Physics of Nanoscale Devices
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Lecture - 45
MOSFET: Landauer Transport

Hello everyone as you know we have been discussing the theory of transport for MOSFETs, which is in other words known as the Landauer Transport formalism for the MOSFETs.

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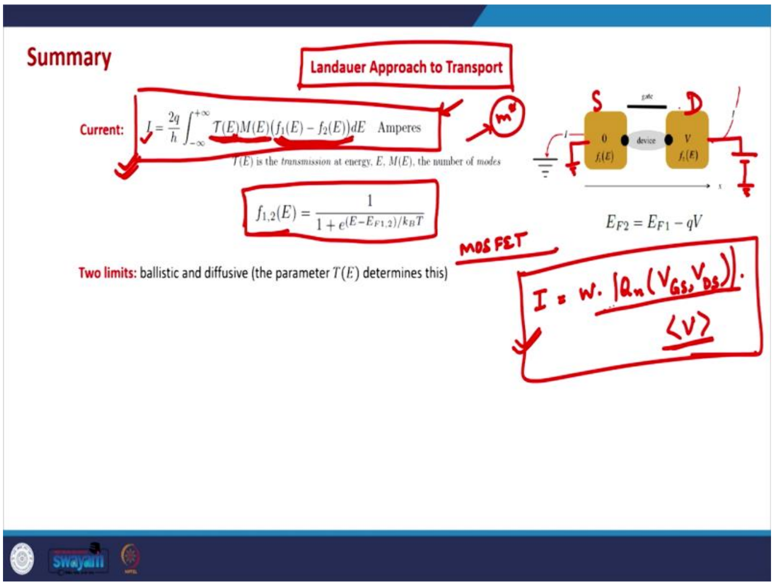

Let us quickly review what we have seen. And if you recall that according to the Landauer approach to the transport according to the Landauer formalism for the transport, the current is given by this expression this is the general form of the current in any device. And we had a proper derivation of this in our discussion of the general model of transport which is essentially the Landauer's model of transport.

And in this the current is dependent on the number of modes in the channel the transmission coefficient and this f_1 - f_2 which is also the forcing function for the device. So, we derived this expression for a two terminal device like this in which one of the terminals is grounded second terminal is connected to a certain voltage, this terminal is grounded.

And in this device this is the general form of the current that we obtain by using the Landauer formalism for transport. So, there is this small thing that we need to keep in mind, that this Landauer transport model this is not a purely quantum mechanical model this is a semi classical model. Although we use quantum mechanics extensively in this we use the density of states of electrons which comes from quantum mechanics as we have seen earlier.

We use the Fermi distribution of electrons in the contacts, we use even we use a bit of quantum mechanics in calculating this transmission coefficient. But still it is assumed that electrons are like particles we assume that electrons travel in wave packets which can be approximated by a particle of effective mass m*.

So, in a way the quantum mechanics is packaged in various parameters here for example, this in effective mass or in number of modes in this parameter modes in density of states in all these things, we try to account for the quantum mechanical nature of the electrons in the channel. But please keep in mind that this is not a purely quantum mechanical treatment of transport that becomes quite tedious for devices like this.

We will have a short discussion on that as well towards the end of this course. But the strength of this formalism is that, it can account for the trans it can account for the transport in modern day MOSFETs particularly the nano MOSFETs. The MOSFETs whose channel length is few 10s of nanometers and without going into too much of the mathematical details of the quantum mechanics ok.

Now, using this model we are able to calculate the IV characteristics, we are able to calculate the various electrical properties of the devices. Specially the nano devices even in ballistic regime and that is the strength of this kind of formalism for the transport. It was just to sort of quickly remind you it was originally given by it was originally given by Rolf Landauer then further it was developed by Supriyo Datta and Mark Lundstrom ok.

So, this is the current expression in this Landauer formalism for the transport and this is what is also what is what we use in MOSFETs as well, so this is what we will also use in the MOSFETs. Just to quickly sort of recall that in MOSFETs the general form of the current is given by this expression times the velocity.

So, by comparing this equation with this equation, we can calculate various parameters we can also we can calculate in a way we can calculate the charge in the channel we can calculate the velocity average velocity of electrons in the MOSFET ok. So, both of these equations are quite important in our understanding of the electrical characteristics of the MOSFETs ok.

So, apart from this we have this Fermi function which is essentially the probability that a certain state is occupied by electrons at energy E and this is relevant for the contacts for the source and drain contacts ok. It tells us about up to which level the states are occupied in the contact and how many states will be filled in the channel.

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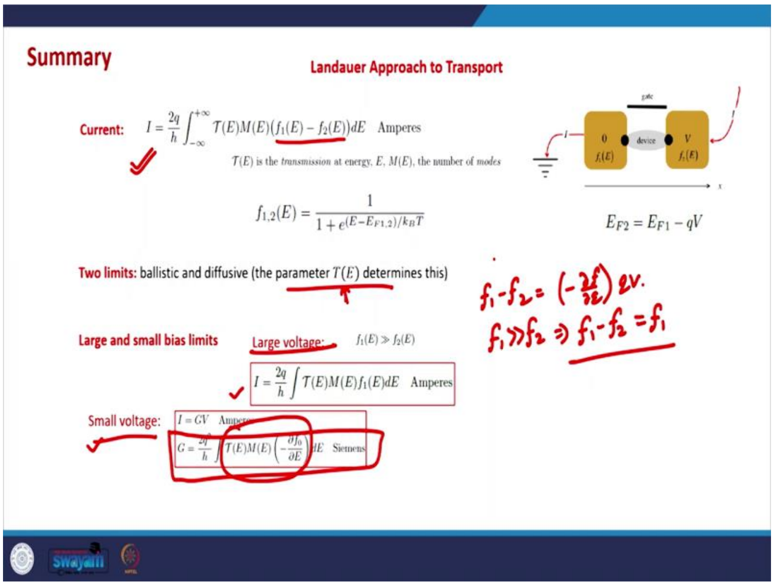

So, apart from this what we have seen is we also saw that that this parameter transmission coefficient this parameter T(E) this accounts for the scattering in the device. And by using this by sort of accounting for scattering or collisions of electrons in the channel by this parameter, we can extend this formalism to the macroscopic formalisms as well ok.

Now, this current expression changes for two different limits of voltages. One is when the voltage is very small in that case, f_1 - f_2 can be approximated by using Taylor series expansion it can be approximated by (-δf/ δE) times q V where V is the applied voltage.

And if the applied voltage is large in that case f_1 becomes significantly larger than f_2 and which means that f_1 - f_2 can be approximated by just f_1 ok. So, this is what we use and in the small voltage limit we can calculate the conductance of the device using this expression and inside the integral this parameter becomes the conductance function as well ok.

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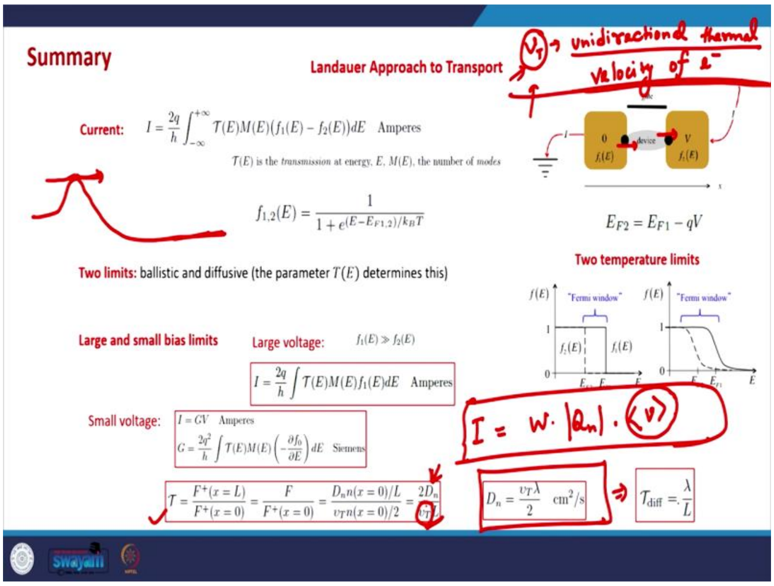

Further in the last class we saw that, that this transmission coefficient is given by the ratio of the flux outgoing from the device, the flux of electrons, coming out of the device on the right terminal divided by the flux of electron entering flux of electrons entering in the left side of the device on the left terminal of the device.

So, in the last class we did an explicit derivation of that and we calculated that the transmission coefficient is given by $2D_n/Lv_T$. Where now this v_T is an interesting parameter v_T is known as the unidirectional thermal velocity of electrons it is and generally it is an equilibrium parameter generally the unidirectional thermal velocity is calculated in equilibrium.

And in equilibrium as all of us know that in the device there is no net flow of electrons, the number of electrons going to the right side is equal to the number of electrons going to the left side. So, the net velocity of electrons is 0, but if we only account for the electrons going to the right side there will be a net velocity and that is known as the unidirectional thermal velocity.

And this is the average velocity of electrons going to the going from the source side of the device to the drain side of the device, from the left side to the right side ok. And since this is the average velocity we need to take proper average while calculating this velocity and this is an important parameter in MOSFETs.

Because if you remember the current equation the current equation is W times average charge times this average velocity. So, this average velocity is the velocity of electrons at the top of the barrier and this is highly dependent on the unidirectional thermal velocity. Because at because as you might remember that MOSFET is a barrier control device and this is the kind of barrier that is there in the MOSFETs.

And this point the point at the top of the barrier is an extremely important point and at this point the electric field is very weak. The effect of the drain voltage is extremely small ideally it should not be there, the effect of the drain voltage at the top of the barrier should not be there.

And this point only depends on the gate voltage, that is what it what determines ah. But the electric field in this direction is negligible or almost equal to 0. So, the velocity with which electrons crosses this point electrons will cross this point will be dependent on the unidirectional thermal velocity of the electrons. And that essentially ultimately governs the IV characteristics of the MOSFET as well.

So, this parameter is an extremely important parameter and we will do a proper analysis of this parameter as well apart from this, in the transmission coefficient calculation there is this parameter called D_n which is known as the diffusion constant and this is given by $\lambda v_T/2$, where λ is the mean free path of the electrons and which ultimately gives us the transmission coefficient to be λ /L ok.

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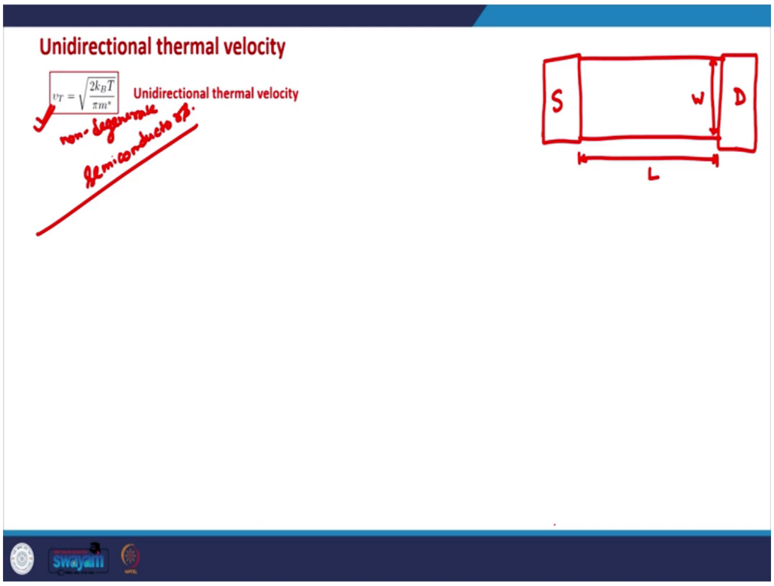

So, with this let us try to calculate the unidirectional thermal velocity of electrons in a in equilibrium ok. So, as usual we take a 2D device 2D channel. So, which means that there is a source contact there is a drain contact and then we have a 2D channel the channel has certain length it has certain width.

This is the final expression of the unidirectional thermal velocity that we obtain for non degenerate semiconductors non degenerate semiconductors ok. So, we will see how we come to this expression and before that let us try to understand the electron flow from the source to the drain side.

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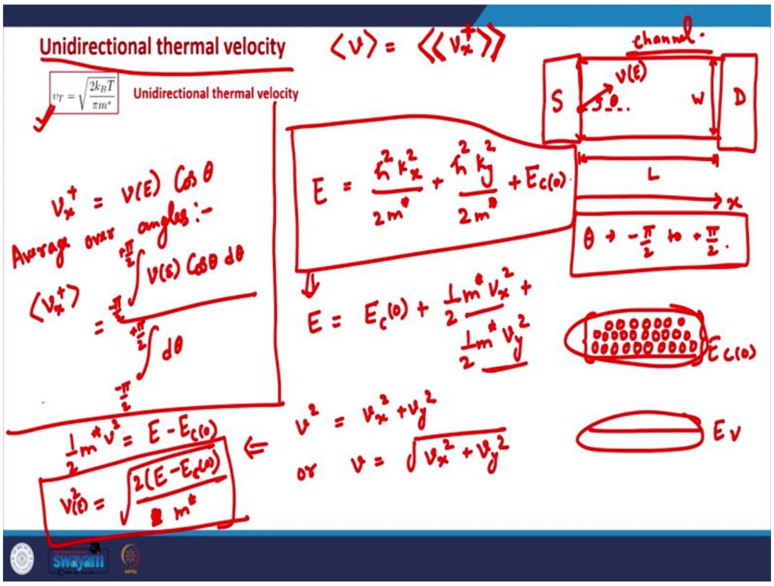

So, the electron that starts from the source terminal right at the interface of the channel in the source. If you recall our discussion on the in the general model of transport this the electron may start at any angle with respect to the X-axis, if the X-axis is along the channel length.

So, it may start with at any angle θ. So, this is angle θ with respect to the X-axis. So, this theta may vary from right from this angle which is –π/2 to π/2. So, this θ may vary from -π/2 to π/2.

And if you recall our discussion then you would remember that most of the conducting electrons are sitting right at the bottom of the conduction band. So, if this is the band diagram of the material this is the valence band this is the conduction band and most of the conducting electrons are sitting right at the bottom of the conduction band.

Which means that the E k energy the E k shape is in most of the cases if you remember, the k p model discussion that the E k relationship is parabolic near the bottom of the conduction band.

Which means that for the electrons sitting at the bottom of the conduction band or for the whole sitting at the top of the valence band, the E k relationship is a parabolic relationship. Which means that for a 2D material the E k relationship might will look something like this, $k_y^2/2m^*$.

So, this will be the energy of electrons apart from the this if this is Ec_0 energy this will be the total energy of the electrons sitting in sort of electrons traveling in the device, from the source to the drain side ok. And if we try to plot this for the parabolic bands it turns out that the velocity also. So, if we write it in we write this in terms of the kinetic energy of the electrons this energy can rewritten can be rewritten as $Ec_0 + (½)m^* v_x^2 + (½)m^* v_y^2$.

So, where this term accounts for the kinetic energy due to the x component of the velocity and this term account for the kinetic energy due to the y component of the velocity. So, for a given energy the total velocity will be or this will be the magnitude of the total velocity and if we see from here it turns out that. So, this velocity is $\sqrt{2(E-E_C)/m^*}$.

So, this velocity is independent of the angle at which it start from the source side. So, that is the point here that the velocity depends only on the energy and not on the angle. As I told you earlier that unidirectional thermal velocity is like the average velocity of electrons in the channel.

And this average actually needs to be taken over two quantities one is, so since is it is a unidirectional velocity, so we need to only account for electrons traveling in the + x direction. And first and first average is over the various angle through which the electron will be travelling from the source to the drain. And the second average will be over the energy states in the channel.

So, in order to calculate the unidirectional thermal velocity of electrons in the channel we need to take the average of velocity over angle and over various energy states in the channel ok. So, if the electron starts with velocity v or better v(E) from the source terminal at an angle θ, v_x^+ will be v(E) $\cos\theta$ ok.

So, if we take average over angle average over various angles. So, to say it will be the average velocity $\cos\theta d\theta$ divided by $d\theta$ and the average is being taken from $-\pi/2$ to $\pi/2$ ok. And this v(E) which is essentially which comes from here it is independent of the angle. So, ultimately it can be taken out of the integral and what is left is if we just remove everything.

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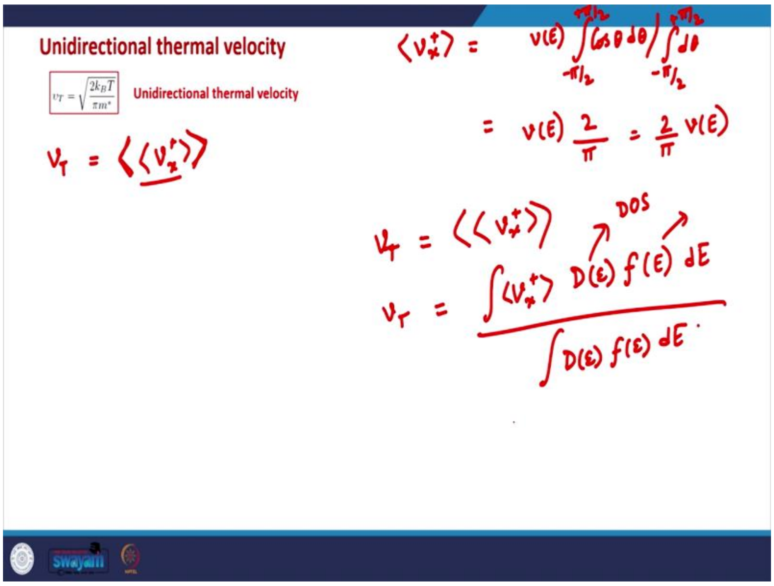

The average velocity in x direction over the where the average is taken over angles is just ∫v(E)cosθdθ divided by ∫dθ where this is from –π/2 to π/2. So, ultimately if you remember this discussion this turns out to be 2/π v(E). So, this is the average velocity in positive x direction where average is taken over the all possible angles in the channel.

Now, this unidirectional thermal velocity is the double average velocity we need to take average over various angles at the same time we also need to take average over various energy states.

So, this finally, this unidirectional thermal velocity will be the average of this average velocity now the average needs to be taken over the various energy states in the channel. So, which means that this will be $\langle v_x^+ \rangle$ D(E) f(E) times dE divided by D(E) f(E)dE. And since we are doing this calculation at equilibrium at equilibrium the source Fermi level and the drain Fermi level will be the same. So, we can take a general Fermi level which is generalized Fermi level so to say.

And this unidirectional thermal velocity will be given by this expression. So, let us do a bit of maths here this. So, if you remember D(E) is the density of states f(E) is the Fermi function ok and we are considering a 2D channel here.

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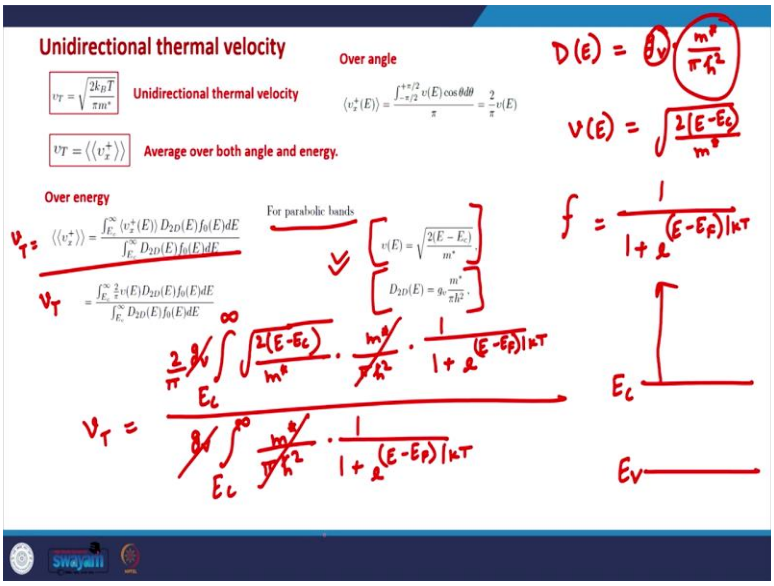

So, this is the unidirectional thermal velocity and in a 2D channel this D(E) is $(g_v m^*/\pi\hbar^2)$, where g_v is the valley degeneracy we can ignore it for the moment. So, this is the essentially the density of states for a 2D channel and this v(E) quantity as we just saw this is $\sqrt{2(E-E_C)/m^*}$. And Fermi function all of us know it is $1/[1 + \exp\{(E-E_F)/kT\}]$ ok.

So, this velocity just to sort of highlight it again and this density of states assume these expressions only for the parabolic bands. Please remember that and as we have just seen that at the bottom of the conduction band generally the E k relationship can be assumed to be a parabolic relationship, so this approximation holds true and.

So, now putting everything in this expression, so this is the unidirectional thermal velocity. So, if we put things here what it turns out to be is v_T is equal to $2/\pi$ comes out and this integration now needs to be taken from the bottom of the conduction band to the all possible energy state. So, this is the bottom of the conduction band this is the top of the valence band and electrons are sitting in the conduction band. So, we need to integrate over all possible energies in the conduction band.

So, this integration is from Ec to ∞, and v(E) is $\sqrt{2(E-E_C)/m^*}$, density of states is g_v comes out $(m^*/\pi\hbar^2)$. $1/[1 + \exp\{(E-E_F)/kT\}]$. In the denominator we have Ec to ∞, $(g_v m^*/\pi\hbar^2)$. $1/[1 + \exp\{(E-E_F)/kT\}]$.

So, this g_v and g_v we can cancel out $m^*/\pi\hbar^2$ they are they can also be taken out of the integral and can be cancelled. So, ultimately what we are left with is we are left with.

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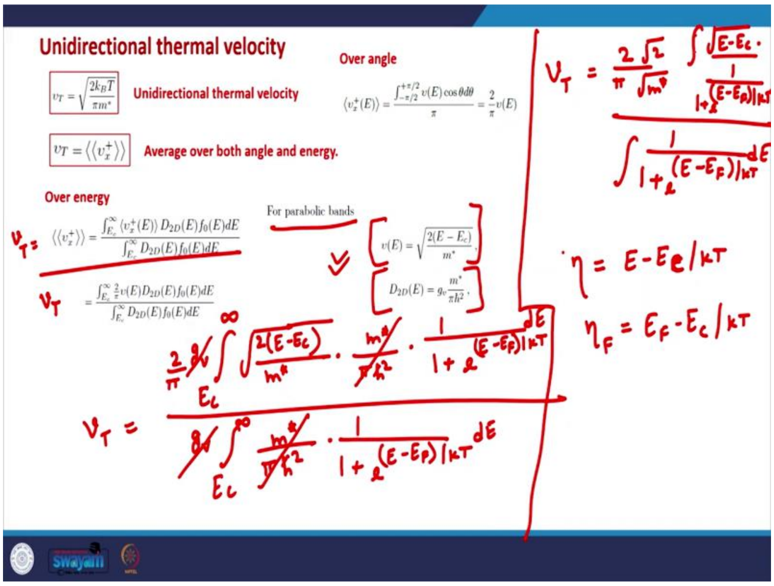

v_T is equal to $(2/\pi)\sqrt{(2/m^*)}$ and in the integral we have $\sqrt{E-E_C}$ times 1 /[1 + exp{(E-E_F)/kT}].

Let me write it in better way 1 + exp{(E-E_F)/kT} divided by integral 1 divided by exp{(E-E_F)/kT} and this is dE this integration is taken over energy. Now these kind of integrals should remind you about the Fermi Dirac integrals. So, if you quickly recall in the Fermi Dirac integrals this parameter η is defined as $(E-E_C)/kT$ and this parameter η_F is defined as $(E_F-E_C)/kT$ ok.

And using these parameters by making these replacements we can convert these integrals to the Fermi Dirac integrals. And if you have a closer look here we have $\sqrt{E-E_c}$. So, which means that there will be a $\sqrt{\eta}$ in the numerator and with the exponential $(E-E_F)/kT$ is there, so it will be $\exp(\eta - \eta_F)$ ok.

So, and the order of the Fermi Dirac integral actually depends on the power of η in the numerator.

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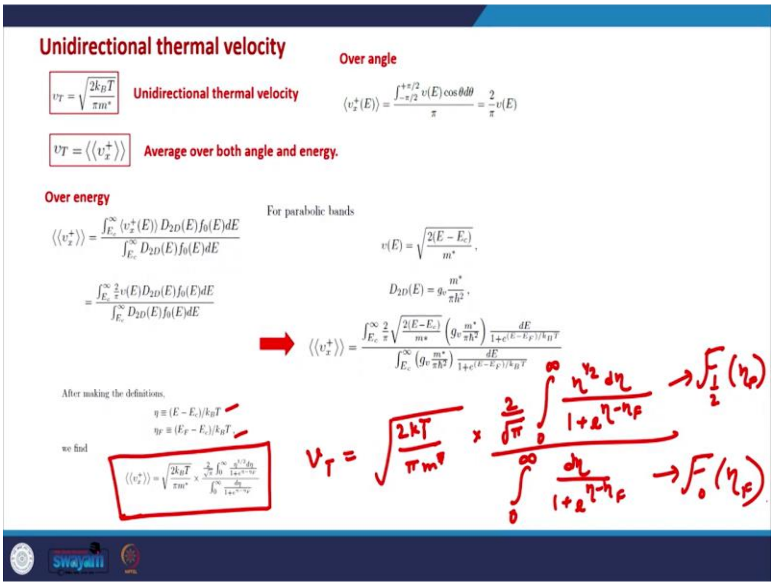

So, ultimately let me have a cleaner picture here. So, by making this replacement η to be (E – Ec)/kT and η_F to be (E_F – Ec)/kT finally, this is the expression that we obtained.

This unidirectional thermal velocity becomes [$\sqrt{(2kT/\pi m^*)}$]. So, please just a small point this k_B and k is essentially the same thing it is the Boltzman constant at some places it is just written as k and at other places it is written as k_B. So, please do not have a confusion here.

And finally, here what we have is $\eta^{1/2}d\eta$ divided by 1 + exp(η - η_F). Similarly in denominator here we have dη divided by 1 + exp(η - η_F). So, this numerator is the Fermi Dirac integral of order half and this denominator is the Fermi Dirac integral of order 0 ok.

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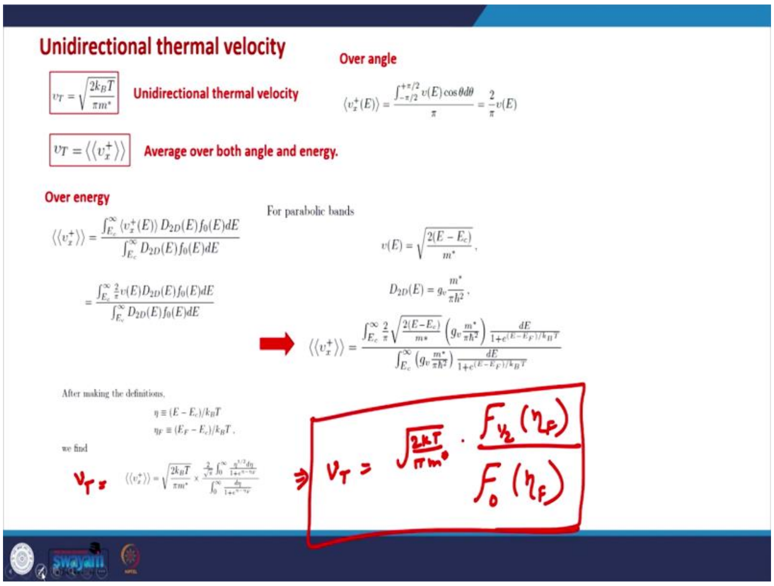

So, which means that this unidirectional thermal velocity ultimately turns out to be $[\sqrt{(2kT/\pi m^*)}]$ $[F_{1/2}(\eta_F)/\ F_0(\eta_F)]$ Fermi Dirac integrals alright. This is essentially written here.

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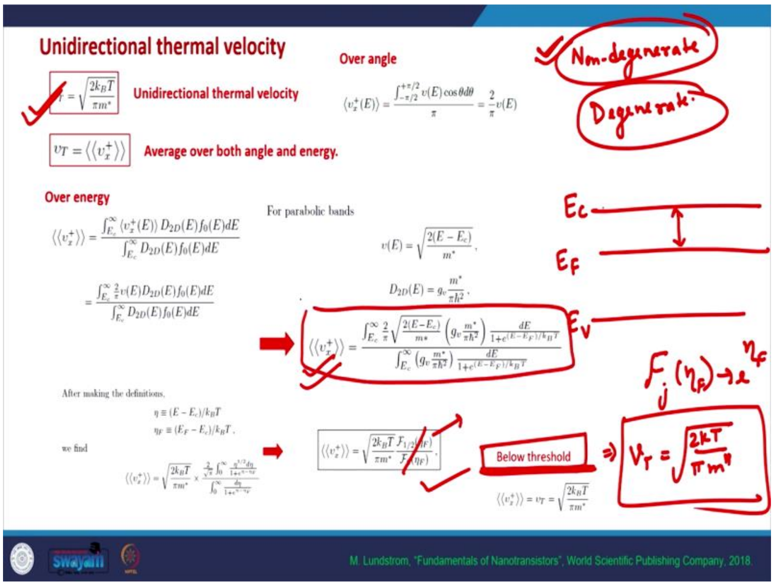

There is a another point that I would like to remind you here that generally there are two kind of semiconductors one is the non degenerate and degenerate semiconductors. So, non degenerate and degenerate semiconductor degenerate semiconductors are heavily doped non degenerate semiconductors are lightly dropped. One of the major differences is that in the non degenerate semiconductors this Fermi level E_F is significantly far away from the bottom of the conduction band.

In the degenerate semiconductors this E_F is quite close to the E_C or sometimes it is above E_C as well. So, in non degenerate semiconductors and in MOSFETs below threshold whenever the barrier the channel charge is not there which means the barrier has not been lowered significantly in that case.

These Fermi Dirac integrals of any order the Fermi Dirac integral of order can be approximated by exponential to η_F. So, the order does not matter when the semiconductor is non degenerate and the MOSFET is below threshold. You can do this small exercise we have already we already saw this in our discussion of the general model of transport.

So, in these two conditions generally we take non degenerate semiconductors in our devices and below threshold both of these numerator and denominator Fermi Dirac integrals in numerator, we have the Fermi Dirac integral of order half in denominator, we have Fermi Dirac integral of order 0. Both of them boils down to exponential η_F, which means that the unidirectional thermal velocity will be given by $\sqrt{(2kT/\pi m^*)}$. So, both of them will cancel out this value which is the result that we actually had shown in the beginning.

So, this is how we calculate the unidirectional thermal velocity in the MOSFETs and this is an extremely important parameters please remember that because this is quite crucial while calculating the current in the MOSFETs and also while trying to understand the saturation in the MOSFETs as we will see in the coming classes.

So, I would recommend you to go back and do this calculation yourself this is not an extremely difficult calculation it is it just had it this calculation has many terms. So, it might look a long calculation, but it is not a difficult one. So, please go back and do this and in the coming class we will start discussing the other concepts.

So, thank you for your attention, see you in the next class.