# Physics of Nanoscale Devices
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## Lecture - 47
## Ballistic MOSFET

Hello everyone, in today's class we will study about the Ballistic MOSFET, the electrical characteristics of the ballistic MOSFET and as all of you know that we have concluded our discussion on the Landauer's model of transport for the MOSFETs.

And that was pretty much a generalization of the of what we had studied for a two terminal device during our discussion of the general model of transport. So, general model of transport and Landauer's model of transport both of them are actually the same, it was given by Landauer and it was further generalized by Datta and Lundstrom ok.

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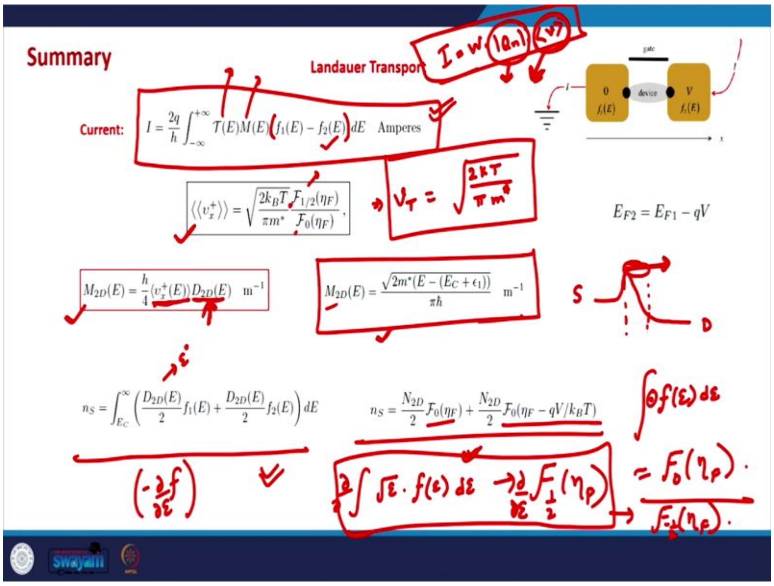

**Summary**

**Landauer Transpor**

$I \cdot W (Q_n) \langle v \rangle$

**Current:** $I = \frac{2q}{h}\int_{-\infty}^{+\infty} T(E)M(E)\left(f_1(E) - f_2(E)\right)dE$ Amperes

$\langle\langle v_x^+ \rangle\rangle = \sqrt{\frac{2k_BT}{\pi m^*}}\frac{\mathcal{F}_{1/2}(\eta_F)}{\mathcal{F}_0(\eta_F)}$,

$\rightarrow v_T = \sqrt{\frac{2kT}{\pi m^*}}$

$E_{F2} = E_{F1} - qV$

$M_{2D}(E) = \frac{h}{4}\langle v_x^+(E)\rangle D_{2D}(E)$ m$^{-1}$

$M_{2D}(E) = \frac{\sqrt{2m^*(E-(E_C+\epsilon_1))}}{\pi\hbar}$ m$^{-1}$

$n_S = \int_{E_C}^{\infty}\left(\frac{D_{2D}(E)}{2}f_1(E) + \frac{D_{2D}(E)}{2}f_2(E)\right)dE$

$n_S = \frac{N_{2D}}{2}\mathcal{F}_0(\eta_F) + \frac{N_{2D}}{2}\mathcal{F}_0(\eta_F - qV/k_BT)$

$\int \varepsilon f(\varepsilon)\, d\varepsilon$

$\left(-\frac{\partial f}{\partial E}\right)$

$\frac{\partial}{\partial E}\int \sqrt{E}\cdot f(E)\,dE \rightarrow \frac{\partial}{\partial E}\mathcal{F}_{\frac{1}{2}}(\eta_F) \rightarrow$

$= \frac{\mathcal{F}_0(\eta_F)}{\mathcal{F}_{-\frac{1}{2}}(\eta_F)}$

So, let us quickly review what we have seen so far in last few classes, we have seen that that in the Landauer transport model this is the current equation that we obtain and this equation has a few important quantities few important parameters. One is the number of modes in the channel, second is the transmission coefficient of the channel and third is the difference between the Fermi functions of the source and the drain.

So, that is quite important this term and that is also the term that drives the current that is also sort of the driving factor because if $f_1$ is equal to $f_2$ in that case the current becomes 0 and there is no current essentially and the equilibrium is there. So, further we saw that in our discussion of the MOSFET. So, this is this is a pretty general expression of the current, the second expression is w times average charge times velocity.

And so this velocity is actually what we are concentrating at the moment and further on we will talk more about the charge which will come when we discuss the electrostatics of the MOSFET. So, in this discussion in our discussion on the MOSFET this velocity is the velocity at which the charge in the channel is moving. But there is a situation in the MOSFET in the MOSFET if you if we plot the barrier profile this is how it looks like.

This is the source side this is the drain side and this is the energy landscape in the MOSFET and this is the point or this is not a very precise I would say more precise is that its right start from this side. So, it is very close to the source and actually this is the point where the electric field is very small and this is the point from where in a way electrons start their journey to the drain side and a drain current is set up in the device.

So, in this kind of energy landscape where is this velocity that we are concerned about? What is this velocity here? Because as we have seen that the electric field at various positions in the channel, so this is the channel region from here to here sort of in this the electric field is different at different positions at the top of the barrier the electric field is minimum. So, to say it is almost 0 and as we go towards the drain or towards the drain side the electric field becomes larger. So, this velocity may also not be in a MOSFET, the velocity may also not be actually the same everywhere in a way ok.

Because especially in the in a diffusive transport based MOSFET in a long MOSFET what happens is velocity is dependent on the electric field. So, in order to do proper calculation of the current we cannot separately calculate the velocity and separately charge, actually they need to be considered together and actual calculation of the MOSFET involves the calculation of the drift diffusion equation and the Poisson's equation.

And if we are doing the quantum mechanical treatment as well the quantum mechanical equation of the transport self consistently. So, that way we are able to get the entire picture, but this discussion of ours in which we have taken we have tried to understand the Landauer transport model and applied to the MOSFET this gives a very good set of tools to us, this gives various concepts that help us understand the IV characteristics of the model MOSFETs very well.

And here this velocity is actually the velocity at the top of the barrier that we are mostly concerned about. That is because at the top of the barrier the influence of the drain is minimum and we can say that the charge mostly depends on the gate voltage and then by calculating the velocity and the charge we can calculate the current in this, ok.

So, and at top of the barrier the average velocity in the x direction is calculated has been calculated in this way, this is the expression of the average velocity the average has been taken over various angles and all the energy states in the channel.

Actually ballistic MOSFET this the notion of the density of states will also change actually while doing the while doing rigorous calculation of the IV characteristics we need to consider the local density of states of the MOSFET at the top of the barrier, ok.

So, in the Landauer transport just now coming back to the Landauer transport model apart from the average velocity which gives us the unidirectional thermal velocity of electrons to be $[\sqrt{(2kT/\pi m^*)}]$, this value plays quite an important role.

And we will see in our coming discussion that the average velocity right at the beginning of the channel or the top of the barrier is heavily is related with $v_T$ intricately. So, apart from the velocity we have the number of modes in the channel which is quite an important parameter and this is this we have already derived from our in our general model of transport and this is how it looks like.

So, it is given by (h/4) average velocity over angle or the velocity average in +x direction where average is taken over angle times the density of states in the channel. In most of our analysis we have considered a 2D channel and that way all the calculations have been done. But on the similar lines the calculations can be done for a 1D channel or a 3D channel as well.

So, for a 2D channel this is the expression for the number of modes if we put the density of states for a 2D channel in the expression, ok. Second, the average or the steady state charge in the channel is given by this expression and while we put the density of states for a 2D channel and the Fermi function this is what we obtain ok. The density of states for a 2D channel is independent of the energy. So, this is independent of the energy and that is why we obtain this kind of expression for the charge.

So, I would just halt for a moment here and tell you about an observation about these calculations. In these calculations you might have observed that there are various orders of Fermi Dirac integrals that are coming in the expressions ok. So, the thing is that wherever we just have, let us say we just have integral of Fermi function dE and something sitting here which is not a function of energy in that case we generally obtain the Fermi Dirac integral of order 0 after simplification.

So, for example, in this expression we are obtaining the Fermi Dirac integrals of order 0 in these expressions because the density of state is independent of the energy, so that is why it is coming like this. When we have let us say number of modes in the current expression, so in that case we will have something like this into Fermi function, this will be Fermi Dirac integral of order half this will give us the Fermi Dirac integral of order half ok. This is just a small observation, so that once you look at the expressions you can know what kind of thing is going on there.

And if we have for example, something like this if we have typically if we have a derivative of the Fermi function in that case the derivative; the derivative comes outside of the integral if we have a derivative and this becomes a Fermi Dirac integral of order -r because the derivative reduces the order of the Fermi Dirac integral by 1. So, these are some small observations about the calculations.

Because sometimes you might feel that that these Fermi Dirac integrals are some are non intuitive parameters and that might create a bit of confusion while understanding these equations. So, just by looking at the order of the Fermi Dirac integral you can know where this is coming from ok.

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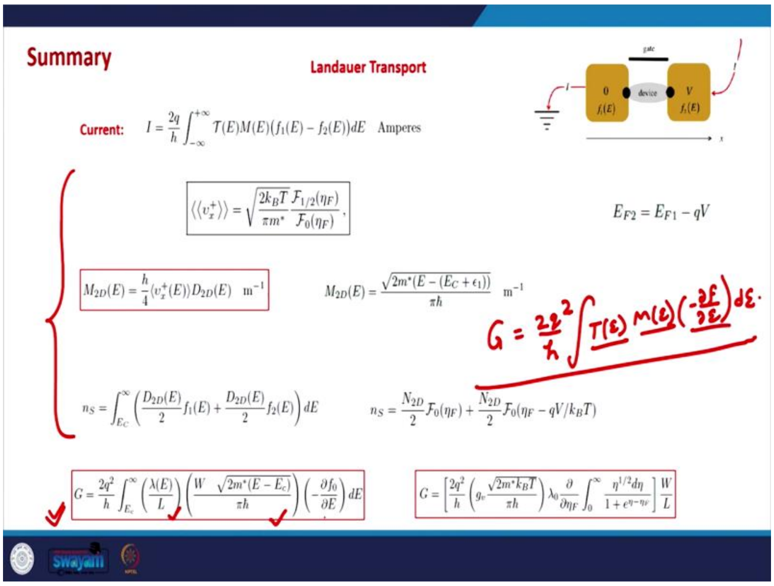

**Summary**

**Landauer Transport**

**Current:** $I = \frac{2q}{h}\int_{-\infty}^{+\infty} T(E)M(E)(f_1(E) - f_2(E))dE$ Amperes

$$\boxed{\langle\langle v_x^+ \rangle\rangle = \sqrt{\frac{2k_BT}{\pi m^*}\frac{\mathcal{F}_{1/2}(\eta_F)}{\mathcal{F}_0(\eta_F)}},}$$

$$E_{F2} = E_{F1} - qV$$

$$\boxed{M_{2D}(E) = \frac{h}{4}\langle v_x^+(E)\rangle D_{2D}(E) \quad \mathrm{m}^{-1}}$$

$$M_{2D}(E) = \frac{\sqrt{2m^*(E-(E_C+\epsilon_1))}}{\pi\hbar} \quad \mathrm{m}^{-1}$$

$$G = \frac{2q^2}{h}\int \underline{T(E)}\,\underline{M(E)}\left(\underline{-\frac{\partial f}{\partial E}}\right)dE.$$

$$n_S = \int_{E_C}^{\infty}\left(\frac{D_{2D}(E)}{2}f_1(E) + \frac{D_{2D}(E)}{2}f_2(E)\right)dE$$

$$n_S = \frac{N_{2D}}{2}\mathcal{F}_0(\eta_F) + \frac{N_{2D}}{2}\mathcal{F}_0(\eta_F - qV/k_BT)$$

$$\boxed{G = \frac{2q^2}{h}\int_{E_c}^{\infty}\left(\frac{\lambda(E)}{L}\right)\left(\frac{W\sqrt{2m^*(E-E_c)}}{\pi\hbar}\right)\left(-\frac{\partial f_0}{\partial E}\right)dE}$$

$$\boxed{G = \left[\frac{2q^2}{h}\left(g_v\frac{\sqrt{2m^*k_BT}}{\pi\hbar}\right)\lambda_0\frac{\partial}{\partial\eta_F}\int_0^{\infty}\frac{\eta^{1/2}d\eta}{1+e^{\eta-\eta_F}}\right]\frac{W}{L}}$$

So, by using all these concepts we can calculate the conductance of the device and this is how it looks like the conductance expression is at actually ($2q^2$/h) ∫T(E)M(E) (-δf/ δE) dE. And for a small I would say for a near equilibrium transport for small applied voltage in the linear region this is how the expression looks like and T(E) is essentially λ/L, M(E) is this and (-δf/ δE).

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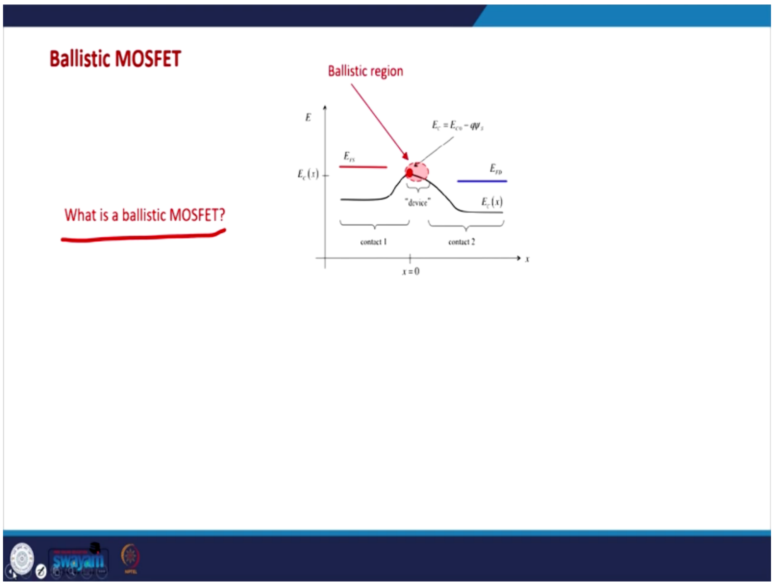

**Ballistic MOSFET**

What is a ballistic MOSFET?

Ballistic region

So, now using the ideas from the Landauer model of transport we will now try to understand the ballistic MOSFET and this is the first question that we ask and that is, what is the ballistic MOSFET? And I guess by now it should be clear to all of us that ballistic MOSFET is the MOSFET in which there is no scattering in the channel ok, which means that the electron which enters into the channel from the source side directly goes to the drain side.

But please remember this as well that the source and the drain regions are large and there is no scattering in the channel, but does not mean that there is no scattering in the source or in the drain.

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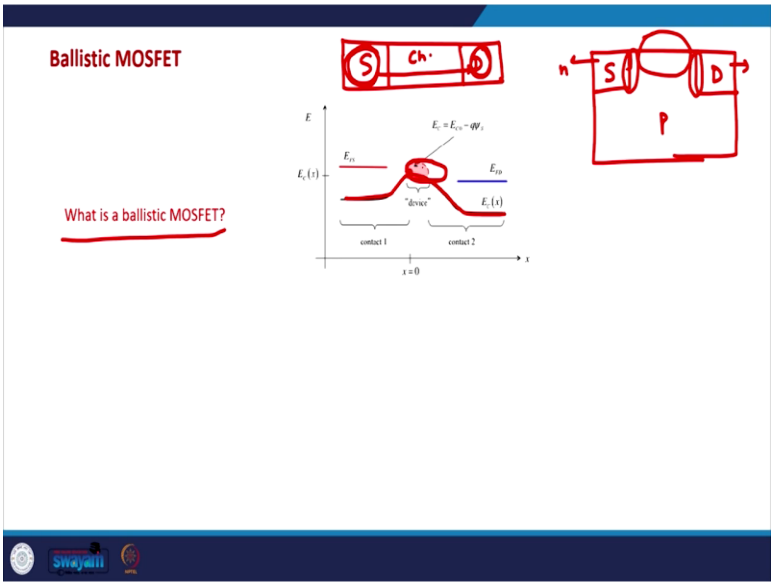

Ballistic MOSFET

What is a ballistic MOSFET?

So, essentially what happens is this is how it looks like there is a source, there is a drain and there is a channel region, we are just looking at the channel region of the MOSFET and the electron starts from the source directly goes to the drain side. So, there is no scattering in the channel, but electrons undergo lot of scattering in the source and in the drain. So, almost always an equilibrium like situation is maintained in the source and the drain region.

So, just remember this part apart from this is how the barrier in that ballistic MOSFET looks like, this is just like any just like barrier in any other MOSFET. And as we just discussed that the most important region of the barrier is the one where or this region right from the top of the barrier up to the right side. So, just to stop for a moment and just to sort of highlight one more point here that in a MOSFET this is how the geometry of the MOSFET is it is typically in an N MOSFET this is a P type material the source and the drain are n type materials.

So, in a way there is as there is a p n junction at the source and the channel region at in this region and there is a p n junction in this region and depending on the doping of the channel and the contacts this depletion width will be there. So, the actual in a way the actual active region of the MOSFET is this region and this is what is shown here, this is the active device of the active device region in the MOSFET and in this region there is no scattering.

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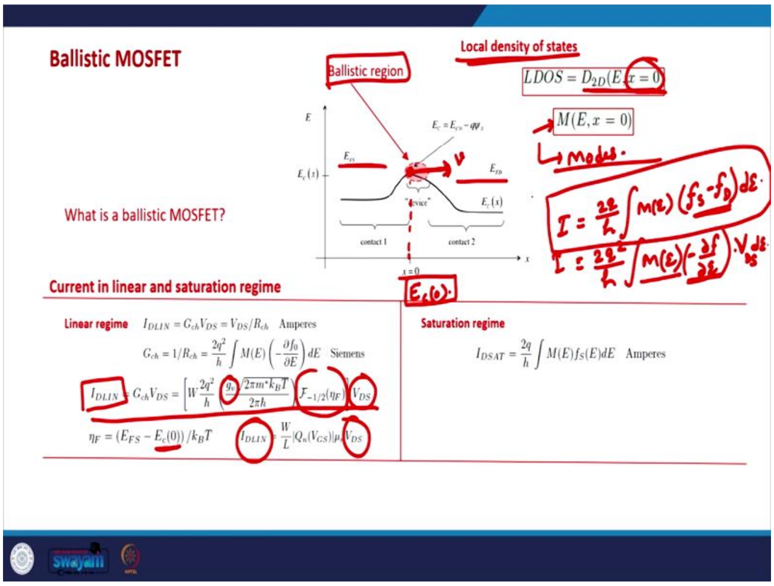

**Ballistic MOSFET**

What is a ballistic MOSFET?

**Local density of states**

$LDOS = D_{2D}(E, x = 0)$

$M(E, x = 0)$

↳ Modes.

$I = \frac{2q}{h}\int M(E)(f_S - f_D)\,dE$.

$I = \frac{2q^2}{h}\int M(E)\left(-\frac{\partial f}{\partial E}\right) \cdot V_{DS}\,dE$.

**Current in linear and saturation regime**

**Linear regime** $I_{DLIN} = G_{ch}V_{DS} = V_{DS}/R_{ch}$ Amperes

$G_{ch} = 1/R_{ch} = \frac{2q^2}{h}\int M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$ Siemens

$I_{DLIN} = G_{ch}V_{DS} = \left[W\frac{2q^2}{h}\left(g_v\frac{\sqrt{2\pi m^* k_B T}}{2\pi\hbar}\right)\mathcal{F}_{-1/2}(\eta_F)\right]V_{DS}$

$\eta_F = (E_{FS} - E_c(0))/k_B T$

$I_{DLIN} = \frac{W}{L}|Q_n(V_{GS})|\mu V_{DS}$

**Saturation regime**

$I_{DSAT} = \frac{2q}{h}\int M(E)f_S(E)dE$ Amperes

So, and in a MOSFET generally what we have seen is that the velocity at this top most point, the velocity here is quite important because by just by calculating the charge and the velocity at this point we can calculate the current in the MOSFET. So, we are mostly concerned about or mostly will be focused about this single point which is the top of the barrier in the MOSFET. And here in order to better understand the electrons behavior we need what is known as the local density of states which means the number of electrons per unit energy right at the top of the barrier.

So, it is generally represented as the density of states as a function of energy, but at x equal to 0. So, this x equal to 0 is the point which is the point of top of the barrier ok and right at this point we also need what is known as the number of modes in the MOSFET. So, this is the situation with us we have a source Fermi function we have a drain Fermi function and in between we have a ballistic region where there is no scattering, but there is a scattering in the source and the drain region.

So, in equilibrium equal amount of electrons are coming from the source and the drain side. So, that is why the current is 0, but as soon as some voltage is applied on the drain side there is this $E_{FD}$ and $E_{FS}$ are misaligned. $E_{FD}$ goes down if the applied voltage is positive and in that case there is a current which starts in the device. So, the current in the linear regime, so the expression of the current is if you recall it is $(2q/h) \int M(E)(f_S - f_D)dE$ is this is the general expression of the current and in the linear regime the applied voltage is very small.

So, in that case this $(f_S - f_D)$ can be approximated by. So, it is approximated by $(-\delta f/ \delta E)$ times qV. So, q we have already taken outside and only the V is there. So, this is what we have, where V is the $V_{DS}$ in this case which is the voltage between the drain and the source. So, if we simplify this expression by putting the number of modes and taking the derivative of this is what we obtain. We obtain W times $(2q^2/h)$ $g_v$ is the valley degeneracy and $[\{\sqrt{(2\pi m^* k_B T)}\}/ 2\pi\hbar]$ Fermi Dirac integral of order minus half $F_{-1/2}(\eta_F)$.

So, this is what we obtain in the linear region and as you can see that this linear region the current is directly proportional to the applied voltage. So, it is a linear relationship between the current and the applied voltage on the drain terminal as is expected from the linear regime. This parameter $\eta_F$ is $E_{FS}$ minus $E_{c0}$ and just to take a note here this $E_{c0}$ is this exactly this point the top of the barrier point essentially, this is since this is x equal to 0. So, the conduction band minima at this point is represented as $E_{c0}$.

So, generally this is the point where we focus our attention because at this point we can in a way make a resolution between the effect of the gate and the effect of the drain on the current ok. So, this is pretty straight forward thing in the ballistic MOSFET this is the current in the linear regime, in the saturation regime starting with this expression. So, starting with this expression in the saturation regime the applied voltage is relatively large.

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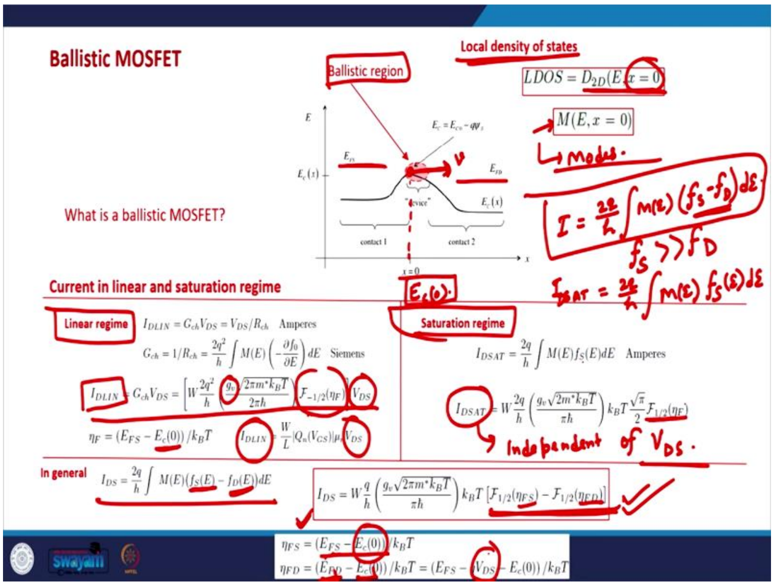

## Ballistic MOSFET

What is a ballistic MOSFET?

Local density of states

$LDOS = D_{2D}(E, x = 0)$

$\rightarrow M(E, x = 0)$

↳ Modes.

$I = \frac{2q}{h}\int M(E)(f_S - f_D)dE$

$f_S >> f_D$

$I_{SAT} = \frac{2q}{h}\int M(E) f_S(E) dE$

Current in linear and saturation regime

Linear regime $I_{DLIN} = G_{ch}V_{DS} = V_{DS}/R_{ch}$ Amperes

$G_{ch} = 1/R_{ch} = \frac{2q^2}{h}\int M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$ Siemens

$I_{DLIN} = G_{ch}V_{DS} = \left[W\frac{2q^2}{h}\left(\frac{g_v\sqrt{2\pi m^* k_BT}}{2\pi\hbar}\right)\mathcal{F}_{-1/2}(\eta_F)\right]V_{DS}$

$\eta_F = (E_{FS} - E_c(0))/k_BT$ $\quad I_{DLIN} = \frac{W}{L}|Q_n(V_{GS})|\mu V_{DS}$

Saturation regime

$I_{DSAT} = \frac{2q}{h}\int M(E) f_S(E) dE$ Amperes

$I_{DSAT} = W\frac{2q}{h}\left(\frac{g_v\sqrt{2m^* k_BT}}{\pi\hbar}\right)k_BT\frac{\sqrt{\pi}}{2}\mathcal{F}_{1/2}(\eta_F)$

↳ Independent of $V_{DS}$.

In general $I_{DS} = \frac{2q}{h}\int M(E)(f_S(E) - f_D(E))dE$

$I_{DS} = W\frac{q}{h}\left(\frac{g_v\sqrt{2\pi m^* k_BT}}{\pi\hbar}\right)k_BT\left[\mathcal{F}_{1/2}(\eta_{FS}) - \mathcal{F}_{1/2}(\eta_{FD})\right]$ ✓ ✓

$\eta_{FS} = (E_{FS} - E_c(0))/k_BT$

$\eta_{FD} = (E_{FD} - E_c(0))/k_BT = (E_{FS} - V_{DS} - E_c(0))/k_BT$

So, which gives us $f_S$ is greater than $f_D$ which means that now the current in the saturation regime $I_{DSAT}$ can be written as (2q/h) $\int M(E)f_S(E)dE$. So, so this M(E) is the. So, since if we are mostly we are considering a 2D channel. So, this for a 2D channel this is how it looks like and after doing the calculations it is this times this mathematical factor times Fermi Dirac integral of order half in this case.

And as you can see that the current here is does not have a direct dependence on the applied voltage which implies that the current has saturated, it is no longer independent of the or no longer dependent on the $V_{DS}$ or it is independent of $V_{DS}$, ok. So, these are the extremas of the current; one is the linear an extremely small applied $V_{DS}$, one is the saturation regime where the large $V_{DS}$ is there.

For a general expression we need to start from this and do the, so as expected we can calculate the current right from this expression and the general expression of the current will be by putting the number of modes which will now have Fermi Dirac integral of order plus half. But there will be two Fermi Dirac integrals please remember that one is because of the source Fermi function and second is because of the drain Fermi functions and that is shown here ok.

So, I hope these mathematical expressions are now making sense I would again repeat it just to emphasize its importance that the exact understanding of ballistic MOSFETs or modern MOSFETs is better done when we do the calculations self consistently when we do the Poisson equation, the semi classical equation of transport and Poisson equation of electrostatics and solve them self consistently.

Here we are first trying to understand the semi classical transport, the ballistic transport, Landauer model of transport and later on we will see the electrostatics. And then doing themselves consistently can also be easily or naturally that will be a natural consequence of both of these discussions that will be understandable once we understand these things.

So, even then these equations are quite important tool to understand the IV characteristics of a given MOSFET ok. So, that is the main point here that they are quite an important they play quite an important role in understanding the; in understanding the any experimental characteristics or any characteristics that we obtained from the simulations.

So, just to sort of highlight this that $\eta_{FS}$ here is $E_{FS}$ - $E_{c0}$ and $\eta_{FD}$ is $E_{FD}$ -$E_{c0}$ where $E_{FS}$ is the Fermi level of the source contact and $E_{FD}$ is the Fermi level of the drain contact and the difference between $E_{FS}$ and $E_{FD}$ is essentially q times $V_{DS}$, $E_{c0}$ is again the conduction band minima at the top of the barrier point.

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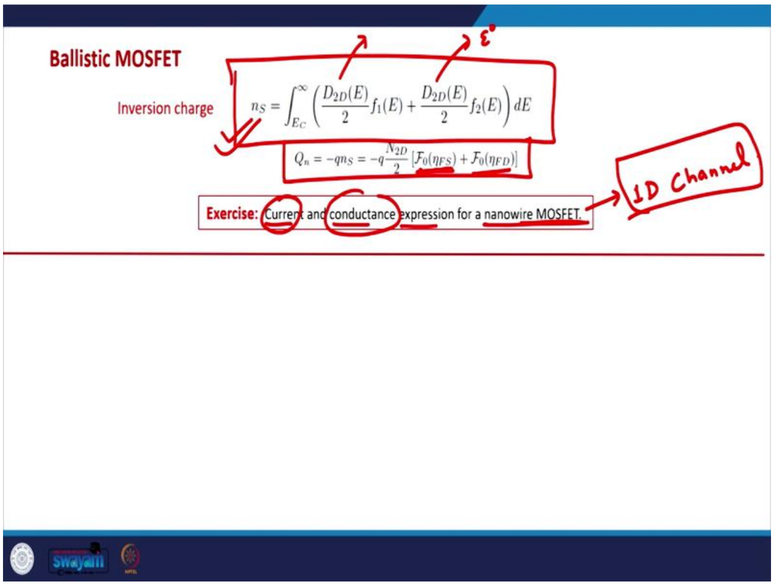

**Ballistic MOSFET**

Inversion charge

$$n_S = \int_{E_C}^{\infty} \left( \frac{D_{2D}(E)}{2} f_1(E) + \frac{D_{2D}(E)}{2} f_2(E) \right) dE$$

$$Q_n = -qn_S = -q\frac{N_{2D}}{2}\left[\mathcal{F}_0(\eta_{FS}) + \mathcal{F}_0(\eta_{FD})\right]$$

**Exercise:** Current and conductance expression for a nanowire MOSFET.

So, finally, the charge in a ballistic MOSFET can be calculated using the this expression which is pretty general expression. And here since the density of state of a 2D channel is independent of the energy, it means that it will now have this integral will have Fermi

Dirac integral of order 0. So, this will be the charge expression. Now, all these calculations have been done for the for a 2D channel and in 2D channel typically what you have? You must have observed by now is that in the charge expression we obtain the Fermi Dirac integrals of order 0.

In the current expression in the linear regime we have the Fermi Dirac integral of order minus half in the saturation regime we have the Fermi Dirac integral of order plus half. So, these are the various orders of integrals of Fermi Dirac integrals that we encounter in a 2D channel. So, now, this is again for you to do this is like an homework exercise for all of you to do in order to have a better understanding of this kind of maths.

Now, I would recommend you to derive the current and conductance expressions for a nanowire MOSFET. So, nanowire MOSFET can be assumed to be a 1D channel. So, assuming the density of states of a 1D channel, now please do all these calculation the calculation of the current and the conductance also for the charge in the channel, that will give you proper insights on how these calculations are done.

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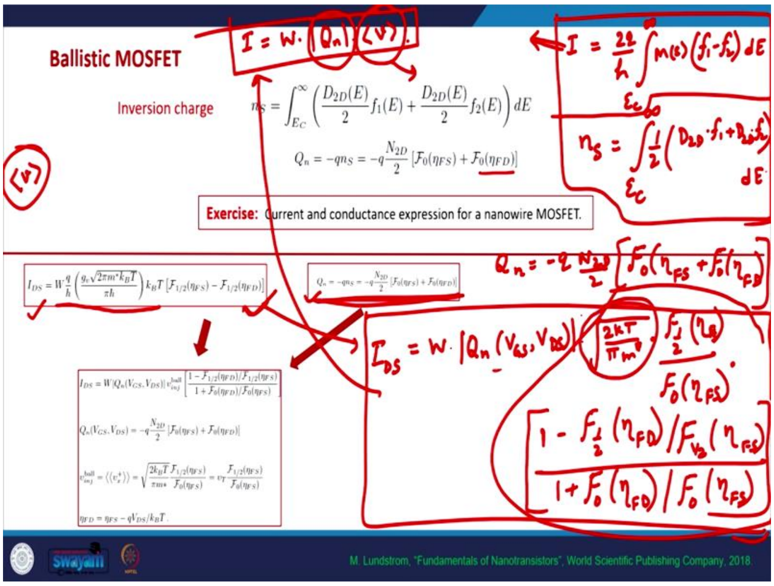


**Ballistic MOSFET**

Inversion charge

$$n_S = \int_{E_C}^{\infty}\left(\frac{D_{2D}(E)}{2}f_1(E)+\frac{D_{2D}(E)}{2}f_2(E)\right)dE$$

$$Q_n = -qn_S = -q\frac{N_{2D}}{2}\left[\mathcal{F}_0(\eta_{FS})+\mathcal{F}_0(\eta_{FD})\right]$$

**Exercise:** Current and conductance expression for a nanowire MOSFET.

$$I_{DS} = W\frac{q}{h}\left(\frac{g_v\sqrt{2\pi m^* k_B T}}{\pi\hbar}\right)k_B T\left[\mathcal{F}_{1/2}(\eta_{FS})-\mathcal{F}_{1/2}(\eta_{FD})\right]$$

$$Q_n = -qn_S = -q\frac{N_{2D}}{2}\left[\mathcal{F}_0(\eta_{FS})+\mathcal{F}_0(\eta_{FD})\right]$$

$$I_{DS} = W|Q_n(V_{GS},V_{DS})|\,v_{inj}^{ball}\left[\frac{1-\mathcal{F}_{1/2}(\eta_{FD})/\mathcal{F}_{1/2}(\eta_{FS})}{1+\mathcal{F}_0(\eta_{FD})/\mathcal{F}_0(\eta_{FS})}\right]$$

$$Q_n(V_{GS},V_{DS}) = -q\frac{N_{2D}}{2}\left[\mathcal{F}_0(\eta_{FS})+\mathcal{F}_0(\eta_{FD})\right]$$

$$v_{inj}^{ball} = \langle\langle v_x^+\rangle\rangle = \sqrt{\frac{2k_BT}{\pi m*}}\frac{\mathcal{F}_{1/2}(\eta_{FS})}{\mathcal{F}_0(\eta_{FS})} = v_T\frac{\mathcal{F}_{1/2}(\eta_{FS})}{\mathcal{F}_0(\eta_{FS})}$$

$$\eta_{FD} = \eta_{FS} - qV_{DS}/k_BT.$$

$$I = W\cdot|Q_n|\langle v\rangle.$$

$$I = \frac{2q}{h}\int_{E_c}^{\infty} M(E)(f_1-f_2)\,dE$$

$$n_S = \int_{E_c}^{\infty}\frac{1}{2}(D_{2D}\cdot f_1 + D_{2D}\cdot f_2)\,dE$$

$\langle v\rangle$

$$Q_n = -q\frac{N_{2D}}{2}\left[F_0(\eta_{FS} + F_0(\eta_{FD})\right]$$

$$I_{DS} = W\cdot|Q_n(V_{GS},V_{DS})|\sqrt{\frac{2kT}{\pi m^*}}\,\frac{F_{\frac{1}{2}}(\eta_{FS})}{F_0(\eta_{FS})}\cdot\left[\frac{1-F_{\frac{1}{2}}(\eta_{FD})/F_{1/2}(\eta_{FS})}{1+F_0(\eta_{FD})/F_0(\eta_{FS})}\right]$$

M. Lundstrom, "Fundamentals of Nanotransistors", World Scientific Publishing Company, 2018.

So, in summary what we have so far is that by using this expression for the current (2q/h) ∫M(E)($f_1 - f_2$)dE, limits from $E_c$ to ∞, and by using this expression for the charge density (1/2)[ $D_{2D}(E)f_1$ + $D_{2D}(E)f_2$ ]dE. So, by using these two expressions we can understand the steady state charge and steady state current in the device ok.

And ultimately while solving these integrals it is converted in Fermi Dirac integrals of various orders. So, this analysis is in a way I would say complete, but we have also seen in the beginning that the current expression, the general current expression is something like this. So, if we try to reformulate this current expression in this form that will give us better understanding of how much channel charge is there and what is the average velocity of the electrons in the MOSFET at the top of the barrier, ok.

So, that is what we do we try to we have this is the general expression of the current, it has a constant factor times Fermi Dirac integrals of order half and this is the expression for the charge. Now, if we try to reformulate this thing in this form, then what we need to do is. So, the charge is essentially $Q_n$ is -q times $N_{2D}/2$ $[F_o(\eta_{FS}) + F_o(\eta_{FD})]$.

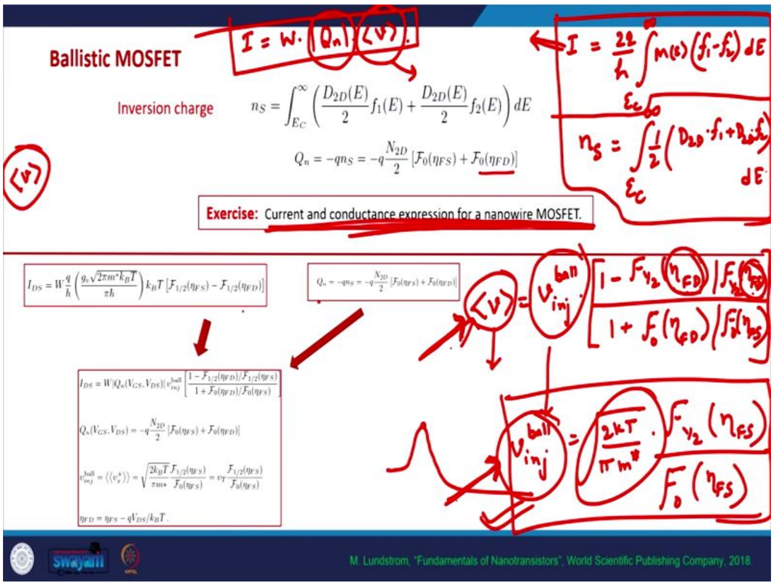
And this is the expression for the current that we obtained from the Landauer model. So, what we do here is that we now in the current expression in this particular expression of the current $I_{DS}$ is we take W outside and this $N_{2D}$ e is can also be expanded and that will give us $Q_n$ ($V_{GS}$, $V_{DS}$) because this is a function of $V_{GS}$ time $V_{DS}$. And here what we obtain is $[\sqrt{(2kT/\pi m^*)}]$ times Fermi Dirac integral of order half divided by Fermi Dirac integral of order 0 times this factor comes in play.

Now, 1 minus Fermi Dirac integral of order half $\eta_{FD}$ times divided by Fermi Dirac integral of order half $\eta_{FS}$ divided by one plus Fermi Dirac integral of order 0 $\eta_{FD}$ divided by Fermi Dirac integral of order 0 $\eta_{FS}$. So, if what we do is we divide this expression $I_{DS}$ expression by $Q_n$ and then see what happens on the right side and then we take the $Q_n$ to the right side and this is what we obtain.

So, we obtain $I_{DS}$ is equal to W times $Q_n$ times this factor here. So, this entire factor after $Q_n$ it can be considered to be the average velocity because if we compare this expression to this expression above. So, the current is effectively the current is this and if we compare this to this expression the average velocity this part this will be essentially this entire thing here ok.

So, this is the average velocity right at the top of the barrier in a ballistic MOSFET and as you can see here that apart from the unidirectional thermal velocity components is let us may be cleanly right here this average velocity ok.

(Refer Slide Time: 30:45)



## Ballistic MOSFET

Inversion charge $n_S = \int_{E_C}^{\infty}\left(\frac{D_{2D}(E)}{2}f_1(E) + \frac{D_{2D}(E)}{2}f_2(E)\right)dE$

$Q_n = -qn_S = -q\frac{N_{2D}}{2}\left[\mathcal{F}_0(\eta_{FS}) + \underline{\mathcal{F}_0(\eta_{FD})}\right]$

**Exercise:** Current and conductance expression for a nanowire MOSFET.

$I_{DS} = W\frac{q}{h}\left(\frac{g_v\sqrt{2\pi m^* k_BT}}{\pi\hbar}\right)k_BT\left[\mathcal{F}_{1/2}(\eta_{FS}) - \mathcal{F}_{1/2}(\eta_{FD})\right]$

$Q_n = -qn_S = -q\frac{N_{2D}}{2}\left[\mathcal{F}_0(\eta_{FS}) + \mathcal{F}_0(\eta_{FD})\right]$

$I_{DS} = W|Q_n(V_{GS}, V_{DS})|\,v_{inj}^{ball}\left[\frac{1-\mathcal{F}_{1/2}(\eta_{FD})/\mathcal{F}_{1/2}(\eta_{FS})}{1+\mathcal{F}_0(\eta_{FD})/\mathcal{F}_0(\eta_{FS})}\right]$

$Q_n(V_{GS}, V_{DS}) = -q\frac{N_{2D}}{2}\left[\mathcal{F}_0(\eta_{FS}) + \mathcal{F}_0(\eta_{FD})\right]$

$v_{inj}^{ball} = \langle\langle v_x^+\rangle\rangle = \sqrt{\frac{2k_BT}{\pi m*}}\frac{\mathcal{F}_{1/2}(\eta_{FS})}{\mathcal{F}_0(\eta_{FS})} = v_T\frac{\mathcal{F}_{1/2}(\eta_{FS})}{\mathcal{F}_0(\eta_{FS})}$

$\eta_{FD} = \eta_{FS} - qV_{DS}/k_BT$.

$I = W\cdot|Q_n|\langle v\rangle$.

$I = \frac{2q}{h}\int_{E_c}^{\infty} M(E)(f_1 - f_2)\,dE$

$n_S = \int_{E_c}^{\infty}\frac{1}{2}(D_{2D}\cdot f_1 + D_{2D}f_2)\,dE$

$\langle v\rangle$

$\langle v\rangle = v_{inj}^{ball}\cdot\left[\frac{1-F_{1/2}(\eta_{FD})/F_{1/2}(\eta_{FS})}{1+F_0(\eta_{FD})/F_0(\eta_{FS})}\right]$

$v_{inj}^{ball} = \sqrt{\frac{2kT}{\pi m^*}}\cdot\frac{F_{1/2}(\eta_{FS})}{F_0(\eta_{FS})}$

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So, the average velocity will be ballistic injection velocity times $[1 - (F_{1/2}(\eta_{FD})/F_{1/2}(\eta_{FS}))]/[1 + (F_0(\eta_{FD})/F_0(\eta_{FS}))]$ . So, this is the average velocity of the electrons right at the top of the barrier. So, this if this is the barrier in the MOSFET, this is the average velocity of the electrons when a $V_{DS}$ voltage has been applied in the device.

So, as you can see that it depends on both $\eta_{FD}$ and $\eta_{FS}$ where this ballistic injection velocity is this velocity which is $v_T$ times which is $[\sqrt{(2kT/\pi m^*)}]$ that is the thermal velocity times Fermi Dirac integral order plus half Fermi Dirac integral of order 0 $\eta_{FS}$. So, this is what it is, this is the unidirectional thermal velocity, this is the ballistic injection velocity.

Or, in a way if we just leave it to the source this is the velocity with which electrons are being pumped from the source side without any influence from the drain side this is what the ballistic injection velocity actually mean. And when in addition to the source voltage we have a non-zero drain voltage as well in that case the average velocity right at the top of the barrier is a function of the of both $\eta_{FD}$ and $\eta_{FS}$.

So, it depends both on the source voltage and also on the drain voltage ok. So, in the transport theory essentially now we have reached to the average velocity at the top of the barrier which is I would say the most important conclusion in the MOSFET transport. So, this is what we will discuss in more detail in the coming class and till then I would like you to do this calculation the current conductance and the charge calculation for a nanowire MOSFET and try to think more about this average velocity in the channel at x equal to 0 point.

The ballistic injection velocity is essentially whenever we have equilibrium there is no effect of the drain in the system this is just because of the source, so that is the ballistic injection velocity it also is an important parameter ok. So, that is all for this class see you in the next class.

I thank you for your attention during the class.