Turbo machinery Aerodynamics

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Lecture No. # 10
Tutorial 02: Three Dimensional Axial Flow Compressor

We have been talking about various kinds of flows through axial flow compressor. We have done lectures on two-dimensional flow features, the two-dimensional flow theories and then we went on to extend that to three-dimensional flows, the simple three-dimensional flow theories, and then, of course, we did a full three-dimensional flow theory in a last lecture.

Today we will try to take a look at using the simple three-dimensional flow theories that we have done, essentially extension of the two-dimensional flow theory that you have done earlier and how that extended flow theory can be used to solve some simple problems. You see some of the simple problems that are available in the textbooks or those which can be created to get the hang of the basics of the subject, elements of the subjects can be solved with the help of, you know, simple three-dimensional flow theories which are extended two-dimensional flow theories. The more comprehensive 3-D flow theories that we have done indeed requires a large computing capability, and hence strictly speaking cannot really be, you know, solve through simple classroom lectures or simple problems that one encounters in textbooks or even the specialized books on a turbo machinery. We will be covering some of those aspects later on in this lectures series, where full 3-D flow theories are attempted to be solved. All the solution methods through various computational techniques would be discussed in some detail.

So, today we will go back to our simple flow theories that we had done and those as I mentioned are the extension of the 2-D flow theories, and those simple 3-D flow theories we will try to invoke in solving some standard problems. Now, these problems are essentially designed kind of problems. In this course, we are talking about turbo machineries which are specific products; you know, we are not talking about basic physics or basic mathematics; we are talking about specific products and that product are turbo machinery components, compressors and turbines. If you need to analyze those things, you need to design them first, you need to create them, only after the created you can really analyze them.

So today, the theories that we are going to do essentially are used for design and immediate pose design, performance analysis to cater to the design that has been carried out. So, the simple three 3-D flow theories are essentially used for various design purposes. So, let us take a look at those flow theories very quickly. So, first we will do a couple of problems, solutions of those problems, and then later on I will leave you with some two or three unsolved problems which you can solve on your own using the theories that we have done. Let us take a quick look at those theories.

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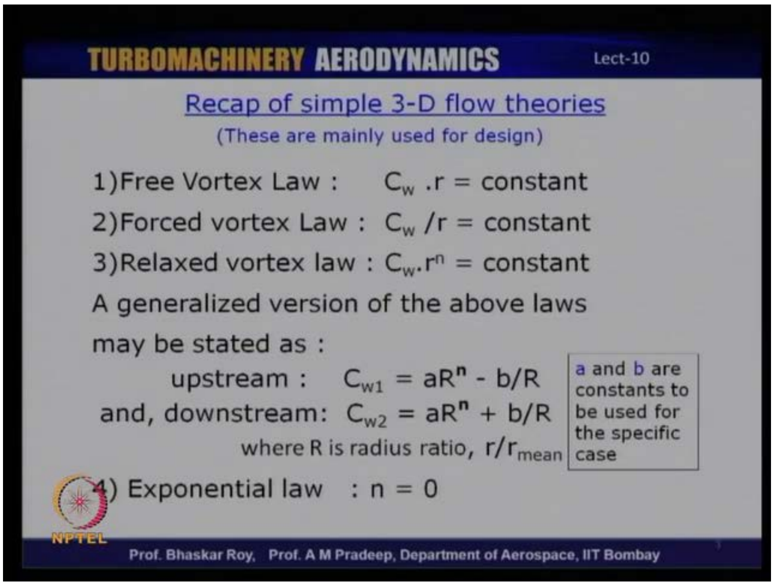

As I mentioned, these theories are essentially used for design and immediate pose design analysis. So, the simple theory is essentially that we have already put together can be now listed here. The first one of course is a free vortex law which was derived from the simple radial equilibrium equation and that leads as C w into r equal to constant. The corresponding opposite or theoretically the opposite of that is the force vortex law which in some books is also referred to as a solid body law, which means the flow essentially behave like a solid body and that is given as C w by r equal to constant.

The relaxed free vortex law or the generalized free vortex law - a vortex law was put together as C w into r to the power n equal to constant, where n is a parameter that could vary from minus 1 which is the forced vortex law to 1 which is free vortex law and any value in between the value of n could also be as we have seen more than 1. Now, in many textbooks and many other literature, a generalized version of the above laws is written down as for upstream flow C w1 equal to a R to the power n minus b by R, and downstream of the rotor C w2 would be a R to the power n plus b by R. Now, R here is the radius ratio which is small r by small r means, small r mean is the actual radius at the mean radius of the blade, and r of course is radius at any location on the blade from root to tip. Now, capital R is a ratio of these two parameters. And a and b are the two constants to be used for the specific case, they would indeed vary from one compressor rotor to another. So, before designing or applying these laws to a particular compressor, the values of a and b need to be found out or prescribed before this comprehensive version of the simple 3-D flow law can be applied.

Now, as we see here, the earlier stated laws in 1, 2, 3 have been generalized, and now they have applied upstream and downstream that is the flow before going into the blade can be subject to vortex law, and when it comes out at the rear of the blade, it can be again subject to vortex law and as we see here, the two laws are slightly different from each other; one is a R to the power n minus b by R and another is a R to the power n plus b by R, and this of course stands to reason, because upstream the value of C w1 is normally rather small are or rather low, and it acquires a large value of C w2 after going through the rotor, and hence that value is expected to be of a much higher order, because work has been done work has been supplied and the downstream flow actually carries that extra work, and indeed as we know C w 2 minus C w 1 is a measure of the work that has been put in into the flow through the compressor blades.

So, these are the simplified generalized vortex laws that are applicable to compressors, which we are going to use in the problems that we are going to encounter in today's lecture. A fourth law which we have stated also is stated as exponential law and it simply says that n that is written down in the earlier, you know simplified versions of the law is 0 and that gives us another law, earlier we are talking that n could be minus 1 to plus 1 or even more than 1 somewhere in between n is 0, and that value of a 0 actually caters to another law and that law is referred to as exponential law. And we shall see that this exponential law sometimes in some literature is also referred to as a law that creates or caters to creation of constant reaction blading. Now, we have seen this earlier in our lecture that some of the blades could have constant reaction from root to the tip of the blade of a stage and that constant reaction could be some value like 0.5 which is the more popular value of a constant reaction blading. However, if one goes into this theory a little more in detail or one can try to get into the numeric's of a particular problem solution, one would probably see that exponential law does not necessarily always aided to constant reaction blading.

So, constant reaction blading is not synonymous with exponential law. One can have a constant reaction blading, that is a separate issue and one can design one quite carefully to create constant reaction, but that does not necessarily cater to exponential law which is derived from the vortex law where all kinds of assumptions were need and those assumptions would not be valid, if you go for a pure constant reaction blading. So, constant reaction blading are not exactly in line with all the assumptions made in creating the vortex laws. So let us keep that in mind that the exponential law which have stated here at the end does not necessarily lead to constant reaction blading. Having stated the various laws that cater to creation of compressor blades and immediate post design analysis, let us take a look at a couple of problems, that would use a some of these laws and essentially through the numerical numbers would show you what happens if you apply this law or that law, what happens to the various parameters through the blades, how what happens to the blade shape, the blade geometry and the flow parameters through the blade. So, let us do a couple of problems to exemplify the laws that we have just stated here.

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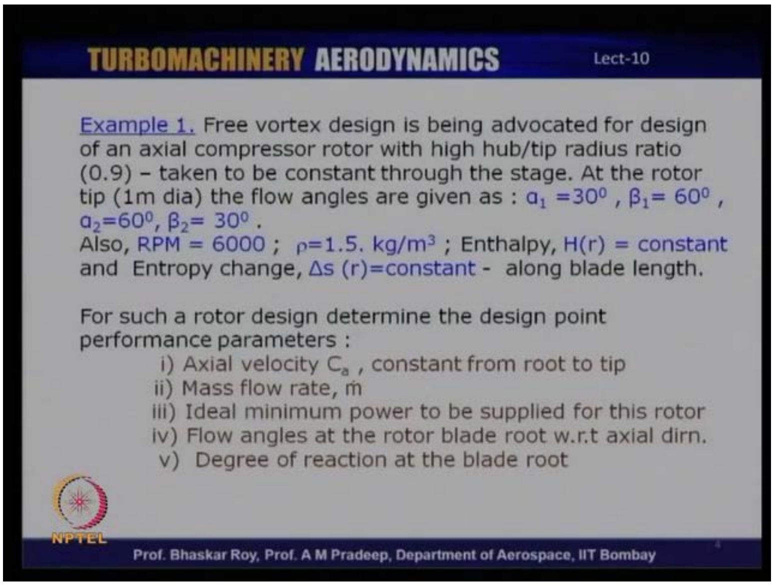

The first problem that we take as an example is a application of free vortex design that is being advocated for design of an axial compressor rotor, and this has a high hub to tip ratio of 0.9, we have talked about this earlier before and 0.9 is definitely high hub to tip ratio blade. And this high hub to tip ratio is taking to the constant through the stage that means the the high hub to tip ratio is constant from leading edge of the rotor till that the trailing edge of the rotor and indeed till the trailing edge of the stator. So, it is a constant radius hub and tip stage that we are advocating. Typically, this would be valid for a last stage of an axial flow compressor or one before last stage, and such a stage indeed have a high hub to tip ratio of the order of 0.9, and they indeed do have constant hub and tip radius. So, the problems being stated here is reasonably a realistic problem.

Now, at the rotor tip the diameter is one meter, and the flow angles given are alpha 1 equal to 30 degree, beta 1 equal to which is related flow angle 60 degree at the exit to the rotor, alpha 2 the absolute flow angle is 60 degree and the related flow angle beta 2 is 30 degree. Now, we can see here of course that the blade is essentially of a symmetric velocity triangle through which, would immediately mean that the tip of the blade has a degree of reaction 50 percent or 0.5.

Now, this compressor is been designed for rotating speed of 6000 rpm and the density of the working medium is 1.5 which indicates that 1.5 kg per meter cube which indicates that this is probably somewhere towards the rear of a compressor stage and definitely not the first stage in the rear where the density of the air has already gone up to somewhat higher values. Or it could be indicative of a working medium which is not air, but something heavier than air. Also stated in the problem are that the enthalpy H is constant along the radius from the root to the tip. And this is typically what we had also assume for free vortex theory if you remember. The entropy change across the rotor from root to tip is also held constant. So, the losses that we have done in our earlier lectures, essentially are cater to the entropy change, and this can also cater to the efficiency of the or isentropic efficiency of the compressor rotor and the compressor stage. And this entropy change is now being held constant from root to tip along the blade length.

Now, for this rotor the problem requires you to determine at the design point at which all those values are prescribed the performance parameters, that means you try to find out what is actual velocity, which is again prescribed to be constant from root to tip. Then, the mass flow rate which is passing through the blade and that needs to be found out what should be the mass flow rate that the compressor should process, quite often mass flow rate is indeed a primary parameter for which a compressor is designed. So, mass flow rate here is being asked to be found out given the blade geometry. Then, the ideal minimum power to be supplied to this rotor, this means no efficiency is coming into picture. We are assuming which is 100 percent efficiency and as a result the turbine that is being used to supply power or any other prime mover has to supply a minimum amount of power to do this compression job.

So, what is the minimum power or what is the power that the compression is doing. So, that is the minimum power that you need to supply. So, that it needs to be also found out in many actual designs that power may be available that power may be available or sometimes you need to find out how much power you need to be supplied with to do a prescribed amount of compression. And the next thing you need to find out of course, other flow angles at the blade root, flow angles have been prescribed here only at the tip. Now, you need to find out the flow angles at the root then at the mean, and then of course you need to find out the degree of reaction. We have defined degree of reaction and if you remember degree of reaction essentially a two-dimensional parameter. So, this problem and as I mentioned, in today's lecture we are doing problem we are which in which we are using extended two-dimensional flow theories; three-dimensional theories - simple three-dimensional theories, in which many of the two-dimensional concepts are also being used and used to solve the problems at hand. So, degree of reaction is a two-dimensional definition, but this problem wants you to find the value of degree of reaction at the blade root. So, let us see whether we can find a solution to the problem given the problem statement as it is.

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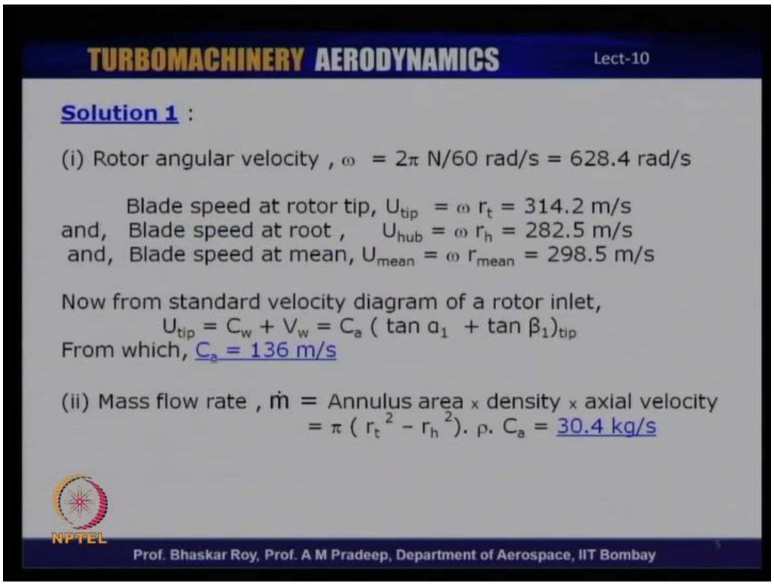

Now, given the problem that the rpm is 6000 rpm, first thing is we can find out the angular velocity and that is 628.4 radiance per second. Now, this angular velocity normally is to be held constant from root to tip. So, with with the use of this angular velocity, we can now find the actual blade speed - the solid body blade speed at tip root and mean. So, at the tip its omega into r tip and that would be 314.2 meters per second, at the hub it is omega into r hub and there is 282.5 meters per second and at the mean that is exactly the mean between hub and the tip, the U mean is omega into r mean and that would be to equal to 298.5 meters per second. So, these are the three solid body blade speeds with which the blades are actually rotating and with them you are not have to find out what kind of work they can actually do.

Now, from the standard velocity diagram that we have done in all lectures, indeed you can have compressors with some unusual kind of velocity diagrams, but we are not doing those things at this moment, though specially the transonic ones we are not bothered about those things in this particular problem. So, we will take some standard velocity diagrams of a rotor inlet. In which U tip would typically be C w plus V w, now at the inlet it would be U tip 1 or U 1 and that would be C w 1 plus V w 1. Now, this would be then C a in to tan alpha 1 plus tan beta 1 at the tip. Now, we have already prescribed in this problem that the value of C a is constant from root to tip. They already found U tip above which is 314.2 meters per second, and at the tip the value of alpha and beta are already prescribed. And if you can use those values of 30 and 60 degrees and put them over here, you can directly find the value of C a and the direct calculation gives you C a equal to 136 meters per second. So, that is the actual velocity, constant from root to tip that the flow through this compressor blade is being designed for. So, that is actual velocity.

Now, the mass flow through the compressor rotor or compressor that we have at hand is indeed directly found from using the continuity and that is the annulus area into the actual velocity which gives the volume flow rate and that into density is the mass flow rate. Now, the annulus area of this particular compressor is phi equal to r tip square minus r hub square and that is the area annulus area, all compressors is essentially have annulus area subtended between the tip and the hub, and that into the actual velocity C a is the volume flow rate through this compressor, and that into the density prescribed value is indeed would be the mass flow rate, and that then turns around to be if you put in all the values are available, the mass flow rate turns out to be 30.4 kilograms per second. So, that is a mass flow rate that has been asked for and now we can we see that it can be directly found, having found the actual velocity in the earliest step.

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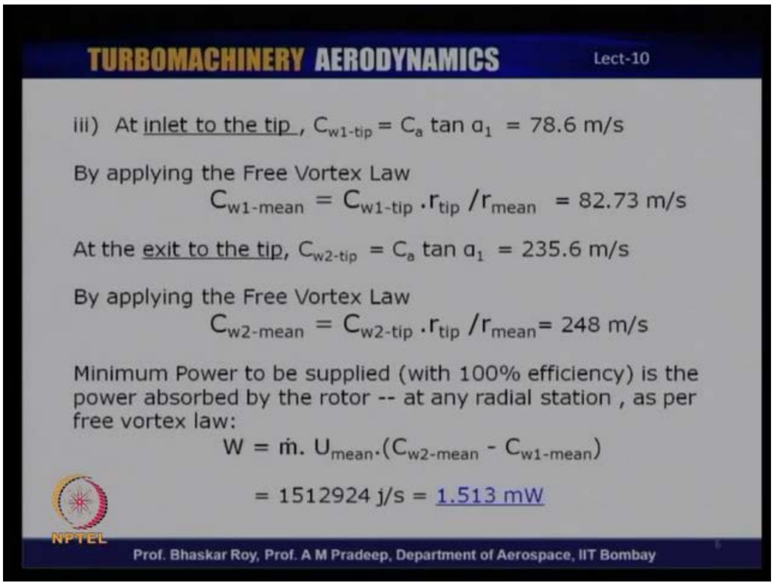

The next things to be found are the whirl components of the tangential components of the velocities. Now, at the inlet at the tip C w1 tip can be found out from C a into tan alpha 1 at the tip. And that comes out to be C w1 tip 78.6 meters per second. Now, if you apply the free vortex law, which is simplified 3-D flow theory. If you apply the free vortex law, you can find the value of C w1 mean, and as a result of which you can see that the value at C w1 mean would be 82.73 meters per second, applying C w into r equal to constant. At the exit to the tip, the C w2 is C a into tan alpha 2, and this would be 235.6 meters per second. Now, this can be used to find again using free vortex law. The C w 2 mean that is at the mean radius and C w 2 mean would then be again applying the free vortex law of C w into r equal to constant, and that could come out to be 248 meters per second. So, we can find C w 1 mean and C w 2 mean that means before the rotor and after the rotor in both the stations applying free vortex law. A free vortex law can be applied between this at the inlet to the rotor at the exit to the rotor or some people sometimes use it in the middle of the rotor at the mean passage. But in this problem we we are not doing that we are applying it at the inlet and at the exit, and having applied that we have got the values of C w 1 mean and C w 2 mean.

From this, we can now find that the minimum power to be supplied that means with hundred percent efficiency of the compressor turbine, and the supply shaft, intermediate shaft between the turbine and compressor if all that is hundred percent, then the power absorbed by the rotor is indeed that the power is required for doing the compression job, and that is typically as for free vortex law is is supposed to be constant from root to tip. So, if you find the value at the tip or at the mean anywhere, it would be a valid everywhere, and that work done is m dot into U mean into C w 2 mean minus C w 1 mean and that gives us a value of 1.513 mega watts. So, that is the kind of a power you would need to activate this particular compressor for that kind of a mass flow, and given the prescribed rotating speed of 6000 rpm. So, given all the parameters we can now see that the amount of work that would be necessary to activate this compressor is 1.513 mega watts.

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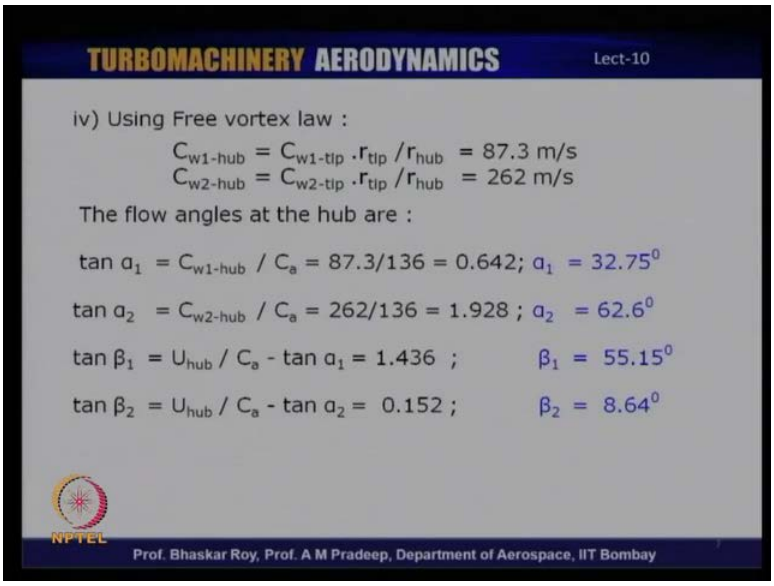

Now, if you apply the free vortex law, you can find the value of C w1 hub and C w2 hub that means the C w values before the rotor and after the rotor using the free vortex law of C w r equal to constant, and that comes out to be 87.3 meters per second, and C w 2 would be 262 meters per second. Now, using all these values, we can now use the velocity triangles that you have done in quite a great detail in the earlier lectures. And those velocity triangles simply yield that the flow angles at the hub tan alpha 1 equal to C w1 hub by C a equal to 87.3 by 136 which is the actual velocity, and that would be point 0.642 and hence the alpha 1 is 32.75 degrees; alpha 2 which is the absolute flow angle at the exit to the rotor would then be C w2 hub divided by C a, and that 262 as we are found earlier divided by 136, and that value of tan alpha 2 is 1.928 and that yields alpha 2 of 62.6 degrees.

The corresponding value of tan beta 1 comes out to be again using the values available at U hub and that is 1.436 which is a value of beta 1 of 55.15 degrees; corresponding time beta 2 if you calculate the same way comes out to be a small value of 8.64 degrees. Now, you can see here that at the exit to the rotor the value of beta 2 is rather small; at the entry the value of beta 1 is rather high. Now, that is the angle at which that is a relative flow angle with which the flow is going into the rotor blade, and that is what is often referred to as the blade stagger angle. That is angle at which the blade would need to be oriented to get the blade in working order. So, beta 1, beta 2 are the relative flow angles at which the blade rotor blades would have to be oriented to get the work done. So, those are the values that we get from using the free vortex law that has been prescribed in this problem.

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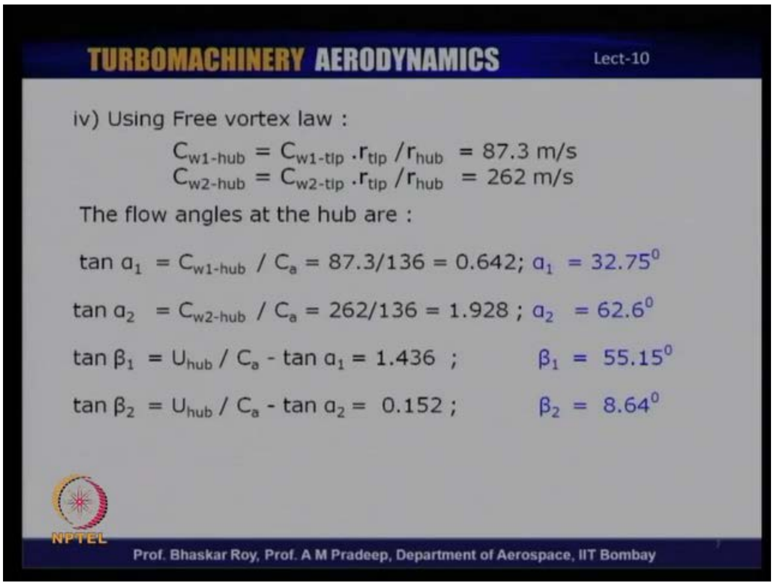

The next step in this problem is to find out the degree of reaction at the hub as required. And degree of reaction and the hub can be found out by using the degree of reaction relation that we have done in the lecture series, lectures earlier, and that is tan beta 2 hub minus tan alpha 1 hub into C a by 2 U hub. All these values have already been calculated in the earliest steps, and if you just put in all the values, you get a degree of reaction R x hub equal to 0.382. As you can see here, the degree of reaction at the hub is much lower than that at the tip where it was prescribed as 0.5.

So, at the hub it is expected that the degree of reaction in a free vortex design would be lower and as we have stated in our lectures earlier that one needs to keep an eye on this value, because the value of degree of reaction at the hub, if one is not careful especially in a free vortex design could indeed go pretty close to 0 or indeed if one is not a very careful it could go below 0 that means it could be negative, and in negative degree of reaction compressor blade is quite useless as a compressor, because it is now starting to behave like a turbine. So, degree of reaction negative is certainly not acceptable and in free vortex design, there is a tendency for the degree of reaction near the hub to become very low quite close to 0, and if one is not careful, it could become less than 0. So, that is why in any design very quickly one needs to find out what is the degree of reaction at the hub. Because that is where it could go rather low, rather dangerously low and that is normally not acceptable. So, these are some of the things that one needs to be careful about when one using the free vortex design.

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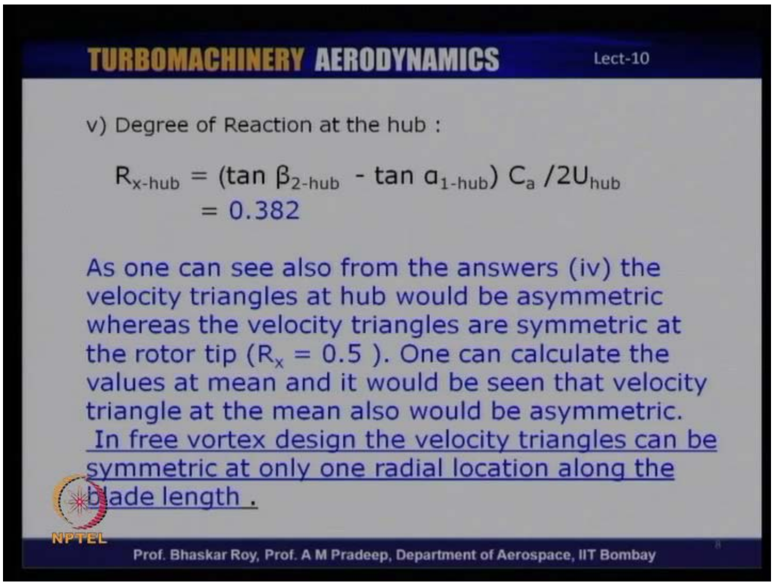

The other issue is that the free vortex design actually shows that it can be symmetric at only one radial location along the blade length. In this problem, it was prescribed that degree of reaction R x is equal to 0.5 or 50 percent at the tip which means the diffusion split between rotor and stator is 50-50 at the tip. There are many designers who would like to do that at the mean, if you do that at the mean as we have discussed before if you remember, at the tip the value of degree of reaction would be much higher than 0.5, but at the hub it could go rather dangerously close to 0. This particular design proposes that it is 0.5 at the tip, and as a result of which at the hub it is very safe, and it is about it is 0.382.

So, the value of degree of reaction is another issue which needs to be kept an eye on even one is using free vortex design degree of reaction, as we know is essentially a two-dimensional parameter, but this is where it helps the three-dimensional blade design that it tells you when the design may possibly be going towards the wrong direction, if you do not keep an eye on the degree of reaction. And of course a symmetric blading, the ideal the original symmetric blading design or 50 percent reaction is possible in free vortex design only at one place. If you want symmetric blading from root to tip and people have done such design earlier, you have to go for a constant reaction blading. So, in this problem that was not prescribed, it was prescribed as a free vortex design.

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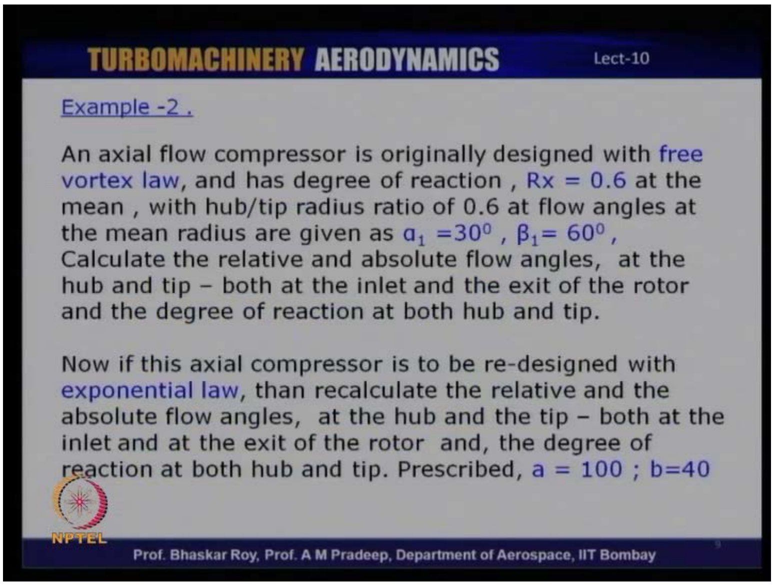

Let us go to the next problem, the example 2. And in this the axial flow compressor is originally designed with free vortex law, and this free vortex law has a degree of reaction 0.6 at the mean, with hub to tip ratio of 0.6, with flow angles at the mean radius given are alpha 1 equal to 30 degree and beta 1 equal to 60 degrees. What is required first is that you calculate the relative absolute flow angles at the hub and the tip, that completes the so called blade is geometry blade design, and both at the inlet and the exit, and to complete the whole thing you calculate the degree of reaction at both hub and tip. As we have seen those are the critical values that need to be checked out at the beginning of the design itself.

So, this problem states that you should find out those values as per free vortex law to begin with. Now, if this axial compressor is to be redesigned with exponential law which we have discussed earlier, then it is prescribed that you recalculate the relative and the absolute flow angles at the hub and at the tip both at the inlet and at the exit of the rotor, and then corresponding degrees of reaction at both hub and tip for this exponential law, it is prescribed at the value of a is 100 and the value of b is four 40 that is prescribed for this exponential law to be applied for redesign of this original compressor blade which was designed with free vortex law. Let us see how we can proceed to solve this problem.

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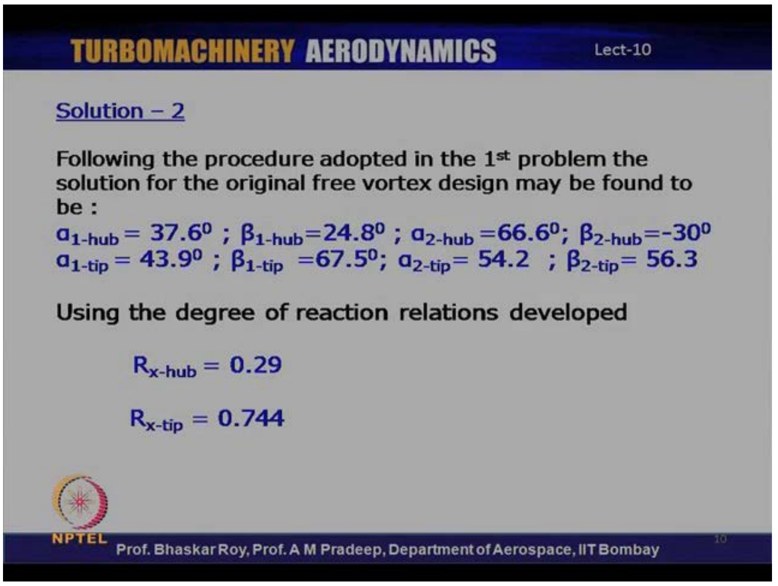

Now, the original free vortex law designed can be directly applied, as we had done in the first problem, and the procedure that was adopted in the first problem can be adopted again to find the original free vortex design of this problem, that gives us alpha 1 is equal to 37.6 degrees, beta 1 at the hub is 24.8 degrees, alpha 2 at the hub is equal to 66.6 degrees and beta 2 at the hub is equal to minus 30 degrees. Now, this is something that can happen that beta 2 at the hub can become minus which means it is gone on the other side of the actual direction, whereas the other flows are on the other side of the actual direction. The values of the tip are alpha 1 tip is equal to 43.9 degrees, beta 1 tip is equal to 67.5 degrees, alpha 2 at the tip is 54.2 degrees that is at the rear of the rotor and beta 2 at the tip reality flow angle is 56.3 degrees.

Now, we can see here that beta 1 in the hub is very low that is 24.8 degrees, beta 1 at the tip is 67.5 degrees and those are the relative flow angles, and also essentially are close to the stagger angle of the blades. So, at the tip the stagger angle is very high, and the blade is set at a high angle, and at the hub the blade is more actual actually set. So, those are the typical flow angles that you can get of typical flow free vortex design and those angular settings are also indicative of the kind of values, you may expect out of a typical free vortex design. At the hub, the values as you can see at beta 2 at the hub has gone negative, it can indeed go negative, because the hub airfoil at the is typically of a very high camber and it takes the flow from positive flow angle to negative angle; whereas at the tip the you can see here the beta 1 tip is 67.5, beta 2 tip is 56.3 which indicates that delta beta at the tip is only about 11 degrees 11.2 degrees. That means it is a low camber airfoil, whereas at the tip it was 54 degrees of camber. So, at the hub it was 54 degrees of camber.

So, typically you would in a free vortex design, you would have a hub camber that is highly camber blade, at the tip it is a flattered blade. So here in this problem, the tip camber is 11 degrees and hub camber is about 54 degrees. So, also it tells us that the beta 1 and beta 2 values have changed a lot from root to the tip of the blade which is indicative that the blade would be highly twisted. And that is also typical of free vortex that blades do tend to be highly twisted blades. That is something which the early designers found out and that is one of the reasons, the other vortex laws were created. So, that the blades are not always very highly loaded in a highly twisted, because highly twisted blades do get structurally very highly loaded. The high twist creates structural loading and that is a bit of a problem in modern compressor. So.. But that is typical of free vortex design that you get a highly twisted blade. The camber is very high at the hub rather low at the tip and you end up getting a highly twisted blade.

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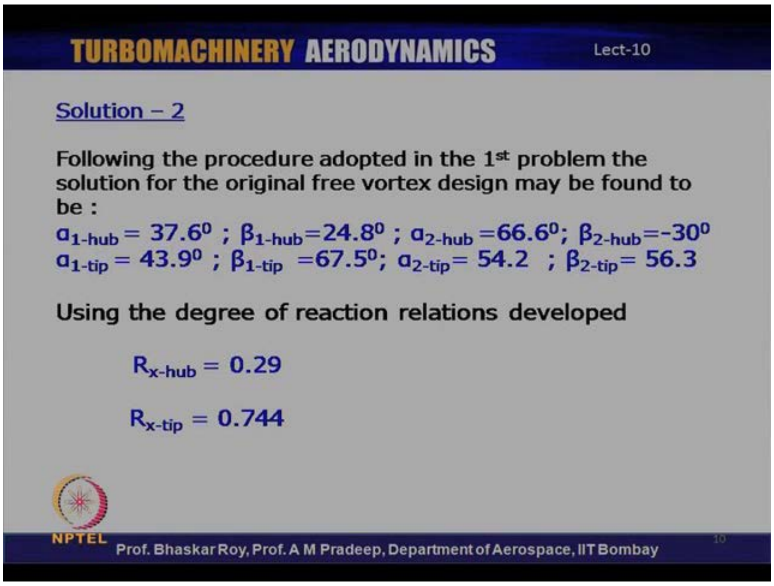

Let us now look at the degree of reaction. In this particular problem, the degree of reaction prescribed was at the mean, and using those prescribed values we get the free vortex degree of reaction at the hub 0.29 which is a good value, and at the tip it is 0.744 which is again a good value. So, in the two degrees of reaction that we get are safe values at the hub and at the tip. So, these are the free vortex values of the original blade design. What we need to do now is recalculate these values using the exponential law.

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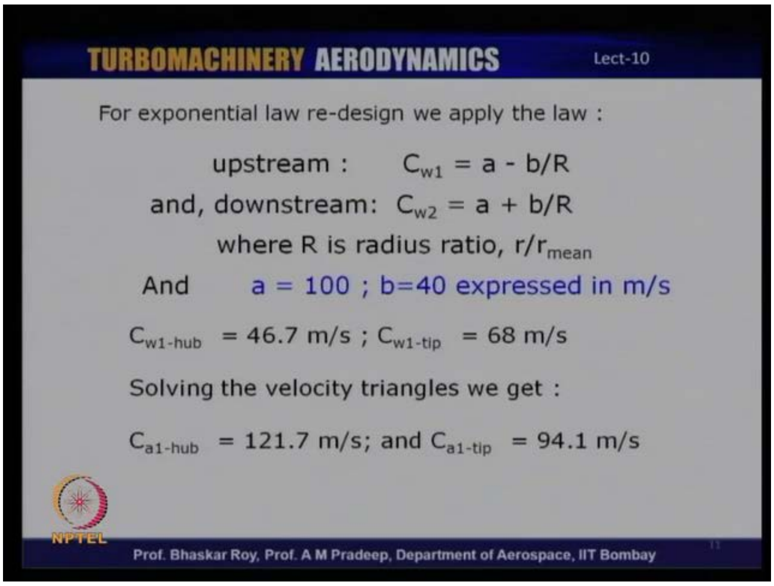

Now, if we go for the exponential redesign, the law can be stated as C w1 equal to a minus b by R upstream of the blade, and C w2 equal to a plus b by R downstream of the blade where capital R here is the radius ratio which is r by r mean that is related to the mean radius of the blade, and the values a and b given are 140, these can actually be a dimensional values are expressed in terms of meters per second as per the law that is stated here, which directly gives us the values of C w1 and C w2 in meters per second, because R is a non-dimensional parameter, it is a radius ratio really.

Using this law now from the prescribed values that were given to us in the problem statement C w1 hub would be equal to 46.7 meters per second, C w1 tip is 68 meters per second which are comparatively low values at the inlet to the rotor. C a1 hub is 121 meters per second and C a1 tip is 94.1 meters per second by solving the velocity triangles. Now, as we can see here C a1 is not constant, in free vortex design it is normally assumed and it is prescribed, and it is held constant from hub to tip. The moment you are weird away from the free vortex law and using other kinds of law C 1 C a is not constant anymore. It varies from hub to the tip and you can solve the velocity triangles to find those values.

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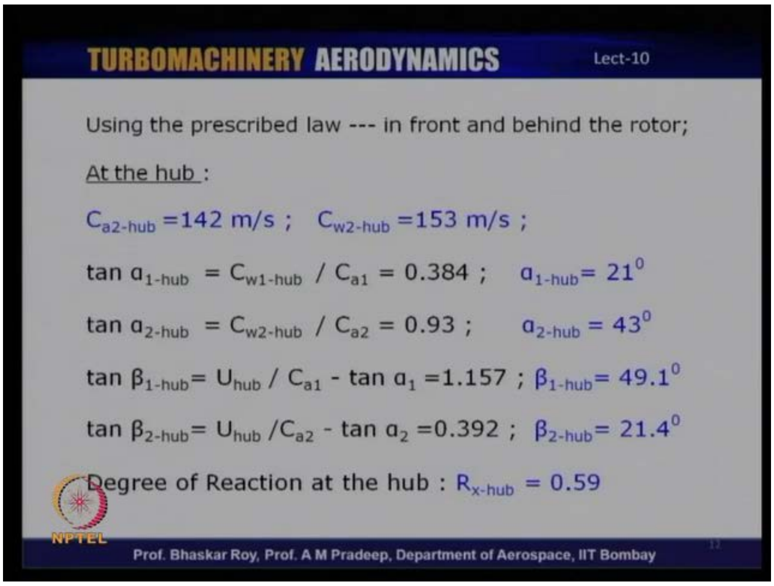

Now, using these values and using the prescribed law in front and behind the rotor, one can find at the hub C a2 hub that is behind the rotor 142 meters per second, C w2 hub is now higher than C w1 is 153 meters per second. Now, using all these values of velocities the whirl component and the actual components, we can find the angles. Now, tan alpha 1 hub is equal to C w1 hub divided by C a1, and this would yield an alpha 1 hub of 21 degrees. And tan alpha 2 hub is C w2 hub minus C a at by C a2 and this yields alpha 2 hub of 43 degrees; tan beta 1 hub is equal to U hub by C a1 minus tan alpha 1 and this yields a beta 1 hub of 49.1 degree; and tan beta 2 hub yields a value of 21.4 degree using similar relation.

So, as we can see here now, the relative flow angles beta 1 and beta 2, beta 1 is very high at the inlet, but it is comparatively low at the exit. And now we can see here beta 2 at the hub is actually positive, it is no more negative and this is what it has done. So, one can say that the flow turning here is of the order of 27 degrees a little more than 27 degrees not as high as we had seen free vortex design, where it was almost double of the order of 54 degrees. Hence using the exponential law, the degree of reaction at the hub R x hub is equal to 0.59 which is a very safe and good degree of reaction, and correspondingly we can use the prescribed exponential law and find the values at the tip.

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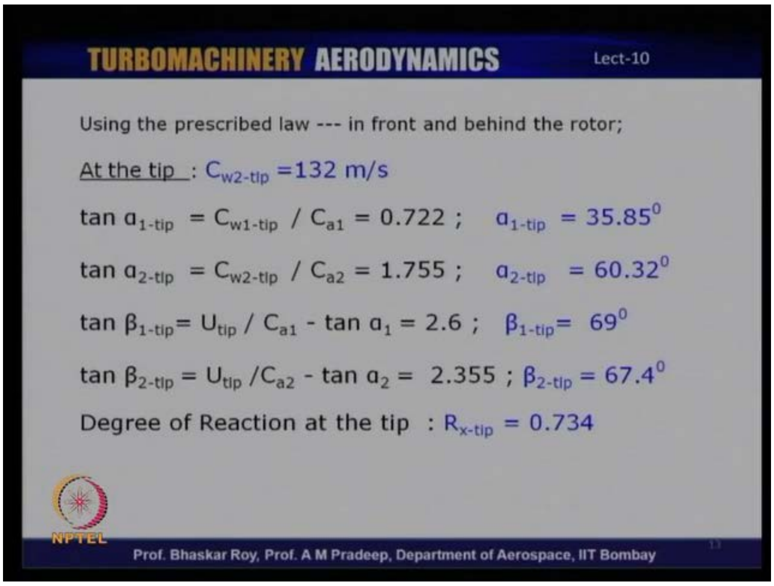

The C w2 tip it would be 132 meters per second that is a whirl component. Correspondingly now, tan alpha 1 tip would be C w1 tip by C a1, and that would yield a value of alpha 1 tip equal to 35.85 degrees, tan alpha 2 tip is equal to C w2 tip divided by C a2 and that would yield a value of 60.32 degrees, tan beta 1 tip U tip by C a1 minus tan alpha 1 that gives a value of 2.6, and beta 1 tip of 69 degrees, that is a very high beta 1, but that is typical of compressor rotor blade where the relative flow angle at the tip is the highest flow angle. Beta 2 tip correspondingly as we see here now comes out to be 67.4 degrees which indicates that the camber at the the tip is very very small of the order of just about 2 degrees all it less than that.

So, the degree of reaction at the tip is Rx tip is equal to 0.734 which essentially means that you have much more of a diffusion going on in the tip. The work done at the tip is decided by the camber is decided by the U tip which is rather high. And more of that work is now being converted to pressure inside the tip blade passage, and much less would be left for the stated to diffuse. So, degree of reaction of the order of 0.7 or higher indicates that rotor is participating in the diffusion of the flow more than that in the stator. So, the degree of reaction at the tip has been found to be 0.735.

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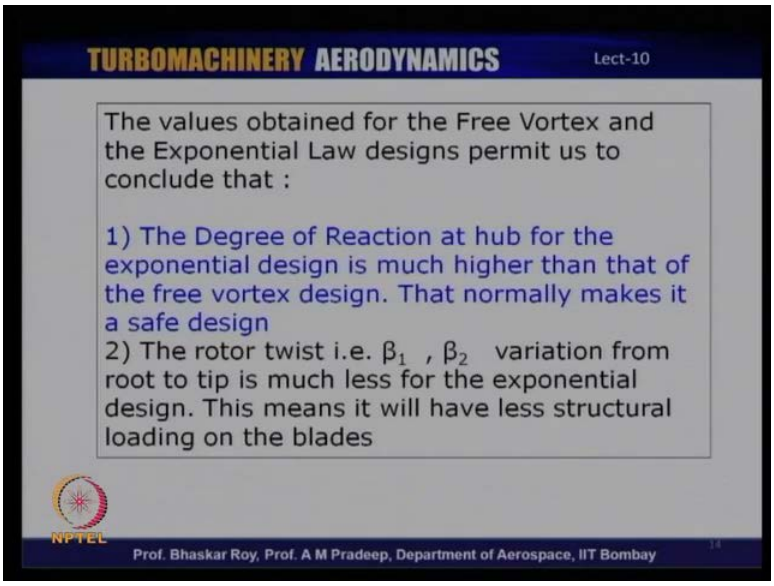

So, the values obtained for of original free vortex design and the redesign exponential law permits us to now to make a few remarks - concluding remarks of this particular problem solution. The degree of reaction at the hub for the exponential design is much higher than that of the free vortex design and that makes it very safe design. There is no way that particular blade would encounter negative degree of reaction under any operating condition. The rotor twist that is the values of beta 1 and beta 2, and the variation from root to tip is much less for the exponential design. Now, this means that the blade is much less twisted. Now, this is what I was talking about that free vortex design tends to be a high twisted blade. People resort to exponential design and other kinds of vortex laws to bring the twist down, because highly twisted blade can get very highly stressed under the aerodynamic loading of the on the blade surfaces.

So, the structural loading of the blade is now substantially reduced and this is true of the modern compressor blades where the pressure rise is higher, blades are indeed aerodynamically highly loaded, and high aerodynamic load in combination with blade twist would create very high stress levels on the blade solid body, and that would essentially could be prohibitive from the design point of view. So, exponential law essentially relieves those stresses by reducing the twist of the blade. So, this is how the second problem has been solved and we have given a redesign blade, and its blade geometry as a part of this solution.

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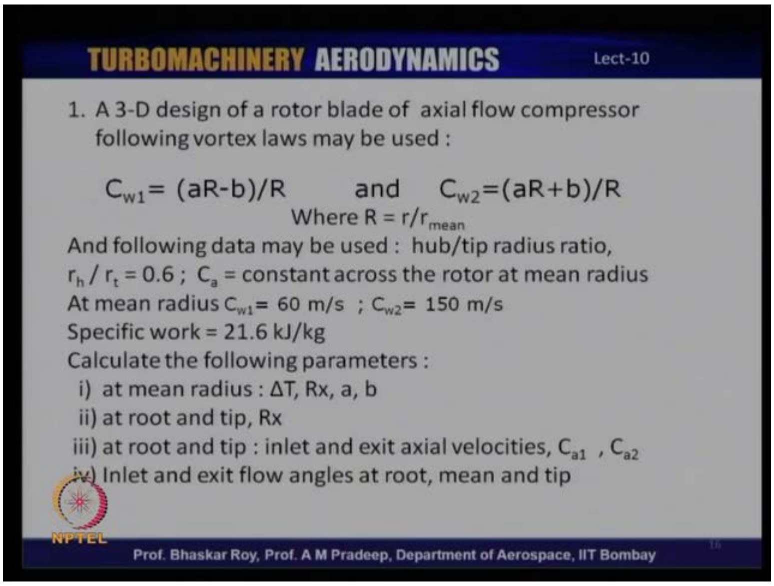

Now, I will leave you with a few problems which you can attempt to solve on your own on the basis of the theories that we have done, following the examples that we are given in this lecture. So, I will leave you with a few tutorial problems for your own exercise. The first problem is a 3-D design of a rotor blade of axial flow compressor following the following vortex laws may may be used; first is C w1 equal to a R minus b by R that is upstream of the blade, and the other law is downstream of the blade would be C w2 equal to a R plus b divided by R where R is indeed the radius ratio with respect to the mean radius.

The following data may be used. The hub to tip radius ratio is prescribed as 0.6, which is a moderate hub to tip ratio. C a which is the constant across the rotor from the inlet to the exit at mean radius, and the mean radius C w 1 is 60 meters per second, C w2 is 150 meters per second, the specific work that is prescribed and is being supplied or is going to be supplied is 21.6 kilo joules per kg, and that is the work that you would be supplied with. With these prescriptions calculate the following parameters, at mean radius delta t equal delta t that is the temperature rise through this compressor, the degree of reaction Rx, the values of a and b that are given in the laws, they are to be now found out. At the root and the tip find out the degree of reaction, at the root and the tip find out the inlet and exit axial velocities is C a1 and C a2; that is at the inlet and at the exit of the rotor. And the inlet and the exit flow angles at the root mean and tip. That means all the alphas and betas at the root mean and tip at the exit and at the inlet to the blades. So, those are all the parameters that would complete the entire blade design, and you have to find out given here the work that is being supplied and the vortex laws that have been prescribed. So, that is the first problem. Let us take a look at the second problem.

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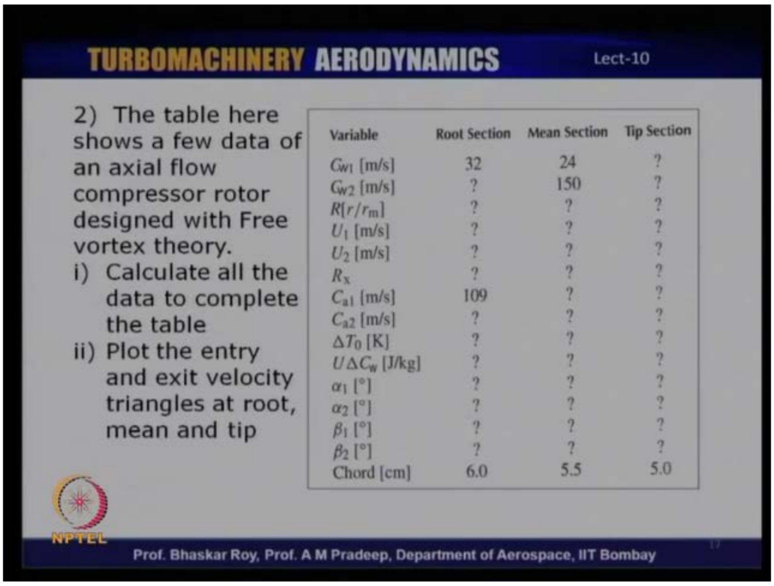

Now, the in the second problem, the table a table is being given here, it shows a few data of an axial flow compressor rotor which is been designed with free vortex theory. Given a few data, you are to calculate all the data to complete the table. So, few data are given with those data can you complete the entire table, actually the problem is very similar to the earlier problem, calculating all the alphas and betas, and C a 1 and U 1 and so on, and so far. So, all the flow velocities, all the flow angles, and then the work done, and then of course the degree of reaction; so, all those values that we have been calculating essentially has now been put in a tabular form, and you are asked to complete the table. You can also plot the entry and exit flow velocity angles at the root, mean and tip and that will give you an idea, what kind of blade you are getting out of this particular problem.

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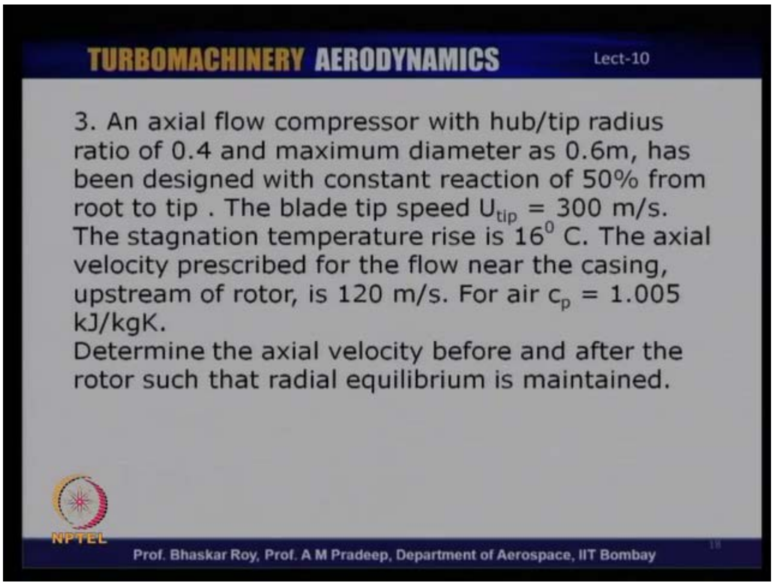

The third and the last problem that I will leave you with is an axial flow compressor with hub to tip ratio of 0.4 which you will notice is a somewhat lower hub to tip ratio as we have discussed earlier in our lectures. And the maximum diameter for this compressor is 0.6 meters, and this has been designed with a constant reaction of 50 percent of 0.5 from root to tip. So, this is a constant reaction design which we have discussed before in the lecture and a little bit today also. And this has the blade tip speed of U tip of 300 meters per second. The stagnation temperature pressure rises 16 degrees centigrade which means it is a comparatively lowly loaded compressor. The actual velocity prescribed for the flow near the casing upstream of the rotor not exactly at the leading edge, but it will upstream is 120 meters per second. For air you can take the value of C p as 1.005 kilo joules per kg K as we have probably done in all the earlier lectures and the earlier tutorial that you have done.

The problem asked you to determine the actual velocity before and after the rotor in such a manner that the radial equilibrium is maintained or obtain. So, that is how the 3-D flow is brought into this particular problem. That you are to maintain radial equilibrium and if you are to maintain radial equilibrium, can you get the actual velocity before and after the rotor, and as I was stating earlier, if the axial velocity is constant across the rotor as it is assumed in a free vortex design, you cannot get a constant reaction from hub to tip. So, the constant reaction design is an intrinsically a contradictory to constant axial flow across the blade. So, this problem exempli exemplifies that particular understanding and asks you to actually calculate those values to essentially formalize that particular understanding.

So, I hope you will be able to sit down and do the calculations using the same steps that we have done in the earlier examples, and you should have no problem getting the answers. The accuracy of the answers depends on the accuracy of the some of the earlier parameters like C p and all that you use. So, I hope you will be able to solve the problems and get your answers; the trend of the answers are already been given in the two examples that we have taken. So, you should be able to quickly make out whether you are getting the answers that are reasonably correct answers. With this we come to the end of today's lecture that is the solution of the problems and problems for your exercise.

In the next class, we will be moving towards some of the other issues of compressors which are related to compressor instability, flow distortion in the compressor which are indeed extremely vexing issues and we shall look at the physics of the problem in some detail in the next few lectures, trying to look at issues that are three-dimensional and much more than three-dimensional, their distortions and sometimes they are born out of or leading to instability in the compressors. Those are the issues we shall be doing in the next few lectures.