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Lecture - 43 MOSFET: Transport - I

Hello everyone. As you may recall that we have been discussing the traditional approach of deriving IV characteristics of a MOSFET and in the last class we could have a relationship between I_{DS} , V_{DS} and V_{GS} . We were trying to calculate the electric field across the channel in the MOSFET that is due to V_{DS} and V_{GS} using the traditional approach.

So, we will complete that discussion today and today hopefully we will start discussing the transport theory in MOSFET and specifically transport in nano-MOSFETs which is the MOSFETs nowadays ok.

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So, let me quickly review what we have been discussing. So, as you know we are discussing the electrical characteristics of a MOSFET and a MOSFET looks like this. The general form of the current is like this, in this expression as you can see that the current depends on the charge in the channel and the velocity with which this charge is moving through the channel ok.

And we need also need to multiply by the width to account for the total current in the channel. And so, this becomes the basic starting point, this becomes the starting point of MOSFET IV characteristics in any case. So, in even in traditional case this is the starting point and even in our nano-MSOSFET case, ballistic MOSFET case this is also the starting point to analyse the IV characteristics of the MOSFET ok.

So, as you might by now remember that this current depends on electrostatics which governs the charge in the channel and the transport, which essentially accounts for the velocity of the electrons in the channel. So, that is why we need to understand these two things in order to properly understand the electrical characteristics of the MOSFET. Just to sort of recall, the MOSFET is a barrier controlled device, there is a barrier in the channel because of the geometry of the MOSFET.

And this is the energy barrier for the electrons and this barrier can be controlled by the gate voltage, the height of the barrier can be controlled by the gate voltage and barrier symmetry can be controlled by the drain voltage. So, the; so this is the effect of the gate voltage on the barrier and this is how the effect of the drain voltage looks like on the barrier.

So, please remember that this is the barrier in the energy landscape of the electrons and because of this barrier structure, this gate terminal becomes the control terminal ok. Because now this can reduce the barrier the gate can reduce the barrier or increase the barrier ok.

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So, we also saw in our discussion of the IV characteristics that generally it is assumed that when the gate voltage is less than the threshold voltage, the charge in the channel is negligible. So, the charge in the channel starts appearing only when V_{GS} is greater than V_T ok and generally this part of the device, especially this part of the device the gate oxide and channel part of the device this is this can be treated like a capacitor.

Because there is a conducting plate, gate plate there is a conducting channel and in between we have an insulator or a dielectric. And the voltage in access to the threshold voltage which means this voltage V_{GS} - V_T , this is the voltage which is actually creating charge in the channel.

So, if the capacitance of this capacitor, this gate oxide channel capacitor is Cox and then we can say that that the charge in the channel is -Cox times ($V_{GS} - V_T$), because we are considering the charge on the other plate of the capacitor. So, that is why this negative sign is there. So, this gives us the charge in the channel when drain voltage is 0 and if in addition to the gate voltage we are also applying a drain voltage in the device. So, we have put a battery on the drain terminal as well, generally the source is assumed to be grounded we connect a battery to the gate terminal as well ok.

So, this battery on the drain terminal is known as the V_{DS} voltage source and the battery on the gate terminal is the V_{GS} voltage source. So, when there is a V_{DS} voltage, in that case the potential at any point in the channel will also be there because of this V_{DS} . So, as we know that on the source side the voltage is 0, on the drain side the voltage is V_{DS} or V_{DD} , sometimes it is written as V_{DD} as well, and along the channel the voltage drops from 0 to V_{DS} ok.

So, at any arbitrary point in the channel. So, this is the channel direction is the x direction. So, at any arbitrary point in the channel, the voltage can be written as V(x) ok. So, in that case, if in addition to the gate voltage we also have a drain voltage in that case the channel charges Qn is dependent on both V_{GS} and V_{DS}, it is -Cox (V_{GS} - V_T - V(x)).

So, this is how the charge is accounted for in the channel and the velocity at which this charge moves this depends on the mobility times the electric field. And so that is, this is true in the case of long channel MOSFETs, most of the times this is true because in long channel MOSFETs we can properly define the mobility of electrons. In ballistic MOSFETs as we have already seen that this notion of mobility is not well defined and we need to start with the basic principles.

So, in terms of the density of states and the number of modes, even this mobility can be represented. In the case of ballistic MOSFETs this notion of mobility is not a fundamental notion we need to start with the density of states and the scattering properties of the channel and then we can define the mobility. So, but in long channel MOSFETs this velocity at which this charge moves, depends on this.



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And in linear regime this we have seen already, in linear regime this electric field is V_{DS}/L and that is why this current can be given as, from this starting from this expression the current can be given by this expression in the linear regime. We also saw that that the saturation regime can be there or can happen because of the first because of the velocity saturation and second because of the pinch off.

So, when the saturation is happening because of the velocity saturation, in that case this the current will be given by this value. Because instead of this v, average v here we can just directly put v_{sat} and instead of Qn we can put this value essentially. And when this is happening because of the classical pinch off, in that case the current can be represented in this way, this we have already seen in the last class.

And as you can see from the from these two expressions that I_{DS} is independent of the V_{DS} . In our previous discussions we also could derive the current the general current which looks something like this, $\mu_n C_{ox}$ (W/L) ($V_{GS} - V_T$). $V_{DS} - V_{DS}^2/2$.



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So, with this we started analyzing the electric field in the channel and in order to analyze electric field in the channel we take the help of the current expression and in this expression if this V_{DS} is there, then instead of L' we generally have L. So, please consider this to be L this is not L' because when we are considering V_{DS} which is the voltage that is applied on the drain terminal, in that case this should be L ok.

So, this prime is not there in that case and. So, this is the expression of the current and the current at any arbitrary point in the channel at any point x in the channel can be written in this way. From the basic equation of the current and since in steady state the current is uniform, the current is constant throughout the channel.

We can put these two expressions of the current equal to each other and this gives us the, if we integrate this equation from x equal to 0 to x equal to any arbitrary value of x, this gives us an this quadratic equation in V(x). So, we obtain a quadratic equation in V(x) ok. Now, if we can calculate V(x), which means the potential at any arbitrary point in the channel. So, we can also calculate the electric field by this basic relationship or this will be $-\delta V/\delta x$ ok.

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So, this is the way we generally solve and this I gave you as a small homework to do, to calculate the V(x) at any arbitrary point x in the channel. So, if we have in a quadratic equation in this form, then the solution of this quadratic equation can be written from the basic maths formula $[-b \pm \sqrt{b^2-4ac}]/2a$.

So, in this case a is half, b is $-(V_{GS} - V_T)$ and c is this entire thing. So, if we do this calculation, I am not going to do this step by step because it is a simple calculation.

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This is the expression of potential at any arbitrary point x in the channel and this is L here ok. This looks a bit complicated. But if we make proper approximation. So, for example, if we analyze in linear region, when we apply a small V DS in linear region, please remember that the applied drain voltage is extremely small. In that case this V DS square will be even smaller and this term can be ignored.

So, what is left inside the square root here is $[2 (V_{GS} - V_T) V_{DS} / (V_{GS} - V_T)^2](x/L)$. So, this $(V_{GS} - V_T)$ will be cancelled. So, what is left is $[(1 - 2V_{DS})/(V_{GS} - V_T)](x/L)$. And now since in linear region this V_{DS} is a very small, let us assume that this is a very small voltage that is applied.

So, we can have this binomial expansion of this square root and the binomial expansion of the square root, when this x is small, then this 1 - $x^{1/2}$ can be written as and rest of the terms can be generally ignored. All these rest of the terms can be ignored ok.

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So, using this expansion this square root is now can be taken away and what is left inside is V(x) is (V_{GS} - V_T) into $[1 - (1 - 2V_{DS})/2(V_{GS} - V_T) (x/L)]$ ok. So, 2 and 2 go away, 1 and 1 sorry this yes, this 1 and 1 will go away and what is left is V(x) is (V_{GS} - V_T) times this minus becomes +V_{DS}/L into x/(V_{GS} - V_T). So, this will also go away and so V(x) is essentially V_{DS}/L times x.

So, the electric field which is $-\delta V/\delta x$ will be $-V_{DS}/L$, which is what we also assumed in the beginning. So, for small applied V_{DS} values for the small drain voltages, which means in the linear regime of the IV characteristics, this electric field is constant throughout the channel and the voltage drops linearly across the channel. Which is we also, which is what we also have observed previously.

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And in the saturation region for example, in the pinch off region we need to put. So, at the onset of the pinch off and the onset of the pinch off happens when V_{DS} is exactly equal to V_{GS} - V_T and at this voltage if we put V_{DS} value, then this simplifies to be to this point and it means that the electric field can be given by this expression.

So, as you can see that this is not this is now in the saturation region the electric field is no longer constant, it is a non-linear thing and generally if you remember the barrier in the MOSFET this is how it looks like. So, the electric field is extremely small at this point, at x equal to 0, this is a very small value almost 0 and it changes abruptly as we go away from the top of the barrier to the drain side. This is the source side this is the drain side electric field changes non-linearly from the top of the barrier to the drain side.

So, please remember that this is a, this is a key concept, please remember this thing about the MOSFET. This is the energy landscape in the MOSFET in the channel, starting from the source to the drain side ok in this case a drain voltage has been applied. So, that is why this source and the drain energy level source and drain conduction band energy levels are nonaligned with each other, are far apart from each other and this height can be controlled by the gate voltage ok.

So, this essentially concludes our discussion of the electric field and the takeaway here is that, if we need to calculate the electric field in a device we need to start like this with the basic equation of the currents and in the linear region the electric field almost, electric field is almost constant, voltage drops linearly, but in the saturation regime the voltage drop is also non-linear and the electric field is also non-linear ok.



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So, this essentially come concludes our discussion of the classical or traditional approach of doing MOSFET electrostatics. But please remember that in this kind of discussion, in this IV treatment of MOSFET we assume that the MOSFET is a long channel or a at least it is not a ballistic MOSFET, because we are using the notion of mobility, we are using the notion that velocity is dependent on the mobility and the electric field.

But all these notions actually break away, if we go down to the ballistic limits and there we need to again think more fundamentally and that is what we will do in the coming discussion. So, just to some things up this is what we have seen so far, we have seen that the MOSFET is a barrier control device, which can be the barrier can be controlled by the gate and the drain voltage.

The current in a very in the most general form is defined in this way and we need to understand the electrostatics and the transport in the MOSFET.

This is the barrier in the channel, this point the point at the beginning of the barrier very close to the source or just at the source side, this is under the strong control of the gate and weak influence of the drain. So, generally in long channel MOSFETs this point is shielded from the drain side, but in nowadays in the short channel MOSFETs, what

happens is because of the drain voltage even this barrier might be slightly lowered and that effect is known as the drain induced barrier lowering.

And this effect is conspicuously visible in the sub threshold characteristics of the MOSFET. So, the and while discussing the MOSFET characteristics this is what we saw that, for a better understanding of the characteristics we generally plot the output characteristics which is a relationship between I_{DS} and V_{DS} and we plot the transfer characteristics, which is the relationship between I_{DS} and V_{GS} .

The transfer characteristics tells us about the control of the gate terminal on the MOSFET current. While we zoom this part, by plotting the log of drain current as a function of the gate current we see this kind of plot and in this we can see the we can precisely define the gate controlled in terms of sub threshold swing.

And the sub threshold swing is defined as the quantity or the amount of gate voltage that is required to change the drain current by a factor of 10, below the threshold voltage. When the gate voltage is below the threshold voltage in that regime this is the amount of the gate voltage that is required to change the current by a factor of 10.

And the unit of this is millivolt per decade and there is a fundamental limit in MOSFETs on this quantity and that is 60 millivolt per decade. Apart from this we analyzed the current in the MOSFET, in terms of the flux of electrons going from the source side to the drain side and from the drain side to the source side.

But that analysis is still not complete because in that analysis we did not put the expression of the charge explicitly and also the velocity explicitly. We after that we did a traditional or we derived the traditional IV characteristics of the MOSFET and this is how it looks like. And we basically analyzed the characteristics of the MOSFET in linear regime and in the saturation regime. And from this current expression itself we can actually derive the electric field and the voltage at any arbitrary point in the channel.

But this treatment will not hold true in the case of ballistic MOSFETs or nano-MOSFETs. So, that is why if we need to properly understand the transport theory of the MOSFET, before going into the transport theory, let me just give you a glimpse of how the electrostatics of the MOSFET looks like.

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So, as you know that the electrostatics is an extremely important component of understanding the MOSFET physics and it primarily governs the charge in the channel, also has influence on the velocity actually. Strictly speaking velocity depends on electric field to an extent and so we cannot properly demarcate that, this is this comes under the ambit of electrostatics and this comes under the ambit of transport theory.

Actually, there is an overlap between the two, but broadly speaking the electrostatics actually governs the charge in the channel and the transport governs the velocity of the electrons in the channel. So, in generally in traditional MOSFETs in the long channel MOSFETs which looks like this. If you closely see that, the distance between the source and the drain. So, this the channel length or this channel length this is in long channel MOSFET it can be 100s of nanometers or sometimes a 1 micrometer or so.

So, this distance is of the order of 100s of nanometers or generally of the order of 1 micrometer and this distance if you see this is only a few nanometer, the channel thickness is also a few nanometer. So, even in long channel MOSFETs, the oxide thickness is few nanometers and also the channel thickness is, channel the thickness of the channel charge is also few nanometers.

So, if you see that in this direction, we apply a gate voltage and generally this body is grounded. So, within a few nanometers a lot of things happen. So, generally the distance through which this voltage drops in this direction is less as compared to the distance

through which this applied voltage on the drain drops in this direction, which means in other words that in traditional MOSFETs the electric field in this direction is high as compared to the electric field in this direction.

So, that is precisely the reason that in traditional MOSFETs while sort of trying to understand the charge and trying to do electrostatics, generally only 1D electrostatics is done and that is done normal to the channel in this direction, which is conventionally is the y direction in the MOSFETs.



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But in short channel MOSFETs when this is only few tens of nanometers in that case the electric field in this direction also becomes significant, it becomes of the same order as the electric field in this direction. And in that case we need to consider the electrostatics in the short channel MOSFETs, sorry the 2D electrostatics in the short channel MOSFETs.

So, that is how typically this elect this treatment of electrostatics change, while we go to nano MOSFETs or ballistic MOSFETs. We in very small MOSFETs, we need to consider the 2D electrostatics and in large MOSFETs we can generally do calculation using one d electrostatics and that will give us fairly good results.

So, while dealing with 2D electrostatics, we need to consider the electrostatics in this direction as well as in this direction, both in x direction and in y direction. So, this is

tilted picture of the MOSFET. And the most fundamental equation in the electrostatics is this equation, this is the Poisson's equation in a, very general Poisson's equation. So, if there is a charge distribution in the space somewhere, if this ρ is the charge distribution then the relationship between the charge distribution and the electric displacement vector which is this vector D here, is given by this fundamental relationship.

So, by using this relationship we can find out the potential due to a charge distribution and we can also find out the charge due to an applied voltage or due to an applied electric field. So, generally in electrostatics of the MOSFETs, generally this equation is solved ok. This is just an expanded version of this equation, in this instead of the electric displacement factor we have this potential field and this epsilon is the dielectric constant of the semiconductor through which we are trying to do the electrostatics.

So, generally in electrostatics only this is the focal point of our discussion. Generally, we try to solve this equation in the MOSFET for various regimes and for various applied voltages on the gate and on the drain side. So, we will come to this in a bit, we will come to this in a bit just this is just to sort of give you a glimpse. First let us try to understand the transport in the nano-MOSFETs. So, as you know that we have already discussed the general model of transport in the beginning in the first half of this course.

Transport	Landauer Approach to Transport	
Current:	$I = \frac{2q}{h} \int_{-\infty}^{+\infty} T(E)M(E)(f_1(E) - f_2(E))dE \text{Amperes}$ $I = \underbrace{\left(\begin{array}{c} 2g \\ 2g $	$\frac{1}{\overline{z}} + \frac{1}{\int_{\lambda(k)}^{k \times c} \int_{\lambda(k)}^{k \times c} \frac{v}{f_{\lambda}(k)}},$ $E_{F2} = E_{F1} - qV$
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And so, we will borrow heavily from that and in that discussion if you recall we generally take a two-terminal device, a device which is which looks like this. There is a

source, there is a channel and there is a drain. Generally, the source is grounded there is there might be a voltage on the drain terminal ok.

So, for this kind of device, for the two terminal device we have already done or we have we have tried to understand how the IV characteristics look like in this case, when a certain voltage is applied on the drain terminal. And what happens in the ballistic case, what happens in the diffusive case, what are the expressions of the current and the conductances, the notion of modes in the channel comes about. So, all these things we have already discussed.

So, we will borrow heavily from there, from that discussion and we will try to contextualize all those things in the context of a MOSFET ok. So, we will discuss the model of transport for now for nano-MOSFETs for small MOSFETs, which are present nowadays, which are pretty common in devices nowadays.

So, if you remember that, that the basic equation of the current that comes from the general model of transport is this and this is the case for a two-terminal device in which the Fermi function on the first terminal is f_1 , the Fermi function of the second terminal is f_2 . The number of modes in the device is M and T(E) is the transmission coefficient and we are considering the all possible ranges of energy through which the electrons can travel. And then we have this constant which is there.

So, before beginning this part, let me ask you to revise the general model of transport and in the next class we will start this discussion for the MOSFETs. So, I would recommend you to please go back and study the general model of transport, various concepts there, various basic equations there and then it would be easier to understand things in this part ok. So, that is all for the day.

Thank you for your attention see you in the next class.