now calculate the angle circle, the rotor exit; now this specific work of turbine, which is W T is C P is delta T not, which can be express in terms of U into delta C W that is U into C a tan alpha 2 plus tan alpha 3. In which case, we know the angles at the inlet tan alpha 2 is known U is known C a is known and delta T not is also known therefore, tan alpha 3 is C P times delta T not by U C a minus tan alpha 2, so all these parameters are known to us simply substitute that in we get alpha 3 as 4.54 degrees.

Similarly, U by C a is also equal to tan beta 3 minus tan alpha 3 and therefore, tan beta 3 is equal to 1, by phi plus tan alpha 3 and therefore, beta 3 is 48.54 degrees. So, we are calculated all the angles that are involved starting from the rotor entry, alpha 2 and alpha beta 2 and rotor exit, the alpha 3 and beta 3; so you see that to be able to solve in fact, all three problems that we have now solved, requires you to have good understanding of the velocity triangles, because of that is where that is basically, the starting point of solving these questions, if necessary that, you can constant the velocity triangle, clearly understanding the different components of velocity, which has been specified, and then use that information to proceed and solve the problem, based on what is already given to even in the problem like some questions, you have mean blade speed; some questions may specify axial velocity or some angles so on.

So, in this question we need basically, solved we calculated the pressure ratio, we are also calculated all the angles that are involved and in this case of course, it is also given towards that in nozzle is operating under choked conditions. So, I have one more question for you in this is different from the all the questions we have solved, in the sense that this question requires as to find the number of stages that are required for operating a certain turbines, it is a multi stage turbine, how do you calculate the number of stages; if you know the pressure drop across the entire turbine, how can you estimate the number of stages that are involved.

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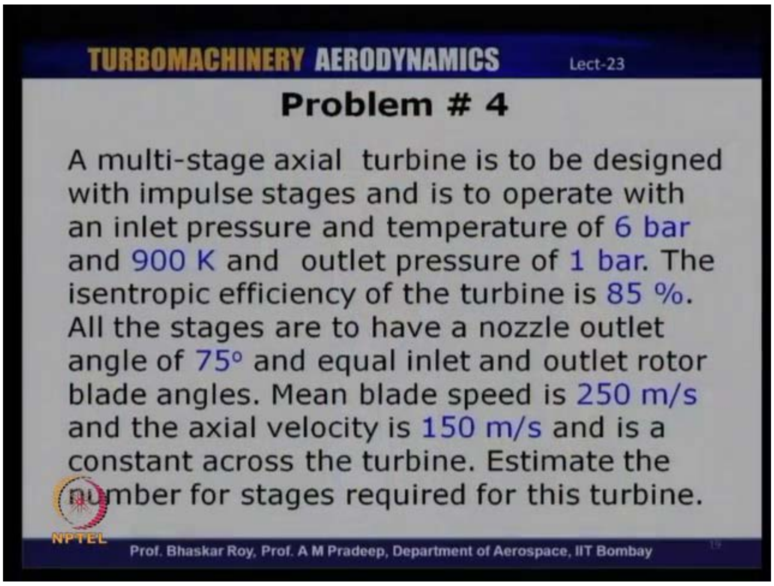

TURBOMACHINERY AERODYNAMICS Lect-23

Problem # 4

A multi-stage axial turbine is to be designed with impulse stages and is to operate with an inlet pressure and temperature of 6 bar and 900 K and outlet pressure of 1 bar. The isentropic efficiency of the turbine is 85 %. All the stages are to have a nozzle outlet angle of 75° and equal inlet and outlet rotor blade angles. Mean blade speed is 250 m/s and the axial velocity is 150 m/s and is a constant across the turbine. Estimate the number for stages required for this turbine.

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So, this is a problem statement is the following; a multi stage axial turbine is to be designed with impulse stages and is to operate with an inlet pressure and temperature of 6 bar and 900 kelvin and outlet pressure of 1 bar. The isotropic efficiency of the turbine is 85 percent, all the stages are to have a nozzle angle outlet of 75 degrees, and equal inlet and outlet rotor blade angles; the mean blade speed is 250 meters per second and the axial velocity is 150 per second and is a constant across the turbine; estimate the number of stages required for this turbine; so this is an impulse turbine, and so it is needless to say that the rotor inlet and outlet angles are going to be equal. It is also given that we have the inlet pressure in temperature for the nozzle entry, we have been given the isotropic efficiency and the nozzle exit angle, we also have the mean blade speed and axial velocity; so lot of parameters that are involved in the velocity triangles are given to us, but we need to calculate the number of stages that are required for getting this kind of pressure ration that is involved in this particular turbine.

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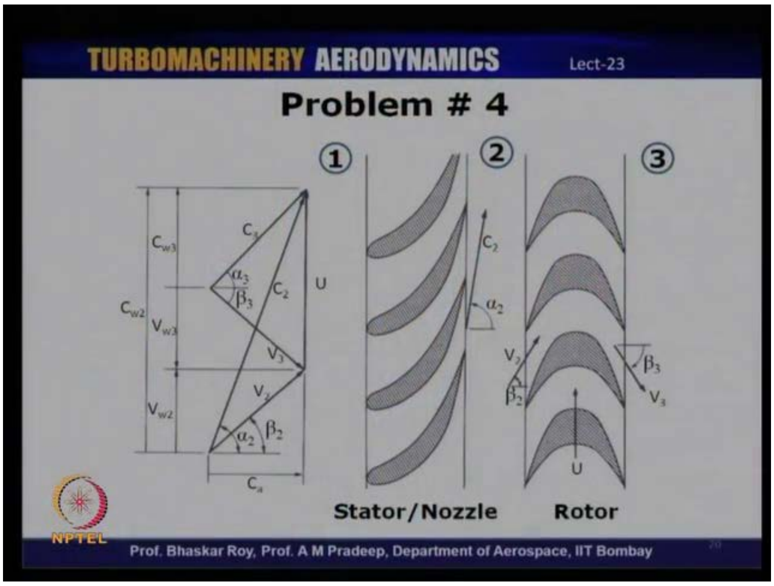

TURBOMACHINERY AERODYNAMICS Lect-23

Problem # 4

Stator/Nozzle Rotor

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So, let us take a quick look at the velocity triangle, because we will need to come back to this little later; an impulse turbine would involve a velocity triangle like what is shown here; C 2 is the flow exiting nozzle at an angle of alpha 2, in this case, it is given to us; V 2 is the relative velocity entering the rotor at an angle of beta 2 and it leaves the rotor at velocity V 3, which is equal to 2, and an angle beta 3, which is equal to beta 2, the absolute velocities at the rotor exit are is equal to C 3, which is in fact equal to V 3 and V 2 for constant axial velocity. So, in this case it is indeed specified that the axial velocity is constant, so it means that C 3 is equal to V 3 which is in turn equal to V 2.

So, we will come back to this velocity triangle as we proceed to solve this equation; so first part of the question, we will what we will try to do is that since, we know the pressure drop across overall pressure drop is known; let us calculate the overall stagnation temperature drop across the turbine; and then what we will do is to estimate the stagnation temperature drop in one stage therefore, the overall temperature drop divided by temperature drop in one stage will give us the number of stages, an estimate of the number of stages required.

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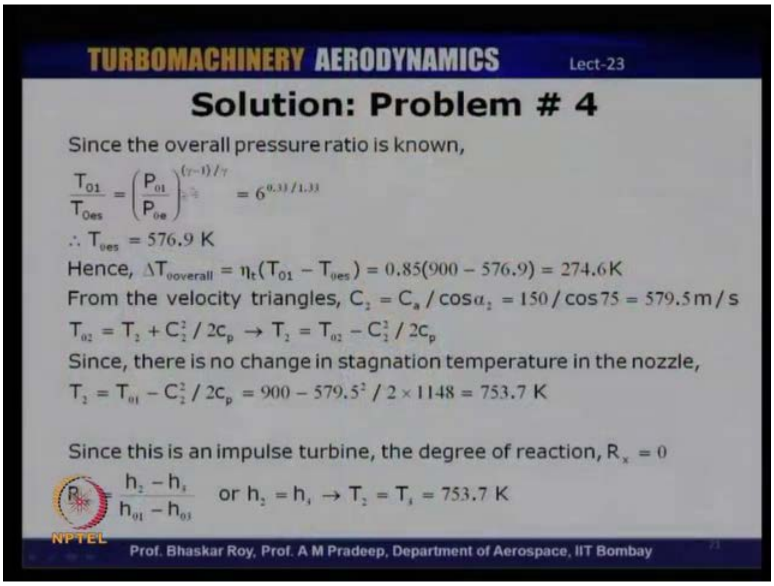

TURBOMACHINERY AERODYNAMICS Lect-23

Solution: Problem # 4

Since the overall pressure ratio is known,

$$\frac{T_{01}}{T_{0es}} = \left(\frac{P_{01}}{P_{0e}}\right)^{(\gamma-1)/\gamma} = 6^{0.33/1.33}$$

$\therefore T_{0es} = 576.9$ K

Hence, $\Delta T_{0overall} = \eta_t(T_{01} - T_{0es}) = 0.85(900 - 576.9) = 274.6$K

From the velocity triangles, $C_2 = C_a / \cos\alpha_2 = 150 / \cos 75 = 579.5$m/s

$T_{02} = T_2 + C_2^2 / 2c_p \rightarrow T_2 = T_{02} - C_2^2 / 2c_p$

Since, there is no change in stagnation temperature in the nozzle,

$T_2 = T_{01} - C_2^2 / 2c_p = 900 - 579.5^2 / 2 \times 1148 = 753.7$ K

Since this is an impulse turbine, the degree of reaction, $R_x = 0$

$R_x = \frac{h_2 - h_3}{h_{01} - h_{03}}$ or $h_2 = h_3 \rightarrow T_2 = T_3 = 753.7$ K

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So, the overall pressure ratio is given as 6 therefore, T 0 1 by T not E S, where T not E S is the stagnation temperature, isotropic at the exit of the turbine is equal to P 0 1 by P 0 E raise to gamma minus 1 by gamma. Therefore, the stagnation temperature at the exit of the turbine is equal to 576.9 kelvin, this is isotropic temperature; so the actual temperature overall will be equal to the isotropic temperature difference multiplied by the isotropic efficiency, which is 85 percent in this case; so delta T not overall, which is the stagnation temperature across the entire turbine is equal to eta T, which is the isotropic efficiency multiplied by T 0 1 minus T naught E S, T 0 1 is given as 900, T naught E S is calculated as 576.9 kelvin, so the overall temperature drop in the turbine is 274.6 kelvin.

So, now we come back to the velocity triangles; we know from the inlet velocity triangles C 2 equal to C a divided by cos alpha 2, C 2 is this, cos alpha 2 is C a divided by C 2 and therefore, C 2 is C a by cos alpha 2, C a is known, alpha 2 is also known therefore, C 2 is equal to 150 divided by cos 75 that is 579.5 meters per second. Now, this stagnation temperature at the inlet is known T 0 2, which is equal to T 0 1; and therefore, static temperature at the nozzle exit T 2 is equal to T 0 2 minus C 2 square by

2 C P; therefore, T 0 1, which is equal to T 0 2 is 900 kelvin, C 2 we just now calculated 579.5 and therefore, we can calculate T 2 that is 753.7 kelvin.

Now, it is given specifically that this is an impulse turbine stage, which means that degree of reaction is basically 0; so when the degree of reaction is 0, it implies that the degree of reaction, we have defined as H 2 minus H 3 divided by H 0 1 minus H 0 3; since, the degree of reaction is 0, we have H 2 is equal to H 3, implies the static temperature should be same T 2 and T 3 should be the same and that is equal to 753.7 kelvin, because there is no change in static conditions across rotor, the entire pressure drop has actually take place in the nozzle nothing changes in the rotor T 2 is equal to T 3.

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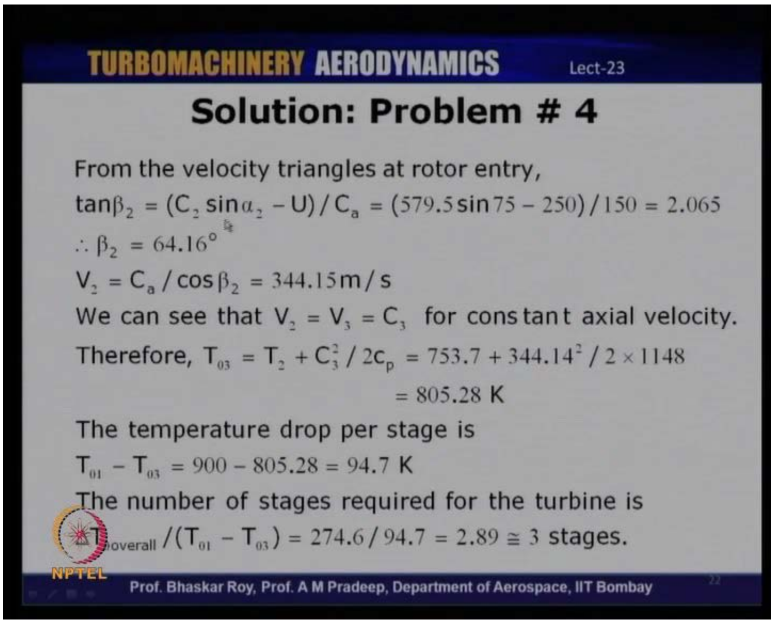

TURBOMACHINERY AERODYNAMICS Lect-23

Solution: Problem # 4

From the velocity triangles at rotor entry,

$\tan\beta_2 = (C_2 \sin\alpha_2 - U)/C_a = (579.5 \sin 75 - 250)/150 = 2.065$

$\therefore \beta_2 = 64.16°$

$V_2 = C_a / \cos\beta_2 = 344.15 m/s$

We can see that $V_2 = V_3 = C_3$ for constant axial velocity.

Therefore, $T_{03} = T_2 + C_3^2/2c_p = 753.7 + 344.14^2/2 \times 1148$

$= 805.28$ K

The temperature drop per stage is

$T_{01} - T_{03} = 900 - 805.28 = 94.7$ K

The number of stages required for the turbine is

$T_{overall}/(T_{01} - T_{03}) = 274.6/94.7 = 2.89 \cong 3$ stages.

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Now, at the exit of the velocity triangle at rotor entry well, let us look at rotors entry first; we have calculated alpha 2, which is known to us; let us calculate beta 2 as well, tan beta 2 we know is C 2 sin alpha 2 minus U by C a, beta 2 is C 2 sin alpha 2 that is this component minus U divided by C a; so that is C 2 sin alpha 2 minus U divided by C a, and that comes out to be beta 2 give comes out to be 64.16 degrees. And therefore, from this, we can calculate the relative velocity V 2, and that is C a cos beta 2, it is 344.15 meters per second; for constant axial velocity in an impulse turbine, we have seen that V 2 is equal to V 3 which is equal to C 3; and therefore, we can calculate the stagnation temperature at the rotor exit T 0 3 in terms of, because T 3 and T 2 are same T

0 3 is T 2 plus C 3 square by 2 Cp that is 753.7 plus 344.14 square by 2 in to C p that is 805.28 kelvin.

Therefore, the per stage temperature drops, stagnation temperature drops is T 0 1 minus T 0 3, and that is 900 minus 805.28 94.7 kelvin. We have already calculated the overall temperature drop, which was 274.6 that divided by 94.7comes out to be 2.89 and that is to be approximated to the next integer that is three stages, so we have we are required three stages for achieving this kind of a pressure drop in an generating this work output from this kind of impulse turbine, which has this efficiency and mean blade speed and so on. So, this is one of ways of estimating the number of stages that are required, basically they are calculating the overall temperature drop and calculating the temperature drop per stage diving the 2 will give us the number of stages required.

So, this completes this particular problem as well, we have solved so far 4 distinct problems, which have to do with different types of turbines, how to analyze velocity triangle and solved the problem using these velocity triangles. So, what I will do now, is to leave you with exercise problems, which you can solved based on what we have solved in today’s lecture, as well as the discussions we had in the last few lectures, I will also give the final answers of these questions, so you can check after you have solved these problems.

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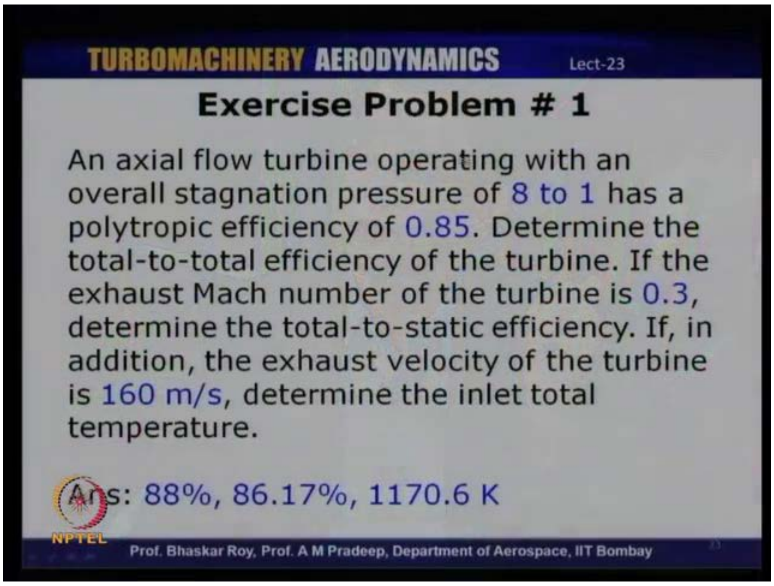

Exercise Problem # 1

An axial flow turbine operating with an overall stagnation pressure of 8 to 1 has a polytropic efficiency of 0.85. Determine the total-to-total efficiency of the turbine. If the exhaust Mach number of the turbine is 0.3, determine the total-to-static efficiency. If, in addition, the exhaust velocity of the turbine is 160 m/s, determine the inlet total temperature.

Ans: 88%, 86.17%, 1170.6 K

So, the first exercise problem is a following an axial flow turbine operating with an overall stagnation of 8 is to 1 has a polytrophic efficiency of 0.85, determine, the total to total efficiency of the turbine, if the exhaust Mach number of the turbine is 0.3, determine the total to static efficiency. If in addition, the exhaust velocity of the turbine is 160 meter per seconds, determine the inlet total temperature the answers to these questions are 88 percent that is total to total efficiency total to static efficiency is 86.17 percent, inlet total temperature is 1170.6.

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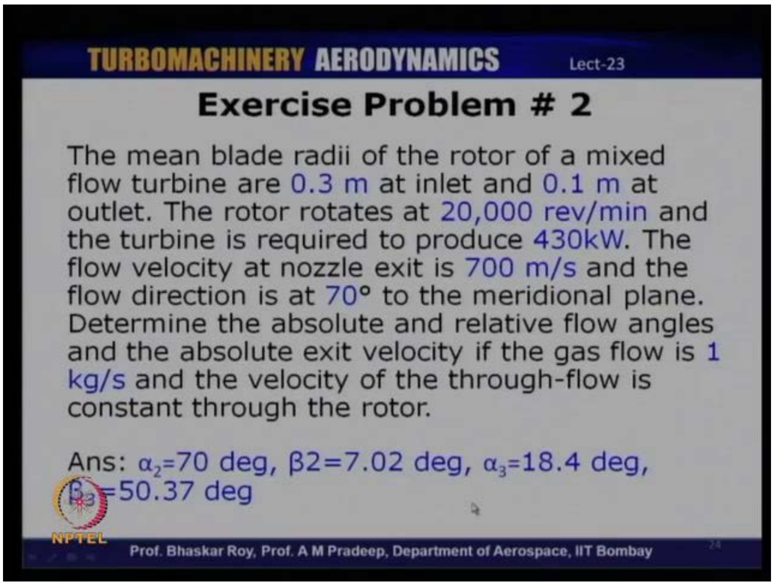

TURBOMACHINERY AERODYNAMICS Lect-23

Exercise Problem # 2

The mean blade radii of the rotor of a mixed flow turbine are 0.3 m at inlet and 0.1 m at outlet. The rotor rotates at 20,000 rev/min and the turbine is required to produce 430kW. The flow velocity at nozzle exit is 700 m/s and the flow direction is at 70° to the meridional plane. Determine the absolute and relative flow angles and the absolute exit velocity if the gas flow is 1 kg/s and the velocity of the through-flow is constant through the rotor.

Ans: α_2=70 deg, β2=7.02 deg, α_3=18.4 deg, β_3=50.37 deg

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The second exercise problem is the mean blade radii of the rotor of a mixed flow turbine is 0.3 meter at the inlet and 0.1 at the outlet; the rotor rotates at 20000 revolutions per minute and the turbine is required to produce 430 kilo watts; the flow velocity at nozzle exist is 700 meters per second and the flow direction is at 70 degrees to the meridional plane. Determining the absolute and relative flow angles and the absolute exist velocity. If the gas flow rate is 1 kg per second and also the velocity of the through flow is constant through the router, so the angles are alpha 2 is 70 degrees beta 2 is 7.02 degrees alpha 3 18.4 degrees and beta 3 is 50.37 degrees

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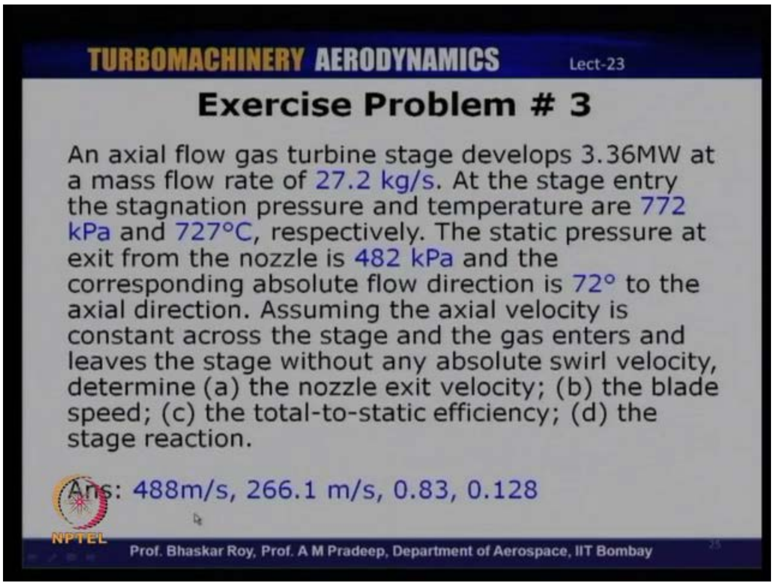

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Exercise Problem # 3

An axial flow gas turbine stage develops 3.36MW at a mass flow rate of 27.2 kg/s. At the stage entry the stagnation pressure and temperature are 772 kPa and 727°C, respectively. The static pressure at exit from the nozzle is 482 kPa and the corresponding absolute flow direction is 72° to the axial direction. Assuming the axial velocity is constant across the stage and the gas enters and leaves the stage without any absolute swirl velocity, determine (a) the nozzle exit velocity; (b) the blade speed; (c) the total-to-static efficiency; (d) the stage reaction.

Ans: 488m/s, 266.1 m/s, 0.83, 0.128

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The third question is an axial flow gas turbine stage develops 3.36 mega watts at a mass flow rate of 27.2 kg per second; at the stage entry, the stagnation pressure and temperature are 772 kilo Pascal and 727 degree Celsius respectively; the static pressure at exit from the nozzle is 482 kilo Pascal and the corresponding absolute flow direction is 72 degrees to the axial direction; assuming axial velocity is constant across the stage, and the gas enters and leaves the stage without any absolute swirl velocity. Determine, part (a) the nozzle exit velocity; part (b) the blade speed part (c) the total to static efficiency and (d) the stage reaction; answers are 488 meters per second, ythe blade speed is 266.1 meter per second, total to static efficiency 0.83 and the stage reaction is0.128.

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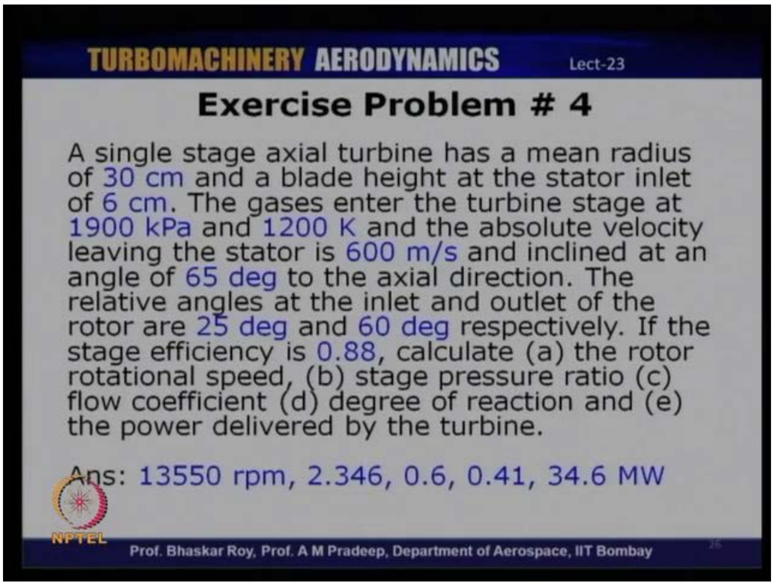

TURBOMACHINERY AERODYNAMICS Lect-23

Exercise Problem # 4

A single stage axial turbine has a mean radius of 30 cm and a blade height at the stator inlet of 6 cm. The gases enter the turbine stage at 1900 kPa and 1200 K and the absolute velocity leaving the stator is 600 m/s and inclined at an angle of 65 deg to the axial direction. The relative angles at the inlet and outlet of the rotor are 25 deg and 60 deg respectively. If the stage efficiency is 0.88, calculate (a) the rotor rotational speed, (b) stage pressure ratio (c) flow coefficient (d) degree of reaction and (e) the power delivered by the turbine.

Ans: 13550 rpm, 2.346, 0.6, 0.41, 34.6 MW

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And last question is the single stage axial turbine has a mean radius of 30 centimeters and a blade height of 6 centimeters; the gas enters the turbine stage at 1900 kilo Pascal and 1200 kelvin, and the absolute velocity leaving the stator of nozzle 600 meters per second and inclined at 65 degree to the axial direction; the relative angles at the inlet and outlet of the rotor are 25 degrees and 60 degrees respectively; if the stage efficiency is 0.88, calculate part (a) the rotor rotational speed part (b), stage pressure ratio part (c) flow coefficient (d) degree of reaction and part (e) the power delivered by the turbine; here, the answers are rotational speed is 13550 rpm, the pressure ratio is 2.346, flow coefficient is 0.6, degree of reaction is0.41, power delivered is 34.6 megawatts.

So, these are four different problems that I have put up as an exercise for you, and I hope with what we have discussed in last few classes as well as today’s tutorial, you will be able to solve these problems, based on these discussions. So, we will continue our discussion on axial turbines further in basically, looking at the 3D flows in further lectures. And, I hope you will be able to understand the basic looking of a three-dimensional flow in axial turbine based on the fundamental that we have discussed based on two-dimensional flow in axial flow turbines. So, we will continue discussion on some of these topics in the coming lectures.