Physics of Nanoscale Devices
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Lecture - 46
Landauer Transport and Ballistic MOSFET

Hello everyone, today we will discuss or we will complete our discussion on the Landauer Model of Transport in the context of MOSFETs and then we will discuss some of the ideas related to the Ballistic MOSFET particularly ok. So, this is going to be an interesting discussion because how do we understand the IV characteristics in ballistic MOSFET and how do we understand the saturation of IV characteristics in ballistic MOSFET.

That is what we hope we will possibly understand during the course of this discussion may be in this class or in the coming class ok.

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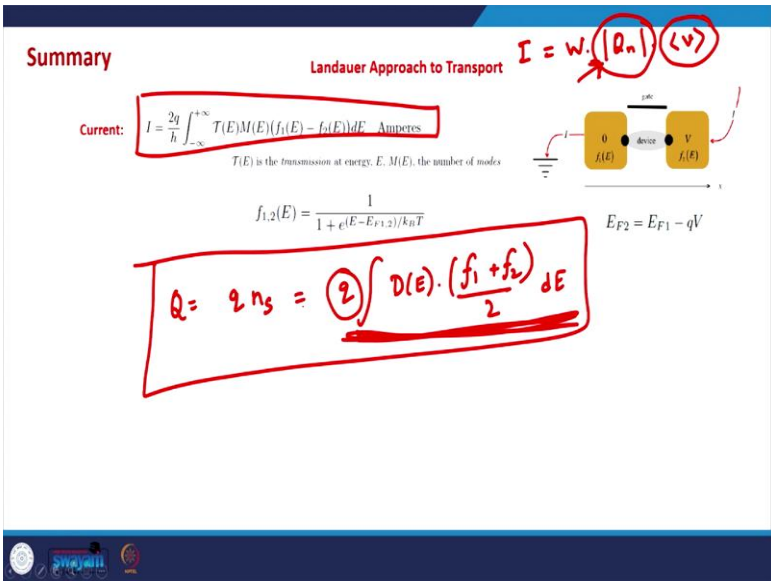

Summary

Landauer Approach to Transport

$I = W.\boxed{|Q_n|}\boxed{\langle v \rangle}$

Current: $\boxed{I = \frac{2q}{h}\int_{-\infty}^{+\infty} T(E)M(E)(f_1(E) - f_2(E))dE \quad \text{Amperes}}$

$T(E)$ is the *transmission* at energy, E, $M(E)$, the number of *modes*

$$f_{1,2}(E) = \frac{1}{1 + e^{(E - E_{F1,2})/k_BT}}$$

$E_{F2} = E_{F1} - qV$

$$\boxed{Q = q n_s = \boxed{2}\underline{\int D(E)\cdot\left(\frac{f_1 + f_2}{2}\right)dE}}$$

So, let me quickly review what we have seen so far. So, in the Landauer formalism of transport this is the current equation that we start with and this equation was although derived for a two terminal device in our discussion, but this is equally applicable to the MOSFET device as well and while we try to use this equation in the context of MOSFET we will generally compare this to our conventional equation this equation.

And that way we can try to understand what is the velocity average velocity with which the charge carriers are moving in the MOSFET and what is the total charge in the MOSFET. This charge can also be calculated from our expression of charge which is which looks like this in the steady state. So, this is the charge in the steady state it also comes from the general model of the transport and this is generally can be written as D(E) times $(f_1 + f_2)/2$.

So, this is the total number of charge carriers at steady state in the channel q is the charge in this case it will be negative. So, we will use a negative sign while doing the discussion of the charge in the MOSFET.

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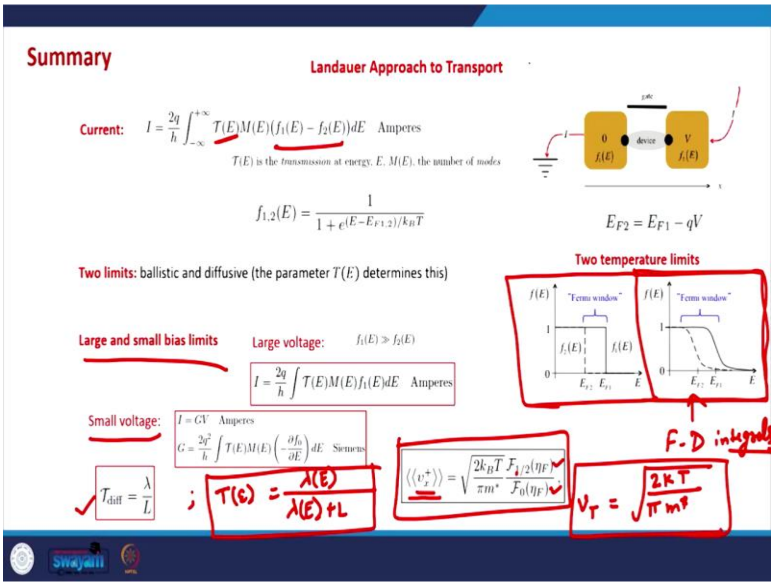

Summary

Landauer Approach to Transport

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$$E_{F2} = E_{F1} - qV$$

Two limits: ballistic and diffusive (the parameter $T(E)$ determines this)

Two temperature limits

Large and small bias limits

Large voltage: $f_1(E) \gg f_2(E)$

$$\boxed{I = \frac{2q}{h}\int T(E)M(E)f_1(E)dE \quad \text{Amperes}}$$

Small voltage:

$I = GV$ Amperes

$G = \frac{2q^2}{h}\int T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$ Siemens

$$\boxed{T_{\text{diff}} = \frac{\lambda}{L}} \; ; \; \boxed{T(E) = \frac{\lambda(E)}{\lambda(E) + L}}$$

$$\boxed{\langle\langle v_x^+ \rangle\rangle = \sqrt{\frac{2k_BT}{\pi m^*}}\frac{\mathcal{F}_{1/2}(\eta_F)}{\mathcal{F}_0(\eta_F)}} \quad \boxed{V_T = \sqrt{\frac{2KT}{\pi m^*}}}$$

F-D integrals

So, as you know that in our Landauer approach of transport generally this you might recall from our previous discussion as well that generally we try to understand the I V characteristics for small biases or in the linear regime.

Because a small biases in the linear regime of the I V characteristics of the MOSFET and for large biases which is the saturation regime and at extremely low temperatures this entire analysis becomes very easy it is it becomes pretty easy to understand and at higher temperatures since the Fermi window the idea of Fermi window or this term $f_1 - f_2$ this becomes quite complex and that is why we need to use the Fermi - Dirac integrals.

In order to account for the scattering in the channel we use a transmission coefficient parameter which is T(E) parameter and this parameter from our derivation this turns out to be λ /L, where λ is the mean free path and more precisely it is T(E) is $\lambda(E)/[\lambda(E)+L]$. And in our previous class we discussed that the unidirectional thermal velocity is the velocity which is defined as the velocity of electrons going from the source side to the drain side.

And it is averaged over angle and averaged over various energy states and this is what is the expression of the double average and unidirectional thermal velocity is generally defined as the velocity in the case of when we consider the non degenerate semiconductor and this is the expression of the unidirectional thermal velocity because this Fermi- Dirac integral in the numerator and in the denominator they will cancel out each other when we are considering a non degenerate semiconductor ok.

So, this is what we have and this is also the situation below threshold in the MOSFET because below threshold not much charge is there in the channel which means that still the conditions like non degenerate semiconductor conditions are maintained and there is one interesting aspect here which is that for the electrons sitting in the conduction band.

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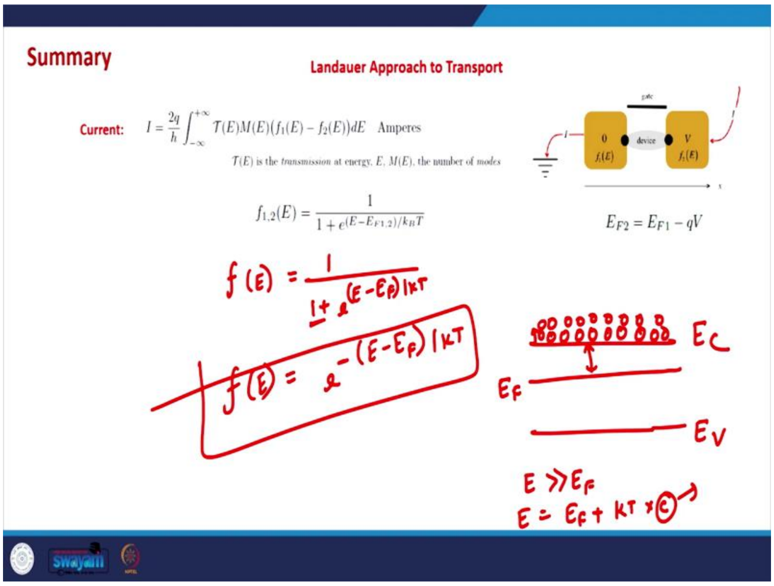

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$$E_{F2} = E_{F1} - qV$$

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$$

$$\boxed{f(E) = e^{-(E-E_F)/kT}}$$

$E \gg E_F$

$E = E_F + kT \times c$

In the case of non degenerate semiconductor the electrons sitting in the conduction band will look something like this we will have electrons here ok and for the non degenerate semiconductor this E_F the Fermi level is significantly far away from the bottom of the conduction band ok. And the energy of electrons which is just above the bottom of the conduction band is larger than E_F and it is quite significantly larger than E_F or it is many kT times or what we can write is E is E_F + kT times c where c is a good number c is maybe 9, 10 or maybe more than that. So, in that case this Fermi Dirac distribution function this in this exponential term becomes large and as compared to 1 and it can be approximated by a the Maxwell distribution function.

So, this Fermi - Dirac distribution function can be approximated by the Maxwell distribution function in the case of non degenerate semiconductors which is actually the case in most of the practical devices and that is why these Fermi - Dirac integrals can be approximated by exponentials and that is why we can cancel out the term in the numerator and the denominator and that is why this thermal velocity is defined as unidirectional thermal velocity is just defined as the this thing without the Fermi - Dirac integrals.

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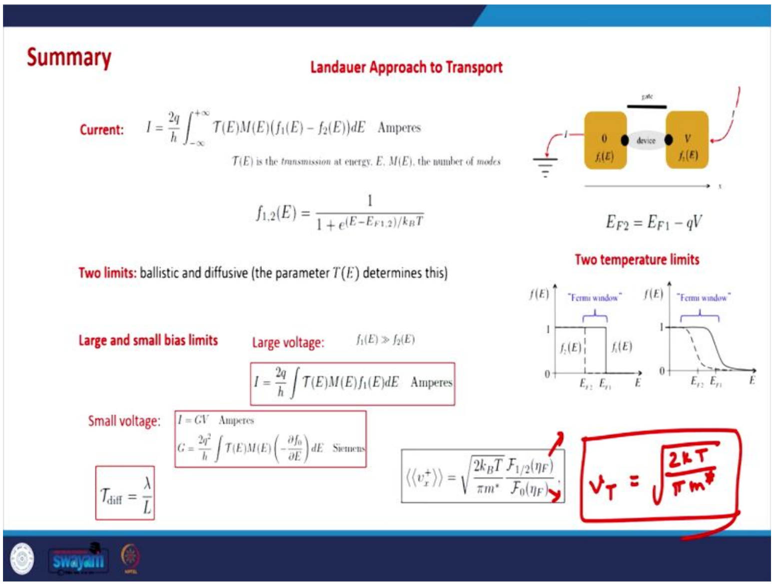

Summary

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Two limits: ballistic and diffusive (the parameter $T(E)$ determines this)

Two temperature limits

Large and small bias limits **Large voltage:** $f_1(E) \gg f_2(E)$

$$\boxed{I = \frac{2q}{h}\int T(E)M(E)f_1(E)dE \quad \text{Amperes}}$$

Small voltage:

$$\boxed{\begin{aligned} &I = GV \quad \text{Amperes} \\ &G = \frac{2q^2}{h}\int T(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE \quad \text{Siemens}\end{aligned}}$$

$$\boxed{T_{\text{diff}} = \frac{\lambda}{L}}$$

$$\boxed{\langle\langle v_x^+ \rangle\rangle = \sqrt{\frac{2k_BT}{\pi m^*}}\frac{\mathcal{F}_{1/2}(\eta_F)}{\mathcal{F}_0(\eta_F)}}$$

$$\boxed{v_T = \sqrt{\frac{2kT}{\pi m^*}}}$$

Because this is the velocity at equilibrium which means we are assuming that the below threshold conditions are prevailing and below threshold conditions are the conditions which are quite similar to the non degenerate case conditions. However, when current is flowing through the device in that case we cannot make this approximation we cannot approximate the Fermi - Dirac integrals by the exponentials and we need to explicitly solve for them ok.

In some cases although in order to have a better understanding we try to actually approximate the Fermi - Dirac integrals by the exponentials. So, I hope this idea of the unidirectional thermal velocity is clear and a related idea is the idea of the injection velocity ballistic injection velocity that will become apparent in the discussion of the ballistic MOSFET.

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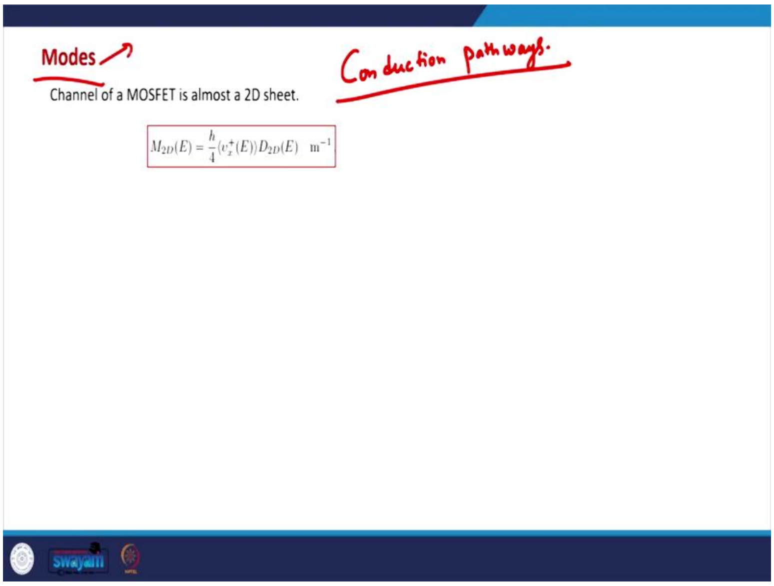

Modes

Channel of a MOSFET is almost a 2D sheet.

$$\boxed{M_{2D}(E) = \frac{h}{4}\langle v_x^+(E)\rangle D_{2D}(E) \quad \mathrm{m}^{-1}}$$

Conduction pathways.

Before going to the topic of the ballistic MOSFET let us quickly revise the idea of modes in the Landauer transport model. In the modes are defined as the conduction pathways and this we have I guess repeated many times by now that modes are defined as the conduction pathways and the reason is that when electrons travel through the channel they travel through the energy states, but because of the finite lifetime of electrons in these energy states.

These energy states are broadened and that results in a broadened pathway in the channel which is different from the energy states or this parameter m which is the modes it becomes different from the density of states and that is precisely the reason that we need to introduce a new parameter in the general model of transport which is known as the modes in the channel ok.

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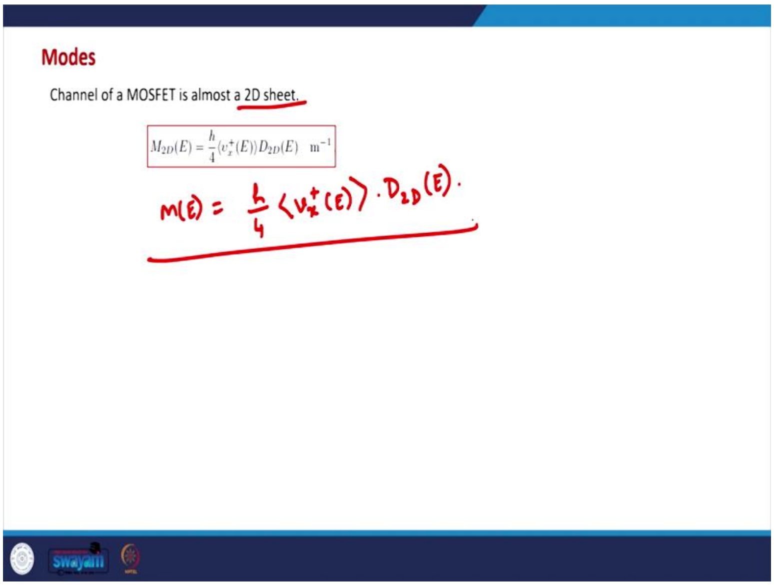

Modes

Channel of a MOSFET is almost a 2D sheet.

$$\boxed{M_{2D}(E) = \frac{h}{4}\langle v_x^+(E)\rangle D_{2D}(E) \quad \mathrm{m}^{-1}}$$

$$M(E) = \frac{h}{4}\langle v_x^+(E)\rangle . D_{2D}(E).$$

So, for a 2D channel the modes are defined as M(E) is equal to (h/4) times average velocity in x direction times the density of states of the device for a 2D channel.

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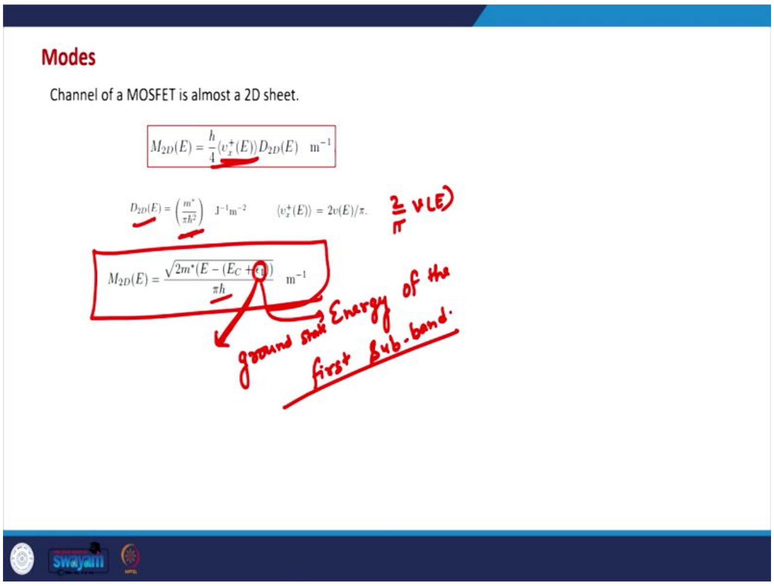

Modes

Channel of a MOSFET is almost a 2D sheet.

$$\boxed{M_{2D}(E) = \frac{h}{4}\langle v_x^+(E)\rangle D_{2D}(E) \quad \mathrm{m}^{-1}}$$

$$D_{2D}(E) = \left(\frac{m^*}{\pi\hbar^2}\right) \quad \mathrm{J}^{-1}\mathrm{m}^{-2} \qquad \langle v_x^+(E)\rangle = 2v(E)/\pi. \qquad \frac{2}{\pi}v(E)$$

$$\boxed{M_{2D}(E) = \frac{\sqrt{2m^*(E-(E_C+\epsilon_1))}}{\pi\hbar} \quad \mathrm{m}^{-1}}$$

ground state Energy of the first sub-band.

And for a 2D channel we can do this calculation explicitly. The density of states is given by m*/$\pi\hbar^2$ and the average velocity this average is taken over the angles this will be given by (2/π)v(E) and if we put these expressions together then this is what we obtain. So, the modes in a 2D channel is 2 m* (E-(E_C+$€_1$)) divided by $\pi\hbar$, where this $€_1$ is the energy or the ground state energy of the first sub band.

So, actually a 2D material is practically a quasi 3D material because there is a finite thickness of the material. So, there is a confinement in the third direction as well and because of that sub bands arise in a 2D material and that results in this. So, that modifies the ground state of the electrons and so that is why we need to account for this parameter €1 as well.

So, this is the parameter that also appears in the current expression and while explicitly calculating the current in the channel we will use this expression actually.

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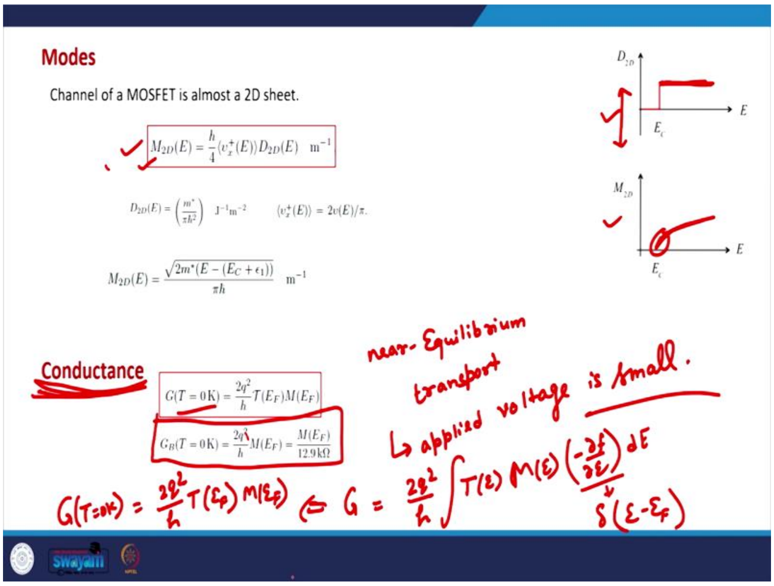

Modes

Channel of a MOSFET is almost a 2D sheet.

$$\boxed{M_{2D}(E) = \frac{h}{4}\langle v_x^+(E)\rangle D_{2D}(E) \quad \text{m}^{-1}}$$

$$D_{2D}(E) = \left(\frac{m^*}{\pi\hbar^2}\right) \quad \text{J}^{-1}\text{m}^{-2} \qquad \langle v_x^+(E)\rangle = 2v(E)/\pi.$$

$$M_{2D}(E) = \frac{\sqrt{2m^*(E-(E_C+\epsilon_1))}}{\pi\hbar} \quad \text{m}^{-1}$$

Conductance

$$\boxed{G(T=0\text{K}) = \frac{2q^2}{h}T(E_F)M(E_F)}$$

$$\boxed{G_B(T=0\text{K}) = \frac{2q^2}{h}M(E_F) = \frac{M(E_F)}{12.9\,\text{k}\Omega}}$$

near- Equilibrium transport

↳ applied voltage is small.

$$G(T=0K) = \frac{2q^2}{h}T(\mathcal{E}_F)M(\mathcal{E}_F) \Leftarrow G = \frac{2q^2}{h}\int T(\mathcal{E})M(\mathcal{E})\underbrace{\left(-\frac{\partial f}{\partial \mathcal{E}}\right)}_{\delta(\mathcal{E}-\mathcal{E}_F)}d\mathcal{E}$$

And this is the difference between the density of states and the modes as is clear, they are quite different from each other in a 2D material the density of state is independent of the energy of the state it is the same for all possible energies in the conduction band.

But the number of modes as you can see it is low for energies close to the bottom of the conduction band and it increases as we go away go far away from the bottom of the conduction band which means that very few conduction pathways are there at the bottom of the conduction band in a 2D channel. Although so these two ideas are related, but physically and intuitively they have quite different implications.

So, this is just to quickly review the idea of modes. So, we will use this idea of modes in a while calculating the current in the MOSFET particularly in ballistic MOSFET or even in a diffusing MOSFET using the Landauer model of transport. So, once we have this idea of modes with us we can calculate the conductance of the channel as well conductance of the device as well.

And if you recall our discussion the conductance expression can very cleanly be calculated for near equilibrium transport, which means near equilibrium means that the applied voltage is not so high. So, it is a small applied so, at small applied voltage is the conductance is given by $(2q^2/h) \int T(E)M(E)\,(-\delta f/\delta E)\,dE$ and at T equal to 0 Kelvin this expression assumes a very clean form because this derivative here this changes to a delta function.

And so, this integral vanishes and which means that at T equal to 0 Kelvin this turns out to be $(2q^2/h) \int T(E)M(E)\,\delta(E - E_F)\,dE$ this is delta function at the Fermi energy. So, it becomes transmission coefficient at energy E_F times the modes at energy E_F ok and if we make this calculation explicitly and for a ballistic conductor T(E) is 1 and this $2q^2/h$ is 1/(12.9 kΩ).

So, the conductance is equal to the number of modes divided by 12.9 kilo ohms which means that the conductance of a single mode is 1/12.9 kilo ohms or thus unit of the conductance is the inverse of the ohm this ohm comes in the numerator. When the temperature is not 0 Kelvin at room temperature in practical situations what happens is that this current expression then takes the integral in the current expression is converted to the Fermi - Dirac integral.

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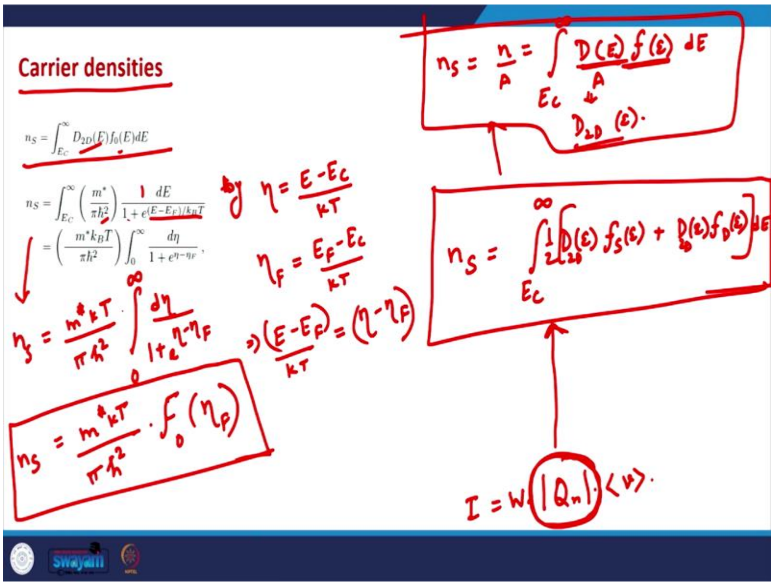

Carrier densities

$$n_S = \int_{E_C}^{\infty} D_{2D}(E) f_0(E)\, dE$$

$$n_S = \int_{E_C}^{\infty} \left(\frac{m^*}{\pi\hbar^2}\right) \frac{1\; dE}{1 + e^{(E-E_F)/k_BT}}$$

$$= \left(\frac{m^* k_BT}{\pi\hbar^2}\right) \int_0^{\infty} \frac{d\eta}{1 + e^{\eta-\eta_F}},$$

by $\eta = \frac{E - E_c}{kT}$

$\eta_F = \frac{E_F - E_c}{kT}$

$\Rightarrow \frac{(E - E_F)}{kT} = (\eta - \eta_F)$

$n_S = \frac{m^* kT}{\pi\hbar^2} \cdot \int_0^{\infty} \frac{d\eta}{1 + e^{\eta-\eta_F}}$

$$\boxed{n_S = \frac{m^* kT}{\pi\hbar^2} \cdot \mathcal{F}_0(\eta_F)}$$

$$\boxed{n_S = \frac{n}{A} = \int_{E_c}^{\infty} \frac{D(E)}{A} f(E)\, dE}$$

$\Downarrow$

$D_{2D}(E).$

$$\boxed{n_S = \int_{E_c}^{\infty} \frac{1}{2}\left[D_{2D}(E) f_S(E) + D_{2D}(E) f_D(E)\right] dE}$$

$I = W \cdot |Q_n| \cdot \langle v \rangle.$

So, this idea of number of modes and the idea of conductance is I guess I hope clear. Now, this. Now, we come to this an important idea which is the carrier densities or number of charge carriers in the channel in the Landauer transport model and if you recall in an arbitrary semiconductor the number of electrons or the number of charge carriers in the conduction band is given by this integral from Ec to ∞, D(E) number of electrons per unit area will be the charge density will be and this is actually the $D_{2D}(E)$ parameter.

In a device in a two terminal device and also in a MOSFET the charge carriers or the charge carrier density will be given by this parameter $(½)[D_{2D}(E)f_S(E) + D_{2D}(E)f_D(E)]dE$ So, this will be the carrier density in the channel. And if you remember our general form of the current this is the general form of the current in the MOSFET. So, this charge in the channel the charge density in the channel is actually obtained by this expression here ok.

So, please keep this in mind this is an important parameter and it depends on the electrostatics of the MOSFET that we will study in coming discussions. But a straightforward way to calculate the carrier density the charge density the number of electrons per unit area in the channel is by using this expression ok. So, in order to calculate this expression let us first see how this expression actually looks like.

This is the expression for a general semiconductor when there is when it is in contact with a with or when it has a Fermi function f(E) and density of states $D_{2D}(E)$. So, in that case in a 2D case this is how it looks like $D_{2D}(E)$ is $m^*/\pi\hbar^2$ and the Fermi function is $1/[1 + \exp\{(E-E_F)/kT\}]$.

Now this integral is again transformed into a Fermi - Dirac integral by replacing the variables like this $(E - E_C)$, η is defined as $(E-E_C)/kT$ and η_F is defined as $(E_F-E_C)/kT$. So, which means that $\{(E-E_F)/kT\}$ is actually $\eta - \eta_F$.

And this is the exponent in the Fermi function. So, this exponent $(E-E_F)/kT$ can be replaced by $\eta - \eta_F$ and this D(E) can be replaced by kT times dη. So, finally, this carrier density n_s turns out to be $m^*kT/\pi\hbar^2$ square limits transform from 0 to ∞, $d\eta/[1 + \exp(\eta - \eta_F)]$ and if you remember the Fermi - Dirac the form of the Fermi - Dirac integral this is the Fermi - Dirac integral of order 0.

So, this becomes ($m^*kT/\pi\hbar^2$) $F_0(\eta_F)$, Fermi - Dirac integral of order 0. So, this is the number of electrons in the channel per unit area, this is the density of electrons in the channel at a finite temperature. Now by using or by having a generalization of this expression we can calculate the electrons in the MOSFET as well or the carrier density charge density in the MOSFET as well using this equation.

This equation and this equation and this is again please remember that this is an important parameter because this tells us about how much charge is there in the channel while the MOSFET is conducting ok.

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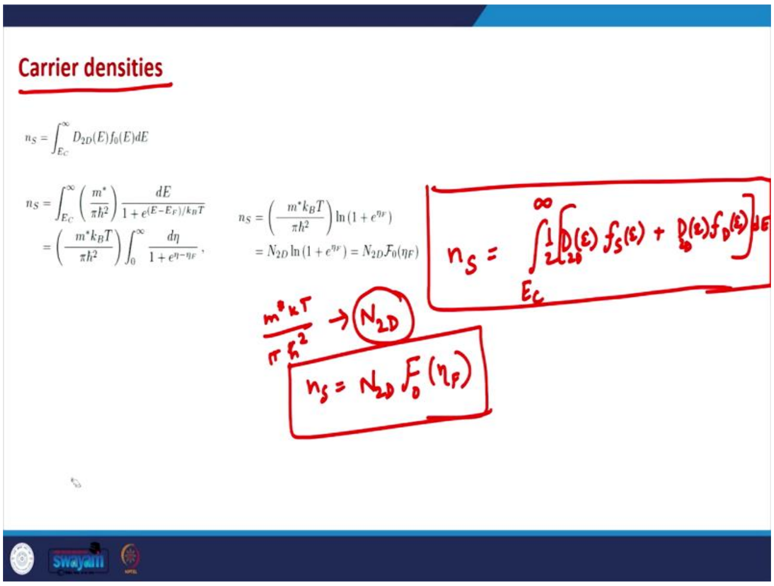

Carrier densities

$$n_S = \int_{E_C}^{\infty} D_{2D}(E) f_0(E) dE$$

$$n_S = \int_{E_C}^{\infty} \left(\frac{m^*}{\pi\hbar^2}\right) \frac{dE}{1 + e^{(E-E_F)/k_BT}}$$
$$= \left(\frac{m^* k_BT}{\pi\hbar^2}\right) \int_0^{\infty} \frac{d\eta}{1 + e^{\eta-\eta_F}},$$

$$n_S = \left(\frac{m^* k_BT}{\pi\hbar^2}\right) \ln\left(1 + e^{\eta_F}\right)$$
$$= N_{2D} \ln\left(1 + e^{\eta_F}\right) = N_{2D} \mathcal{F}_0(\eta_F)$$

$$\boxed{n_S = \int_{E_C}^{\infty} \frac{1}{2}\left[D_{2D}(E) f_S(E) + D_{2D}(E) f_D(E)\right] dE}$$

$$\frac{m^* kT}{\pi\hbar^2} \rightarrow N_{2D}$$

$$\boxed{n_S = N_{2D} F_0(\eta_F)}$$

So, generally this parameter $m^*kT/\pi\hbar^2$ is defined as N_{2D}, where N_{2D} is the modified density of electrons. So, this carrier density becomes capital N_{2D} times Fermi - Dirac integral of order 0 ok.

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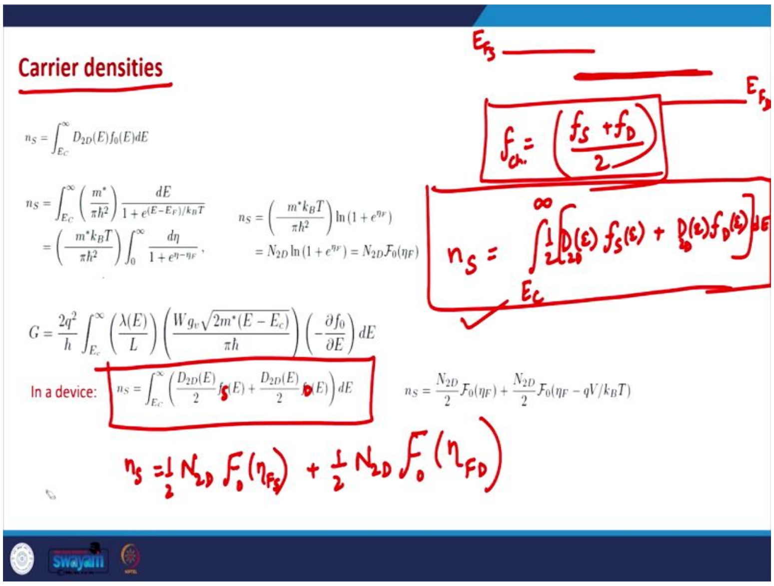

Carrier densities

$$n_S = \int_{E_C}^{\infty} D_{2D}(E) f_0(E) dE$$

$$n_S = \int_{E_C}^{\infty} \left(\frac{m^*}{\pi\hbar^2}\right) \frac{dE}{1+e^{(E-E_F)/k_BT}} = \left(\frac{m^* k_B T}{\pi\hbar^2}\right) \int_0^{\infty} \frac{d\eta}{1+e^{\eta-\eta_F}},$$

$$n_S = \left(\frac{m^* k_B T}{\pi\hbar^2}\right) \ln\left(1+e^{\eta_F}\right) = N_{2D} \ln\left(1+e^{\eta_F}\right) = N_{2D} \mathcal{F}_0(\eta_F)$$

$$G = \frac{2q^2}{h} \int_{E_c}^{\infty} \left(\frac{\lambda(E)}{L}\right) \left(\frac{W g_v \sqrt{2m^*(E-E_c)}}{\pi\hbar}\right) \left(-\frac{\partial f_0}{\partial E}\right) dE$$

In a device: $\boxed{n_S = \int_{E_C}^{\infty} \left(\frac{D_{2D}(E)}{2} f_S(E) + \frac{D_{2D}(E)}{2} f_D(E)\right) dE}$ $\quad n_S = \frac{N_{2D}}{2} \mathcal{F}_0(\eta_F) + \frac{N_{2D}}{2} \mathcal{F}_0(\eta_F - qV/k_BT)$

E_{FS}

E_{FD}

$$\boxed{f_{ch} = \left(\frac{f_S + f_D}{2}\right)}$$

$$\boxed{n_S = \int_{E_C}^{\infty} \frac{1}{2}\left[D_{2D}(E) f_S(E) + D_{2D}(E) f_D(E)\right] dE}$$

$$n_S = \frac{1}{2} N_{2D} \mathcal{F}_0(\eta_{FS}) + \frac{1}{2} N_{2D} \mathcal{F}_0(\eta_{FD})$$

Now, it gives us an expression for the conductance as well. So, in a device the charge the carrier density is defined as this which is essentially this. So, this means just to quickly review what it means this means that the source contact the left hand side contact is trying to fill the channel up to the source Fermi source Fermi level.

And the drain contact is trying to fill the channel up to the drain Fermi level and in steady state the channel will be can be assumed to be filled up to the halfway of the two Fermi level. So, for example, if the source Fermi level is here E_{FS} is here, the drain Fermi level is here, it can be assumed that in steady state the channel Fermi function will be in the midway of the two Fermi levels.

Or the channel Fermi function can be assumed to be the average of the source and the drain Fermi functions. So, this f of the channel can be assumed to be the $(f_S + f_D)/2$. So, this calculation we have already done because this $D_{2D}(E)f(E)/2$ is essentially N_{2D} times Fermi - Dirac integral of order 0.

So, this first term the carrier density in the channel is (1/2) N_{2D}, because this (1/2) will come out Fermi - Dirac integral of order 0 η_{FS}, where η_F please remember that it should be actually η_{FS}, + (1/2) N_{2D} Fermi - Dirac integral of order 0 η_{FD}. Because now this parameter η_F will be different for the source contact as compared to the drain contact.

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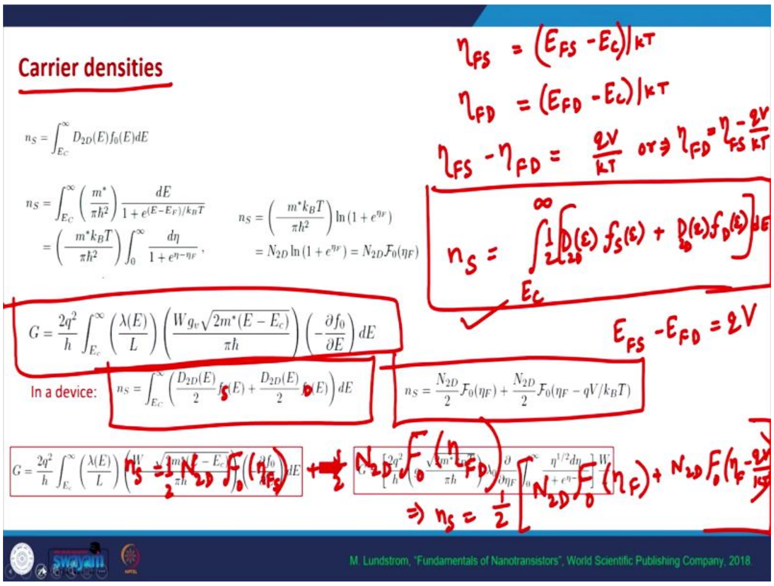

Carrier densities

$$n_S = \int_{E_C}^{\infty} D_{2D}(E) f_0(E) dE$$

$$n_S = \int_{E_C}^{\infty} \left(\frac{m^*}{\pi\hbar^2}\right) \frac{dE}{1+e^{(E-E_F)/k_BT}} = \left(\frac{m^* k_B T}{\pi\hbar^2}\right) \int_0^{\infty} \frac{d\eta}{1+e^{\eta-\eta_F}},$$

$$n_S = \left(\frac{m^* k_B T}{\pi\hbar^2}\right) \ln\left(1+e^{\eta_F}\right) = N_{2D} \ln\left(1+e^{\eta_F}\right) = N_{2D}\mathcal{F}_0(\eta_F)$$

$$\boxed{G = \frac{2q^2}{h} \int_{E_c}^{\infty} \left(\frac{\lambda(E)}{L}\right) \left(\frac{W g_v \sqrt{2m^*(E-E_c)}}{\pi\hbar}\right) \left(-\frac{\partial f_0}{\partial E}\right) dE}$$

In a device: $\boxed{n_S = \int_{E_C}^{\infty} \left(\frac{D_{2D}(E)}{2} f_S(E) + \frac{D_{2D}(E)}{2} f_D(E)\right) dE}$ $\boxed{n_S = \frac{N_{2D}}{2}\mathcal{F}_0(\eta_F) + \frac{N_{2D}}{2}\mathcal{F}_0(\eta_F - qV/k_BT)}$

$$G = \frac{2q^2}{h} \int_{E_c}^{\infty} \left(\frac{\lambda(E)}{L}\right) \left(\frac{W \sqrt{2m^*(E-E_c)}}{\pi\hbar}\right) \left(-\frac{\partial f_0}{\partial E}\right) dE$$

$$\eta_{FS} = (E_{FS} - E_c)/kT$$

$$\eta_{FD} = (E_{FD} - E_c)/kT$$

$$\eta_{FS} - \eta_{FD} = \frac{qV}{kT} \text{ or } \Rightarrow \eta_{FD} = \eta_{FS} - \frac{qV}{kT}$$

$$\boxed{n_S = \int_{E_c}^{\infty} \frac{1}{2}\left[D_{2D}(E) f_S(E) + D_{2D}(E) f_D(E)\right] dE}$$

$$E_{FS} - E_{FD} = qV$$

$$n_S = \frac{1}{2} N_{2D} \mathcal{F}_0(\eta_{FS}) + \frac{1}{2} N_{2D} \mathcal{F}_0(\eta_{FD})$$

$$\Rightarrow n_S = \frac{1}{2}\left[N_{2D}\mathcal{F}_0(\eta_F) + N_{2D}\mathcal{F}_0\left(\eta_F - \frac{qV}{kT}\right)\right]$$

M. Lundstrom, "Fundamentals of Nanotransistors", World Scientific Publishing Company, 2018.

So, this η_{FS} can be defined as $(E_{FS} - E_c)/kT$ and η_{FD} is defined as $(E_{FD} - E_c)/kT$ and which ultimately gives us and please remember that $E_{FS} - E_{FD}$ is q times the applied voltage qV. So, which means that $\eta_{FS} - \eta_{FD}$ is qV times kT or η_{FD} is essentially $\eta_{FS} - qV/kT$.

So, this η_{FD} can be replaced. So, if we assume that η_{FS} is η_F. So, this η_{FD} can be assumed to be $\eta_F - qV/kT$. So, that is what it finally, becomes this becomes N_{2D} Fermi - Dirac integral of order 0 η_F + Fermi -Dirac integral of order 0 $\eta_F - qV/kT$. So, this is the steady state charge density in the device in the MOSFET according to the Landauers formalism.

And similarly, this conductance can also be calculated in this way ok. So, we have already seen the calculation of the conductance, this is quite a general this is just a generalization of the Landauer model of transport that we discussed for a two terminal device to the MOSFET device. So, I would recommend all of you to do this calculation on your own we have already done this.

So, this will be a good exercise on your part. So, this essentially concludes our discussion of the main points of the Landauer model of transport in MOSFETs. So, we are now ready to understand the electrical characteristics of the ballistic MOSFET and just to quickly review the ballistic MOSFET cannot be understood in terms of the basic in terms of the conventional conduction theory of electrons which is based on the Drude's model.

So, that is why we need this Landauer model of transport and since we have discussed the main points of this new transport model we are now ready to understand the ballistic MOSFET.

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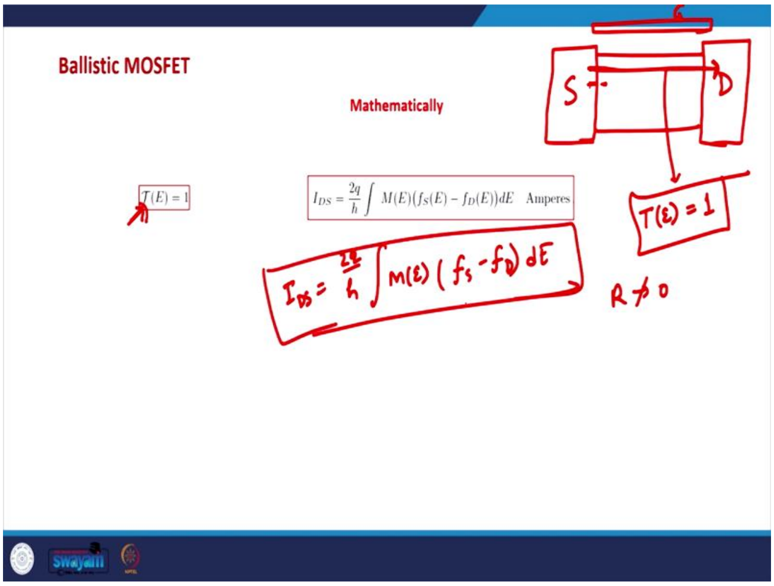

Ballistic MOSFET

Mathematically

$\boxed{T(E) = 1}$

$$\boxed{I_{DS} = \frac{2q}{h}\int M(E)(f_S(E) - f_D(E))dE \quad \text{Amperes}}$$

$$\boxed{I_{DS} = \frac{2q}{h}\int M(E)\,(f_S - f_D)\,dE}$$

$\boxed{T(E) = 1}$

$R \neq 0$

So, in a ballistic MOSFET the first term the first thing that comes to our mind is that there is a left contact, source contact, there is a channel, there is a right contact in the device, there would be a third contact as well which is also known as the gate contact or the control terminal.

So, in a ballistic MOSFET the electron starts from the left contact and directly goes to the right contact without any collision in the channel. So, it travels without collisions, which means that the transmission coefficient is 1, there is no collision no scattering in the channel. So, the transmission factor is 1.

So, in all the expressions that we have discussed so far in the Landauer model of transport we need to put T(E) to be equal to one while we are dealing with a ballistic MOSFET and there are many interesting ideas in the ballistic MOSFET. So, since there is no collision in the channel the question is where is the resistance coming from and this question we have addressed in detail, this resistance is coming from the contact source contact and the or from the interface between the between a bulk contact and a ballistic channel.

So, the energy or the resistance is at the interface of the channel in the contact, the energy is dissipated in the channel dissipated in the contact not in the channel. So, all these things are all these ideas must be clear because there might be a false notion that in ballistic MOSFETs the resistance will be 0, that is not the case the resistance is there and that is there because of the interface between the source and the channel and the drain and the channel.

So, the current expression is by putting T(E) is equal to 0 in Landauer formalism the current expression becomes (2q/h) M(E) modes times $(f_S – f_D)$ times dE this is the drain current in a ballistic MOSFET.

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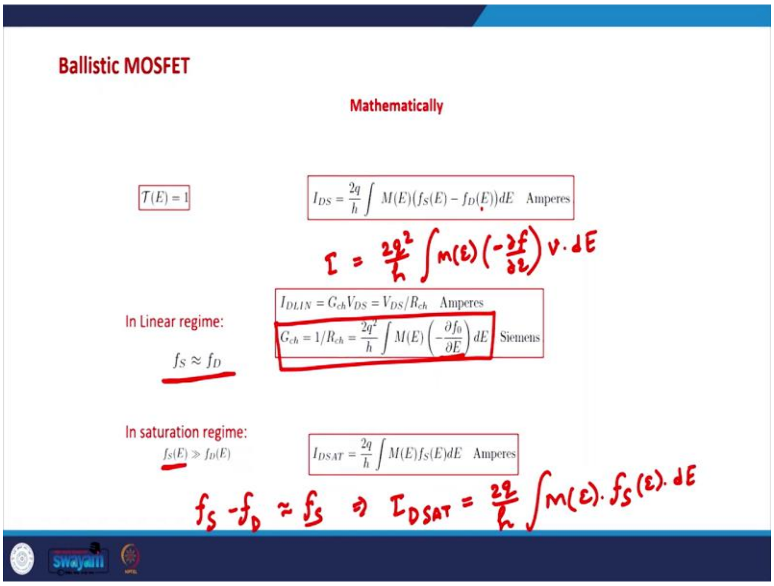

Ballistic MOSFET

Mathematically

$\boxed{T(E) = 1}$

$$\boxed{I_{DS} = \frac{2q}{h}\int M(E)\left(f_S(E) - f_D(E)\right)dE \quad \text{Amperes}}$$

$$I = \frac{2q^2}{h}\int M(E)\left(-\frac{\partial f}{\partial E}\right)V \cdot dE$$

In Linear regime:

$f_S \approx f_D$

$$I_{DLIN} = G_{ch}V_{DS} = V_{DS}/R_{ch} \quad \text{Amperes}$$

$$\boxed{G_{ch} = 1/R_{ch} = \frac{2q^2}{h}\int M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE} \quad \text{Siemens}$$

In saturation regime:

$f_S(E) \gg f_D(E)$

$$\boxed{I_{DSAT} = \frac{2q}{h}\int M(E)f_S(E)dE \quad \text{Amperes}}$$

$$f_S - f_D \approx f_S \Rightarrow I_{DSAT} = \frac{2q}{h}\int M(E) \cdot f_S(E) \cdot dE$$

So, in the linear regime when the applied voltage is very less in that case the source Fermi function is very close to the drain Fermi function.

And in that case this $(f_S – f_D)$ can be approximated by $(-\delta f/ \delta E)$ times qV. So, the current in the linear regime actually becomes $(2q^2/h)$ M(E) times $(-\delta f/ \delta E)$ times V times dE and from here we can define the conductance of the MOSFET which is $(2q^2/h) \int M(E)(-\delta f/ \delta E)dE$.

And in the saturation regime, which is when a high voltage has been applied on the drain terminal in that case the source Fermi function or the source Fermi level is well above the drain Fermi level and in that case this difference between the source and the drain

Fermi functions can be approximated by the just the source Fermi function and it means that in saturation the drain current in the MOSFET is $(2q/h) \int M(E) f_S(E) dE$ ok.

So, these are the equations to start with while trying to understand the ballistic MOSFET.

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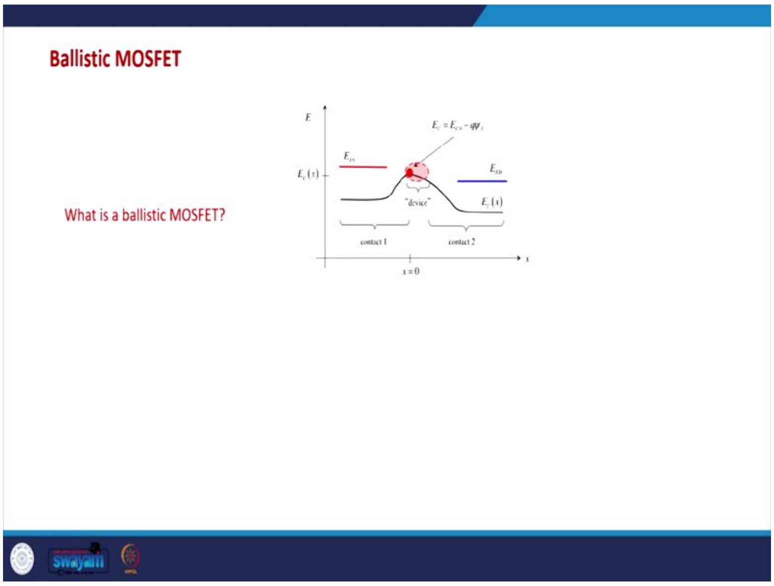

Ballistic MOSFET

What is a ballistic MOSFET?

So, now the question is how does the transport happen in a ballistic MOSFET? So, that is the first question that we will ask here how does electron travel through this barrier in the MOSFET and I will first let you think about this and then in the next class we will start with this discussion how the electron is essentially transported in a ballistic MOSFET first intuitively and second mathematically.

And then we come across an interesting fact that the current in ballistic MOSFET also saturates even though there is no velocity saturation because of the scattering and there is no pinch off. So, called pinch off according to the conventional theory, but still there is a saturation and there is a different reason for the saturation that we will see here ok.

So, please think about the electron movement across the barrier in a ballistic MOSFET and we will start from this point in the next class.

Thank you for your attention see you in the next class.