Physics of Nanoscale Devices
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Lecture - 48
Ballistic Injection Velocity

Hello everyone in today's class if you recall we, in the last class we stopped at the concept of the injection velocity in a MOSFET. In today's class we will discuss that concept in greater detail. So, that is I would say the most important idea in the MOSFET transport that is to understand the injection velocity right at the top of the barrier in the MOSFET.

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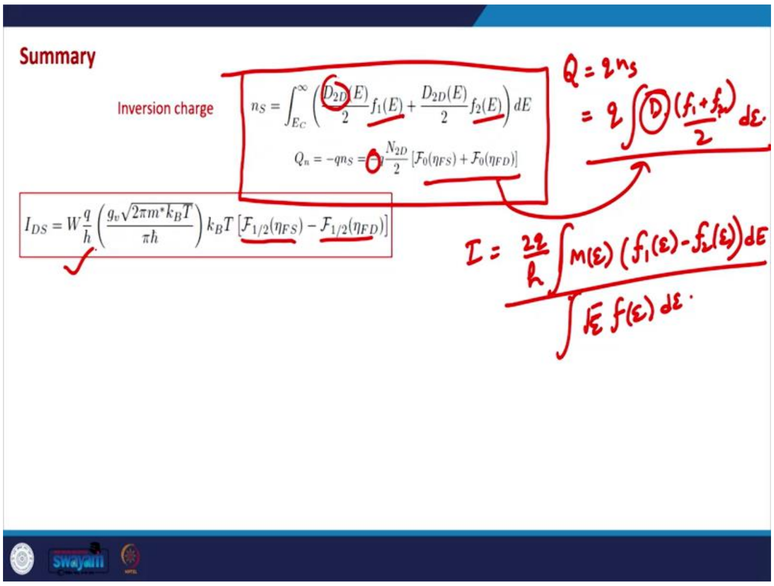

And in this case, we are considering a ballistic MOSFET and we are trying to understand the injection velocity in a ballistic MOSFET. Before going into those details let us quickly review what we have seen so far. So, what we have seen is that in steady state in a ballistic MOSFET the charge in the channel which is the inversion charge is given by this expression and this comes from the Landauer transport model, this is actually another form of this expression which is q times $\int D\ (f_1 - f_2)/2\ dE$, where this negative sign is there.

Because the charge involved is the electrons and D is the density of states which is represented by D_{2D} here, f_1 is f1(E) f_2 is f2(E). So, this is this the inversion charge can be calculated from this simple expression and the current can be calculated if you recall from this expression for a ballistic MOSFET ok. And as you can see, as we now by now we have seen this in more details in quite a lot of details, that depending on the factor in the in this integral we will obtain Fermi-Dirac integrals of various orders when we simplify this integral ok.

So, Fermi-Dirac integral of order plus half is obtained when we have square root of E times Fermi function in the integral ok.

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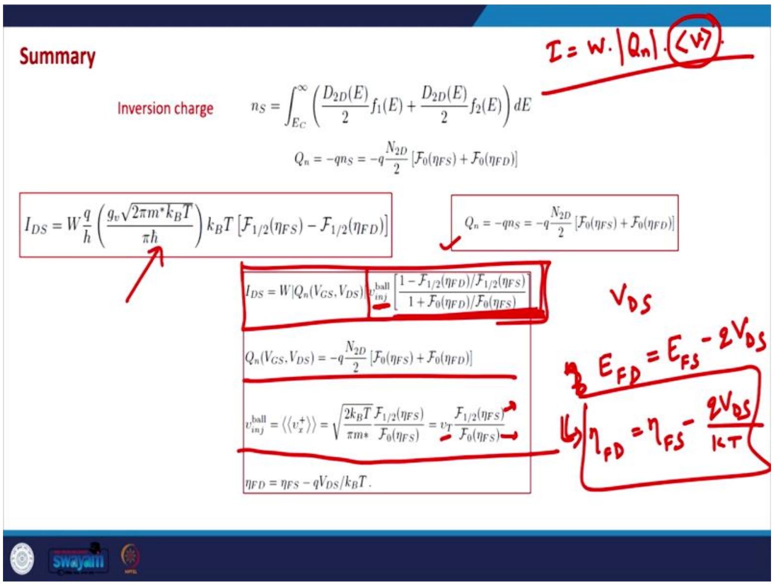

So, this is the current expression that we obtain and the charge expression is this and if we try to understand this expression in our more general form of current expression, which is given by this expression I is equal to W times Q_n times average velocity.

So, in this case this is what we obtain and from here what you can see is at the average velocity right at the beginning of the channel is this ballistic injection velocity times this factor which is a complicated looking factor involving various Fermi-Dirac integrals in it. So, this is the charge, the ballistic injection velocity is just the thermal unidirectional thermal velocity times Fermi-Dirac integral of order plus half divided by Fermi-Dirac integral of order 0.

And when a V_{DS} voltage is applied across the system then in that case this E_{FD} is E_{FS} - qV_{DS} and as a consequence of this η_{FD} is $\eta_{FS} - (qV_{DS}/kT)$. So, now, this average velocity is this total this expression and this is what we will try to understand today, this is the central topic of today's discussion.

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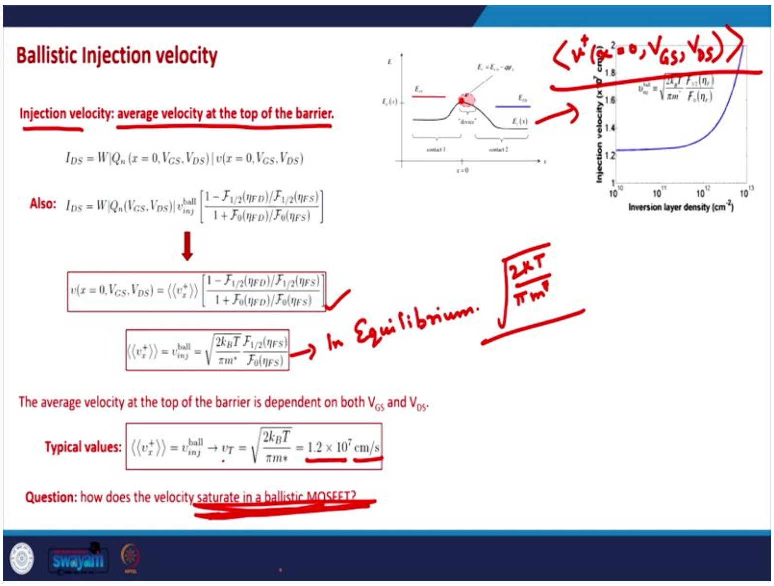

So, in a ballistic MOSFET this is how the energy landscape looks like, this is the top of the barrier point and the injection velocity is essentially the average velocity right at the top of the barrier, which can also be written as average velocity in positive x direction at x equal to 0 point when we have applied a V_{GS} and a V_{DS} voltage to the system.

So, this is a more correct way or more elaborate way to write down the injection velocity in MOSFETs. So, as we have seen that this is how it looks like, just before going into the details of its discussion, here this ballistic injection velocity which is the velocity defined in equilibrium which means that no drain voltage has been applied, what is the typical order of that velocity, let us try to see that ok.

So, if we do this calculation and if you just try to put various constants in this expression, [$\sqrt{(2kT/\pi m^*)}$], you would realize that the order of this velocity is around 1.2 times 10 to the power 7 centimeter per second. So, this is quite higher velocity and generally the velocity saturates in MOSFETs.

And one of the reasons of velocity saturation is the scattering in the MOSFET as. So, typically in long channel MOSFETs the velocity is dependent on the electric field in a linear way. So, as the electric field increases the velocity increases, but after a certain electric field because of the increased collisions as well the velocity saturates, but the velocity also saturates in this case as well ok.

And in even in the ballistic MOSFET and in the ballistic MOSFET there is no collision in the channel so that is the central topic of today's discussion; how does the velocity saturates in a ballistic MOSFET and what is the underlying physical principle of that. So, here we show the injection velocity as a function of inversion layer density, as a function of the charge density. And as you can see that it is actually as the charge increases inversion layer density increases the injection velocity also increases.

Because of these various these Fermi-Dirac integrals involved in the expressions and this is a simulation of these expressions ok.

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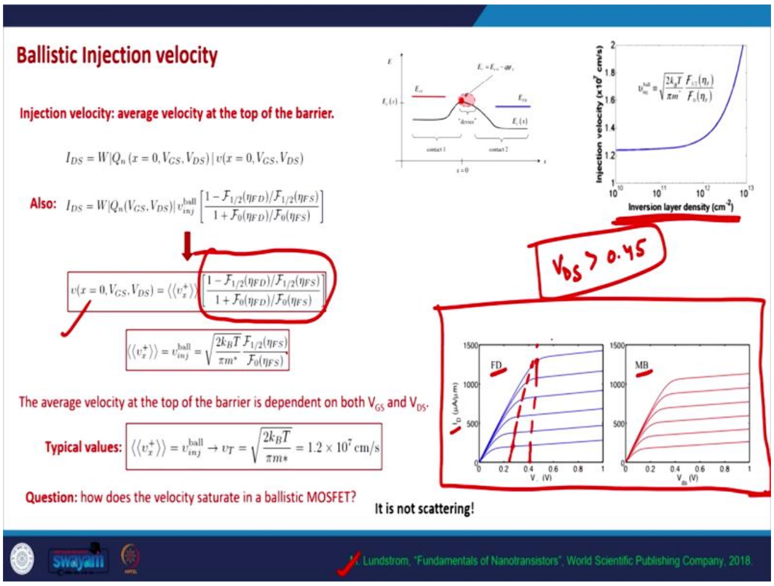

Yeah; so this is the main topic here that and this is a real simulation for a ballistic MOSFET and taken from this reference, where it is taken in which it is taken from a paper and.

So, this calculation has been done by assuming the Fermi-Dirac statistics in the ballistic MOSFET and this has been done by assuming the Maxwell-Boltzmann statistics. As you can see that although their values are different, but they are fairly similar to each other. Although note this is not exactly the same, but they can explain the behavior of electrons pretty much in the same way and as you can see that the current is saturating after a certain drain voltage, which is in this order which is it starts from for low gate voltages it starts from 0.25 to 0.3.

But typically when the gate voltage is also high, it starts from 0.4 volts and above. So, from 0.4 volts and above drain voltage when V_{DS} is 0.4 volts and above or maybe 0.45 volts in that case the current is getting saturated and this is from a real simulation of a ballistic MOSFET and this is what we will be trying to understand today, why is this saturation taking place even though there is no scattering in the channel.

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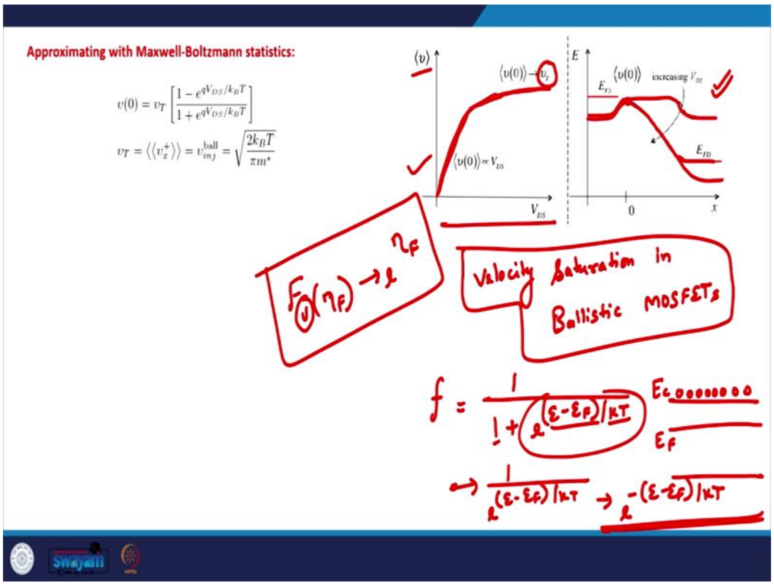

And this is the plot of the average velocity as a function of the drain voltage and as you can see, that initially the velocity is linearly dependent on the drain voltage, but for higher drain voltage it starts saturating and it approaches the unidirectional thermal velocity. So, this is precisely the reason that this current is getting saturated after a certain drain voltage.

So, now we need to understand what is the reason for the velocity saturation in ballistic MOSFETs, just think about it, give it a thought for a moment and then we will go into the details of this idea. This is the energy picture of the device, when the V_{DS} the drain voltage is 0 this is how the barrier looks like. And when we have applied a certain V_{DS} this is how the barrier or this is how the bottom of the conduction band looks like when we are applying a drain voltage to the system.

So, this expression, this particular expression which gives us the velocity at x equal to 0 for an for a V_{GS} and V_{DS}, if we assume the Maxwell-Boltzmann statistics and let me just quickly tell you why we can do that.

So, as you can see that the distribution of electrons is governed by the Fermi-Dirac distribution and this is how it looks like, f is $1/[1+\exp((E-E_F)/kT)]$ and in a typical semiconductor when the semiconductor is non degenerate in that case the Fermi level is reasonably far away from the bottom of the conduction band.

So, for the electrons sitting in the conduction band this quantity $E-E_F$ is many kTs is multiple kT value. So, this parameter becomes larger as compared to 1.

So, ultimately this can be approximated by this one can be removed from here by kT. So, it becomes $\exp(-(E-E_F)/kT)$ and this is the Maxwell-Boltzmann distribution. So, for non-degenerate semiconductors and specially in the MOSFETs when we are not applying high drain voltages, high current is not flowing in that case assuming a back Maxwell-Boltzmann distribution for electrons is not a very bad idea.

It is quite reasonable approximation and if we do that then the Fermi-Dirac integrals simplify pretty much and Fermi-Dirac integral of order j actually can be approximated by $\exp(\eta_F)$. So, this order of the Fermi-Dirac integral is not relevant when Maxwell-Boltzmann statistics is there in the picture. So, if we do that then all these Fermi-Dirac integrals here, they can be approximated by the exponentials.

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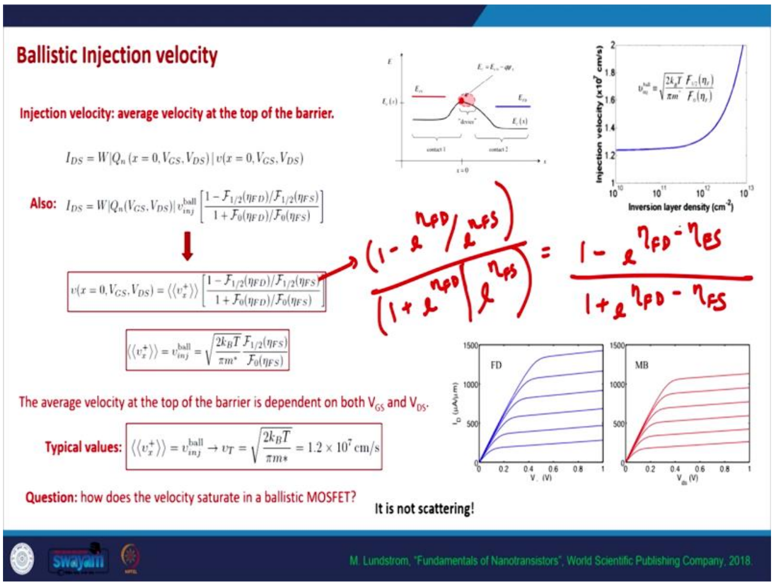

And as you can see that on the numerator here, on the numerator we have a Fermi-Dirac integral order half η_{FD}.

So, it can be approximated by $[1 – (F_{1/2}(\eta_{FD})/ F_{1/2}(\eta_{FS}))] = 1 – \exp(\eta_{FD} – \eta_{FS})$, and the denominator is $[1 + (F_0(\eta_{FD})/ F_0(\eta_{FS}))] = 1 + \exp(\eta_{FD} – \eta_{FS})$.

So, this actually simplifies to. So, as you are aware that η_{FS} - η_{FD} is $– (qV_{DS}/kT)$. So, this simplifies to exponential this velocity injection velocity at x equal to 0 is thermal velocity $[\sqrt{(2kT/\pi m^*)}]$ times $[1 – \exp(qV_{DS}/kT)]/ [1 + \exp(qV_{DS}/kT)]$.

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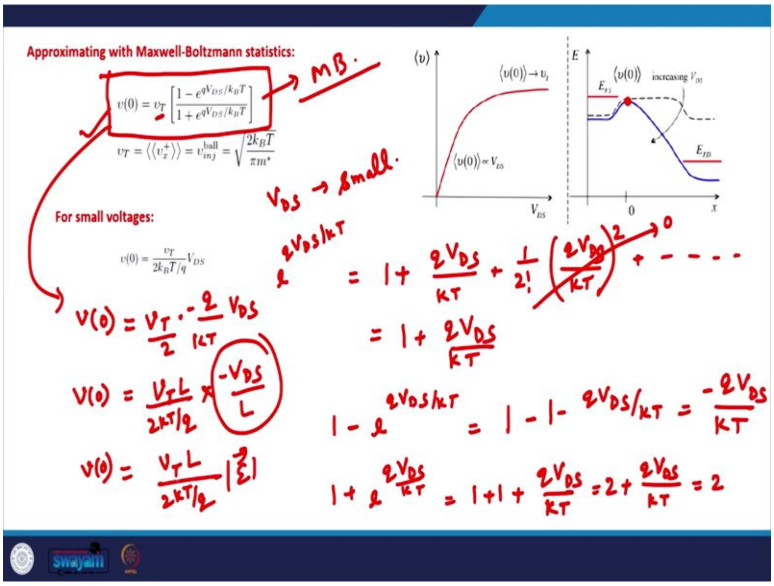

So, this is what becomes of the expression for the injection velocity while we assume the Maxwell-Boltzmann statistics, which is a fair assumption at low biases particularly and in equilibrium.

So, for small voltages if this V_{DS} is small is very small in that case this exponential thing here is qV_{DS}/kT, if we expand this, this is $1 + qV_{DS}/kT + (1/2!)(qV_{DS}/kT)^2$ and higher order terms. Even the second order terms can be ignored. So, this can be approximated by $1 + qV_{DS}/kT$. So, $[1 - \exp(qV_{DS}/kT)]$ is essentially $-qV_{DS}/kT$.

So, it is just $-qV_{DS}/kT$ and the denominator in this expression which is $1 + qV_{DS}/kT$, this is $1 + 1 + qV_{DS}/kT$. So, it is $2 + qV_{DS}/kT$ and this V_{DS} is very small as compared to kT by q, so it is just 2. So, this thing here, this velocity for small voltages for small V_{DS} it is v_T times in the numerator we have $-qV_{DS}/kT$, in the denominator we just have a factor of 2.

So, this velocity is actually v_T times 2kT/q times $-V_{DS}$ here ok. So, if we multiply and divide by length here, this is the this becomes the electric field in the device. So, it is $[v_TL/(2kT/q)]\bar{E}$ and if we compare this to our conventional expression of the velocity conventional relationship between velocity and electric field, which is that the velocity is mobility times the electric field.

So, this the first term here which is $[v_TL/(2kT/q)]$ that will just be the ballistic mobility or that can assume to be the ballistic mobility in the system.

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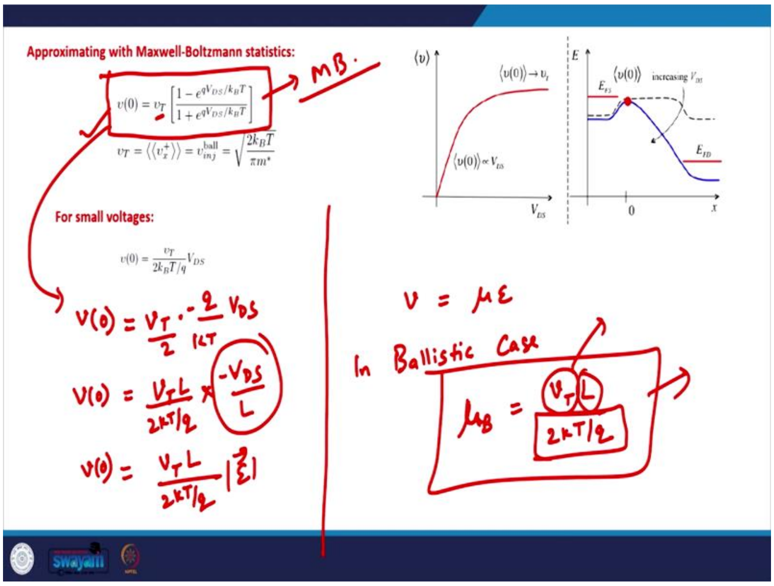

So, if we compare this to this in the ballistic case this μ_B, will be equal to [$v_T L/(2kT/q)$]. So, this is an important expression for the, if we need to if somebody asks you what is the ballistic mobility for a given ballistic MOSFET, this is how we calculate that we first calculate the unidirectional thermal velocity.

And then by if we know the channel length and then these are just the constants using all this, we can calculate the ballistic mobility of the channel.

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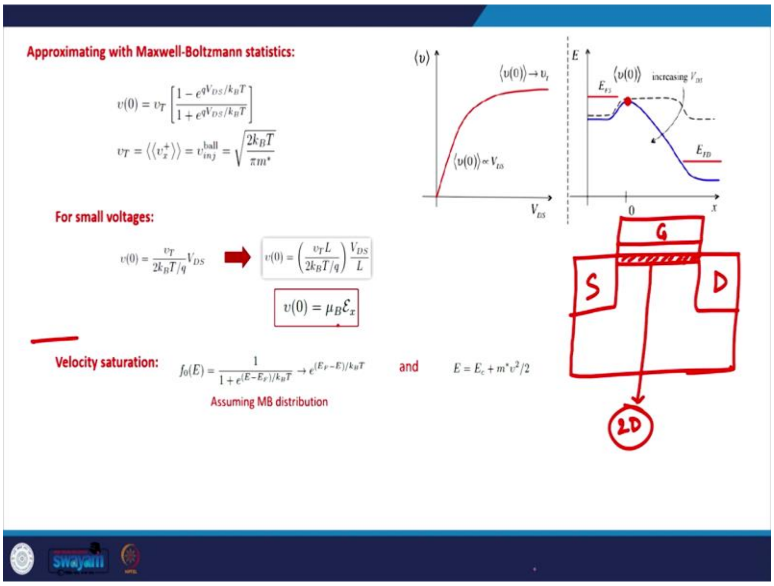

So, this is this I would say that this was a small site discussion here, but what our central aim is that is to understand the velocity saturation in the ballistic MOSFETs why this is happening.

So, for that we need to understand the velocity from a microscopic point of view we need to understand the distribution of velocity in the device. So, the device is something like this, in an actual MOSFET the source the drain is there, even the gate is there and there is a small channel region here which is just like a 2D device this channel is almost like a 2D channel.

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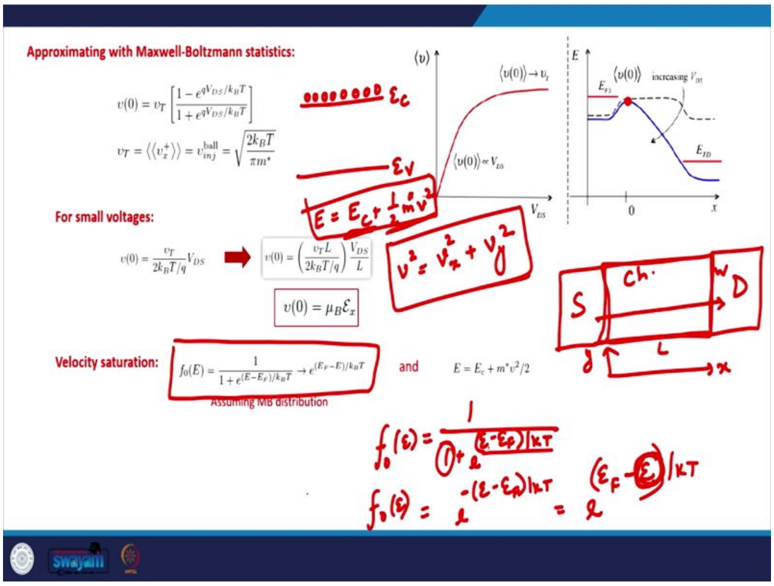

So, that is why most of our analysis which is for a 2D channel will hold true in this case. So, our analysis is for a device something like this, in which there is a source, there is a drain and the channel is a 2D channel, even in an actual MOSFET the channel is a quasi 2D entity there. So, this is the length, this is the width, this is let us say x direction this is let us say the y direction in this.

So, we need to in order to understand the velocity saturation, we need to understand the distribution of velocities whenever an electron starts from the source side to go to the drain side what is its v_x what is its v_y and how these velocities are distributed for various electrons in this device. So, that is what we need to understand and we will start with the Fermi-Dirac distribution function in order to understand that.

So, the Fermi-Dirac distribution function says that this is how the electrons are distributed in various energy levels in the device. And just now, we saw that that for a non-degenerate semiconductors we can approximate this factor is large. So, this one can be ignored.

So, ultimately this becomes exp(-(E-E_F)/kT) or exp((E_F-E)/kT) ok. So, this is what becomes of the Fermi-Dirac distribution, when we are considering the non-degenerate semiconductors because this parameter is many kT value.

So, this one can be ignored and it boils down to the Maxwell-Boltzmann distribution of the electrons. Now, if we look at just at the top of the channel at x equal to 0 and try to understand the velocity here, we will come to that. So, what is this e here this is the total energy of the electrons and if we draw the bands this is the valence band, this is the conduction band.

And the total energy of any electron that is sitting in the conduction band is E is Ec + (1/2)mv^2 where this m is m* which is the effective mass and v is a 2D velocity. So, it can be broken down in its components. So, v^2 is v_x^2 + v_y^2 ok.

So, the total energy of the electron that is sitting in the conduction band is the potential energy which is the bottom of the conduction band plus the kinetic energy. And if we put this energy value in the Fermi-Dirac distribution function here which is which becomes a Maxwell-Boltzmann distribution.

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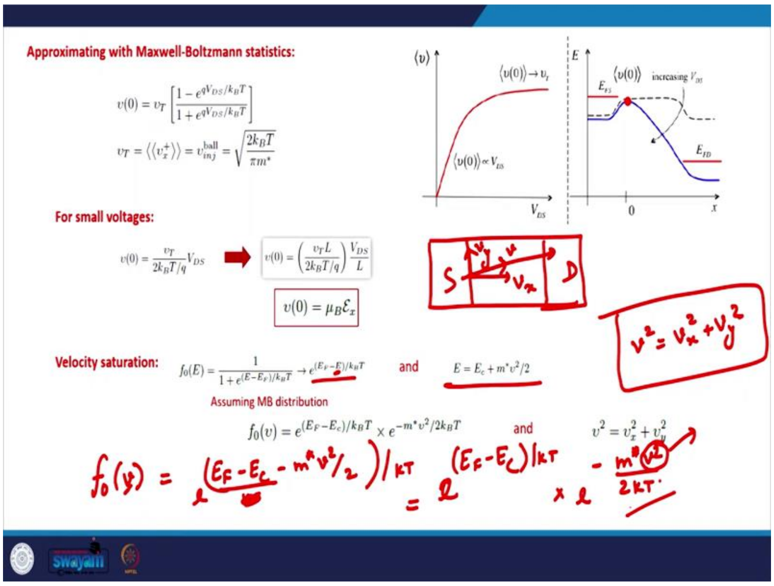

So, if we put E is Ec + (1/2)m*v^2 in this function what we obtain is the distribution of various velocities.

So, what we can say is that the distribution of velocities is exp((E_F-E)/kT) or E is being replaced by Ec + (1/2)m*v^2. So, this will be Ec - (1/2)(m*v$^{2/}$2kT). This kT is the common denominator here by kT. So, this can be this constant part can be taken out. So, which is exp((E_F-Ec)/kT) times exp(-m*v$^{2/}$2kT).

And now this v^2 is actually the 2-dimensional velocity. So, this v^2 is $v_x^2 + v_y^2$. For an arbitrary electron which is starting in the channel. So, if there is this arbitrary electron it will have this is the velocity at any angle it will have a v_x component and it will have a v_y components, this is the net velocity, this is source, this is drain and this is the channel.

So, this is how we obtain the distribution of velocities in the ballistic MOSFET. And what is the meaning of this distribution of velocities? So, this is in a way the probability that the that an electron will have a velocity v in the channel.

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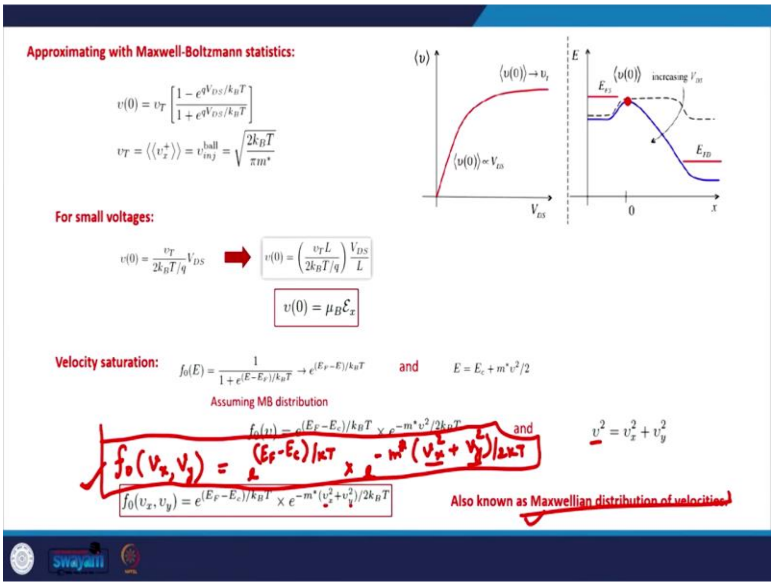

And if we simplify this further, it will be this v^2 can be written down as $v_x^2 + v_y^2$. So, this velocity distribution function can be written as $f_o(v_x , v_y)$ is equal to $\exp((E_F\text{-}Ec)/kT)$ times $\exp(-m^*(v_x^2 + v_y^2)/2kT)$.

So, this velocity distribution function which is also known as the Maxwellian distribution of velocities is the probability that a certain electron will have the v_x velocity in x direction and v_y velocity in y direction, that will be given by this function. That is why it is known as the velocity distribution function also known as the Maxwellian distribution of velocities. Because the Fermi-Dirac distribution has boiled down to the Maxwell-Boltzmann distribution in this case.

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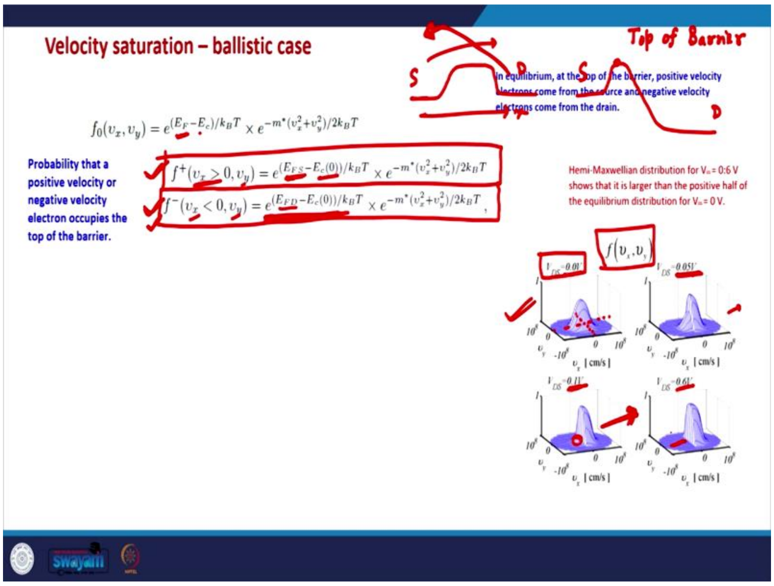

So, now we can see how this saturation thing is coming in picture in our case. So, in a MOSFET typically, if we just draw the barrier in the MOSFET. This is how the barrier is without any V_{DS}, this is the source side this is the drain side and this is how the barrier becomes when a drain voltage is applied in the system.

So, there will there would be some electrons that will come go from source to drain and there would be some electrons that would go from drain to source. So, some electrons will be travelling in positive x direction and some electrons will be traveling in negative x direction and the channel is along the x direction.

So, we will be focused more on the v_x component of the velocity and for the electrons travelling from source to drain side or positively travelling electrons or electrons travelling in +x direction this E_F will be E_{FS}, because the source is the electron where they are coming from. And the electrons that are coming from the drain side the drain contact will be the origin of the electrons. So, instead of E_F we will have E_{FD} ok.

So, this will be the Maxwellian distribution of velocities for electrons travelling in plus x direction from source to drain and this will be the Maxwellian distribution of velocities for electrons travelling from drain to source, the only difference is in E_{FS} and E_{FD} ok. So, this tells us about something important and here in generally in MOSFET we are concerned only about this point which is the top of the barrier.

So, for this point Ec becomes E_{c0} and these functions will give us the probability that a positive velocity electron is there. So, this will be the probability that an electron with velocity v_x and v_y is travelling in +x direction and this will be the probability that an electron is travelling in -x direction with velocity v_x and v_y.

Now, let me show you some something important here. So, this is the plot of the Maxwellian distribution of velocity for various drain voltages. So, this is the plot for V_{DS} equal to 0 volts, this is for us very small V_{DS} 0.05 volts this is slightly larger V_{DS} 0.1 volts and this is a very high V_{DS} 0.6 volts and this is the key to understand the velocity saturation in the ballistic MOSFET.

So, when there is no drain voltage V_{DS} is 0 volts, in that case this distribution function $f_o(v_x , v_y)$ in this function both positive components and negative components are equal and as you can see in this 3D plot here that that this is exactly a symmetric distribution of velocities. And so, the number of electrons or the electrons having positive velocity, the probability of electrons to have positive velocity and the probability to of electrons to have negative velocity is the same.

So, the net velocity is 0 in that sense and that is the equilibrium condition. When a small V_{DS} is applied, in that case this E_{FD} goes down a little bit and that essentially reduces this E_{FD} - E_{c0}. This actually this screwed things are now screwed now f+ and f- are not the same. And as you can see here that now more electrons have positive velocity as compared to the negative velocity.

So, there is a net current in the MOSFET, for slightly larger V_{DS} the negative velocity electrons drop down abruptly and only the positive velocity electrons are there. And finally, for higher values this totally diminishes the negative component totally goes away and only the positive component is there and this is the situation, when the negative component is totally gone away.

This is the case when the velocity is saturated because even on increasing V_{DS} further, the velocity the net velocity will not increase. Because the net velocity is the difference between the positive and the negative functions here. So, at a certain V_{DS} this negative component goes to 0 and that essentially saturates the velocity in the MOSFETs.

So, I will let you think more about this and in the next class we will conclude this discussion and we will start with the electrostatics part of the MOSFET and that will put everything together so that this MOSFET theory becomes clear. So, that is all for the day.

Thank you for your attention, see you in the next class.