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# **Lecture - 44 MOSFET: Transport - II**

Hello everyone, today we will continue our discussion on the theory of electron transport through the MOSFET and as you might have might recall that in our previous class we concluded the discussion on the traditional way of deriving the IV characteristics of the MOSFET and we started with the with understanding the electron transport theory in the nano MOSFETs ok.

(Refer Slide Time: 01:00)



So, let me quickly summarize what we have seen so far and you might have seen this slide earlier as well. So, the basic understanding of the MOSFET can be done in terms of this energy barrier device. So, the MOSFET can be visualized as a device which has a barrier in the energy landscape in the channel.

So, the electron sees a barrier in the channel and by changing the voltage on the gate terminal this barrier can be manipulated and similarly the by changing the voltage on the drain terminal, we can create a symmetry around the barrier and facilitate the current flow.

So, this is this plot which you see here this is the plot of energy as a function of x and the energy is the conduction band energy the bottom of the conduction band energy here. And this is an extremely important plot I would say in order to properly understand the physics of the MOSFET ok.

So, please keep this in mind that when we have a gate voltage and a drain voltage applied on a MOSFET, this is how the energy landscape of the MOSFET looks like. And this point here which is the point right at the beginning of the channel this point which is also the peak of the barrier is an extremely important point. And generally it is under the weak influence of the drain voltage and under the strong influence of the gate voltage ok.

And that is the essence of functioning of the MOSFET because if this point is also under the influence of the drain voltage in that case, there would be a huge variability in the system. Because this DIBL effect will be there and the threshold voltage will not be properly defined cannot be properly defined. It cannot be defined just in terms of the gate voltage and that is what this scenario that not what we do not want.

So, we have seen the sub threshold swing, we have seen how do we traditionally define the IV characteristics of the MOSFET in terms of  $I_{DS}$  in terms of  $V_{DS}$  and  $V_{GS}$ . And how do we calculate the electric field in a traditional MOSFET from the current expression ok. But we also saw that this generally this equation is not valid if we go to nano MOSFETs or ballistic MOSFETs.

Because in that case this notion of mobility is not defined and we need to we might need to consider the ballistic transport. So, we need to invoke the general model of transport in order to properly understand the MOSFET physics.

## (Refer Slide Time: 04:07)



So, that is what we will see and that, this transport theory is also known as the Landauer transport theory of the MOSFET. Because it was initiated by Rolf Landauer he was the first person to formulate or to calculate the quantum of conductance in a MOSFET.

And then this theory was developed by Professor Supriyo Datta and Professor Mark Lundstrom from the Purdue University and that is how we also try to understand this in this part of the course ok. So, generally in this approach what we have is we have a left contact in between we have the device region or the channel region and on the right side we have a right contact.

So, this is source, this is drain and this is the channel region. This is the case with the two terminal device, but in a MOSFET in addition to these two terminals the source and the drain there is a third terminal as well which is known as the gate terminal ok. So, the way things now can be defined here is that if we plot the electronic energy states in this device on the left side.

So, generally we assume that the source and the drain are bulk regions they are pretty much like bulk regions. So, we can define the Fermi levels for the source and the drains. So, we can define Fermi level for the source  $E_{F1}$  let us say and Fermi level for the drain EF2 or sometimes EFD as well. The channel is a small channel in modern day MOSFETs it is only just few tens of nanometers.

So, we need to define the electronic energy states in the channel and the distribution of electronic energy states is defined by the density of states in the channel. And since the channel is small these states can be discrete state states as well and at some places there might be some gap in the energy landscape some of the energies may be disallowed for the electrons. So, for example, this range of energy is let us say disallowed.

Generally the voltage on the source side is fixed this is grounded. So, this  $E_{F1}$  also stays fixed and generally a drain voltage is applied in the system. So, we put a battery sometimes here and that is how we can change and that is why this  $E_{F2}$  can change and this  $E_{F2}$  may come down. So, if a positive voltage is applied  $E_{F2}$  will come down and this will change by  $-qV_D$  ok.

So, this is the E<sub>F2</sub> after the voltage has been applied on the second terminal ok. So, this is the difference between  $E_{F1}$  and  $E_{F2}$ .

(Refer Slide Time: 07:32)



Now, this gate voltage as you might have guessed by now, by changing the voltage on the gate terminal we can change electric field or potential in the channel and if the potential is changed in the channel it means that the potential energy of the electrons can also be changed.

So, what this does is this gate voltage here this can shift these electronic states up or down, so that is the major influence of the gate voltage. The when the gate voltage is applied this these energy states that are there in the channel all of them, they can either be shifted upwards or they can be shifted downward. So, if you are applying a positive voltage, they will be shifted downwards and if you applying we are applying a negative voltage, they will be shifted upwards ok.

So, that is the basic qualitative mechanism that is what is happening here and the source and the drain terminals are bulk terminals or quasi bulk terminals. So, we can define the Fermi functions in these contacts  $f_1$  and  $f_2$  and Fermi functions tell us about the electronic distribution in the each of these contacts. Generally it is assumed that that because of the inelastic scattering in the contacts.

Even when there is a current in the system even when these Fermi functions these distributions are maintained ok. So, that is an assumption and that is why we can use the notion of Fermi functions even when the current is flowing through the system. However, just to sort of point it out here if the number of electrons that are traveling from the source into the channel is huge or is significant and this number is comparable to the total number of electro free electrons in the source terminal.

In that case this current might change the distribution significantly and in that case we cannot use the these distribution functions when the current is flowing in the system. But generally that is not the case the source and the drain contacts are like, bulk contacts there are there is a huge number of electrons sitting in these contacts.

And the even if there is a current it does not change the distribution significantly and that is why we can use the notion of the Fermi functions in the even in the steady state, when the current is flowing through the system.

So, that is what happens and there is a basic point that I would just like to remind you which we have also discussed several times during our discussion on the general model of transport. That what happens is when we apply a voltage on the drain terminal this the drain contact tries to bring the channel in equilibrium with the drain contact with itself and the source contact tries to bring the channel in equilibrium with the source contact or with itself.

What it means is that this source contact tries to fill all the electronic states in the channel up to source Fermi level up to this energy level. And similarly the drain contact tries to fill all the electronic states in the channel up to this energy level. So, the energy states in between these energy states are the most interesting energy states, because the source is trying to fill them and the drain is trying to empty them, trying to take electrons out of them.

And that is how a current is maintained in the device. So, in the intermediate range of the energy states the energy states that lie between  $E_{F1}$  and  $E_{F2}$  in those energy states the current conduction actually happens in a way. So, that is what it is and that is why in the current conduction this difference of the Fermi functions this difference  $f_1 - f_2$  becomes the driving point driving factor basically.

And if you recall our discussion on the general model of transport the current is given by this equation, where the current is a constant times, the number of modes in the device and the difference of the Fermi functions integrated over all possible energy values. And if there is a scattering in the channel if the channel is a diffusive channel in that case we also need to take into account the transmission coefficient ok.

So, this is a very straight forward expression in a way because its, it takes into account this factor this parameter  $M(E)$  and what is  $M(E)$ ?  $M(E)$  is the number of modes in the channel. And what are the modes if you recall from our discussion on the general model of transport, modes are like conduction pathways in the channel they are like lanes in the channel through which electrons travel in the channel.

So, it depends on the number of lanes in the channel times the difference in the Fermi functions and this is become this becomes the driving force actually. And this parameter T(E) accounts for the scattering in the channel. So, ultimately if we need to if we need to find out the IV characteristics of a MOSFET or a nano device or a ballistic MOSFET this is the equation to start with actually ok.

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So, this is the equation where we need to start with in nano devices. And for that we need to know the parameter  $M(E)$  we need to know the parameter  $T(E)$  and we obviously, need to know the difference in the Fermi functions in the device. So, the Fermi functions are defined as so for the left contact this is the basic definition of the Fermi function : 1/  $[1 + \exp{ (E - E_{F1}) / kT}].$ 

And similarly f<sub>2</sub> Fermi function is defined as  $1/[1 + \exp{((E - E_{F2})/kT)}]$ . So, the Fermi function just to sort of remind you is the probability that a state electronic state at energy E is occupied by the electron, that is the probability and that is given by the Fermi function. So, this  $f_1$  tells us about the probability that a state in the left contact is occupied by the electron and  $f_2$  tells us about the probability that the state in the right contact is occupied by the electron.

#### (Refer Slide Time: 15:21)



So, in a way these are also the probability functions as well and in this in the current equation this parameter  $T(E)$  is the transmission coefficient and it is also a function of energy E. Because at each different energy levels the transmission might be different, similarly the modes might also be a function of energy generally it is a function of energy.

So, generally while discussing the this model of transport this Landauer model of transport we first we see we try to understand what happens in ballistic limit and what happens in diffusive limit. And finally, if we can understand things in the diffusive limit we can extend these this diffusive limit idea to the long channel MOSFETs as well.

So, what we can in principle do is we can calculate the IV characteristics of the MOSFET from here if we properly know the this  $T(E)$ ,  $M(E)$  and  $f_1 - f_2$  dE and if we integrate over proper energy range. Then we can extend this IV characteristics to the traditional IV characteristics ok.

But we cannot do vice versa we cannot we cannot generalize the traditional IV characteristics and account for the ballistic regime, so that is why this treatment is more fundamental. And I would also like to remind you one more point here, that this approach this is not a purely quantum mechanical approach as well. Because in this approach also the electrons are assumed to be I would say balls or like particles still they are assumed to be particles and but at the same time we are considering the quantized states of the electrons we are considering the Fermi dirac distribution of the electrons we are considering the effective mass.

So, we are not solving the Schrödinger equation between the contact and the channel and between the source and the channel and the channel and the drain. We are assuming that it is like a particle it may enter the channel or scatter back. So, that way it is not a purely quantum mechanical treatment a pure quantum mechanical treatment, will involve the solution of the Schrödinger equation in each case ok in a particular in a given case we would need to solve the Schrödinger equation for the.

But this treatment is pretty much is fairly general as compared to the traditional treatment and it can explain most of the electrical characteristics of the modern day MOSFETs. The MOSFETs which are a few 10s of nanometers in size whose channel length is maybe 10 nanometer in size ok.

So, so now, generally in MOSFETs if you recall there is a linear regime of operation and there is a saturation regime of operation. In linear regime the applied voltage on the drain terminal the applied voltage on this terminal is small and in the saturation regime this voltage is actually large.

So, there are two limits in which we finally, need to discuss this equation one is the small bias limit when the applied voltage is small and one is the large biased limit. So, as if we plot the energy states here the energy states of the source the drain and the channel this is the Fermi function and the, this is the Fermi level  $E_{F1}$  Fermi function  $f_1$  drain side Fermi function  $f_2$  Fermi level E<sub>F2</sub> ok.

So, and this is the applied voltage. So, this voltage is large then  $f_1 - f_2$  is actually equal to almost equal to f 1 and that is also clear from the plots of f 1. So, for example if we take if we plot the Fermi function at 0 Kelvin on the Y-axis, if we have the Fermi function and on the X-axis if we have the energy. So, at 0 Kelvin if this is  $E_{F1}$  this is  $E_{F2}$  and at 0 Kelvin, the Fermi level  $E_{F1}$ ,  $f_1$  will look like this.

So, it will be for all energy values less than  $E_{F1}$  and it will be sorry it will be 1 for all the energy values up to  $E_{F1}$  and it will be 0 above energy values  $E_{F1}$  and this is the Fermi function  $f_1$ .

Similarly, the Fermi function  $f_2$  will be this and this  $E_{F1}$  is extremely larger than  $E_{F2}$ which means that this is far away from this and this is may be close to the origin or somewhere. So, then this  $f_1 - f_2$  can be approximated by just  $f_1$  fairly. So, in that case this current equation in this equation this  $f_1 - f_2$  term can be approximated by the  $f_1$  term.

So, that is the case generally the case in saturation region of the MOSFETs in the large bias limit. And in the small bias limit actually the small bias limit has been discussed a lot during the discussion on the general model of the transport.



(Refer Slide Time: 22:17)

So, you might as well remember that, that in the small bias limit  $f_1$  is very close to  $f_2$  and this difference function can be approximated by a Taylor series expansion of the. So, this can be approximated by this function this is the small bias limit. So, in the linear regime of the MOSFET generally this approximation will be used and in the saturation regime of the MOSFET generally  $f_1 - f_2$  can be approximated just by the  $f_1$  function ok.

So, we are just trying to see how to use general model of transport which we had already discussed in the context of the MOSFET ok. So, yeah this we have already seen that in the small bias limit this difference goes to this value or sorry there is this term q times V as well here. So, instead of dE we have q times V where V is the applied voltage.

And in the small bias limit we can define the conductance in a very clean way because the conductance is defined as the ratio between the current and the voltage I by V. And in the small bias limit there is a very clean relationship between the current and the applied voltage and this conductance is defined in this way.

So, the conductance turns out to be  $2q^2/h$  integration of T(E) M(E) (- $\delta f/\delta E$ ) dE. And this constant  $2q^2$  h is the fundamental sort of fundamental constant and it is also known as the quantum of the conductance ok, we had a pretty comprehensive discussion on the conductance. So, we can you can actually revise that if you need to.

(Refer Slide Time: 24:41)



And finally, we have this Fermi window function actually. So, the Fermi window is generally applied is generally used in the low bias limit. So, this function (-δf/ δE) this is known as the Fermi window function and it generally is the states between the two Fermi levels between  $E_{F1}$  and  $E_{F2}$  which is pretty much clear. So, this plots on the left hand side this plot is the plot of  $f_1$  and  $f_2$  for low temperature on the right hand side this is a plot of  $f_1$  and  $f_2$  at high temperature.

And at low temperature it is pretty much evident that this function  $f_1 - f_2$  which ultimately boils down to this is essentially the states between  $E_{F1}$  and  $E_{F2}$ . And at high temperature this is slightly more complicated and it is the states primarily the states between  $E_{F1}$ .  $E_{F2}$ , but some states out of that range as well ok. And here at high temperature limit if you remember we need to invoke the Fermi Dirac integrals in order to do the proper calculation of the conductance and the conductance functions.

So, with this basic revision we will see what is the transmission energy transmission coefficient at a certain energy or what is the transmission in the context of the MOSFET, although we have had a discussion on the transmission in the general model of transport as well.

(Refer Slide Time: 26:40)



But this number of modes can directly be taken from the, our previous discussion, let us quickly try to understand the transmission in a MOSFET. In a MOSFET generally if this is the channel region of the MOSFET, if you remember in the MOSFET generally the electrons are injected from the source side they travel to the drain side.

And I would say that the transmission is defined as the electrons received on the drain side as divided by the electrons injected from the source side. So, some of the electrons may scatter in the channel they may travel back they may change energy as well in the during the transport from the left contact to the right contact. So, that is why this parameter T is defined as the electrons received on the right side on the right contact defined by the electrons injected on the left contact.

So, the left contact is taken to be  $x = 0$  point the right contact is taken to be  $x = L$  point. And let us see that the flux which is the number of electrons per unit time coming on the left contact is  $F^+(x=0)$  it is the flux at the left contact is this plus means that the electrons travelling in the plus x direction. And the flux received at the right contact is  $F^+(x=L)$ . So, the transmission coefficient of the device will be defined as  $F^+(x=L)/F^+(x=0)$  ok.

And in the diffusive transport case if you remember the derivation there we apply a Fick's law we apply the Fick's law basically which looks like this. So, the flux that is injected on the left contact all of that does not reach to the right contact some of it may come back and. In fact, what happens is that number of excess electrons on the left contact is generally let us say if this is number of excess electrons is this on the left contact. Generally in long channels what happens is the number of excess electrons is on the right side is 0.

So, this if we plot the number of excess electrons as a function of channel length this is how it drops. So, on the left side it is  $n(x=0)$  on the right side it is 0 and this is the channel length. So, the gradient of charge carriers in this devices becomes  $n(x=0)$  /L or this is the gradient this is the negative gradient actually if this is  $x = 0$  point and this is x  $=$ L point.

So, this becomes the gradient of the excess charge carriers in the MOSFET. So, by applying Fick's law the net flux that is reaching to the right side becomes minus D n in terms of the diffusion constant  $D_n$  times dn /dx. So, it becomes  $D_n$  times n(x=0)/L and this is the flux reaching to the right contact ok. So, we have calculated the flux reaching to the right contact in terms of the diffusion constant it turns out to be  $D_n$  times number of excess electrons on the left contact divided by the length of the channel.

And what is the flux on the left contact what is  $F^+(x=0)$ . So,  $F^+(x=0)$  will depend on the velocity of the charge carriers as well.

(Refer Slide Time: 31:27)



So, this quantity is  $F^+(x=L)$ ,  $F^+(x=0)$  will be number of excess charge carriers on the left contact times the velocity at which they are travelling or this velocity is an interesting parameter here this is also known as the unidirectional thermal velocity.

Because the net velocity of electrons is 0 typically 0 because if we consider that a flux is injected but almost half of that may scatter and come back. So, almost half electrons are travelling to the left side half electrons are traveling to the right side and that way the net velocity will be 0, but if we are only considering the flux going to the right side in one direction plus x direction.

Then we need to take this thing which is known as the unidirectional thermal velocity which is the velocity of which is just the velocity of electrons traveling in the plus x direction. And so the flux at x equal to 0 will be  $n(x=0)$  equal to times velocity and we need to divide it by 2 because the number of electrons going to the right side is actually half, if we consider that half of them may scatter and come back to the left side.

So, from Fick's law we have calculated the flux entering to the right contact from our basic intuitive understanding we know what is the flux at the left contact. So, we can find out the transmission coefficient as well, which is the flux entering the right contact divided by the flux of electrons at the left contact.

So, it will be this by this. So, n and n cancel what is left is  $2 D_n$  divided by L times  $v_T$ this will be the transmission coefficient. And if you remember that this  $D_n$  the diffusion constant is generally  $Lv_T/2$  and this comes from the scattering theory which we have not covered in this course and possibly we would not be able to do that.

But let us take it as it is and if we put this then this transmission coefficient in the diffusive limit becomes  $\lambda v_T$  divided by Lv<sub>T</sub>, 2 as well. So, T is essentially  $\lambda/L$  which we have also seen during our discussion on the general model of the transport. So, this is the transmission coefficient in the MOSFETs in diffusive limit, but if we consider the quasi ballistic limit in which the channel length and the mean free path is of the same order in that case this becomes  $\lambda/(\lambda+L)$ .

So this is just a quick refresher of the general model of transport and how we will use those equations in the context of the MOSFET in the coming class we will discuss this interesting parameter this unit direction thermal velocity and then IV characteristics of the ballistic MOSFET. And we will also try to see how the current depends on the voltage in the case of ballistic MOSFET ok.

So, that is all for the for this class

Thank you for your attention and see you in the next class.