

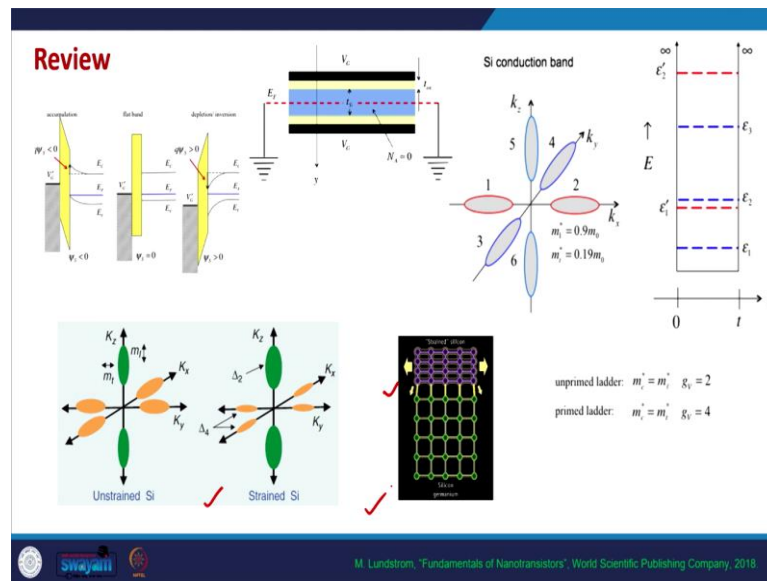
**Physics of Nanoscale Devices**  
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**Lecture - 57**  
**Thermoelectric Effects**

Hello everyone. Today, we will discuss about Thermoelectric Effects and as we have seen in our previous class that even in all the electronic circuits heat is generated. So, it is quite important to understand the relationship between heat and charge currents basically.

So, we will discuss about two important thermoelectric effects in today's class basically Seebeck effect and Peltier effect. Before going into this discussion, let us quickly review what we have seen so far.

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So, so far we have seen quantum confinement in MOSFETs and we also discussed about the strain engineering in silicon specially in nanostructures in ETSOI MOSFETs. And, what we saw was that if we grow the channel silicon on a silicon germanium substrate, in that case we can change the band structure of silicon in such a way that first effective mass is modified such that the mobility is increased and second the intervalley scattering in silicon is reduced.

So, by using strain in proper direction, we can enhance the mobility or injection velocity of electrons in the silicon channels. And that is one of the, I would say one of the very smart ways of improving transistor performance without changing the dimension of the transistor.

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**Introduction**

$$I = -I_x = \frac{2q}{h} \int T(E)M(E)(f_1 - f_2)dE$$

Electrons are particles that carry both charge and heat.

**Voltage difference:**  $(f_1 - f_2) \approx \left(-\frac{\partial f_0}{\partial E}\right) q\Delta V$

**Due to temperature difference:**  $\Delta T_L = T_{L2} - T_{L1} > 0$   $f_1 > f_2$  for energies below  $E_F$   
 $f_1 < f_2$  for energies above  $E_F$

**In near equilibrium case:**  $(f_1 - f_2) \approx f_1 - \left(f_1 + \frac{\partial f_1}{\partial T_L} \Delta T\right) = -\frac{\partial f_0}{\partial T_L} \Delta T_L$

*Handwritten notes:*  $\epsilon_c$ ,  $\epsilon_v$ ,  $n\text{-type: } I < 0 \rightarrow$ ,  $p\text{-type: } I > 0$

M. Lundstrom, and C. Jeong "Near-Equilibrium Transport", World Scientific Publishing Company, 2013

So, also we also started discussing thermoelectric effects and we started with this familiar expression. And, this is the expression for current in a conductor, in a conductor like this is a semi-conductor material which has contact 1 on left side, contact 2 on the right side. The current through this bulk semi-conductor is given by this expression and this we have already seen, this expression essentially comes from the this comes from the Landauer's general model of transport.

So, what we understand here is that  $(f_1 - f_2)$  is the is an important quantity, it is actually the forcing function for the current. So, if  $(f_1 - f_2)$  is 0 in that case the current will also be 0 and there is no electrical conduction in the circuit, but if  $(f_1 - f_2)$  is non-zero in that case there is a non-zero current in the circuit.

And, what we also saw was that  $(f_1 - f_2)$  can become non-zero by number 1 voltage difference, if there is a voltage difference between terminal 1 and terminal 2, in that case  $(f_1 - f_2)$  will be non-zero. So, this is properly understandable from this point, in this case  $V_2$  the voltage on terminal 2 is greater than  $V_1$  which is the voltage on terminal 1. And, we are assuming in this case that the temperature on both terminals is equal.

So, in this case the if  $V_2$  is greater than  $V_1$ , in that case the Fermi level of terminal contact 2 will go down and the Fermi function, Fermi Dirac distribution function on the contact 2 will be shifted to the left side this way. So, as you can see in a small energy range around the Fermi levels  $(f_1 - f_2)$  is non-zero,  $(f_1 - f_2)$  is positive in fact.

And, that way the current is positive which means electrons are flowing from contact 1 to contact 2 and the current is flowing from contact 2 to contact 1. So, the positive current in this particular discussion was assumed to be in  $-x$  direction, because this is the  $x$  direction and we are assuming positive current from contact 2 to contact 1 which is minus  $x$  direction ok.

So, this we have discussed in great detail now that because of the voltage difference there can be a current in the in a conductor. Also due to temperature difference also  $(f_1 - f_2)$  can become non-zero. And, this is the scenario in this case  $V_1$  is equal to  $V_2$  which means that the voltage on both the contacts is equal, but the contact 2 is at higher temperature now  $T_{L2}$  which means the lattice temperature of contact 2 is more than the lattice temperature of contact 1.

And, in that case what happens is that this at higher temperature as all of us know the Fermi function modifies and at higher temperature the Fermi function sort of broadens it, basically becomes non-zero for a wide range of energy values. So,  $f_2$  becomes like this because  $T_2$  is now greater than  $T_1$  and as you can see  $(f_1 - f_2)$  is non-zero is positive in this regime and  $(f_1 - f_2)$  is sorry  $(f_1 - f_2)$  is negative above Fermi level and positive below Fermi level ok.

And, the Fermi level does not shift because of the temperature difference ok. So, what it means is that the current specially this kind of charge current can be induced by two factors: one is the voltage difference and second is the temperature difference between the two contacts.

So, far we have only considered the voltage difference, we have not discussed about the temperature difference in the conductor. So, that is what is the subject matter of thermoelectric devices or thermoelectric effects. So, if there is a temperature difference which means  $\Delta T_L$  which is  $T_{L2} - T_{L1}$ , the difference of temperature of contact 2 and contact 1, if this is greater than 0 in that case in near equilibrium situation near equilibrium.

Means, that this  $\Delta T_L$  is not very large, it is still very small difference. But, the contact 2 is now hotter than contact 1, but it is not too hot. The temperatures are almost equal, there is a small difference in the temperature. In that case  $(f_1 - f_2)$  can be given by  $-(\delta f/\delta T)$  times  $\Delta T$ , where  $\Delta T$  is the temperature difference between the two contacts  $T_{L2} - T_{L1}$ .

So,  $\Delta T$  is actually  $\Delta T_L$  which is this value and so, what we can say here is that for n-type conduct. So, if this semi-conductor is an n-type material which means that the current conduction happens in the conduction band. So, in n-type material this is the conduction band, this is the valence band edges and the Fermi level is close to the conduction band.

And, most of the conduction happens in the conduction band. So, the conduction happens above Fermi level in case of n-type material and if there is a temperature gradient or if there is a temperature difference between the two contacts in an n-type material.

So, we need to focus here because in this regime current conduction happens and in this regime  $(f_1 - f_2)$  is negative. So, that there will be a negative current which means negative current means electrons will flow from contact 2 to contact 1.

And, current will flow from contact 1 to contact 2 or there will be a current in positive x direction ok. Similarly, in a p-type material, the Fermi level is close to the valence band maxima.

(Refer Slide Time: 08:43)

**Introduction**

$$I = -I_x = \frac{2q}{h} \int T(E)M(E)(f_1 - f_2) dE$$

Electrons are particles that carry both charge and heat.

**Voltage difference:**  $(f_1 - f_2) \approx \left(-\frac{\partial f_0}{\partial E}\right) q\Delta V$

**Due to temperature difference:**  $\Delta T_L = T_{L2} - T_{L1} > 0$   $f_1 > f_2$  for energies below  $E_F$   
 $f_1 < f_2$  for energies above  $E_F$

**In near equilibrium case:**  $(f_1 - f_2) \approx f_1 - \left(f_1 + \frac{\partial f_1}{\partial T} \Delta T\right) = -\frac{\partial f_0}{\partial T} \Delta T$

*Handwritten notes:*  
 n-type:  $I < 0 \rightarrow +x$  direction  
 p-type:  $I > 0 \rightarrow -x$  direction

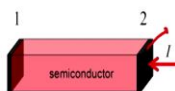
M. Lundström, and C. Jeong, "Near-Equilibrium Transport", World Scientific Publishing Company, 2013

And current conduction takes place at the bottom of at the top of the valence band here and the current conducts due to holes. In this case, we need to and in this case if there is a temperature difference, in this case we need to see  $(f_1 - f_2)$  below Fermi level. And, as you can see  $(f_1 - f_2)$  is positive which means there is a current in minus x direction and minus x direction.

So, depending on the I would say the doping of the semi-conductor, the current direction will be determined when a temperature difference is applied on across the semi-conductor. And, this is very important because let us say if we have a an unknown piece of semi-conductor and in that case we do not know whether it is p-type or n-type in that case what we just need to do is, we need to heat on the on one side.

(Refer Slide Time: 09:50)

**Introduction**



$$I = -I_x = \frac{2q}{h} \int T(E)M(E)(f_1 - f_2)dE$$

Electrons are particles that carry both charge and heat.

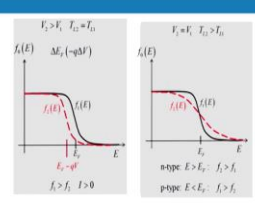
**Voltage difference:**  $(f_1 - f_2) \approx \left(-\frac{\partial f_0}{\partial E}\right) q\Delta V$

**Due to temperature difference:**  $\Delta T_L = T_{L2} - T_{L1} > 0$   $f_1 > f_2$  for energies below  $E_F$   
 $f_1 < f_2$  for energies above  $E_F$

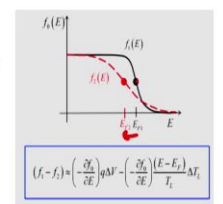
**In near equilibrium case:**  $(f_1 - f_2) \approx f_1 - \left(f_1 + \frac{\partial f_1}{\partial T_L} \Delta T\right) = -\frac{\partial f_0}{\partial T_L} \Delta T$

**Considering both:**  $(f_1 - f_2) \approx \left(-\frac{\partial f_0}{\partial E}\right) q\Delta V - \left(-\frac{\partial f_0}{\partial E}\right) \frac{(E - E_F)}{T_L} \Delta T$

*Near Equ.*



*n-type: I < 0 → +ΔV*  
*p-type: I > 0 → -ΔV*



$(f_1 - f_2) \approx \left(-\frac{\partial f_0}{\partial E}\right) q\Delta V - \left(-\frac{\partial f_0}{\partial E}\right) \frac{(E - E_F)}{T_L} \Delta T$

M. Lundstrom, and C. Jeong, "Near-Equilibrium Transport", World Scientific Publishing Company, 2013

Let us say we will heat the semi-conductor on this on contact 2 and if the current is in minus x direction, in that case the semi-conductor is p-type and if the current is in plus x direction, the semi-conductor is n-type. So, we can find out about the polarity of the or about the doping the nature of the doping in the semi-conductor just by measuring the current due to temperature difference.

So, now what if we have both voltage difference and temperature difference. So, what if we are applying a battery across the semi-conductor and there is also a temperature difference between contact 1 and contact 2. So, in that case we need to combine the contribution of both of these terms this one and this one in  $(f_1 - f_2)$ .

So, in that case  $(f_1 - f_2)$  will be given by  $-(\delta f/\delta E)$  times  $q\Delta V$  from here  $-(\delta f/\delta E)$ . So, this  $-(\delta f/\delta T)$  becomes  $-(\delta f/\delta E)$  into  $(E - E_F/T_L)$  times  $\Delta T_L$ . So, this we derived in our previous class if you recall. So, when there is a voltage difference across the semi-conductor and there is also a temperature difference in that case  $(f_1 - f_2)$  will look like this.

The plots will look like this and as you can see we have applied a positive voltage on terminal 2 which means  $E_{F2}$  is now shifted to the left side. And, also we have heated the contact 2 which means  $f_2$  is now spread out,  $f_2$  is now more broader as compared to  $f_1$ . So, in this case which is a very general case  $(f_1 - f_2)$  will be given or we need to consider both of these terms, while calculating  $(f_1 - f_2)$ .

And, please remember that this is near equilibrium situation, near equilibrium means that the applied voltage is also is quite small, also the temperature difference is also

small; it is we are not applying a large voltage or a large temperature difference. So, with this background what we can say is that the current will conduct because of the two driving factors.

One is the voltage difference and second is the temperature difference across the conductor. And, in order to properly model the current in any conductor, we need to take into account both of these parameters while doing the actual current calculation. With this I think this is a good time to introduce a new kind of current in conductors, because electrons are particles that carry both charge and heat; electrons carry charge as well as heat ok and this we discussed last time.

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The slide is titled "Charge and Heat Currents" and contains the text "The notion of charge current". It displays the equation  $I = -I_x = \frac{q}{h} \int T(E)M(E)(f_1 - f_2) dE$ . A handwritten red box on the right side of the slide contains the equation  $I = \frac{2q}{h} \int T(E)m(E)(f_1 - f_2) dE$ . The slide also features logos for Swajathi and other institutions at the bottom.

So, this is how we define the charge current and in this case this expression becomes the expression for charge current because we have this factor of q here. So, if we remove this factor of q from here, in that case this will be this expression will give us just the particle flux. When there is a difference in the Fermi functions of on between the left contact and the right contact.

So, this is a generic particle flux or I would say the particle current and if we multiply this by charge it becomes charge current and this is what we derive in the derived in the general model of transport. And, this is what we use in calculating the current in all the devices in fact, but as we know that apart from the charge, electrons also carry heat in terms of their in the form of their kinetic energy.

So, when there is a flow of electrons apart from a charge current there will be a heat current as well, it will be as if a heat is also flowing through the material. So, that is what we need to define.

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$\Delta V, \Delta T$

### Charge and Heat Currents

The notion of charge current

$$I = -I_x = \frac{2q}{h} \int T(E) M(E) (f_1 - f_2) dE$$

$$I'(E) = \frac{2q}{h} T(E) M(E) (f_1 - f_2)$$

Electrons are particles that carry both charge and heat

For heat current:  $q \rightarrow (E - E_f)$

$$I'_q(E) = \frac{2(E - E_f)}{h} T(E) M(E) (f_1 - f_2)$$

$$(f_1 - f_2) \approx \left( \frac{\partial f_0}{\partial E} \right) q \Delta V - \left( \frac{\partial f_0}{\partial E} \right) \frac{(E - E_f)}{T_1} \Delta T$$

$$I = G \Delta V + S_T \Delta T$$

M. Lundstrom, and C. Jeong, "Near-Equilibrium Transport", World Scientific Publishing Company, 2013

But, the charge current at a particular energy is given by this value, just by removing the integral and without q, this is the number of particles flowing at a certain energy E. If we put q here, in that case it becomes the charge current actually ok. So, as we have seen that  $(f_1 - f_2)$  is or when there is a gradient in voltage as well as in temperature. So, if across a conductor we have  $\Delta V$  as well as  $\Delta T$ , in that case  $(f_1 - f_2)$  is given by these two terms.

And, if we put these two terms here then the charge current becomes like this. So, this is the general or the most general form of the charge current, both in presence of voltage difference and temperature difference. So, both of these terms need to be taken into account and we will discuss more about it in coming slides in after few minutes. But, let us take a pause and define the heat current as well.

And, in order to define the heat current, we just need to replace q in this expression or in this expression by  $E - E_F$ . And why  $E - E_F$ ? The reason for this is that generally in most of the cases most of the conduction takes place around the Fermi level, specially in metals almost all the conduction takes place at the Fermi level. All the electrons are in



metal a new electron is entered at the Fermi level or any electron exit from the Fermi level itself, Fermi level energy.

Even in semi-conductor, the Fermi level defines in a way the distribution of particles in a material or distribution of particles in the semi-conductor. So, in order to define heat current, the amount of energy that is in access to the Fermi level  $E - E_F$  is the heat in that system ok, that is. So, this comes mainly from the metals or conductors. And, if we just put replace  $q$  by  $E - E_F$  in this expression, it will give us the heat current.

So, heat current will be heat current at a certain energy  $E$  will be  $2(E - E_F)/h$  into integral  $T(E)$  times  $M(E)$  times  $(f_1 - f_2)$  ok. So, please remember that this is not charge current and that is why this is the heat current and this is that is why this, its unit will also be different. So, I will give you few minutes and so, just think about what would be the units of  $I_q$  ok and we will come back to this in coming slides.

So, this is the typical picture of conduction in a semi-conductor in which we have a semi-conductor in this is the semi-conductor. This is the conduction band of the semi-conductor, minima of the conduction band.

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**Charge and Heat Currents**

The notion of charge current

$$I = -I_x = \frac{2q}{h} \int T(E)M(E)(f_1 - f_2)dE$$

$$I(E) = \frac{2q}{h} T(E)M(E)(f_1 - f_2)$$

$$(f_1 - f_2) \approx \left( -\frac{\partial f_0}{\partial E} \right) q\Delta V - \left( -\frac{\partial f_0}{\partial E} \right) \frac{(E - E_F)\Delta T}{T_L}$$

$$I = G\Delta V + S_T\Delta T$$

Electrons are particles that carry both charge and heat.

For heat current:  $q \rightarrow (E - E_f)$

Handwritten notes:  $E_v$ ,  $T_{L2} > T_{L1}$

M. Lundström, and C. Jeong, "Near-Equilibrium Transport", World Scientific Publishing Company, 2013

And, there might be  $E_v$  the valence band maxima and this is the conduction band minima. So, and if so, let us first draw the equilibrium picture, in the equilibrium what

happens is that on left contact we have Fermi level  $E_{F1}$ , on the right contact we have Fermi level  $E_{F2}$ .

And, if we are using an n-type material n-type semi-conductor, the Fermi level in the semi-conductor will be  $E_F$ . And, if it is n-type the Fermi level will be closer to the bottom of the conduction band as compared to the top of the valence band. So, this is the left contact, this is the right contact and this is the semi-conductor in between and this is the equilibrium I would say the equilibrium band diagram of the system.

And, as you can see in equilibrium all the Fermi levels are aligned to each other, there is a single uniform Fermi level in the entire device. But, if there is a voltage applied on contact 2; so, that  $E_{F2}$  goes down there is a if there is a positive voltage on contact 2 and a current will start conducting in the material. There might also be a temperature gradient, there might be so, contact 2 might be heated.

Let us say the temperature of contact 2 is  $T_{L2}$  and the temperature of contact 1 is  $T_{L1}$ , let us assume  $T_{L2}$  is greater than  $T_{L1}$  and in that case there is as a net flow of electrons in the system. So, the way electrons flow is that electrons will start from the left contact jump into the semi-conductor conduction band. And from the semi-conductor conduction band, they will jump to the right contact Fermi level.

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**Charge and Heat Currents**

The notion of charge current

$$I = -I_x = \frac{2q}{h} \int T(E)M(E)(f_1 - f_2)dE$$

$$I(E) = \frac{2q}{h} T(E)M(E)(f_1 - f_2)$$

$$(f_1 - f_2) \approx \left( -\frac{\partial f_0}{\partial E} \right) \Delta V - \left( -\frac{\partial f_0}{\partial E} \right) \frac{(E - E_F)}{T_L} \Delta T$$

$$I = G\Delta V + S_T\Delta T$$

Electrons are particles that carry both charge and heat.

For heat current:  $q \rightarrow (E - E_c)$

$$I_Q^1(E) = \frac{2(E - E_{F1})}{h} T(E)M(E)(f_1 - f_2)$$

$$I_Q^2(E) = \frac{2(E - E_{F2})}{h} T(E)M(E)(f_1 - f_2)$$

$$I_Q = -T_L S_T \Delta V - K_0 \Delta T$$

M. Lundstrom, and C. Jeong, "Near-Equilibrium Transport", World Scientific Publishing Company, 2013

So, in this case the heat current from contact 1 to the semi-conductor is given by  $I'_{Q1}(E)$  at any energy  $E$  and this is  $2(E - E_{f1})/h$  times  $T(E)M(E) (f_1 - f_2)$ . Similarly, heat current from semi-conductor to contact 2 is  $2(E - E_{f2})/h$  times  $T(E)M(E) (f_1 - f_2)$ . So, this is the way we analyze heat current in any device.

When there is this term  $(f_1 - f_2)$  is non-zero and in general if we put if we consider that  $\Delta V$  is non-zero as well as  $\Delta T$  is also non-zero, in that case  $(f_1 - f_2)$  needs to be replaced by these terms in near equilibrium situation. And, in that case the heat current can be written in this general form. So, as you can see heat current is dependent both on voltage difference and temperature difference like the charge current.

So, we will very soon go to these parameters  $G, S, T, T_L, S_T$  and  $K_0$  as well. But, this discussion was just to show you that that apart from charge current, we also need to consider heat current when there is an electron flow through any device. And, heat current is defined by replacing  $q$  by  $E - E_F$  and in that case both charge current and heat current are dependent on the voltage difference and temperature difference ok.

(Refer Slide Time: 22:03)

**Introduction**

$$I = -I_x = \frac{2q}{h} \int T(E)M(E)(f_1 - f_2)dE$$

Electrons are particles that carry both charge and heat.

**Voltage difference:**  $(f_1 - f_2) \approx \left(-\frac{\partial f_0}{\partial E}\right) q\Delta V$

**Due to temperature difference:**  $\Delta T_L = T_{L2} - T_{L1} > 0$   $f_1 > f_2$  for energies below  $E_F$   
 $f_1 < f_2$  for energies above  $E_F$

**In near equilibrium case:**  $(f_1 - f_2) \approx f_1 - \left(f_1 + \frac{\partial f_1}{\partial T_L} \Delta T\right) = -\frac{\partial f_0}{\partial T_L} \Delta T$

**Considering both:**  $(f_1 - f_2) \approx \left(-\frac{\partial f_0}{\partial E}\right) q\Delta V - \left(-\frac{\partial f_0}{\partial E}\right) \frac{(E - E_F)}{T_L} \Delta T$

**n-type:**  $I < 0$   
**p-type:**  $I > 0$

So, so, this analysis we have already done because,  $(f_1 - f_2)$  we have analyzed in our previous class, it is dependent both on voltage difference and temperature difference. Because, from this picture we can see that the Fermi function is both a function of voltage and Fermi level and Fermi level is dependent on the applied voltage. So, it is a function of both temperature and Fermi level and Fermi level is dependent on voltage.

So, that is why the Fermi function becomes a function of both the applied voltage and the applied temperature difference across the conductor ok. So, so far just to quickly summarize we have seen that specially when there is a temperature when we talk about temperature difference across a semi-conductor, we need to understand the notion of that there will be a notion of heat current as well apart from the charge current.

And, both heat current and charge currents are dependent on the applied voltage  $\Delta V$  and applied temperature difference  $\Delta T$ . And, from these ideas will come to we will come to Seebeck effect and Peltier effect later on.

(Refer Slide Time: 23:24)

**Charge Current:**  $I = \int I'(E) dE$

The total current is the sum of the contributions from each energy channel

where the differential current is:  $I'(E) = \frac{2q}{h} T(E) M(E) (f_1 - f_2)$

$(f_1 - f_2) \approx \left( -\frac{\partial f_0}{\partial E} \right) q \Delta V - \left( -\frac{\partial f_0}{\partial E} \right) \frac{(E - E_F)}{T_L} \Delta T$

we obtain:  $I'(E) = G'(E) \Delta V + S_T'(E) \Delta T$

Where:  $G'(E) = \frac{2q^2}{h} T(E) M(E) \left( -\frac{\partial f_0}{\partial E} \right)$

The differential conductance is:

$S_T'(E) = -\frac{2q^2}{h} T(E) M(E) \left( \frac{E - E_F}{q T_L} \right) \left( -\frac{\partial f_0}{\partial E} \right)$

$= -\left( \frac{k_B}{q} \right) \left( \frac{E - E_F}{k_B T_L} \right) G'(E)$

**Handwritten Red Annotations:**

- $I'(E) = \frac{2q}{h} T(E) M(E) (f_1 - f_2)$
- $I'(E) = \frac{2q}{h} T(E) M(E) \left( -\frac{\partial f}{\partial E} \right) q \Delta V$
- Integrating:  $\frac{2q^2}{h} T(E) M(E) \left( \frac{\partial f}{\partial E} \right) \frac{(E - E_F)}{2 T_L} \Delta T$
- $I'(E) = G'(E) \cdot \Delta V + S_T'(E) \cdot \Delta T$

So, let us first analyze how the charge current looks like. So, charge current  $I'(E)$  which is the current at a certain energy  $E$  is given by  $(2q/h) T(E) M(E) (f_1 - f_2)$ ,  $T(E)$  accounts for the scattering transmission coefficient,  $M(E)$  accounts for the modes in the semi-conductor, the conduction pathway is in the channel. And,  $(f_1 - f_2)$  is the forcing function for the current,  $q$  is the fundamental charge unit and if we do not have  $q$ , it just becomes the particle current number of particles flowing per unit time.

So, the total current is the sum of contributions from each energy channel. So, we need to integrate this  $I'(E)$ . So, the total charge current  $I$  is given by the integration of this, the this differential current, this differential current is given by this value. And, as we now know very clearly that  $(f_1 - f_2)$  is  $-(\delta f / \delta E) q \Delta V - (\delta f_0 / \delta E) (E - E_F / T_L) \Delta T$ .

So,  $(f_1 - f_2)$  is dependent both on applied voltage and applied temperature difference. So, if we just put  $(f_1 - f_2)$  in this expression and integrate this what we obtain is we just if we just put  $(f_1 - f_2)$  here,  $I'(E)$  will be  $(2q/h) T(E) M(E)$ ,  $(f_1 - f_2)$  is  $-(\delta f/\delta E)q\Delta V$ ,  $+ (2q/h) T(E) M(E) (\delta f/\delta E) (E-E_F/T_L)\Delta T$ . So, we have just replaced  $(f_1 - f_2)$  by this in the differential current expression.

So, here now this thing is known as the conductance function and this thing is represented as  $S_T$  times  $\Delta T$ . So,  $S_T$  actually is related to the solid coefficient, it is related to the electro thermal diffusion which means electrical diffusion due to thermal energy due to temperature difference. So, the differential current has a contribution both from the conductance function and the this solid diffusion, electro thermal diffusion.

So, in order to calculate the total current; so, this  $G'(E)$  will be  $(2q^2/h) T(E) M(E)$  into  $(-\delta f/\delta E)$  and this electro thermal conductance is  $S_T'$  is  $-(2q^2/h) T(E) M(E) (E-E_F/qT_L) (-\delta f/\delta E)$  ok. So, I leave this to you to sort of figure out to exactly come to this point, this expression. And because, in this case we have been multiplying by  $q$  divided dividing by  $q$  and in that case this is what it becomes negative, negative becomes positive. So, that is why we have a positive term here.

So, the total charge so, this Soret coefficient  $S'_T(E)$  is given by  $(-k_B/q)$  times  $(E-E_F/k_B T_L)$  times  $G'(E)$ . So, even  $S'_T$  can be represented in terms of  $G'(E)$ , even this Soret coefficient is related to the conductance function or the differential conductance of the material.

(Refer Slide Time: 28:26)

**Charge Current:**  $I = \int I'(E) dE$

The total current is the sum of the contributions from each energy channel

where the differential current is:  $I'(E) = \frac{2q}{h} T(E) M(E) (f_1 - f_2)$

$$(f_1 - f_2) \approx \left( -\frac{\partial f_0}{\partial E} \right) q \Delta V - \left( -\frac{\partial f_0}{\partial E} \right) \frac{(E - E_F)}{T_L} \Delta T$$

we obtain:  $I'(E) = G'(E) \Delta V + S_T'(E) \Delta T$

Where:  $G'(E) = \frac{2q^2}{h} T(E) M(E) \left( -\frac{\partial f_0}{\partial E} \right)$

**The differential conductance is:**

$$S_T'(E) = -\frac{2q^2}{h} T(E) M(E) \left( \frac{E - E_F}{q T_L} \right) \left( -\frac{\partial f_0}{\partial E} \right)$$

$$= -\left( \frac{k_B}{q} \right) \left( \frac{E - E_F}{k_B T_L} \right) G'(E)$$

**Integrating:**  $I'(E) = G'(E) \Delta V + S_T'(E) \Delta T$

$$I = G \Delta V + S_T \Delta T$$

*Handwritten notes:*

$$I = \int I'(E) dE$$

$$= \int (G'(E) \Delta V + S_T'(E) \Delta T) dE$$

$$I = (G) \Delta V + (S_T) \Delta T$$

So, finally, after integrating this expression; so, the total current will be the integration of the differential current with respect to energy. And, it becomes  $G'(E) \Delta V + S_T'(E) \Delta T$  integrated over all possible energy pathways. And, this turns out to be  $G \Delta V + S_T \Delta T$ .

So, where  $G$  is the conduct electrical conductance of the system of the conductor of the semi-conductor in under consideration. And,  $S_T$  is the Soret coefficient of the semi-conductor or the electro thermal diffusion coefficient of the semi-conductor.

(Refer Slide Time: 29:15)

**Charge Current:**  $I = \int I'(E) dE$

The total current is the sum of the contributions from each energy channel

where the differential current is:  $I'(E) = \frac{2q}{h} T(E) M(E) (f_1 - f_2)$

$$(f_1 - f_2) \approx \left( -\frac{\partial f_0}{\partial E} \right) q \Delta V - \left( -\frac{\partial f_0}{\partial E} \right) \frac{(E - E_F)}{T_L} \Delta T$$

we obtain:  $I'(E) = G'(E) \Delta V + S_T'(E) \Delta T$

Where:  $G'(E) = \frac{2q^2}{h} T(E) M(E) \left( -\frac{\partial f_0}{\partial E} \right)$

**The differential conductance is:**

$$S_T'(E) = -\frac{2q^2}{h} T(E) M(E) \left( \frac{E - E_F}{q T_L} \right) \left( -\frac{\partial f_0}{\partial E} \right)$$

$$= -\left( \frac{k_B}{q} \right) \left( \frac{E - E_F}{k_B T_L} \right) G'(E)$$

**Integrating:**  $I'(E) = G'(E) \Delta V + S_T'(E) \Delta T$

$$I = G \Delta V + S_T \Delta T$$

$$G = \int G'(E) dE$$

$$S_T = \int S_T'(E) dE$$

These equations are valid in 1D, 2D, or 3D and from the ballistic to diffusive limits.

It is related to the Soret coefficient for electro-thermal diffusion. Note that  $S_T'(E)$  is negative for channels above  $E_F$  and positive for channels below  $E_F$ .

So, this is the most I would say the most general form of the charge current through any conductor and this has contribution both from  $\Delta V$  and  $\Delta T$ . So, we need to consider both

electrical conductance as well as electro thermal diffusion, electro thermal conductance ok. So, this is how things get modified when we consider the thermoelectric effects as well in our system.

(Refer Slide Time: 29:51)

**Heat Current**

$$I_Q'(E) = \frac{2}{h} (E - E_F) T(E) M(E) (f_1 - f_2)$$

- Illustration of heat flow in our generic device.
- Heat is absorbed at contact 1 and emitted at contact 2.
- Heat fluxes at contacts 1 and 2:

$$I_{Q1} = \frac{2}{h} \int (E - E_{F1}) T(E) M(E) (f_1 - f_2) dE$$

$$I_{Q2} = \frac{2}{h} \int (E - E_{F2}) T(E) M(E) (f_1 - f_2) dE$$

Assuming near-equilibrium conditions and using  $(f_1 - f_2) \approx \left( -\frac{\partial f_0}{\partial E} \right) q \Delta V - \left( \frac{\partial f_0}{\partial E} \right) \frac{(E - E_F) \Delta T}{T_L}$

$$I_Q'(E) = -T_L S_T'(E) \Delta V - K_0'(E) \Delta T$$

$$K_0'(E) = \frac{(E - E_F)^2}{q^2 T_L} G'(E)$$

To find the total heat current, we integrate over all of the energy channels

Electronic heat conductance

$$I_Q = -T_L S_T \Delta V + K_0 \Delta T$$

M. Lundstrom, and C. Jeong "Near-Equilibrium Transport", World Scientific Publishing Company, 2013

So, in the similar way the heat current is defined as all of us know, the heat current from contact 1 is defined or if just to sort of recall the differential heat current is given as  $(2/h)$  times  $(E - E_F)$  into  $T(E) M(E) (f_1 - f_2)$ . And, if we need to find out the heat current from contact 1 to the semi-conductor, we need to integrate this over all possible energy channels and, this is what it turns out to be.

Similarly, from contact 2 it is and if we assume the near equilibrium conditions in that case  $(f_1 - f_2)$  is given by this familiar expression, that we have all of us have seen. And, if we put this value of  $(f_1 - f_2)$  in the differential heat current expression in that case, this is what the heat current actually becomes like. So, just replace  $(f_1 - f_2)$  by this right hand side in this equation and see what you get.

So, let us do a quick calculation here  $I_Q'(E)$  is  $(2/h)$  times  $(E - E_F)$  into  $T(E) M(E)$ ,  $(f_1 - f_2)$  is replaced by  $-(\delta f / \delta E) q \Delta V$ ,  $+ (2/h) (E - E_F) T(E) M(E) (\delta f / \delta E) (E - E_F / T_L)$  times  $\Delta T$ . So, this is this becomes the differential heat current in the system. And, if we have a closed look on this side, this coefficient apart from  $\Delta V$ , this is closely related to the Soret coefficient, because this  $S_T$  in our previous derivation is given by this expression.

So, this first term becomes essentially this first term becomes  $-T_L$  times  $S'_T$  times  $\Delta V$  and the second term becomes  $K_0'(E)$  times  $\Delta T$ , where this  $K$  is now known as the electronic heat conductance and this  $K_0$  is known as the electronic heat conductance coefficient. So, in order to find out the total heat current, we need to integrate this differential heat current over all possible energy values.

(Refer Slide Time: 33:13)

**Heat Current**

- Illustration of heat flow in our generic device.
- Heat is absorbed at contact 1 and emitted at contact 2.
- Heat fluxes at contacts 1 and 2:

$$I_{Q1} = \frac{2}{h} \int (E - E_{F1}) T(E) M(E) (f_1 - f_2) dE$$

$$I_{Q2} = \frac{2}{h} \int (E - E_{F2}) T(E) M(E) (f_1 - f_2) dE$$

Assuming near-equilibrium conditions and using  $(f_1 - f_2) \approx \left(-\frac{\partial f_0}{\partial E}\right) q \Delta V - \left(-\frac{\partial f_0}{\partial E}\right) \frac{(E - E_F)}{T_L} \Delta T$

$$I_Q = \int I_Q(E) = \int \left[ T_L S'_T(E) \Delta V - K'_0(E) \Delta T \right] dE$$

$K'_0(E) = \frac{(E - E_{F1})^2}{q^2 T_L} G(E)$  **Electronic heat conductance**

To find the total heat current, we integrate over all of the energy channels

$$I_Q = -T_L S_T \Delta V - K_0 \Delta T$$

where  $K_0 = T_L \left(\frac{k_B}{q}\right)^2 \int \left(\frac{E - E_{F1}}{k_B T_L}\right)^2 G(E) dE$

And, that is why we need to integrate on the right hand side as well and the final expression for the total heat current that we obtain is this. And, this is  $-T_L S_T \Delta V - K_0 \Delta T$ , where this  $K_0$  is given by this expression. And, I will leave you to do this calculation on your own how  $K_0$  turns out to be this value.

This is a very straightforward calculation and you should be able to do it. So, as we have already discussed that the heat current is also dependent both on voltage difference and temperature difference. And, but there is a key distinction here that in heat current  $I_Q$  is  $-T_L S_T \Delta V - K_0 \Delta T$ .

(Refer Slide Time: 34:09)



**Seebeck effect**

- Development of emf due to temp difference in a conductor.
  - Also known as Seebeck voltage

M. Lundström, and C. Jeong, "Near-Equilibrium Transport", World Scientific Publishing Company, 2013

So, this  $S_T$  appears with  $\Delta T$  in with in the case of charge current, if you recall  $S_T$  appears with  $\Delta T$  in the case of charge current, but it appears with  $\Delta V$  in the case of heat current. And, that is an interesting aspect because both of these currents are sort of coupled to each other, that is sort of indicate that.

So, now we have a decent background, a decent mathematical background of both charge current and heat current in conductors. Now, we are in a good shape to define one of the most popularly known thermoelectric effects that is the Seebeck effect. And what is the Seebeck effect? Seebeck effect is essentially the development of an emf due to temperature difference in a conductor.

What it means is if there is a temperature difference in a conductor an emf electromotive force will be developed across the conductor. In other words, a voltage difference will be developed across the conductor and if we have the conductor open circuited in that case, we will have an open circuit voltage due to temperature difference.

This is known as the Seebeck effect which in other words means that if we heat a conductor on one end, there will be a non-zero voltage difference between two of its terminals.

(Refer Slide Time: 36:04)

**Seebeck effect**

$I \propto \frac{\Delta V}{\Delta T}$

- Development of emf due to temp difference in a conductor.

cool  $T_{L1}$   $T_{L2} > T_{L1}$  hot

voltage difference develops to stop current flow

n-type semiconductor

electron flux due to temp gradient

$-V_{oc} + 0$

M. Lundström, and C. Jeong, "Near-Equilibrium Transport", World Scientific Publishing Company, 2013

And, now with the background that we have seen, this is not a not something unfamiliar, because what we have seen is that the charge current is directly proportional to  $\Delta V$  as well as  $\Delta T$ . So, if there is even if we do not have any applied voltage even, if this is 0 and this is non-zero in that case there will be a non-zero current which means the electrons will flow, because of the temperature difference.

And, if the semi-conductor is open circuited in that case this flow of electrons will result in an open circuit voltage which will balance the flow of electrons, because of the temperature difference ok. And that is the root of the Seebeck effect. So, in a way the charge current that we have discussed is actually this Seebeck effect is a natural consequence of this idea. Seebeck effect naturally comes from here even, if we do not have any voltage difference across the semi-conductor which means  $\Delta V = 0$ .

(Refer Slide Time: 37:01)

**Charge Current:**  $I = \int I'(E) dE$

The total current is the sum of the contributions from each energy channel

where the differential current is:  $I'(E) = \frac{2q}{h} T(E) M(E) (f_1 - f_2)$

$$(f_1 - f_2) \approx \left( -\frac{\partial f_0}{\partial E} \right) q \Delta V - \left( -\frac{\partial f_0}{\partial E} \right) \frac{(E - E_F)}{T_L} \Delta T$$

we obtain:  $I'(E) = G'(E) \Delta V + S_T'(E) \Delta T$

Where:  $G'(E) = \frac{2q^2}{h} T(E) M(E) \left( -\frac{\partial f_0}{\partial E} \right)$

**The differential conductance is:**

$$S_T'(E) = -\frac{2q^2}{h} T(E) M(E) \left( \frac{E - E_F}{q T_L} \right) \left( -\frac{\partial f_0}{\partial E} \right)$$

$$= -\left( \frac{k_B}{q} \right) \left( \frac{E - E_F}{k_B T_L} \right) G'(E)$$

Integrating:  $I'(E) = G'(E) \Delta V + S_T'(E) \Delta T$

$$I = G \Delta V + S_T \Delta T$$

$$G = \int G'(E) dE$$

$$S_T = \int S_T'(E) dE$$

These equations are valid in 1D, 2D, or 3D and from the ballistic to diffusive limits.

It is related to the Seebeck coefficient for electro-thermal diffusion. Note that  $S_T'(E)$  is negative for channels above  $E_F$  and positive for channels below  $E_F$ .

In that case, I is given by  $S_T$  times  $\Delta T$  which means there will be a flow of electrons in the conductor. And, if the conductor is open circuited, electrons will accumulate on one side of the conductor creating an open circuit voltage and, that is essentially the Seebeck effect. This open circuit voltage is also known as Seebeck voltage and the way to understand this is that is just from the Fermi function. And, this we have seen in other forms as well.

This is the Fermi function at temperature  $f_1$  is the Fermi function at temperature  $T_{L1}$  and  $f_2$  is the Fermi function at temperature  $T_{L2}$  and  $T_{L1}$  is less than  $T_{L2}$ , let us assume that this end of the semi-conductor is hot and this end of the semi-conductor is cool which means  $T_{L2}$  is greater than  $T_{L1}$ . So, the Fermi function on this side of the semi-conductor will be like this red plot and the Fermi function on the left side of the conductor will be like this.

And, since there is no voltage applied across the semi-conductor  $E_F$  will be the same across the this conductor. So, if this is an n-type semi-conductor in that case, we need to consider the flow of electrons in the conduction band. And, what happens in the conduction band, conduction band is above the Fermi level, above Fermi level  $f_1$  is less than  $f_2$  essentially. So, which means that  $f_2$  is greater than  $f_1$  in conduction band.

So; that means, that the number of electrons or the probability of electrons on the right contact in the conduction band is more. The probability of electrons being found close to the conduction band energies is more on the right contact, the hot contact as compared to the cool contact, cold contact essentially. So, the electrons will try to from the from as

we discussed in the general model of transport, this contact will try to bring this semi-conductor in equilibrium in itself.

So, the electrons will start flowing from hot contact to the semi-conductor and from semi-conductor to the this left contact ok. And, that way electrons will accumulate on this side of the semi-conductor, that will leave a positive voltage on the right the hot contact and a negative voltage on the cold contact. And, that will create this kind of open circuited voltage ok and if we connect any electronic system here that can result in a current as well.

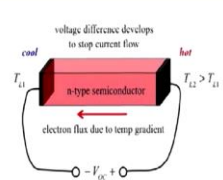
So, that way we can produce an electric current by a temperature difference, just by a temperature difference we can produce electric current. So, essentially converting heat into electricity using semi-conductors. So, that is how the thermoelectric devices are made by using the Seebeck effect, we will not be able to discuss about thermoelectric devices in this course. So, what we will do is we will discuss the basics of Seebeck effect and a related effect known as Peltier effect.

And, essentially most of the devices work on the underlying principle of these two effects. So, if these two effects are clear then we can understand most of the thermoelectric devices mechanism. So, if there is a temperature difference an open circuit voltage will be developed across the semi-conductor and that is known as the Seebeck effect ok.

(Refer Slide Time: 41:12)

**Seebeck effect**

- Development of emf due to temp difference in a conductor.
- Also known as Seebeck voltage



Charge Current

$$I = G\Delta V + S_T \Delta T$$

$$\Delta V = \frac{1}{G}I - \frac{S_T}{G}\Delta T$$

$$\Delta V = R \cdot I - \frac{\Sigma I}{G} \Delta T$$

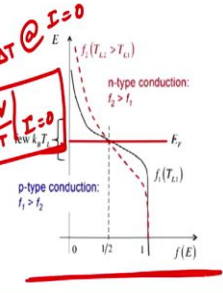
$$\Delta V = R \cdot I - \frac{\Sigma I}{G} \Delta T$$

$$\Delta V = -\frac{\Sigma I}{G} \Delta T \quad (\text{when } I=0)$$

$$S = \frac{\Sigma I}{G}$$

$\Delta V = -S \cdot \Delta T @ I=0$

$S = -\frac{\Delta V}{\Delta T} @ I=0$



n-type conduction:  $T_h > T_c$   
 p-type conduction:  $T_c > T_h$

M. Lundstrom, and C. Jeong, "Near-Equilibrium Transport", World Scientific Publishing Company, 2013

So, from the charge current expression, this is what we have  $I$  is  $G \Delta V + S_T \Delta T$  which means, if we rearrange these terms  $G \Delta V$  is equal to  $I - S_T \Delta T$  or  $\Delta V$  is equal to  $(1/G) I - (S_T/G) \Delta T$ . So, this is the charge current. So, this is what we obtain  $1/G$ ,  $G$  is the electrical conductance, it is  $R I - (S_T/G) \Delta T$ .

Now, if the two ends of the semi-conductor, if the two terminals of the semi-conductor are open circuited, we have not connected anything. There cannot be a flow of current, this  $I$  will be 0 in steady state. So, in that case  $\Delta V$  is equal to  $-(S_T/G) \Delta T$  and this  $\Delta V$  will be the voltage that or the voltage difference that appears across the semi-conductor.

So, this is when  $I$  is 0 in open circuit case and this term is known as the Seebeck coefficient essentially,  $(S_T/G)$  is known as the Seebeck coefficient which is defined as  $S$ ,  $S$  is  $(S_T/G)$ . So, what is the Seebeck coefficient? Seebeck coefficient is essentially becomes from this equation  $\Delta V$  is equal to  $-(S)\Delta T$  at  $I = 0$ . So,  $S$  is  $-\Delta V / \Delta T$  when the current is 0.

So, this is the technical the formal definition of the Seebeck coefficient. This is the ratio of the voltage difference to the temperature difference when the current in the circuit is 0. So, what it also means is when we have a temperature difference across a conductor  $\Delta T$ ,  $\Delta V$  of voltage difference will be produced, when the conductor is open circuited. And, the ratio between  $\Delta V$  and  $\Delta T$  with a negative sign is known as the Seebeck coefficient.

So, this is an important I would say important thermoelectric coefficient and all of us should be very clear about its definition as well, also its physical mechanism. The underlying physical mechanism is essentially this one ok.

(Refer Slide Time: 44:30)

### Seebeck effect

- Development of emf due to temp difference in a conductor.
  - Also known as Seebeck voltage
  - The sign of the Seebeck voltage (hot side voltage - cold side voltage) is positive for n-type conductor.
  - Negative for p-type conductor.
  - This fact can be used to determine the type of a semiconductor.

voltage difference develops to stop current flow

electron flux due to temp gradient

$-V_{oc} + 0$

$I = G\Delta V + S_T\Delta T$

$\Delta V = \frac{1}{G}I - \frac{S_T}{G}\Delta T$

$\Delta V = RI - S\Delta T$

$S = \frac{S_T}{G}$

$S < 0$  for n-type  
 $S > 0$  for p-type

Seebeck coefficient

M. Lundström, and C. Jeong, "Near-Equilibrium Transport", World Scientific Publishing Company, 2013.

So, as we will expect that the sign of the Seebeck voltage which is the  $\Delta V$  voltage is also known as the voltage on the hot side minus voltage on the cold side is positive for an n-type conductor and the Seebeck voltage is negative for the p-type semi-conductor. And, this can also be used to determine the type of semi-conductor that we are using. So, this is this we have also discussed here, also I would also like you to think more about it and also see how electrons will flow in a p-type semi-conductor.

So, in this particular example we have taken an n-type semi-conductor. So, I will recommend all of you to look into the situation when instead of n-type semi-conductor, we take a p-type semi-conductor; in that case how the electrons will flow when the left contact is cool and the right contact is hot ok. So, with this let us maybe finish here today and in the next class we will start with the discussion of Peltier effect ok.

So, thank you all, see you in the next class.