Physics of Nanoscale Devices
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Lecture - 49
Velocity Saturation in Ballistic MOSFET and Electrostatics

Hello, everyone. Today, we will wrap up our discussion on the Velocity Saturation in Ballistic MOSFET – how this happens, how the velocity saturates and then we will start with a new topic that is the topic of MOSFET Electrostatics. Again, an important topic, but I hope this electrostatics part you must have studied already in some of your courses but, still we will try to have a comprehensive review of this entire topic. So, let us quickly see what we have been discussing in the last class.

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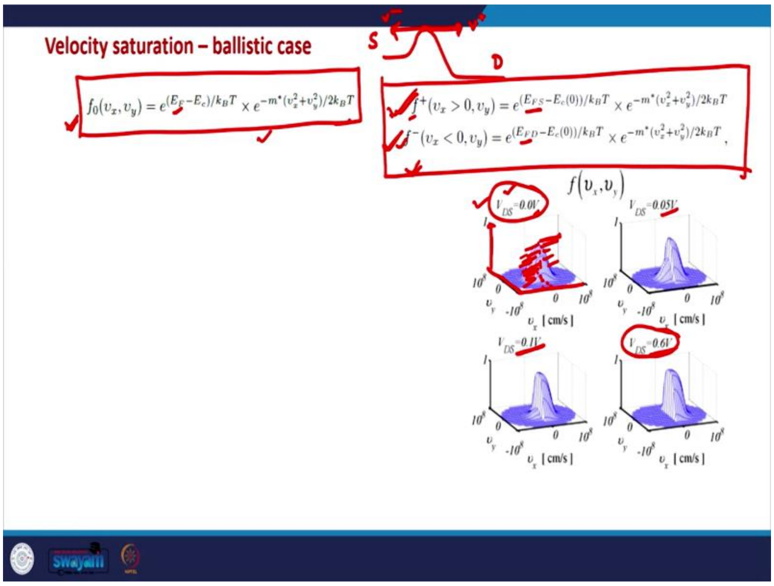

Velocity saturation – ballistic case

$$f_0(v_x, v_y) = e^{(E_F - E_c)/k_BT} \times e^{-m^*(v_x^2+v_y^2)/2k_BT}$$

$$f^+(v_x > 0, v_y) = e^{(E_{FS} - E_c(0))/k_BT} \times e^{-m^*(v_x^2+v_y^2)/2k_BT}$$

$$f^-(v_x < 0, v_y) = e^{(E_{FD} - E_c(0))/k_BT} \times e^{-m^*(v_x^2+v_y^2)/2k_BT},$$

In the last class, we actually started understanding the velocity distribution at the top of the barrier in the MOSFET. So, this is sort of the barrier in a MOSFET, this is the source side, this is the drain side and at the top of this barrier the velocity is extremely important and the distribution of positive velocity and negative velocity is what we are; we were discussing in the last class.

So, we started with the Fermi – Dirac distribution function and if we put proper values of energy value of energy in terms of kinetic energy we get the so called Maxwellian distribution of velocities which is given by this value. And, this can be which is given by this function and this function can further be broken down into two other functions; one is this f+ which is essentially the electrons going to the +x direction and f- which is electrons going to the -x direction.

Which means that f+ essentially means electrons going from source to drain side this way and f- accounts for electrons going from drain to source side this way and that is why in f+ instead of E_F we have E_{FS} and in f- instead of E_F we have E_{FD}. So, if we try to understand how or what is the velocity distribution for the electrons coming from the source side that is given by f+ and the velocity distribution for the electrons coming from the drain side is given by f-.

So, this is how it looks like actually and here what has been done is that this distribution of velocities you we have also seen this plot in the last class as well that this Maxwellian distribution or this f+ f- velocity distribution has been plotted for four different values of the drain voltage. One is the drain voltage of 0 volts second is the drain voltage of 0.5 volts, third is the drain voltage of voltage of 0.1 volts and fourth is the drain voltage of 0.6 volts.

So, this value is pretty huge for a ballistic MOSFET and this is the 0 volts and on the plot we have on this z-axis we have the this function this distribution function f and on the x-axis we have v_x, on y-axis we have v_y. So, this is v_x this is v_y and this is f. So, as you can see that when the drain voltage is 0, it means that both drain and source contacts are symmetric to each other. So, the probability that an electron will come from the source side and the probability that the electron will come from the drain side is equal actually.

So, that is why you see that this distribution of velocities is also symmetric in both in positive and negative direction. So, these are the positive velocities and these are the negative velocities, ok. So, these are the velocities coming from the source side and these are the velocities of the electrons coming from the drain side and since V_{DS} is 0 volts amounts to the equilibrium state.

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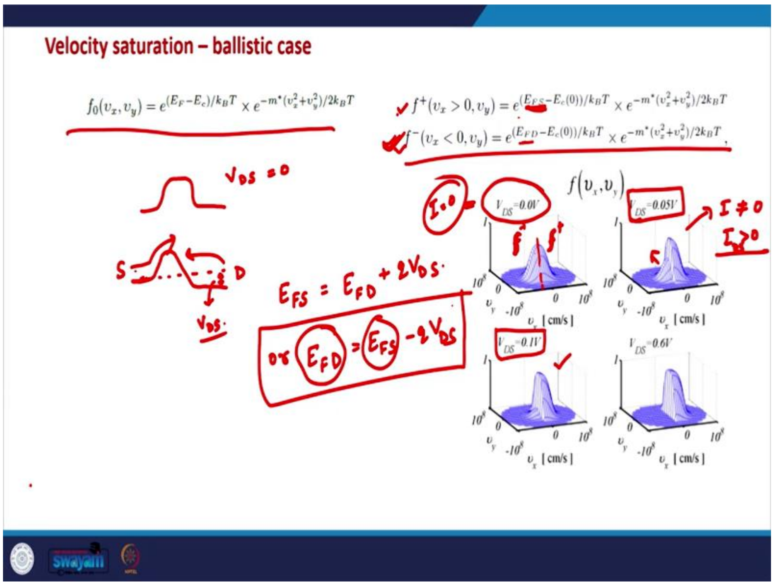

Velocity saturation – ballistic case

$$f_0(v_x, v_y) = e^{(E_F - E_c)/k_BT} \times e^{-m^*(v_x^2+v_y^2)/2k_BT}$$

$$f^+(v_x > 0, v_y) = e^{(E_{FS} - E_c(0))/k_BT} \times e^{-m^*(v_x^2+v_y^2)/2k_BT}$$

$$f^-(v_x < 0, v_y) = e^{(E_{FD} - E_c(0))/k_BT} \times e^{-m^*(v_x^2+v_y^2)/2k_BT},$$

$V_{DS} = 0$

$E_{FS} = E_{FD} + qV_{DS}$.

or $E_{FD} = E_{FS} - qV_{DS}$

$I = 0$

$I \neq 0$

$I > 0$

So, which means that f+ which is the positive half of this distribution and f minus which is the negative half are both equal to each other.

Now, there is an important question that is that comes here and that is that these Fermi functions or in the case of non degenerate semiconductors this boils down to the Maxwell-Boltzmann distribution function in this way. So, these are equilibrium functions and in the equilibrium is achieved by collisions in a material, but in a ballistic MOSFET the collision does not happen in the channel there is no collision in the channel then the question is how is this distribution maintained.

How is it so that f+ is equal to f- and moreover this the shape of f+ and the shape of f- is the proper equilibrium shape. And, let me sort of remind you that you need to go back to the contacts because the contacts are huge and there is lot of scattering that is happening in the contacts.

And this equilibrium is maintained because of the collisions of the electrons in the contacts and that is why this these f+ and f- functions are maintaining their shape in the equilibrium. So, as we can see that the positive velocity distribution and the negative velocity distribution is exactly the same. So, the net velocity will be 0 and the net current will also be 0. So, this case corresponds to a 0 current as expected.

Now, when a slightly when a slight V_{DS} voltage is applied, it means that this is now. So, when the V_{DS} is 0 in that case the barrier looks like this is the case when V_{DS} is 0 when V_{DS} is non-zero in that case this barrier is slightly tilted and this is the source side, this is the drain side. Now, the drain side is slightly below the source side because some V_{DS} has been applied ok.

So, this is the V_{DS}; now with V_{DS} as you can see from this the difference between E_{FS} and E_{FD} or E_{FD} is E_{FS} -qV_{DS}. So, this E_{FD} is smaller than E_{FS} when there is a nonzero V_{DS} applied across the ballistic MOSFET. So, in that case as expected from this equation when E_{FD} is small smaller than E_{FS} in that case this f- values will be smaller than the f+ values. So, that is what we exactly see.

And, here as you can see that the positive half of this distribution function is larger as compared to the negative half of the distribution function of the velocities. So, that is there because now for the negative electrons for the negative velocity electrons the electrons from the drain side the barrier is slightly more than the barrier for the positive velocity electrons.

So, that is why the probability that a electron from the drain side will cross the barrier is now less and that is what is also reflected here. Here you can see that in the negative half we see a smaller value and on in the positive half we see a larger value. So, which means that now there is a net velocity in the system and there is a net current in the device. So, this corresponds to a net current, now the current is greater than 0 volts I_{DS} is greater than 0 volts.

And, as this V_{DS} is increased in that case as expected here that this negative velocity the negative velocity distribution is getting more and more smaller because this E_{FD} is getting small the value of E_{FD} is now more less than what value of E_{FD} is quite below the value of E_{FS}. So, that is why this f+ will be larger as compared to f- and this is what we see. In this V_{DS} is equal to 0.1 volts plot as well.

So, as expected the positive half of this distribution is still intact because there is no change in the as such there is no change in the barrier from the source side to the top of the barrier the height of. So, there is no change in the height of the barrier from source side to the top and that is why this positive half almost remains the same.

However, I would like you to sort of think and look more closely into these pictures because one thing that is happening with increasing V_{DS} is that this negative half is getting diminished it is getting reduced in size at the same time positive half is there it is intact. In fact, it is increasing slightly.

As you as you would see if you look closely here that the height of this total distribution function was now has increased in the positive side as we are increasing the V_{DS} value.

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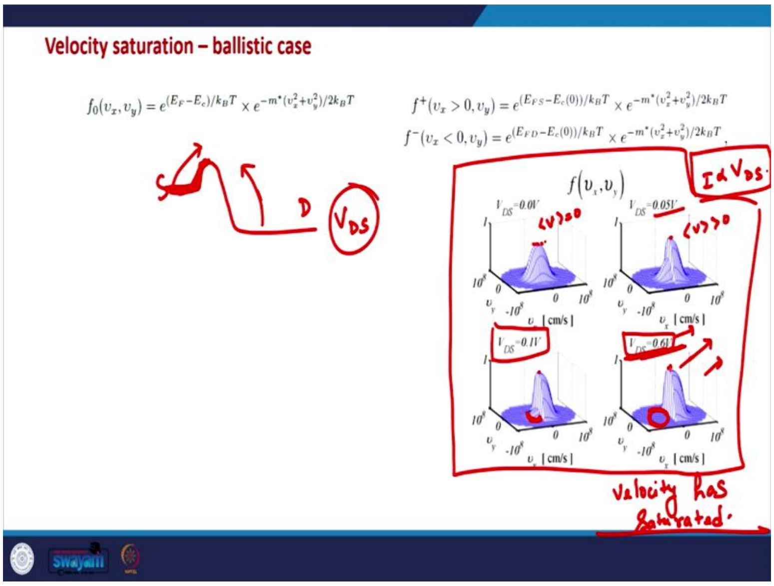

Velocity saturation – ballistic case

$$f_0(v_x, v_y) = e^{(E_F - E_c)/k_B T} \times e^{-m^*(v_x^2+v_y^2)/2k_B T}$$

$$f^+(v_x > 0, v_y) = e^{(E_{FS} - E_c(0))/k_B T} \times e^{-m^*(v_x^2+v_y^2)/2k_B T}$$

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$f(v_x, v_y)$

$I \propto V_{DS}$

Velocity has saturated.

And, at V_{DS} is equal to 0.6 volts at which there is no negative velocity components because now the drain side is quite below the this top of the barrier and it is almost impossible for an electron to go from drain to the top of the barrier and that is why there is no negative velocity distribution seen here.

However, there is a positive velocity distribution and there is this interesting fact that this magnitude of the distribution function has increased slightly. So, this is this will become clear after the close inspection, but leaving that point for a moment and trying to understand the velocity saturation in ballistic MOSFET.

As we are increasing the V_{DS} value as we are increasing the drain voltage in that case what is happening is that the negative velocity electrons are getting less and less in number. So, there is net current in the system and after a certain voltage so, there is a net current and the current is directly proportional to the V_{DS} value. So, and then this is known as the linear region.

At the same time the velocity the net velocity of electrons is also increasing because net velocity here is 0, the net velocity here is positive, the net velocity here is now greater than this case and now this net velocity is maximum. So, at 0.05 volts there is some there are still some negative velocity electrons, at 0.1 volts these negative velocity electrons are extremely small in number.

And, as we increase the voltage slightly above this 0.1 volt value the negative velocity electrons will be almost 0 and at 0.6 volts the negative velocity electrons are almost gone. So, as long as there were negative velocity electrons and with increasing voltage, the negative velocity distribution function was going down, till then the velocity the net velocity of electron was also increasing.

But, as soon as this negative velocity electrons are negligible in number it means that now the velocity has saturated because even if we apply more voltage above this V_{DS} above 0.6 volts even in that case there is even in that case this velocity will not increase further because the net velocity is the difference in velocities of the positive directed electrons and the negatively directed electrons.

So, after 0.6 volts of drain voltage the velocity increase will be 0 because there are no more negative velocity electrons and this velocity has saturated and this actually accounts for the velocity saturation in the ballistic MOSFETs which is quite different from the velocity saturation in long MOSFETs because there the velocity saturates because of the collisions in the channel.

Here it is entirely differently it is because now the barrier from the drain side has become. So, huge that electrons cannot cross the barrier from the drain side. So, only the electrons from the source side are there. So, that means, that and this source to this barrier height is almost constant, it does not change much. So, the velocity will also not change much after a certain drain source voltage ok.

So, I hope this point is clear how the velocity saturates in a ballistic MOSFET. This is an important point and this comes directly from the microscopic inspection of the velocities in the ballistic MOSFET. Now, let us come to the second point which is that as we are increasing the V_{DS} voltage as we are increasing this V_{DS} value in this case even. So, in this case first what we have just discussed is that the negative velocity electrons are going down, but at the same time the positive velocity electrons are becoming slightly or this positive velocity distribution is slightly becoming more in height.

And, what is what could possibly be the reason of this? So, I would ask you to take a moment and think about it why is this height of the positive distribution of velocities increasing with V_{DS}.

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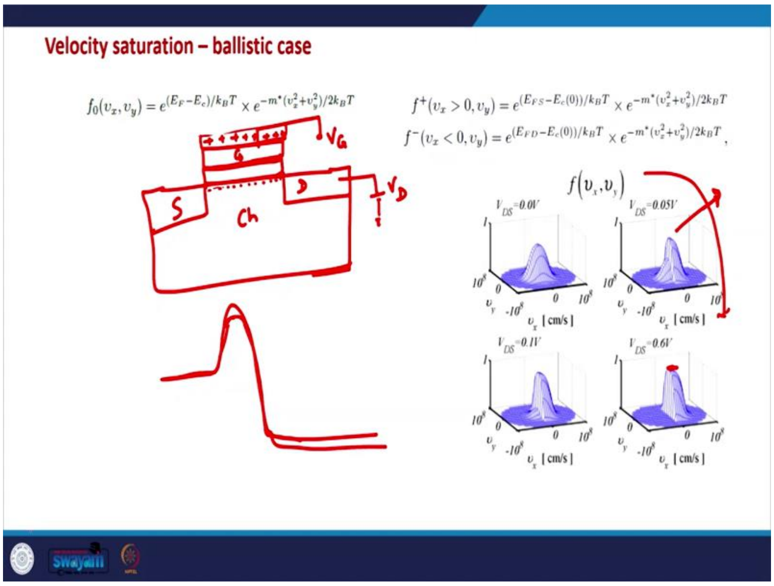

Velocity saturation – ballistic case

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And, the answer to this actually goes back to the design of the MOSFET. In the MOSFET generally what happens is that we have a gate terminal we have an oxide and we have a below the oxide we have the source, the drain and the channel region. So, this is how the MOSFET looks like. We apply certain gate voltage.

So, as soon as the gate voltage is applied it actually results in depositing some or in other words what we can say is if we apply a positive gate voltage, we can say that there is a positive charge that is accumulated on the gate terminal. And, generally the channel charge the negative charge that is induced in the channel because of this positive voltage applied on the gate terminal that depends mostly on the this gate charge or that depends mostly on the gate voltage in a properly designed MOSFET.

So, if the DIBL in the MOSFET is low which means that the effect of the drain voltage on the channel charge is low in that case this charge in the channel depends only on the gate charge. Now, what is happening in this case is that as the negative velocity electrons are reducing in number, it means that the charge is also going down in the channel because the charge in the channel is due to both positive velocity electrons and the negative velocity electrons.

Now, since with a with an increased V_{DS}, this the velocity is increasing net velocity is increasing, but the charge is reducing and in order to compensate for this reduced charge in the channel because this charge depends only on the gate charge. So, there should be a mechanism from where some extra charge should come. So, the number of positive velocity electrons are increased in the channel by reducing the barrier.

So, initially if the barrier is like this if the barrier in the MOSFET looks like this the barrier is reduced slightly because for higher drain voltages.

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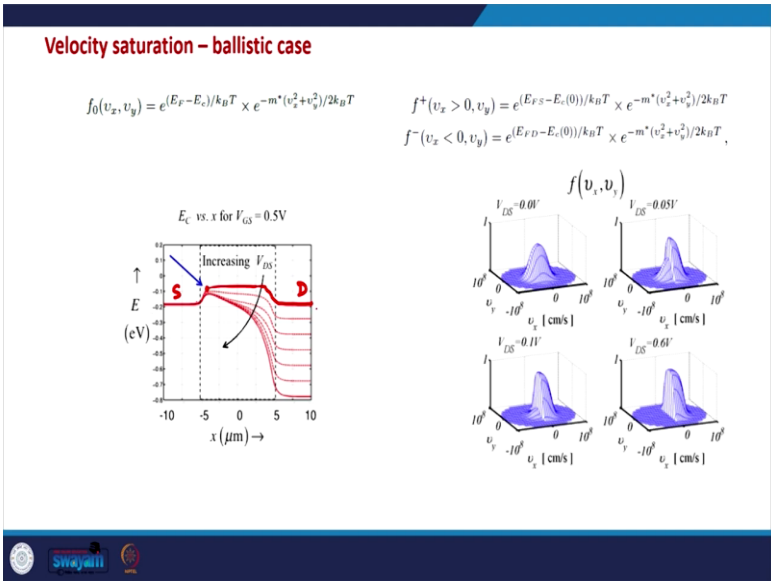

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So, a better a cleaner picture is shown here as you can see here that this is the source region this is the drain region this is the illustration of the barrier ideally this starting point of the barrier should not depend on the drain voltage and it does not depend because that amounts to DIBL in the MOSFETs.

But, as we are increasing V_{DS} which means that this drain Fermi level is going down and that this is the barrier is changing the shape like this initially it was like this then it becomes like this and it becomes like this. So, what is happening is that now because this shape is changing so, the negative velocity electrons are getting removed from the system which means that the now the charge is reducing.

And, in order to compensate for that charge this top of the barrier this peak of the barrier also comes down so that more positive velocity electrons are pumped from the source side in order to neutralize the charge reduction because of the diminishing negative velocity electrons. So, do you see this point that in order to compensate for the charge loss due to the removal of negative velocity electrons.

Now, this barrier height is getting lowered in the channel this is happening automatically this is happen this comes from the electrostatics of the MOSFET and that is why we see slight increase in the height of this positive velocity distribution in the velocity distribution of the electrons in the MOSFET in a ballistic MOSFET.

So, in a ballistic MOSFET along with velocity saturation a slightly lowering in the top of the barrier is also happening and these two are important key points because the these things are fundamentally different from a conventional MOSFET, the way velocity saturates there and the way the barrier is sort of lowered ok.

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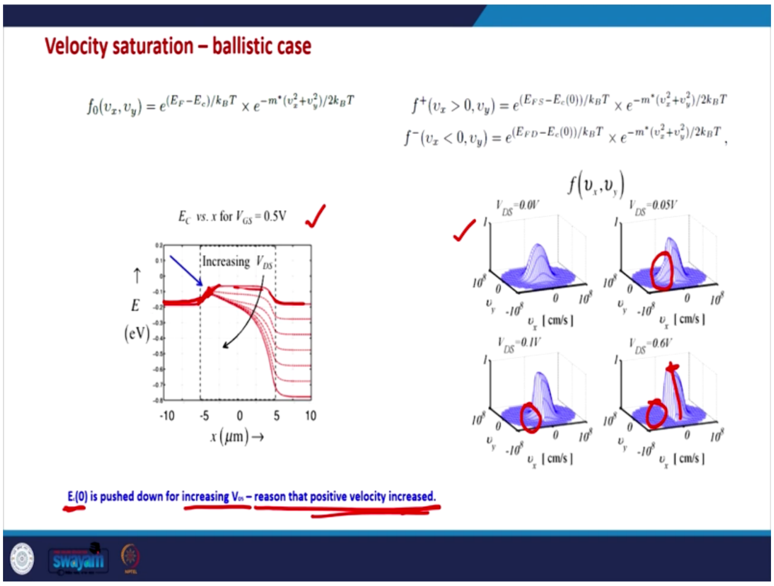

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$$f^-(v_x < 0, v_y) = e^{(E_{FD} - E_c(0))/k_BT} \times e^{-m^*(v_x^2+v_y^2)/2k_BT},$$

$E_c(0)$ is pushed down for increasing V_{DS} – reason that positive velocity increased.

So, I hope this thing is now clear we have already done a mathematical analysis of the velocity saturation, the injection velocity saturation and as you can see this E_{C0} is essentially this profile of this electron profile in the channel this is pushed down for increasing V_{DS} and that is to increase the positive velocity electrons and or in other words it is to increase the charge in the channel or to compensate for the charge loss in the channel because of the absence of the negative velocity electrons.

So, I hope this these two ideas this idea of velocity saturation and this idea of barrier tuning or barrier changing because of this velocity saturation are now clear.

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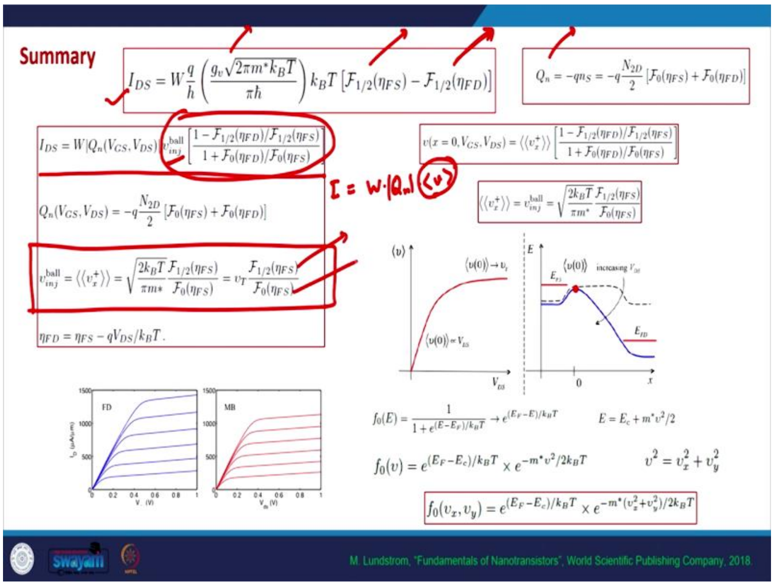

Summary

$$I_{DS} = W\frac{q}{h}\left(\frac{g_v\sqrt{2\pi m^* k_B T}}{\pi\hbar}\right)k_BT\left[\mathcal{F}_{1/2}(\eta_{FS}) - \mathcal{F}_{1/2}(\eta_{FD})\right]$$

$$Q_n = -qn_S = -q\frac{N_{2D}}{2}\left[\mathcal{F}_0(\eta_{FS}) + \mathcal{F}_0(\eta_{FD})\right]$$

$$I_{DS} = W|Q_n(V_{GS}, V_{DS})|\,v_{inj}^{ball}\left[\frac{1-\mathcal{F}_{1/2}(\eta_{FD})/\mathcal{F}_{1/2}(\eta_{FS})}{1+\mathcal{F}_0(\eta_{FD})/\mathcal{F}_0(\eta_{FS})}\right]$$

$$I = W\cdot|Q_n|\langle v\rangle$$

$$Q_n(V_{GS}, V_{DS}) = -q\frac{N_{2D}}{2}\left[\mathcal{F}_0(\eta_{FS}) + \mathcal{F}_0(\eta_{FD})\right]$$

$$v_{inj}^{ball} = \langle\langle v_x^+\rangle\rangle = \sqrt{\frac{2k_BT}{\pi m*}}\frac{\mathcal{F}_{1/2}(\eta_{FS})}{\mathcal{F}_0(\eta_{FS})} = v_T\frac{\mathcal{F}_{1/2}(\eta_{FS})}{\mathcal{F}_0(\eta_{FS})}$$

$$\eta_{FD} = \eta_{FS} - qV_{DS}/k_BT\,.$$

$$v(x=0, V_{GS}, V_{DS}) = \langle\langle v_x^+\rangle\rangle\left[\frac{1-\mathcal{F}_{1/2}(\eta_{FD})/\mathcal{F}_{1/2}(\eta_{FS})}{1+\mathcal{F}_0(\eta_{FD})/\mathcal{F}_0(\eta_{FS})}\right]$$

$$\langle\langle v_x^+\rangle\rangle = v_{inj}^{ball} = \sqrt{\frac{2k_BT}{\pi m^*}}\frac{\mathcal{F}_{1/2}(\eta_{FS})}{\mathcal{F}_0(\eta_{FS})}$$

$$f_0(E) = \frac{1}{1+e^{(E-E_F)/k_BT}} \rightarrow e^{(E_F-E)/k_BT} \qquad E = E_c + m^*v^2/2$$

$$f_0(v) = e^{(E_F-E_c)/k_BT} \times e^{-m^*v^2/2k_BT} \qquad v^2 = v_x^2 + v_y^2$$

$$f_0(v_x, v_y) = e^{(E_F-E_c)/k_BT} \times e^{-m^*(v_x^2+v_y^2)/2k_BT}$$

M. Lundstrom, "Fundamentals of Nanotransistors", World Scientific Publishing Company, 2018.

So, let us summarize of this MOSFET transport theory the especially the ballistic transport theory in MOSFETs. What we have seen is that generally the current in a 2D MOSFET is given by this and this comes from the number of modes. These Fermi – Dirac integrals are direct consequence of the source and drain Fermi functions. In steady state the charge is given by this expression, steady state the current is given by this expression.

And, from these two expressions we can also represent this current in this form and so, this form is pretty much similar to the classical form of the current in which the current is represented like this and from here we obtain the average velocity of electrons at the top of the barrier which is given by this entire expression.

And, the first term in this expression is the ballistic injection velocity which is given by this expression all of them involve Fermi – Dirac integrals, but there is and Fermi – Dirac integrals are quite a complex mathematical objects, but there is a way out for us and that way out is that for non degenerate semiconductors these Fermi – Dirac integrals are simplified to a great extent by their exponentials.

So, after that what we saw is we analyzed this velocity at the top of the barrier and we saw that this velocity saturates as the drain voltage is increased and generally the typical order of this velocity ballistic injection velocity is 10 to the power 7 centimeter per or of that order 10 to the power 7 centimeter per second.

Along with this velocity saturation we also see a current saturation in the ballistic MOSFET and here this these results are the simulation result for a ballistic MOSFET one with the Fermi – Dirac statistics and second with the Maxwell Boltzmann statistics.

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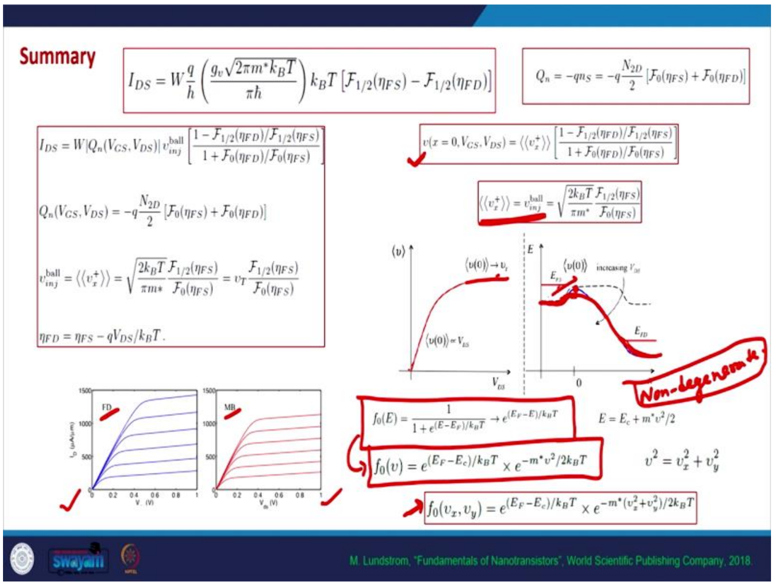

Summary

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$$Q_n = -qn_S = -q\frac{N_{2D}}{2}\left[\mathcal{F}_0(\eta_{FS}) + \mathcal{F}_0(\eta_{FD})\right]$$

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$$Q_n(V_{GS},V_{DS}) = -q\frac{N_{2D}}{2}\left[\mathcal{F}_0(\eta_{FS}) + \mathcal{F}_0(\eta_{FD})\right]$$

$$v_{inj}^{\text{ball}} = \langle\langle v_x^+\rangle\rangle = \sqrt{\frac{2k_BT}{\pi m*}}\frac{\mathcal{F}_{1/2}(\eta_{FS})}{\mathcal{F}_0(\eta_{FS})} = v_T\frac{\mathcal{F}_{1/2}(\eta_{FS})}{\mathcal{F}_0(\eta_{FS})}$$

$$\eta_{FD} = \eta_{FS} - qV_{DS}/k_BT\,.$$

$$v(x=0,V_{GS},V_{DS}) = \langle\langle v_x^+\rangle\rangle\left[\frac{1-\mathcal{F}_{1/2}(\eta_{FD})/\mathcal{F}_{1/2}(\eta_{FS})}{1+\mathcal{F}_0(\eta_{FD})/\mathcal{F}_0(\eta_{FS})}\right]$$

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$$f_0(E) = \frac{1}{1+e^{(E-E_F)/k_BT}} \to e^{(E_F-E)/k_BT}$$

$$E = E_c + m^*v^2/2$$

$$f_0(v) = e^{(E_F-E_c)/k_BT} \times e^{-m^*v^2/2k_BT}$$

$$v^2 = v_x^2 + v_y^2$$

$$f_0(v_x,v_y) = e^{(E_F-E_c)/k_BT} \times e^{-m^*(v_x^2+v_y^2)/2k_BT}$$

M. Lundstrom, "Fundamentals of Nanotransistors", World Scientific Publishing Company, 2018.

Then in order to understand the velocity saturation in MOSFETs we used the Fermi – Dirac distribution function and here we could convert this function to the velocity distribution function or the Maxwellian velocity distribution function for non-degenerate semiconductor case.

So, this non-degenerate semiconductor case is the actually the case most of the times. So, it is not a very bad approximation and by analyzing the velocity and this further can be broken down in the positive velocity components and the negative velocity distribution components and by analyzing these positive velocity distribution and the negative velocity distribution, we could understand that in the ballistic MOSFET the velocity will saturate after a certain time.

Because the negative velocity electrons almost vanishes negative velocity electrons almost vanish from the system and on increasing the V_{DS} further, the velocity will no longer increase ok. And, in order to compensate for that this barrier height may be slightly reduced because of the vanishing negative velocity electrons the charge will also go down, but the charge depends on the gate charge and that this in order to sort of account for that this barrier is lowered so that the positive velocity electrons are increased and the charge is maintained in the channel.

So, this is what happens in a ballistic MOSFET and I hope this is clear to you. So, I would advise all of you to go through this mathematically and also try to understand this intuitively and physically as well. So, with this we finish our transport theory ballistic transport theory or more generally the Landauer transport theory for MOSFETs.

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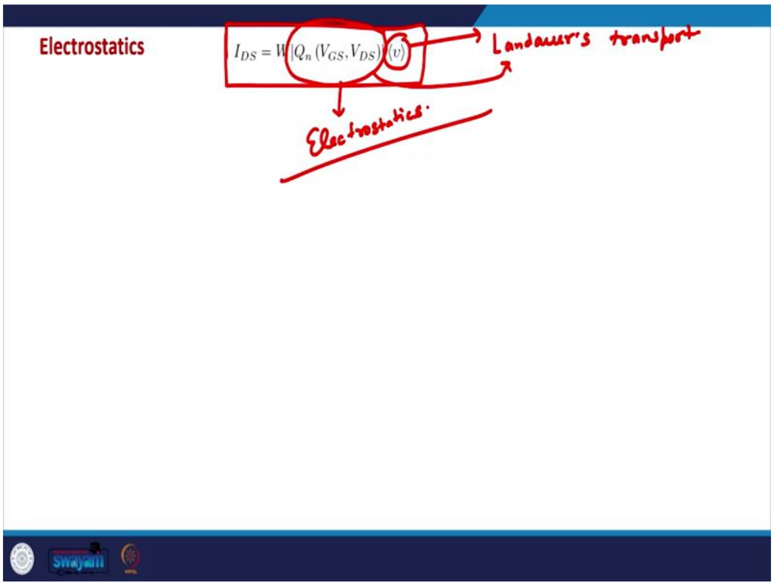

Electrostatics

$I_{DS} = W |Q_n(V_{GS}, V_{DS})| \langle v \rangle$

Now, from this expression in order to properly understand the MOSFET or any device it is clear from this expression that this velocity we have properly understood now from the Landauer's transport. This charge is this charge also comes in discussion of the Landauer's transport, but we have not dealt with the charge in the channel in detailed manner and in order to understand this charge component we need to understand the electrostatics in the MOSFET.

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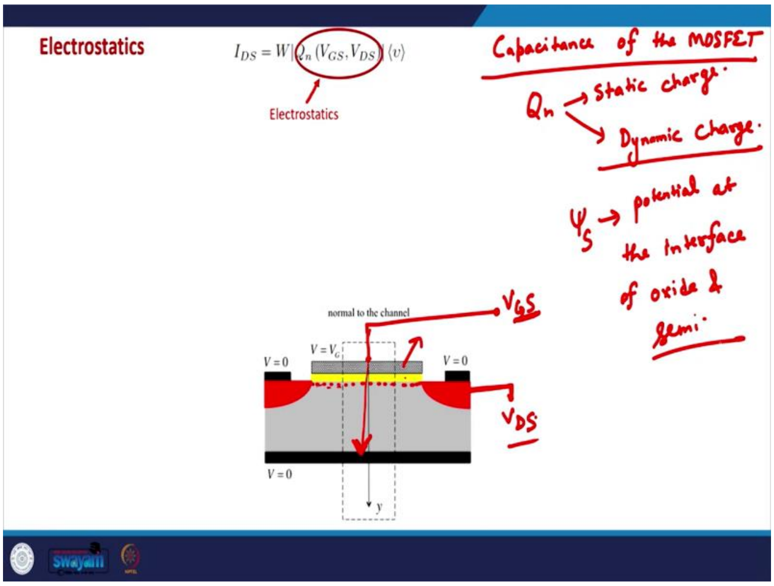

Electrostatics

$I_{DS} = W|Q_n(V_{GS}, V_{DS})|\langle v \rangle$

Electrostatics

Capacitance of the MOSFET

$Q_n \to$ Static charge.

$\searrow$ Dynamic Charge.

$\psi_S \to$ potential at the interface of oxide & semi.

So, that is what we will start now we will start with the MOSFET electrostatics and MOSFET electrostatics means that we are now trying to understand the relationship between the charge in the channel and as a consequence of that current in the channel as a function of the applied gate voltage and as a function of the drain voltage.

Because all of us know that generally there are two voltage terminals in the MOSFET one is we apply voltage on the gate terminal the control terminal and second is we apply voltage on the drain terminal which is the forcing terminal in a way. Because the drain terminal actually forces the electrons to or they it induces a net current in the system.

So, as a function of these two applied voltages or more generally they are written as V_{GS} and V_{DS} we will try to see what is the charge in the device as a function of these voltages. And, electrostatics is not only confined to just studying the charge, there are many more concepts that also come into picture and one is the capacitance of the device.

So, capacitance of the MOSFET capacitance mean generally the capacitance mean the gate capacitance which is because the MOSFET if we look from the gate side it also looks like a capacitor there is a; there is a metal plate there is an oxide there is downside on the other side of the oxide there is a semiconductor.

And, in this semiconductor the charges there might be a positive charge if we apply a negative voltage on the gate terminal or there might be a positive charge if we apply. So, there might be a positive charge if V_{GS} is negative and there might be a negative charge if V_{GS} is positive.

So, this entire system also has a certain capacitance. In fact, it is a voltage dependence capacitor and that is what we also study in the MOSFET electrostatics. The major component is to study the static charge and dynamic charge static charge means the charge that does not move. So, that is the charge because of the dopant atoms because of the fixed dopant atoms and dynamic charges the charge that moves.

So, the dynamic charge is the charge because of the mobile charge carriers that is because of the electrons and holes in the semiconductor. So, with this so, these are certain concepts that we want to study in electrostatics apart from this there is one more important topic that generally we focus upon and that is the quantity known as psi S and psi S is the potential at the interface of oxide and semiconductor.

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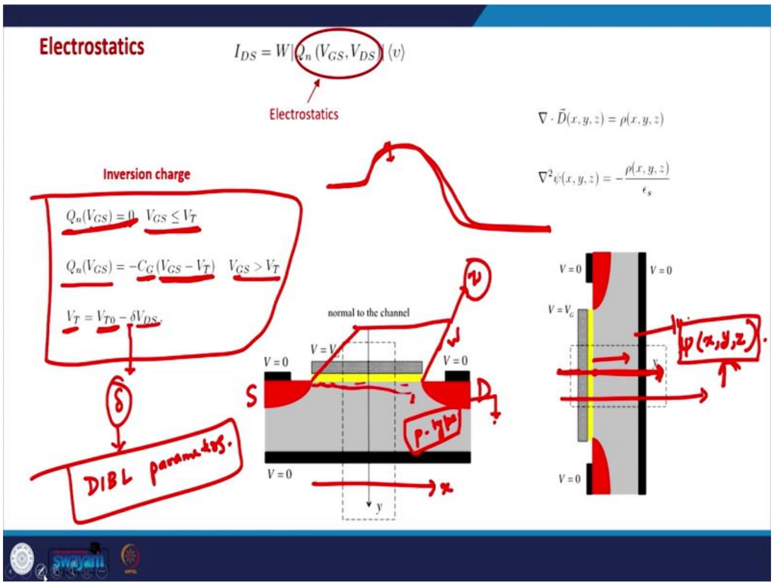

Electrostatics

$$I_{DS} = W|Q_n(V_{GS}, V_{DS})|\langle v\rangle$$

Electrostatics

$$\nabla \cdot \vec{D}(x,y,z) = \rho(x,y,z)$$

$$\nabla^2 \psi(x,y,z) = -\frac{\rho(x,y,z)}{\epsilon_s}$$

Inversion charge

$Q_n(V_{GS}) = 0$ $V_{GS} \le V_T$

$Q_n(V_{GS}) = -C_G(V_{GS} - V_T)$ $V_{GS} > V_T$

$V_T = V_{T0} - \delta V_{DS}$

So, let me sort of remove this and show you this entire picture in this direction the direction from gate to the semiconductor side because of an applied gate voltage there is a certain potential that is created that is that arrives in the semiconductor as well.

So, this other terminal of the semiconductor is generally grounded generally it is put at the ground, but because of the positive voltage on the gate terminal there is a certain positive potential that also is produced here. And, this voltage this is known as the this is known as the potential in the semiconductor and this is generally a function of the all the three parameters x, y and z parameters.

So, x is the direction of the channel from the source to the drain side. So, it is from the source to the drain side y is the direction from the gate to the semiconductor side and z is the direction which is right inside the perpendicular to the screen. So, that will in a way that will be the direction along the width of the semiconductor width of the MOSFET. So, this is the z-direction.

So, generally this potential in the semiconductor is a function of x, y, z because we apply voltage or we apply voltage on gate drain terminal which create electric field in x-direction and we apply voltage on gate voltage which create electric field in y-direction.

So, because of these there might be a complicated potential profile in the semiconductor that we are interested in because this potential in the semiconductor will directly govern the charge in the semiconductor and that. So, in a way this is an extremely important quantity in the MOSFET discussions and this becomes one of the central point of discussion in the electrostatics of the MOSFET.

So, generally with this quick background of the electrostatics and since this electrostatics is I would say quite close to the conventional electrostatics of the MOSFET. So, we will be slightly quick in this discussion in the discussion of electrostatics especially in the beginning of the in the beginning when we discuss the conventional electrostatics of the MOSFET.

So, generally these equations so far these equations have been assumed to be true and what are these equations? So, we assume that. So, this Q_n is the charge in the channel the mobile charge in the channel we assume that when V_{GS} is less than V_T. So, below threshold voltage this mobile charge in the channel is 0.

So, this is typically what we have assume so far and the mobile charge in the channel above V_T depends on the gate capacitance and the difference between V_{GS} and V_T. So, this we have also seen while discussing the IV characteristics of the conventional

MOSFET and V_T the threshold voltage is or V_{T0} which is threshold voltage when there is no drain voltage $-\delta V_{DS}$.

So, this delta parameter is there and this is known as the DIBL parameter. So, generally in MOSFETs what happens is because of the drain voltage the barrier in the channel is reduced. So, generally this is how the barrier looks like and because of the drain voltage this barrier point this top of the barrier is also reduced and that might reduce the threshold voltage as well.

So, threshold voltage just to quickly remind you is the voltage at which the channel is formed with the negative charges and this semiconductor is assumed to be a p-type semiconductor and in order to accumulate negative charges here, these negative charges either would come from the minority carriers in the p-type material or they would come from the source or drain contacts in the in this contact of the or in this device ok.

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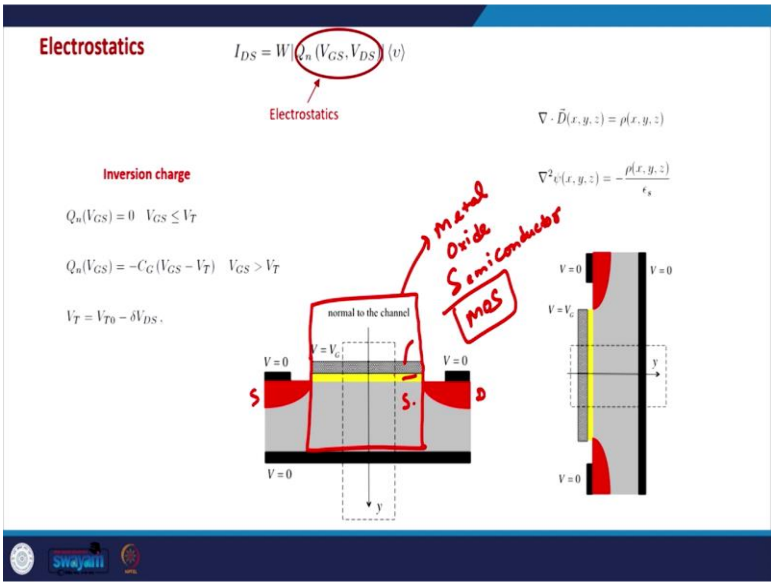

Electrostatics

$$I_{DS} = W|Q_n(V_{GS}, V_{DS})|\langle v \rangle$$

Electrostatics

$$\nabla \cdot \vec{D}(x,y,z) = \rho(x,y,z)$$

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Inversion charge

$Q_n(V_{GS}) = 0 \quad V_{GS} \le V_T$

$Q_n(V_{GS}) = -C_G(V_{GS} - V_T) \quad V_{GS} > V_T$

$V_T = V_{T0} - \delta V_{DS}$.

So, generally in MOSFET electrostatics first generally what happens is generally first we just try to understand the electrostatics of this part this device which is the device without the source and the drain contacts and this device this part of the device is known as the Metal Oxide Semiconductor MOS this is known as the MOS device because here we have a metallic gate, we have an oxide and we have a semiconductor.

In the generally in conventional discussion of electrostatics first electrostatics without the source and the drain contact is done and then finally, the effect of the source and the drain contacts is also understood and the beginning point in electrostatics discussion is or almost always is this equation.

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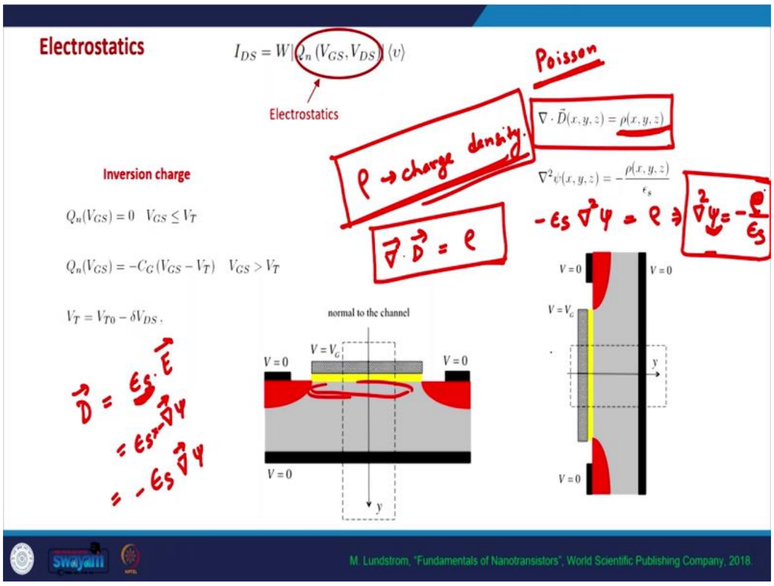

Electrostatics

$I_{DS} = W|Q_n(V_{GS}, V_{DS})|\langle v \rangle$

Electrostatics

Poisson

$\nabla \cdot \vec{D}(x,y,z) = \rho(x,y,z)$

$\rho \rightarrow$ charge density.

$\vec{\nabla} \cdot \vec{D} = \rho$

$\nabla^2 \psi(x,y,z) = -\frac{\rho(x,y,z)}{\epsilon_s}$

$-\epsilon_s \vec{\nabla}^2 \psi = \rho \Rightarrow \vec{\nabla}^2 \psi = -\frac{\rho}{\epsilon_s}$

Inversion charge

$Q_n(V_{GS}) = 0 \quad V_{GS} \le V_T$

$Q_n(V_{GS}) = -C_G(V_{GS} - V_T) \quad V_{GS} > V_T$

$V_T = V_{T0} - \delta V_{DS},$

$\vec{D} = \epsilon_s \cdot \vec{E}$

$= \epsilon_s - \vec{\nabla}\psi$

$= -\epsilon_s \vec{\nabla}\psi$

M. Lundstrom, "Fundamentals of Nanotransistors", World Scientific Publishing Company, 2018.

So, I would recommend you to if you already do not know this I would recommend you to remember this equation because this is one of the most fundamental equations that come from the physics and this is known as the Poisson equation. And, generally the pronunciation is not Poisson it is Poisson generally it is a French name it is pronounced as Poisson.

So, Poisson equation is the starting point for discussion of the electrostatics in the MOSFET. And, what it says is that the relation there is a relationship between the electric field and the charge in a certain area of the space. So, if rho is the charge density in a certain area of space and this here we are discussing the Poisson equation in very general terms.

Then because of this charge distribution and this can be a function of x, y, z so, this can be any distribution of charge in the space because of this charge distribution the electric field vector which is the displacement vector of electric field will be related to it like this. The gradient of the electric field vector is equal to the gradient of displacement vector is equal to the distribution of the charge in the space.

And, generally D is related to E by €_S times E where E is the electric field. So, since we are mostly concerned about the charge in the semiconductor and this €_S is the permittivity of the semiconductor or dielectric constant of the semiconductor this is the relationship and E is related to the potential $-\blacktriangledown\Psi$.

So, this becomes $-\text{€}_S\ \blacktriangledown\Psi$, where psi is the potential distribution or potential function in the semiconductor. So, this equation the Poisson equation will be transformed into if we put instead of D we put $-\text{€}_S\blacktriangledown\Psi$ it will be $-\text{€}_S\blacktriangledown^2\Psi$ is equal to ρ. So, which will be $\blacktriangledown^2\Psi$ is equal to $-\rho/\text{€}_S$.

So, that is what is actually the Poisson equation in the most general form and using this kind of equation this equation we can find out the potential function corresponding to a charge distribution in any system. So, that is what we also do in the semiconductors.

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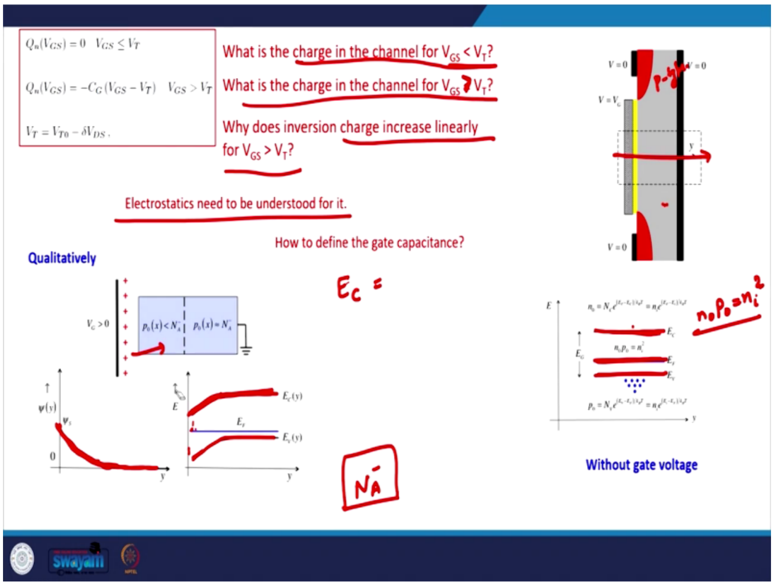

$Q_n(V_{GS}) = 0 \quad V_{GS} \le V_T$

$Q_n(V_{GS}) = -C_G(V_{GS} - V_T) \quad V_{GS} > V_T$

$V_T = V_{T0} - \delta V_{DS}$.

What is the charge in the channel for $V_{GS} < V_T$?

What is the charge in the channel for $V_{GS} > V_T$?

Why does inversion charge increase linearly for $V_{GS} > V_T$?

Electrostatics need to be understood for it.

How to define the gate capacitance?

Qualitatively

$E_c =$

N_A^-

Without gate voltage

$n_0 p_0 = n_i^2$

So, in semiconductors the again just to repeat that in the electrostatics generally the questions that we ask is what is the charge in the channel for below threshold, what is the charge in the channel above threshold it should be above threshold, why does inversion charge increase linearly above threshold. And, in order to answer all these questions we need to understand the electrostatics and the starting point for electrostatics is the Poisson equation.

And, and just to sort of quickly tell you another important aspect while discussing the electrostatics is that the Poisson equation gives us mathematically the potential function as a function of charge distribution. But, a more visual tool a better visual tool to understand electrostatics is the band diagram of the semiconductor.

So, since all we are discussing a p-mos sorry n-mos case in which this substrate is p-type semiconductor and if we flip the device like this and if we plot the band diagram of this material along y direction in that case what we see is that this is how the band diagram of the semiconductor look like because the semiconductor is p type semiconductor. So, we have the valence band the conduction band and the Fermi level is close to the valence band, ok.

And, in a in any semiconductor in equilibrium generally this relationship is always maintained now what happens to this band gap when a positive voltage is applied on the gate terminal. So, a positive voltage on the gate terminal means that now the positive charge in the semiconductor will be repelled. So, it means that so, this p-type. So, this is the p-type semiconductor. So, there are lot of holes in the semiconductor and because of the positive voltage on the gate terminal these holes will be repelled.

So, which means that now these acceptor atoms will be left exposed and generally the acceptor atoms are negatively charged. So, close to the gates close to the interface towards the gate there will be a depletion of holes and a net negative charge in the semiconductor. So, this is how the bands will look like when a positive voltage is applied on the gate terminal ok.

Because this shows that this at the interface so, this is the interface between the oxide and the semiconductor and at the interface this Fermi level is going away from the valence band and it is coming close to the conduction band. So, what it means is that or the for this Fermi level is now right between the valence band and the conduction band.

So, what it means is that the access carriers the holes in the system that were present now have depleted and there is now negative charge or a depletion charge in the system ok. And, the potential corresponding to this is generally will be given by this function because the relationship between the bands and the potential function is always inverse.

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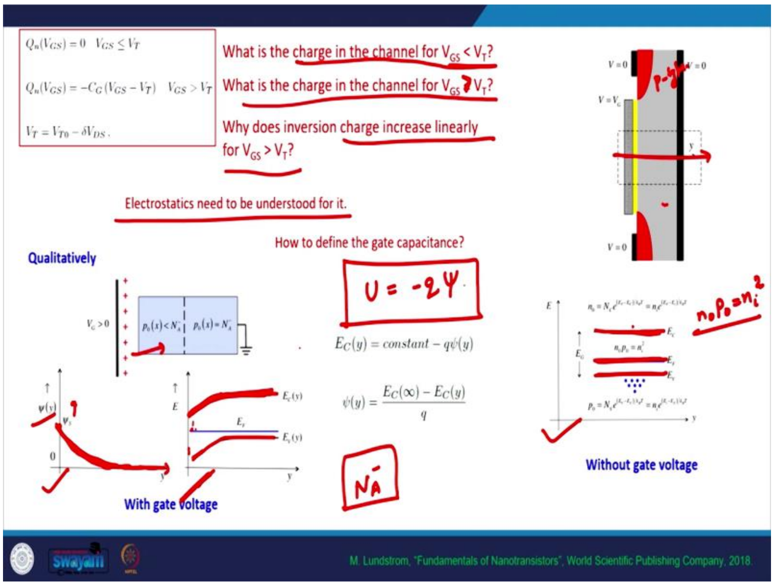

$Q_n(V_{GS}) = 0 \quad V_{GS} \leq V_T$

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$V_T = V_{T0} - \delta V_{DS}$.

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What is the charge in the channel for $V_{GS} > V_T$?

Why does inversion charge increase linearly for $V_{GS} > V_T$?

Electrostatics need to be understood for it.

How to define the gate capacitance?

Qualitatively

$U = -q\Psi$.

$E_C(y) = constant - q\psi(y)$

$\psi(y) = \dfrac{E_C(\infty) - E_C(y)}{q}$

N_A^-

$n_0 p_0 = n_i^2$

With gate voltage

Without gate voltage

M. Lundstrom, "Fundamentals of Nanotransistors", World Scientific Publishing Company, 2018.

Because as you might have seen that generally the bands represent the potential energy the band edges represent the potential energy and the potential energy or potential energy of this of any system or for the electrons is -q times the potential.

So, if the potential energy is reducing which it means that there is a positive potential inside the semiconductor. So, that is why the bending in the bands and the bending in the potential function is always in the opposite direction. So, if the bends are bending in this way this is how the potential function Ψ_y as a function of y will change.

So, these are so, this band diagram of the body band diagram of the semiconductor and this potential function and how these bands bend actually when we apply a gate voltage that is quite an important visualization tool in understanding the MOSFET electrostatics.

So, this is a more this is I would say a better way to understand the relationship between the band edges and the potential function. Generally, the band edge at any point is given as a reference value because the potential energy is always calculated with respect to a reference value minus the potential at that point.

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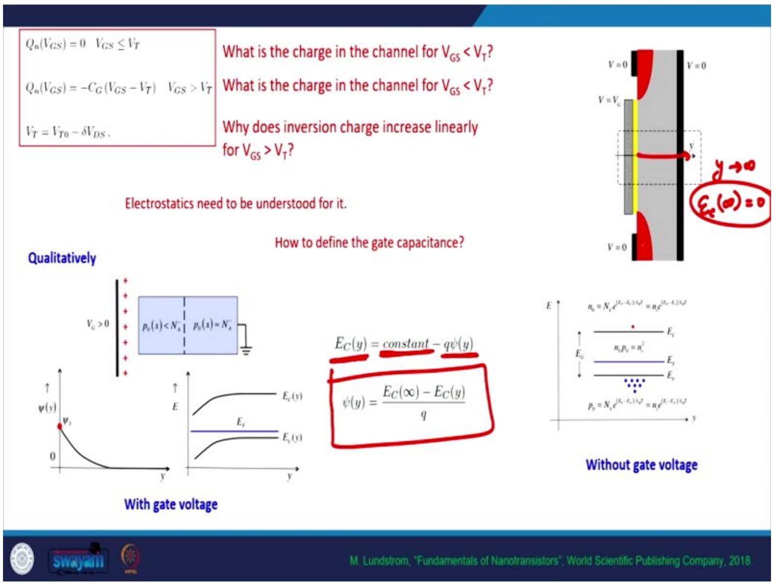

$Q_n(V_{GS}) = 0 \quad V_{GS} \le V_T$

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What is the charge in the channel for $V_{GS} < V_T$?

What is the charge in the channel for $V_{GS} < V_T$?

Why does inversion charge increase linearly for $V_{GS} > V_T$?

Electrostatics need to be understood for it.

How to define the gate capacitance?

Qualitatively

$E_C(y) = constant - q\psi(y)$

$$\boxed{\psi(y) = \frac{E_C(\infty) - E_C(y)}{q}}$$

With gate voltage

Without gate voltage

M. Lundstrom, "Fundamentals of Nanotransistors", World Scientific Publishing Company, 2018.

So, this reference value generally the potential energy deep inside the semiconductor which means in this y direction at as y tends to ∞. So, this $E_C(\infty)$ is assumed to be the reference potential energy and this potential function depends on the potential energy in this way ok.

So, just take a moment and maybe think about it. So, mathematically we need to use the Poisson equation and intuitively we need to understand the band diagram in the semiconductor when we apply any voltage in this on the gate terminal and later on the drain terminal as well. So, that will give us both intuitive idea and the mathematical idea about the electrostatics of the system.

So, in the next class we will start with more elaborate discussion of the electrostatics. Till then I would recommend you to revise the velocity saturation in the ballistic MOSFET and in the and the introduction of the electrostatics of the MOSFET.

So, thanks all of you. See you in the next class.