Physics of Nanoscale Devices

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Lecture - 50

MOS Electrostatics

Hello everyone, today we will start our formal discussion on the MOSFET Electrostatics. And I have written down the name of the topic to be MOS electrostatics which means Metal Oxide Semiconductor electrostatics because, we are only concerned about the metal oxide semiconductor part of the MOSFET for the moment.

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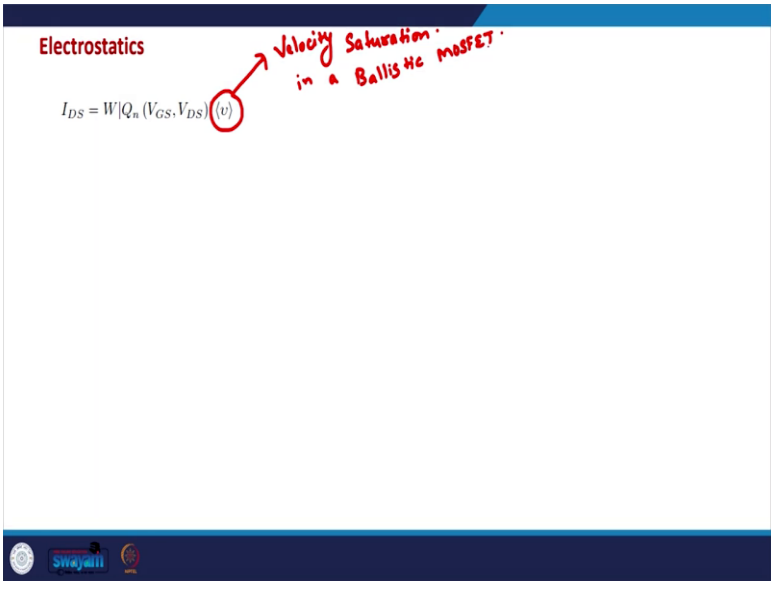

And you might recall that in the previous class we finished our discussion on the Landauer transport theory for the MOSFET; so, which accounts for the velocity saturation in the ballistic MOSFET as well. So, apart from studying the current and the channel charge as a function of various Fermi Dirac integrals and as a function of V_{GS} and V_{DS}, we studied one of the one of the important things that we studied was velocity saturation in a ballistic MOSFET as well.

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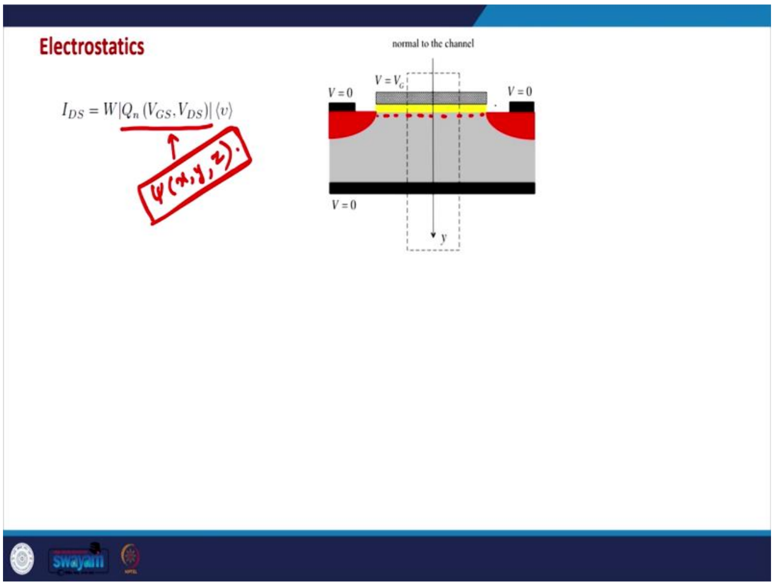

So, I hope this idea is clear by now and we started our discussion of the MOSFET electrostatics which mainly accounts for the charge in the MOSFET and this charge in the MOSFET is directly correlated to the potential in the semiconductor; because the charge in the MOSFET is the charge in this semiconductor channel and that depends on the potential the electrostatics potential in the semiconductor.

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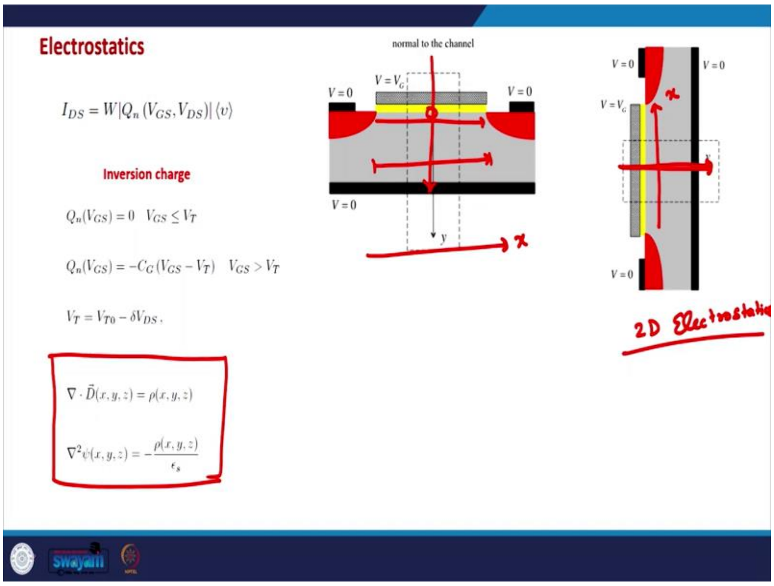

So, in the electrostatics we are quite concerned about the charge, its relationship with the electrostatic potential and that relationship is established by the Poisson’s equation in the; for the MOSFETs. And last in the last class you might have seen that our main emphasis was about discussing the electrostatics in the y direction. And this point I have pointed out one earlier as well, that generally in long MOSFETs in conventional MOSFETs what happens is that the electric field in this direction which means the electric field in x direction is extremely small as compared to the electric field in y direction specially in the channel.

That is the that was the case for the conventional MOSFETs, but in modern MOSFETs since this device length is shrinking. So, the electric field in this direction the x electric field is also becoming comparable is also becoming dominant ok. So, in the long channel MOSFET that is why we were mostly concerned about the electrostatics in the y direction only.

But, in modern day devices we should consider this x direction electric field as well and that is why we should discuss the 2D electrostatics, and that is what we will also study in after this discussion of the basics of the electrostatics ok. So, in the last class we started with the discussion of the electrostatics for conventional MOSFETs.

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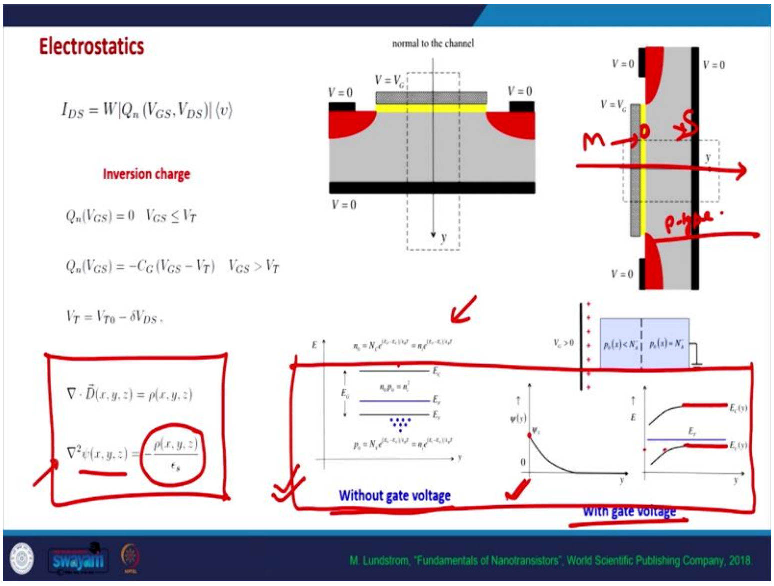

And as you can see that we used the band diagram as a tool to understand the charge and the potential function, potential electrostatic potential in the semiconductor. This is the band diagram of the semiconductor when there is no gate voltage; but, as soon as we apply a gate voltage this is how the band diagram looks like. So, if there is a positive gate voltage it will actually deplete holes from the semi conductor, we are assuming a P type substrate; so, there would be a lot of holes in this substrate.

So, a positive potential on the gate terminal will deplete holes in the semiconductor which will essentially result in a depletion regime. And the bands the P type bands look like this, the Fermi level is quite close to the valence band in the semiconductor. And, but close to the interface between the semiconductor and the oxide, because of the depletion of the holes this Fermi level is now removed from the conduction and valence bands and this is how it looks like.

So, generally these bands are drawn in this direction and these are drawn for this MOS device; M means Metal which is the gate, O means Oxide, and S is the Semiconductor. So, generally these bands are representative of the MOS electrostatics these bands ok, metal oxide semiconductor electrostatics.

And this is the Poisson's equation that gives a relationship between the electrostatic potential and the charge distribution in the space. So; this is a qualitative way to understand this is an intuitive way to understand the MOSFET electrostatics. A more mathematical ways by solving the Poisson equation and that is what we will do right away.

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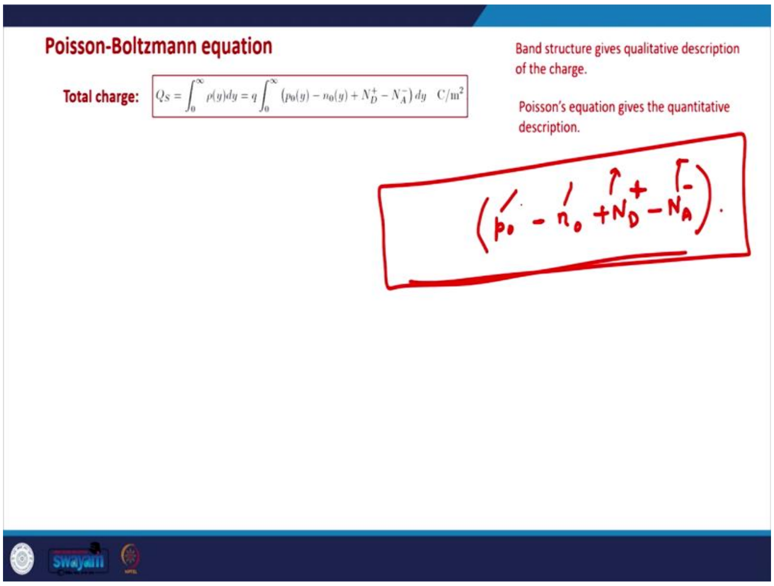

So, the total charge in the semiconductor in any general semiconductor is actually the charge because of the mobile charge carriers, which means the holes and electrons. So, the total charge will be the charge of the holes minus the charge of the electrons and the charge will also be because of the dopant atoms this will be the static charge. So, the donor atoms will contribute as a positive charge and acceptor atoms will contribute as negative charge.

So, this is the charge density in any general semiconductor material. So, if we have an arbitrary doping in a semiconductor, then at any point in the semiconductor the charge is given by the charge of the holes minus the charge of electrons plus the charge of ionized dopants donors minus the charge of the ionized acceptors. Because ionized donors are positively charged and ionized acceptors are negatively charged. So, that is what we also take in this expression and as you know that we are mostly concerned about the MOS electrostatics which is the electrostatics in the y direction.

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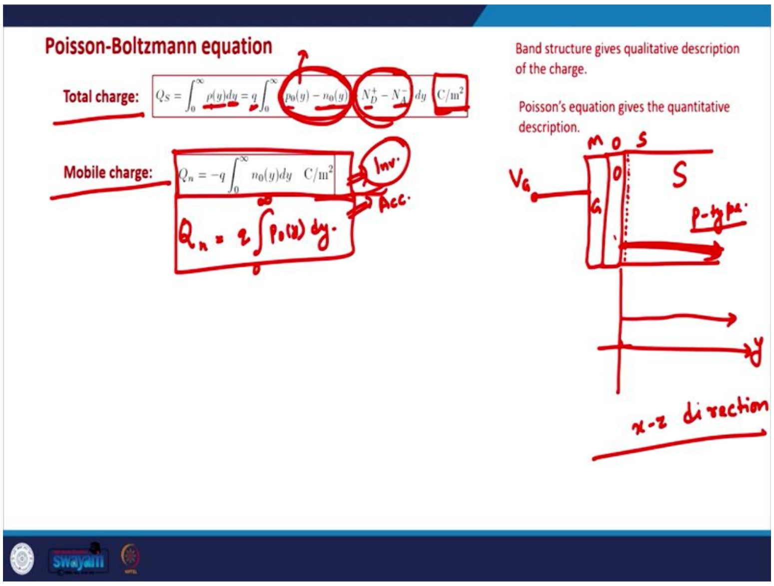

So, that the charge density; so, to say; so, this is the device in the y direction; we have a gate, we have an oxide, and we have a semiconductor ok. Generally, this point is considered to be the y equal to zero point this interface and we study try to study the electrostatics or we try to study the charge and potential in the semiconductor ok.

So, this is metal, this is oxide, this is semiconductor, this is P type. And when a positive voltage is applied on the gate terminal in that case what happens is that a negative charge is accumulated and for very high gate voltage an inversion layer of the electrons is created at the interface. And so, this inversion layer is extremely thin actually this is like a quasi 2D layer. And so that is why the charge or the sheet charge density of this inversion layer is calculated in coulombs per meter square or coulombs per centimeter square ok.

And this is so, in order to see the charge in y direction or in the charge in the this semiconductor as a function of or sheet charge density in the semiconductor, we need to integrate this total charge in y direction.

So, that will give us the total charge per unit area in x z direction which will further help us understand the sheet charge density of the inversion layer. But, at the moment we are trying to understand the total charge that is accumulated in the material, because of the because of the gate voltage which creates electric field in y direction and so that is why we take integration in y direction.

So, the total charge is given as the charge density integrated in y direction; so that will have the unit of coulomb per meter square. And this charge density in a semiconductor is $(p - n + N_D - N_A)$ $(-q)$ is q accounts for the elementary charge. So, this is the number q makes it the coulomb the charge and integrate it from 0 to infinity which means from interface to deep inside the semiconductor.

So, and this is the total charge and the mobile charge in the semiconductor will be; so, these the dopant atoms are the static charge. So, the mobile charge will be just because of these two components; because of the holes, and because of the electrons.

And in inversion layer in inversion of the during the operation of the MOSFET in inversion, the number of holes at the interface are extremely small in number. So, the only charge that is left is the charge because of the electrons. So, that is actually the mobile charge for electrons in the or mobile charge in a MOSFET or in a MOS device in inversion layer.

Similarly, the mobile charge in accumulation would be accumulation means when a negative voltage is applied on the gate terminal and lot of holes are attracted to the interface and it will have accumulation of holes; so, that will be given by $p_o(y)$ dy ok. So, this is how we calculate; so, this is the charge of our interest because these two charge this is in the accumulation, this is in the inversion and generally this inversion regime is the most important regime for a MOSFET device ok.

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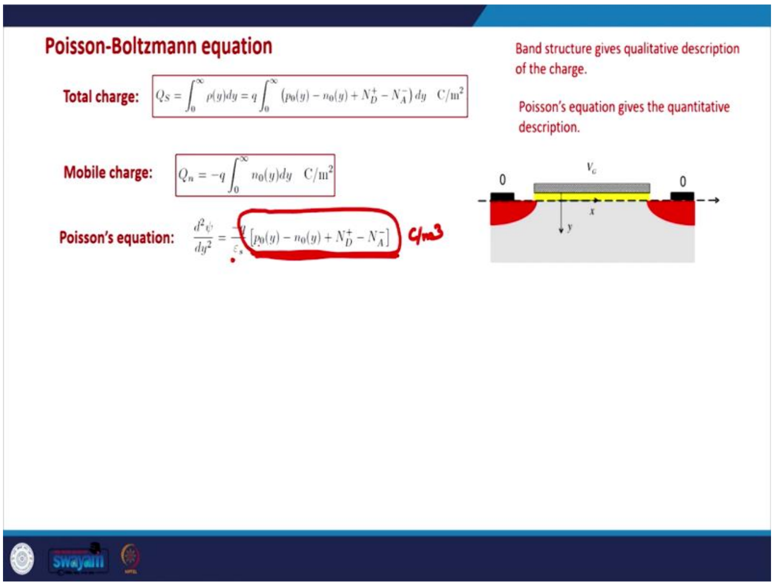

So, we apply the Poisson's equation, the Poisson's equation says that $(d^2\Psi/dy^2) = (-q/€s)$ $(p_o(y) - n_o(y) + N_D^+ - N_A^-)$ or (-charge density)/€s. And the charge density is actually this, this is this charge density q times p_o is in coulomb per meter cube; so, this quantity in this circle is in coulomb per meter cube.

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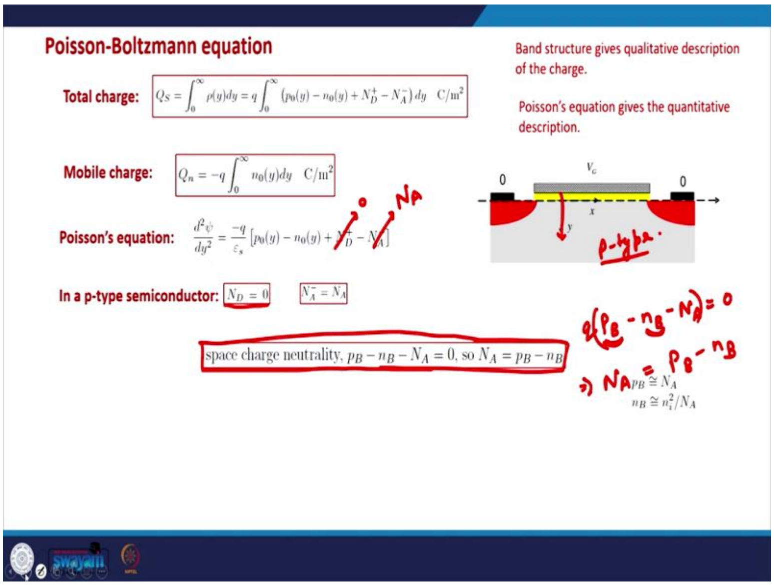

So, this is what we take in the Poisson equation and in a p type semiconductor which is generally the semiconductor in an n MOS which is actually the semiconductor in n MOS, the dopant atoms are not there we can assume that there is no dopant atoms. So, this N_D becomes 0 and we can also fairly assume that N_A^- which is the ionized acceptor atoms. These are equal to the total acceptor atom, which means that the all the acceptor atoms are now ionized; so, this N_A^- is equal to N_A.

And in the bulk which means that deep inside the semiconductor far away from the interface we can assume the space charge neutrality. Space charge neutrality means that the net charge because of the mobile charge and the static charge is 0. So, the charge of holes is p_B - n_B is the electronic charge $-N_A$ is the charge due to acceptor atoms; so, this should be 0 this should be 0.

So, which means that N_A the acceptor atoms are p_B - n_B; where, p_B is the bulk hole concentration which means number of holes per unit volume in the bulk deep inside the semiconductor n_B is the bulk electron concentration bulk electron density in a way, number of electrons per unit volume deep inside the semiconductor.

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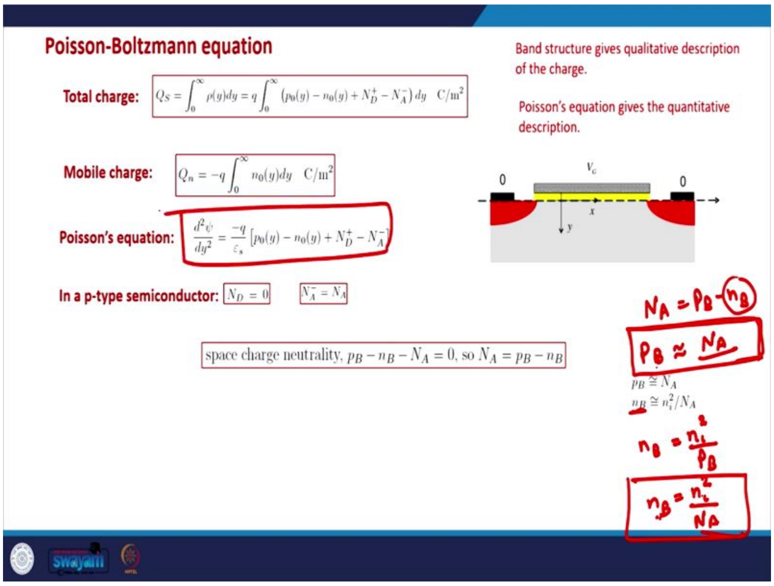

So, but generally in an N type material what happens is; so, this N_A turns out to be p_B - n_B. But, n_B is extremely small as compared to the p_B value; so, it can be assumed with a reasonable accuracy that p_B is almost equal to N_A. So, the number of holes in this bulk of the semiconductor is equal to the number of accepted atoms in the semiconductor.

And this n_B which is number of electrons will be given by this relationship from carrier statistics. So, this will be n_i^2 /N_A ok. So, n_B is the number of electrons in the bulk or the electron density in the bulk p_B is the electron density in the bulk, it is equal to the number of acceptor atoms and n_B is n_i^2 /N_A.

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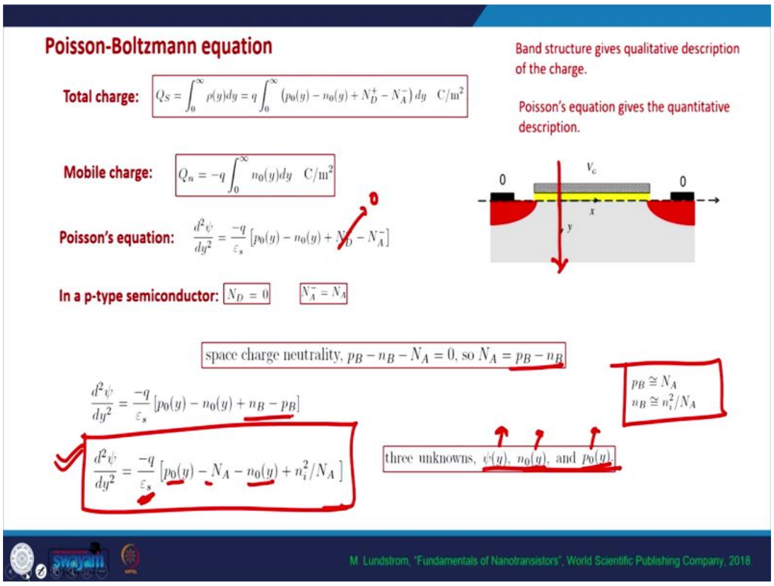

So, now using these values in this Poisson's equation this original Poisson's equation, we will now see that this equation this N_D becomes 0, N_A can be written as N_A, N_A^- can be written as N_A, and N_A can be written as p_B - n_B; so, it becomes n_B - p_B. So, now this n_B and p_B values can be obtained from here. So, this n_B is written as n_i^2 /N_A and p_B is written as - N_A, this is the Poisson equation for a semiconductor for a general semiconductor.

So, at any point y; so, in the y direction this is the direction in which we are trying to understand the electrostatics. The double derivative the second derivative of the electrostatic potential is (-q/€s); where, €s is the dielectric constant of the semiconductor is p. Is number of holes at that point -N_A minus number of electrons at that point + n_i^2/N_A ok.

So, now if you see this equation in this equation we have three unknowns essentially, we have this Ψ(y) is unknown, which means if we apply a certain gate voltage, we need to find out what is the Ψ(y). Also, this $p_o(y)$ and $n_o(y)$ are also unknowns, because we also do not know what are these values.

Because, on application of a of the gate voltage there might be accumulation of holes, there might be depletion of holes, there might be inversion layer; so, these quantities are also unknown. So, there are three unknowns in this equation; so, one equation three unknowns cannot be solved; so, we need two more equations and that is what we dig out.

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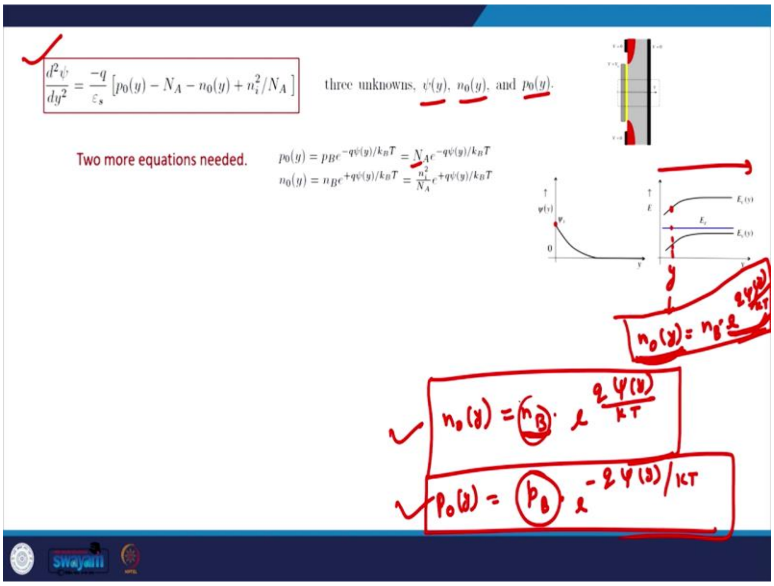

So, the two more equations are they come from the band diagram essentially. So, if this is the potential profile electrostatic potential versus y in the semiconductor, this will be the band profile in the y direction ok. And at any arbitrary point y here; let us say we are considering this point here, at any arbitrary point the number of electrons here which means or number of holes anything, number of electrons here will be number of electrons in the bulk times $e^{q\Psi(y)/kT}$.

Because, if Ψ(y) is positive, the bands will be bend down and in that case number of electrons will increase and this increase is directly exponentially related to the. So, then number or the carrier density is exponentially related to the band bending to the potential that and this comes from the basics of the carrier statistics in the semiconductors this is basic one and one type equation actually.

So, at any point y the number of electrons when there is a nonzero potential at that point is n_B which is the number of potential in the bulk, which means number of putting I am sorry number of electrons in the bulk n_B is the number of electrons or electron density in the bulk, and by bulk we mean that the potential is 0 in the bulk.

So, bulk is a point where the potential is 0; so, $n_o(y)$ is n_B times $e^{q\Psi(y)/kT}$. So, similarly the number of holes at any point is related to the number of holes in the bulk, and bulk is the region where the electrostatic potential is 0, $e^{-q\Psi(y)/kT}$.

So, apart from this Poisson equation we have now these two more equations and these two equations come from the basics of the semiconductor carrier statistics. And now using these three equations we can solve in principle we can solve for the three variables three unknowns, and the three unknowns are $\Psi(y)$ $n_o(y)$ and $p_o(y)$.

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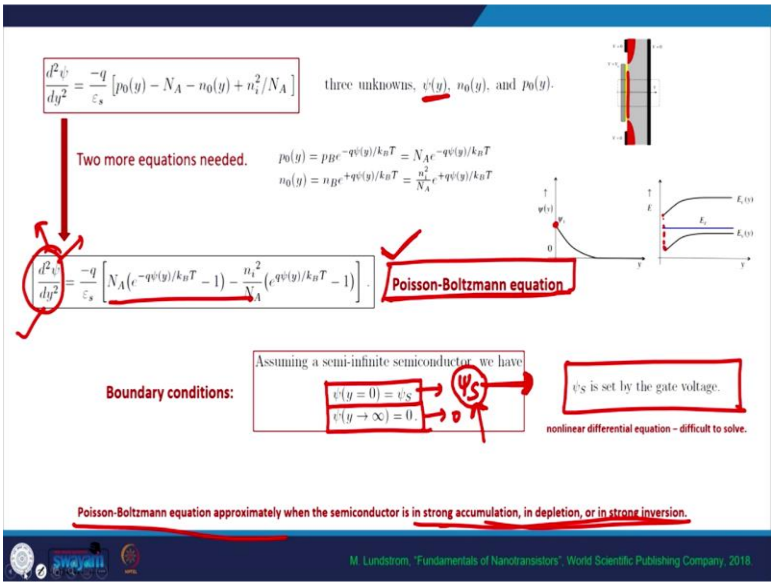

So, that is what we do and p_B is N_A essentially n_B is n_i^2/N_A and if we do if we put all these things in Poisson equation this is what we obtain essentially; $(d^2\Psi/dy^2)$ is $(-q/€s)$ N_A times $(e^{-q\Psi(y)/kT} – 1)$ - n_i^2/N_A $(e^{q\Psi(y)/kT} – 1)$. So, this equation is now the equation in a single variable that is the electrostatic potential in the semiconductor at any arbitrary point $\Psi(y)$ essentially.

And on the left hand side of this equation we have the second derivative of the potential, on the right hand side we have a complex function I would say complex function of $\Psi(y)$. But in principle by solving this equation which is also known as the Poisson Boltzmann equation, because this is a combination of the or this comes from the Poisson and the carrier statistics that is why the name Poisson Boltzmann equation.

This equation tells us about or this equation can precisely give us the electrostatic potential at any point y in the semiconductor when an arbitrary gate voltage is applied to the device ok. And in order to do this we would also need the boundary conditions and the boundary conditions will be that the potential electrostatic potential at right at the interface at y=0 is Ψ_S, and the electrostatic potential deep inside the semiconductor is 0.

So, these with these two boundary conditions and this Poisson Boltzmann equation we can solve for the potential in the semiconductor. And this is one of the main I would say this is one of the main things that we study in the MOSFET electrostatics that is the electrostatic potential inside the semiconductor. So, this equation is not so easy to solve actually by the way, because you can see that we have the second derivative of potential on the left hand side and on the right hand side we have the exponential function.

So, it is quite an involved calculation and generally the solution is done numerically. So, generally we use numerical techniques to software's to solve for the electrostatic potential. But as you can see from this discussion that in order to obtain the electrostatic potential profile in the semiconductor, we need to at least know the this Ψ_S parameter which is the voltage or which is the potential at the interface of the oxide and the semiconductor which is the value of Ψ at this point.

And this is one of the key things that we that is there actually in this in the electrostatics that we need to figure out. Because, once this is there then by using the Poisson Boltzmann equation we can numerically find out the entire distribution of the potential inside the semiconductor.

So, this becomes one of the one of our key things to figure out in our discussion of electrostatics Ψ_S, and this Ψ_S is actually set by the gate voltage it is determined by the gate voltage ok. So, that is how this effect of the gate voltage comes on the electrostatic potential in the semiconductor ok. So, this equation Poisson Boltzmann equation is solved in strong accumulation, depletion, and inversion region with certain approximations.

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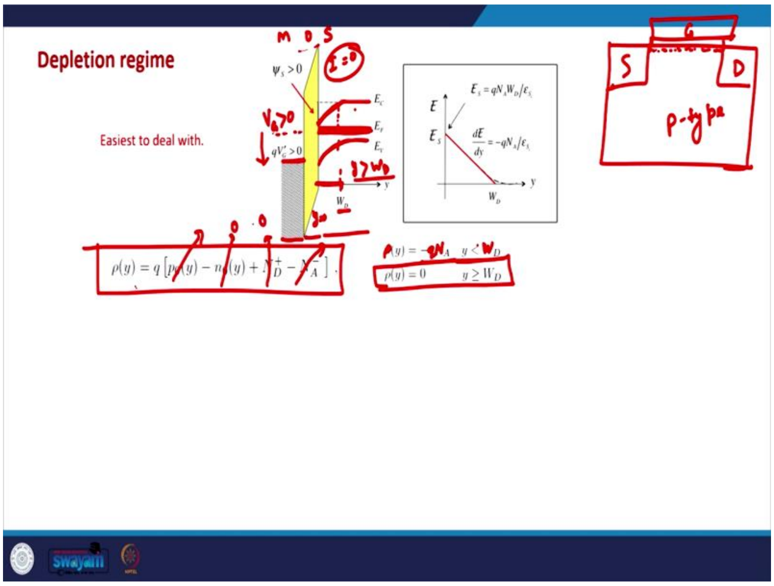

So, that is what we will see and the easiest one is the depletion region. So, the depletion region is the region in an n MOS; so, please remember that in an n MOS; the semiconductor the bulk semiconductor is p type and this is how the n MOS looks like. So, the source, the drain this is a P type semiconductor that is there oxide and gate ok; so, this is oxide. So, when a positive voltage is applied on the gate terminal that ripples the holes and the depletion of holes happen.

And for a more stronger positive voltage for a larger positive voltage, the minority carrier electrons accumulate at the interface and that is known as the inversion region. So, before the inversion, the region of operation this region is known as the depletion region; so, before the electrons are accumulated at the interface of the oxide in the semiconductor before that the this is known as the depletion region of the MOSFET.

And the depletion region is actually easiest to deal with; so, that is why we can we took it first, because we can precisely calculate precisely solve the Poisson equation in this region. This is how it looks like we have applied; so, this is the band diagram, this is the gate side, this is the oxide, and this is the semiconductor.

So, this is metal, this is oxide, this is semiconductor; and, as you can see that we have applied a positive voltage on the gate terminal which has brought down the Fermi level of the gate. Initially the Fermi level of the gate and the semiconductor were assumed to be aligned to each other.

So, this was also flat this oxide conduction band edge was also flat, but because of the positive gate voltage now, the Fermi level of the gate comes down and that results in depletion of the semiconductor, the holes are depleted from here which results in band bending. But please be careful here, because although the conduction band and the valence bands are bending, but the Fermi level is still the still uniform across the semiconductor entire semiconductor. Please take a moment and think about it why the Fermi level is still constant.

Even if we have applied a gate voltage, even in that case the Fermi level is still a uniform energy level across the semiconductor ok. So, the answer to this is that even if we apply a gate voltage, because of the oxide in between there cannot be any flow of electrons. Although on application of gate voltage the charge would like to flow, but this oxide will block the any kind of charge flow; so, that is why there is no net current in the device. So, net current is zero which means that the Fermi level must be uniform across the entire semiconductor.

So, in this in the depletion region, this is the we start with the charge density distribution this is how we write it down. And in the depletion region what happens is that the depletion of holes is there up to a certain width which is known as the depletion width and this is represented as W_D. So, before W_D from $y = 0$ to $y = W_D$ holes are depleted, and for y greater than W_D there is no depletion region it is just a normal semiconductor.

So, the charge distribution looks like this $\rho(y)$ is given as -q times N_A for y less than W_D. We are assuming that all the holes are taken away although in practice or in reality that is not the case there is a distribution of holes, but an ideal approximation is which is also known as the depletion approximation that in the depletion region the mobile charge carriers are not there.

So, which means that the charge density $\rho(y)$ is -q times N_A for y less than W_D, in the depletion width the charge density is this and after the depletion width the net charge will be 0. Because, after the depletion width the mobile charge this holes will neutralize the acceptor atoms, anyway the electrons are very less in number and donor atoms are not there ok.

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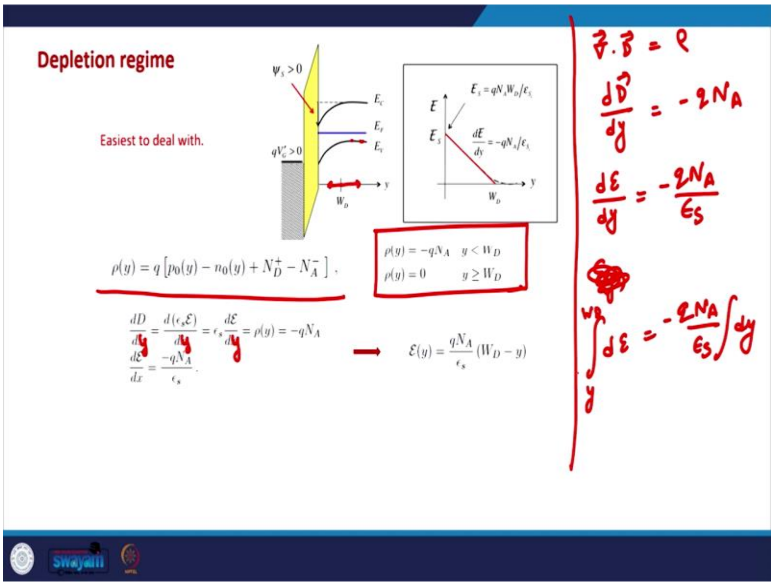

So, finally, this is the key point here that in the depletion region we can approximate the charge distribution by this equation. And the approximation here is that we assume that in the depletion width all the holes have been depleted out of the semiconductor; so, that is why we can write ρ(y) to be -q times N_A. Now, using this, now this Poisson equation becomes quite simple now; so, the Poisson equation will be now ideally it should be y here.

So, ▼.D ; so, we should start with this equation which is equal to charge. So, which means that in y direction dD/dy is equal to -q N_A and D can be written as E times electric field times $€_S$, $-qN_A/€_S$ which means that E(y), E as a function of y or if we integrate on both sides dE is $(-qN_A/€_S)$ dy ok.

Also, because the net charge is 0 in the deep inside the semiconductor beyond the depletion region, the electric field will also be 0. So, the electric field just after W_D is 0 and we need to see what is the electric field inside the depletion region. So, the electric field if we integrate it from an arbitrary point y to W_D; so, at any arbitrary point y the electric field will be E(y) and at W_D the electric field will be 0.

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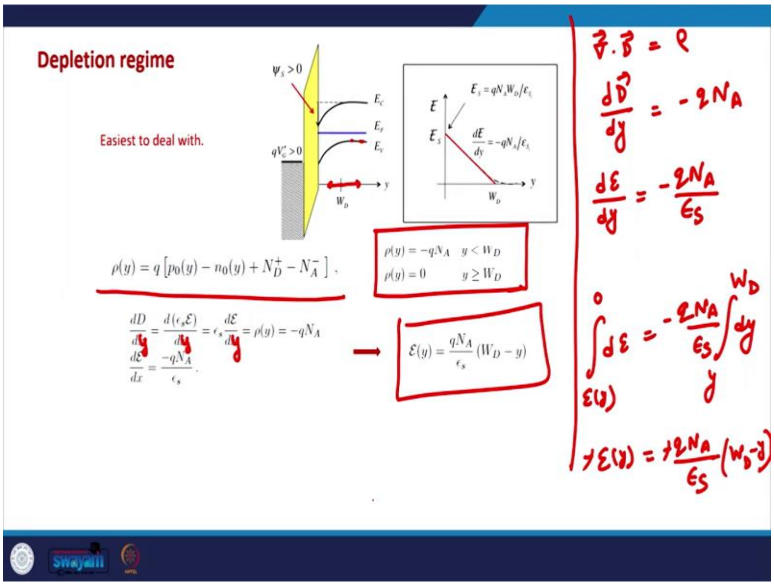

So, the integration limits are from E(y) to 0 and here the integration limits are y to W_D. So, if we integrate it like this on the left hand side we obtain -E(y), on the right hand side we have $(-qN_A/\epsilon_S)$ times $(W_D - y)$, negative sign goes away

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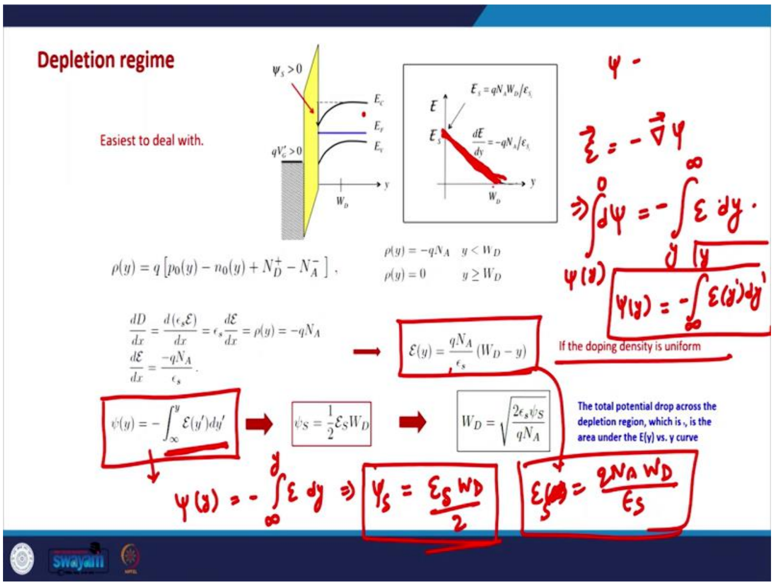

So, this electric field has a function of y is given by this expression which is $(qN_A/\epsilon_S)(W_D - y)$. This is the case in depletion approximation and we are assuming that the doping density is uniform ok; now, with this we can easily calculate the potential distribution as well. The potential function if you recall the relationship between the potential and; so, the electric field is essentially $-\nabla\Psi$.

So, which means that potential can be written as minus ok, potential deep inside the semiconductor is 0; so, at y equal to ∞, y equal to ∞ is generally the point which is deep inside the semiconductor and there the potential is also 0. So, generally this is how it looks like, and the limits are from at any arbitrary point y where the potential is $\Psi(y)$ to infinity where potential is 0; so, $\Psi(y)$ will be equal to $-\infty$ to y, $\int E(y')dy'$.

So, these are the basic equations of electrostatics, I am just reviewing it for the sake of completion of this discussion. Now, you see that in depletion region this electric field is just a function of y, a linear function of y; and if we integrate with this value of the electric field in this equation if we put $-\infty$ to y, Edy.

So, if we put y to be 0; so, this $\Psi(0)$ becomes Ψ_S, and if we assume a linear electric field then we can we obtain this $E_S W_D/2$. If we assume that the electric field or this is the E as a function of y is linear in that case; if E_S is the electric field at the interface at y equal to 0 then this right hand side integral will just be the area under this curve which will be $E_S W_D/2$; so, this is what we obtain.

And from this equation E_S will be $qN_A W_D/\epsilon_S$, because we need to put y equal to 0. Now using these two expressions of E_S; so, in a way we have obtained E_S in two ways, one is from this area under the curve and second is from this integral.

And by equating these two expressions we can find out a relationship between the W_D value and the ϵ_S value. And that is if you do a proper rearrangement this is how it looks like, W_D is equal to $\sqrt{(2\epsilon_S\Psi_S/ qN_A)}$ ok.

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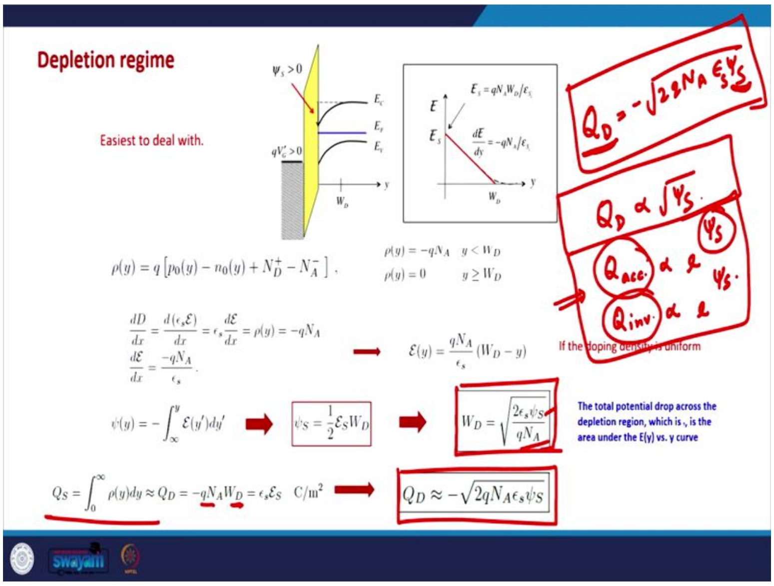

And from W_D we can also calculate the total depletion charge which will be just an integration of the this charge density and it will be $(-qN_A)(W_D)$; so, which will be square root. So, the depletion charge will be $-\sqrt{(2qN_A \epsilon_S \Psi_S)}$.

So, this is the classical electrostatics of the MOSFET, if we look at from the band diagram point of view and this is what we obtain by solving the Poisson equation in the depletion region. And in the depletion region, it is the most easiest to solve the Poisson equation in other regions it is it becomes difficult.

But, one of the main highlights of this discussion is that, that in the depletion region the charge is related to the surface potential this Ψ_S is also known as the surface potential in this way Q_D is directly proportional to square root of Ψ_S. However, in accumulation when we apply a negative voltage Q in accumulation region will be exponentially related to the surface potential. And also in the inversion region, because in inversion also it is the electrons the it is related to the amount of band bending.

And Ψ_S is directly related to the band bending and the accumulation and inversion charges the charge because of the mobile carriers; so, that is why they are exponentially related; but the depletion charge is related in this way. So, yeah just take a note here the about the accumulation and the inversion charges. The depletion charge we have calculated precisely and I would like to think more about I would like you to think more about how we are saying that the accumulation charge and the inversion charge are exponentially correlated to the surface potential.

Please go to the go back to the basics of the carrier statistics in semiconductors and see if there you would find it will be pretty much a straightforward relationship. So, we will stop here and in the coming class we will deal with the various capacitances in the MOSFET and the mobile charges ok.

I thank you for the attention for your attention see you in the next class.