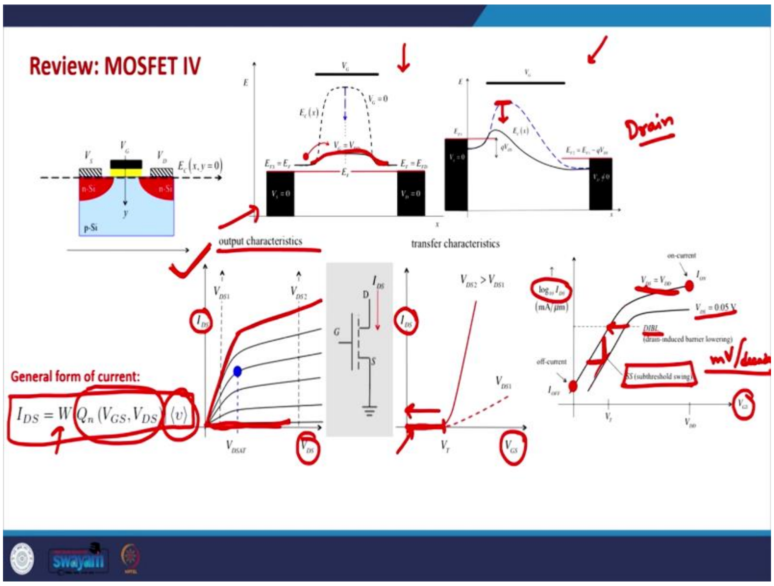
Physics of Nanoscale Devices
Prof. Vishvendra Singh Poonia
Department of Electronics and Communication Engineering
Indian Institute of Technology, Roorkee

Lecture - 42
MOSFET IV Characteristics -Traditional Approach

Hello everyone. As all of you know that in our last class we started with the traditional approach to derive IV characteristics of the MOSFET and as most of you would know that the traditional approach is centered around long channel MOSFETs that was done by keeping long channel MOSFETs in mind. And so, just for the sake of completion of the treatment of MOSFET we will also go through the traditional approach to derive IV characteristics of the MOSFET.

Although we have seen that in the small channel MOSFETs that approach is no longer applicable because the roots model of conduction is no longer applicable in the small channel MOSFETs yeah, but still we would go through the traditional approach just for the sake of completion of this MOSFET treatment.

(Refer Slide Time: 01:25)



Review: MOSFET IV

General form of current:

$$\boxed{I_{DS} = W Q_n(V_{GS}, V_{DS}) \langle v \rangle}$$

Before going into that let me quickly review what we have seen. This is what we have seen so far. The MOSFET geometry is this is the physical form of the MOSFET geometry and in the energy landscape this geometry actually gives us a barrier in the channel and this barrier can be controlled by the gate voltage primarily also this not the barrier, but this landscape of the energy electronic energy can be changed by the drain voltage as well.

So, the effect of the gate voltage is shown in this figure and the effect of the drain voltage is shown in this figure ok. The general form of IV characteristics is written in this way and this is true for any device I would say that the current depends on the charge available for conduction times the velocity at which the charge will move times the width of the device ok.

And what we have seen is that the MOSFET IV characteristics generally broken down in two parts; one is the output characteristics that is the relationship between I_{DS} and V_{DS} and second is the transferred characteristics which is the relationship between I_{DS} and V_{GS}. So, the output characteristics has a linear regime, a saturation regime and also a sub threshold regime.

This sub threshold regime is more clearly I would say present in the transfer characteristics which is basically I_{DS} versus V_{GS} relationship and the plot for V_{GS} less than V_T values is the sub threshold regime.

And if we instead of plotting I_{DS} versus V_{GS} if we plot log I_{DS} logarithmic function of I_{DS} this will give us a better a zoomed in picture of the current in this regime and this is how the current will look like when we have logged in I_{DS} on the y axis and if we plot it as a function of V_{GS} which is there on the x axis.

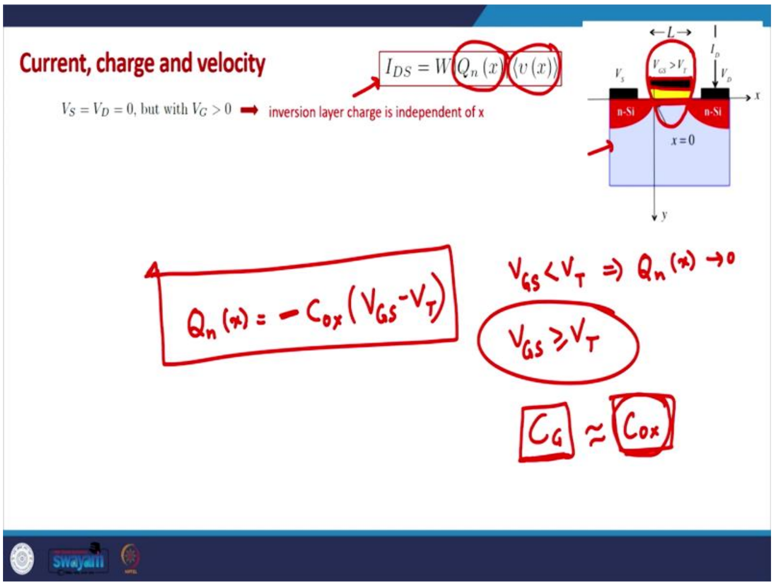
On this we can see that we come across a new MOSFET parameter which is known as the sub threshold swing which is the inverse of the slope of this curve and this tells us about how much control does the gate has on the channel.

So, if the sub threshold swing is less it means that the gate has more control over the channel and if the sub threshold swing is more it is it means that the gate has less control on the channel. The unit of the sub threshold swing is milli volt per decade. So, it is essentially the amount of gate voltage that is required to change the channel current by a factor of 10 ok.

In this we also saw the effect of drain voltage on the IV characteristics especially for small devices and in the small devices the barrier is no longer shielded from the drain terminal electrostatically because now the channel length is extremely small. And if we apply a voltage on the drain terminal it will reduce the barrier in the channel and this is known as the drain induced barrier lowering and this reflects in shifting of the curve to the left side for high drain voltages.

So, if drain voltage is less 0.05 volts the IV curve is on the right side if the drain voltage is high V_{DS} is equal to V_{DD} the drain voltage is shifted to the left side this is the DIBL ideally we would not like DIBL to be present in our devices.

(Refer Slide Time: 05:38)



Current, charge and velocity

$I_{DS} = W \langle Q_n(x) \rangle \langle v(x) \rangle$

$V_S = V_D = 0$, but with $V_G > 0$ ➡ inversion layer charge is independent of x

$$\boxed{Q_n(x) = -C_{ox}(V_{GS} - V_T)}$$

$V_{GS} < V_T \Rightarrow Q_n(x) \to 0$

$V_{GS} \geq V_T$

$\boxed{C_G} \approx \boxed{C_{ox}}$

Now, let us come back quickly come back to the traditional treatment of IV characteristics generally it is assumed that in a MOSFET when the gate voltage V_{GS} is less than the threshold voltage in that case the channel charge is negligible or it is extremely small tends to 0. And the charge appears only when the gate voltage is greater than or equal to the threshold voltage ok.

So, what is a better starting-point than the general expression for the current in this in the general expression of current we need the charge and we need the velocity with which the charge moves. The charge can be derived from the capacitance approximation of the gate channel; this part of the device.

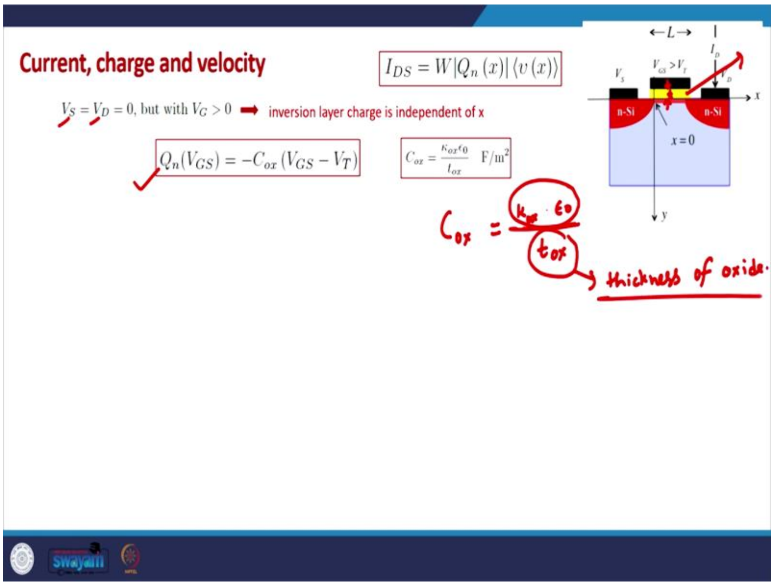
So, this part of the MOSFET can be approximated by a capacitor because we have a conducting gate plate, conducting gate contact we have a conducting channel and in between we have an insulator or the dielectric.

So, if the capacitance of this capacitor is C_G the gate capacitance of the MOSFET which is approximately equal to C_{ox} in most of the cases. So, generally while we do not want to deal with the details of the capacitance in the MOSFET we generally take C_G to be equal to C_{OX}, but ideally C_G includes C_{ox} and the depletion capacitance of the channel.

So, if we have a capacitor with capacity C_G or C_{ox} let us say and we know the voltage applied across it we can easily deduce the charge that appears on the plates of the capacitor.

So, the charge in the channel can be written as to be minus C_{ox} because we are applying a positive voltage on the gate side. So, the charge that will appear on the channel side will be negative V_{GS} - V_T because charge only appears when the gate voltage is above the threshold voltage. So, the voltage that contributes in producing charge in the channel is the voltage in access to the threshold voltage which means V_{GS} - V_T.

(Refer Slide Time: 08:26)



Current, charge and velocity

$$I_{DS} = W|Q_n(x)|\langle v(x)\rangle$$

$V_S = V_D = 0$, but with $V_G > 0$ ➡ inversion layer charge is independent of x

$$Q_n(V_{GS}) = -C_{ox}(V_{GS} - V_T)$$

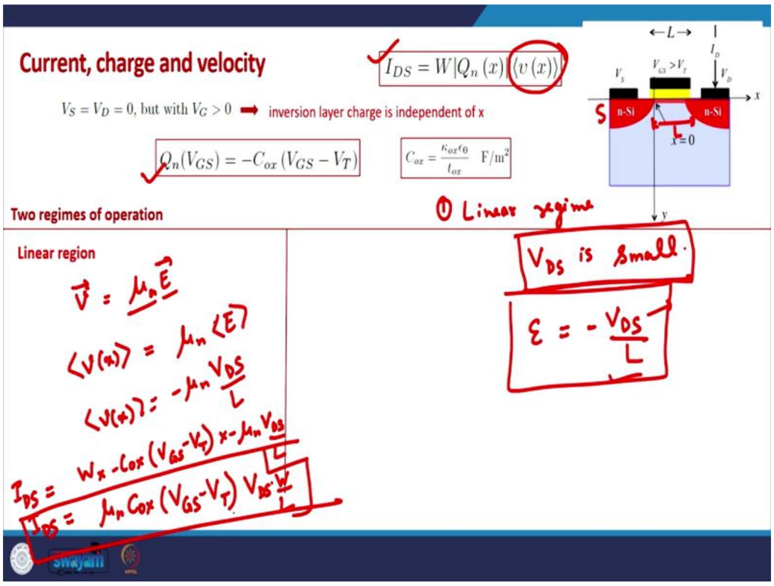
$$C_{ox} = \frac{\kappa_{ox}\epsilon_0}{t_{ox}} \quad \text{F/m}^2$$

$$C_{ox} = \frac{k_{ox}\cdot\epsilon_0}{t_{ox}}$$

→ thickness of oxide.

So, the charge in the channel when the source and the drain terminal are grounded in that case the charge is given as minus C_{ox} times V_{GS} -V_T where C_{ox} is essentially the capacitance of this gate oxide and it is written as k_{ox} times €o divided by t_{ox}. Were in the numerator this is the dielectric constant of the insulator and this is the oxide thickness this is basically this direction this is the t_{ox} thickness of oxide on top of the channel.

(Refer Slide Time: 09:17)



Current, charge and velocity

$I_{DS} = W|Q_n(x)|\langle v(x)\rangle$

$V_S = V_D = 0$, but with $V_G > 0$ ➡ inversion layer charge is independent of x

$Q_n(V_{GS}) = -C_{ox}(V_{GS} - V_T)$

$C_{ox} = \frac{\kappa_{ox}\epsilon_0}{t_{ox}}$ F/m^2

Two regimes of operation

① Linear regime

V_{DS} is Small.

$\mathcal{E} = -\frac{V_{DS}}{L}$

Linear region

$\vec{v} = \mu_n \vec{E}$

$\langle v(x)\rangle = \mu_n \langle E\rangle$

$\langle v(x)\rangle = -\mu_n \frac{V_{DS}}{L}$

$I_{DS} = W \times -C_{ox}(V_{GS}-V_T) \times -\mu_n \frac{V_{DS}}{L}$

$I_{DS} = \mu_n C_{ox}(V_{GS}-V_T) V_{DS} \cdot \frac{W}{L}$

So, there are two regimes of operation broadly one is the linear regime which means the V_{DS} is small in the linear regime ok. So, in the linear regime what would be the velocity of the charge carriers? So, the velocity will be from this basic relationship the velocity of electrons because this is the channel consists of electrons this will be directly dependent on the mobility of electrons and the electric field ok.

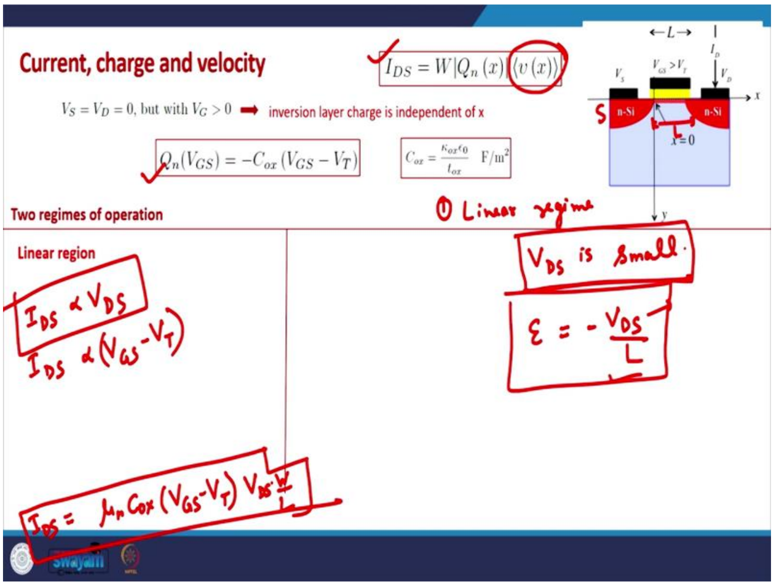
So, the average velocity will be, let us write it as average electric field and for in the linear regime when V_{DS} is small the electric field can be written as, if the channel length is L as we have seen earlier that in the linear regime when V_{DS} is small in that case the electric field is almost constant because V_{DS} varies linearly sorry the drop of voltage is linear in the channel. So, this can be written as $-V_{DS}/L$.

So, it can be assumed that that this voltage V_{DS} drops uniformly across the channel. So, on one side of the channel on the source side we have the 0 volts applied on the drain side we have V_{DS} volts applied and in the linear regime it is assumed that this voltage drops uniformly. So, the electric field is generally $-V_{DS}/L$ and this can be taken as like this ok.

So, we know the charge we know the velocity. So, in this case we can easily find out the current. So, now, the current from this general expression of the current I_{DS} is essentially W times $-C_{ox}$. So, this minus W times $-C_{ox}$ $(V_{GS}- V_T)$ times $-\mu_n(V_{DS}/L)$. So, this turns out to be μ_n C_{ox} $(V_{GS}- V_T)$ into V_{DS} times W/L.

So, this is the traditionally how the current is derived in the MOSFET. This is the expression of current in the linear regime. So, we are at the moment we are looking at how the current would be there in the linear regime and in the saturation regime finally, we will also see a more general expression of the current.

(Refer Slide Time: 12:58)



Current, charge and velocity

$I_{DS} = W|Q_n(x)|\langle v(x)\rangle$

$V_S = V_D = 0$, but with $V_G > 0$ ➡ inversion layer charge is independent of x

$Q_n(V_{GS}) = -C_{ox}(V_{GS} - V_T)$

$C_{ox} = \frac{\kappa_{ox}\epsilon_0}{t_{ox}}$ F/m^2

Two regimes of operation

① Linear regime

V_{DS} is small.

$\mathcal{E} = -\frac{V_{DS}}{L}$

Linear region

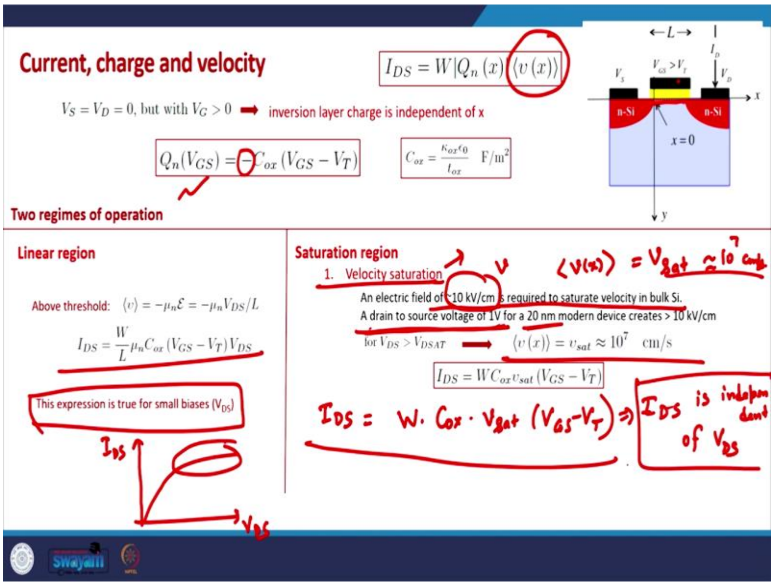
$I_{DS} \propto V_{DS}$

$I_{DS} \propto (V_{GS}-V_T)$

$I_{DS} = \mu_n C_{ox}(V_{GS}-V_T) V_{DS} \cdot \frac{W}{L}$

So, in the linear regime as you can see I_{DS} is directly proportional to V_{DS} and I_{DS} is directly proportional to V_{GS} - V_T ok. So, in the linear regime the current varies linearly with the drain voltage that is why this name is there that is why this regime is known as the linear regime ok.

(Refer Slide Time: 13:30)



Current, charge and velocity

$I_{DS} = W|Q_n(x)|\langle v(x)\rangle$

$V_S = V_D = 0$, but with $V_G > 0$ ➡ inversion layer charge is independent of x

$Q_n(V_{GS}) = -C_{ox}(V_{GS} - V_T)$

$C_{ox} = \frac{\kappa_{ox}\epsilon_0}{t_{ox}}$ F/m^2

Two regimes of operation

Linear region

Above threshold: $\langle v\rangle = -\mu_n\mathcal{E} = -\mu_n V_{DS}/L$

$I_{DS} = \frac{W}{L}\mu_n C_{ox}(V_{GS} - V_T)V_{DS}$

This expression is true for small biases (V_{DS})

Saturation region

1. Velocity saturation

An electric field of ~10 kV/cm is required to saturate velocity in bulk Si.

A drain to source voltage of 1V for a 20 nm modern device creates > 10 kV/cm

for $V_{DS} > V_{DSAT}$ ➡ $\langle v(x)\rangle = v_{sat} \approx 10^7$ cm/s

$I_{DS} = WC_{ox}v_{sat}(V_{GS} - V_T)$

$\langle v(x)\rangle = v_{sat} \approx 10^7$ cm/s

$I_{DS} = W \cdot C_{ox} \cdot v_{sat}(V_{GS} - V_T) \Rightarrow$ I_{DS} is independent of V_{DS}

So, that is true. So, this expression is only true for small biases when the V_{DS} is very small. In the saturation regime when V_{DS} is no longer small and we have seen while discussing the output characteristics I_{DS} versus V_{DS} that this regime is known as the saturation regime and we have also discussed that there could be two reasons for this saturation. One is in the small channel MOSFETs this might be because of the velocity saturation and in the long channel MOSFET this could be because of the pinch off in the channel.

Because in long channel MOSFET if we apply high drain voltage the difference between the gate terminal the voltage difference between the gate terminal and the drain side of the channel is small and the channel is pinched off from the drain side. And that is essentially responsible for the saturation in the current. So, let us consider the case of velocity saturation.

So, as we have seen that and this happens in small channel MOSFETs because when the channel length is very small the electric field becomes extremely high and the velocity actually saturates to a certain value. So, in one of our previous discussions we have seen that typically an electric field of around 10 kilo volt per centimetre is required to saturate the velocity in bulk silicon.

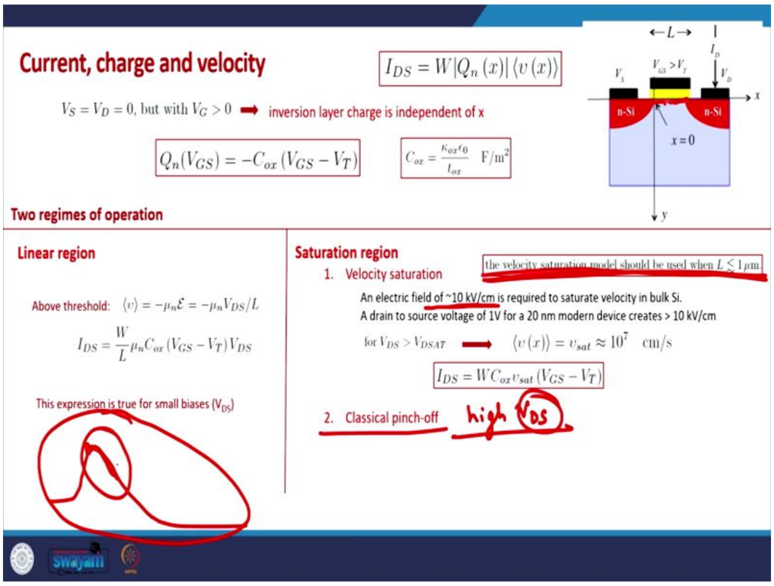
And if we have let us say a 20 nanometre MOSFET or even 50 nanometre MOSFET and if we apply a source voltage a drain to source voltage to be of 1 volt the electric field will be way higher than this value.

So, the velocity would possibly saturate given there is enough scattering in the channel ok. So, generally the velocity saturates to this value. So, the Vx this term becomes the V_{sat} which is typically 10^7 centimetre per second. So, in this case the current we just need to put this Q_n and this velocity.

So, that this I_{DS} essentially becomes W times C_{ox} times V_{sat} into (V_{GS}- V_T) ok. This velocity is generally taken to be negative because the current the direction of the current and the direction of electrons is in opposite direct direction.

So, that is why this negative sign goes away and as you can see that this I_{DS} is independent of from this I_{DS} is independent of V_{DS}. So, that is the case when the saturation is taking place because of the velocity saturation in small channel MOSFETs ok.

(Refer Slide Time: 17:04)



Current, charge and velocity

$$I_{DS} = W|Q_n(x)|\langle v(x)\rangle$$

$V_S = V_D = 0$, but with $V_G > 0$ ➡ inversion layer charge is independent of x

$$Q_n(V_{GS}) = -C_{ox}(V_{GS} - V_T)$$

$$C_{ox} = \frac{\kappa_{ox}\epsilon_0}{t_{ox}} \text{ F/m}^2$$

Two regimes of operation

Linear region

Above threshold: $\langle v\rangle = -\mu_n\mathcal{E} = -\mu_n V_{DS}/L$

$$I_{DS} = \frac{W}{L}\mu_n C_{ox}(V_{GS} - V_T)V_{DS}$$

This expression is true for small biases (V_{DS})

Saturation region

1. Velocity saturation

the velocity saturation model should be used when $L \lesssim 1\,\mu\text{m}$

An electric field of ~10 kV/cm is required to saturate velocity in bulk Si.
A drain to source voltage of 1V for a 20 nm modern device creates > 10 kV/cm

for $V_{DS} > V_{DSAT}$ ➡ $\langle v(x)\rangle = v_{sat} \approx 10^7$ cm/s

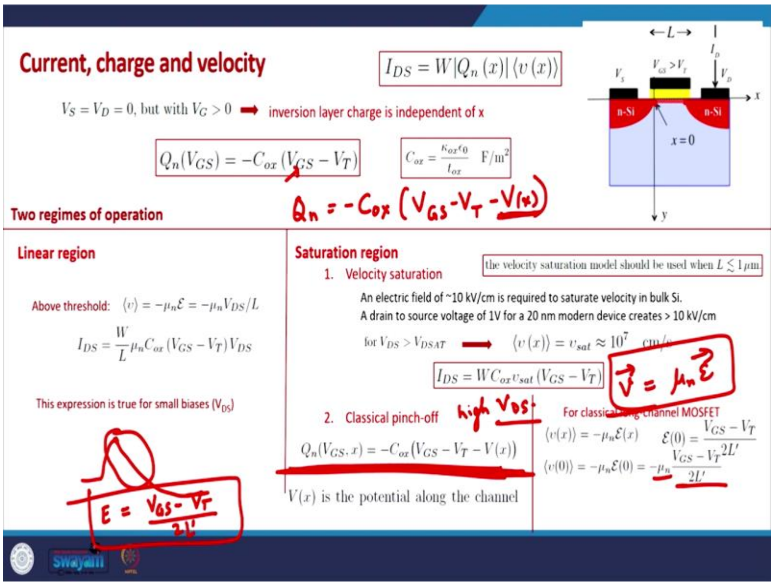
$$I_{DS} = WC_{ox}v_{sat}(V_{GS} - V_T)$$

2. Classical pinch-off high V_{DS}

Generally this small this velocity saturation model that is used when the channel length is smaller than the smaller than 1 micrometre because in that case the electric field becomes higher than this value ok. Now, let us consider the case of classical pinch off classical pinch off means that we have a long channel, but because of the high drain voltage the channel is getting pinched off from the drain side.

So, in this case so, this barrier shape will be and this happens at high V_{DS}. So, for higher V_{DS} as we have also discussed previously that the electric field will be non-linear in the channel specially in this part on one side the electric field will be extremely low especially on the top of the barrier. But it will very quickly become it will very quickly drop abruptly just to the right of the top of the barrier and it is a nonlinearly changing electric field.

(Refer Slide Time: 18:19)



Current, charge and velocity

$I_{DS} = W|Q_n(x)|\langle v(x)\rangle$

$V_S = V_D = 0$, but with $V_G > 0$ ➡ inversion layer charge is independent of x

$Q_n(V_{GS}) = -C_{ox}(V_{GS} - V_T)$

$C_{ox} = \frac{\kappa_{ox}\epsilon_0}{t_{ox}}$ F/m^2

$Q_n = -C_{ox}(V_{GS} - V_T - \underline{V(x)})$

Two regimes of operation

Linear region

Above threshold: $\langle v\rangle = -\mu_n\mathcal{E} = -\mu_n V_{DS}/L$

$I_{DS} = \frac{W}{L}\mu_n C_{ox}(V_{GS} - V_T)V_{DS}$

This expression is true for small biases (V_{DS})

$E = \frac{V_{GS} - V_T}{2L'}$

Saturation region

1. Velocity saturation

the velocity saturation model should be used when $L \lesssim 1\,\mu m$

An electric field of ~10 kV/cm is required to saturate velocity in bulk Si.
A drain to source voltage of 1V for a 20 nm modern device creates > 10 kV/cm

for $V_{DS} > V_{DSAT}$ ➡ $\langle v(x)\rangle = v_{sat} \approx 10^7$ cm/s

$I_{DS} = WC_{ox}v_{sat}(V_{GS} - V_T)$

$\vec{v} = \mu_n\vec{\mathcal{E}}$

2. Classical pinch-off high V_{DS}

$Q_n(V_{GS}, x) = -C_{ox}(V_{GS} - V_T - V(x))$

$V(x)$ is the potential along the channel

For classical long channel MOSFET

$\langle v(x)\rangle = -\mu_n\mathcal{E}(x)$ $\quad \mathcal{E}(0) = \frac{V_{GS} - V_T}{2L'}$

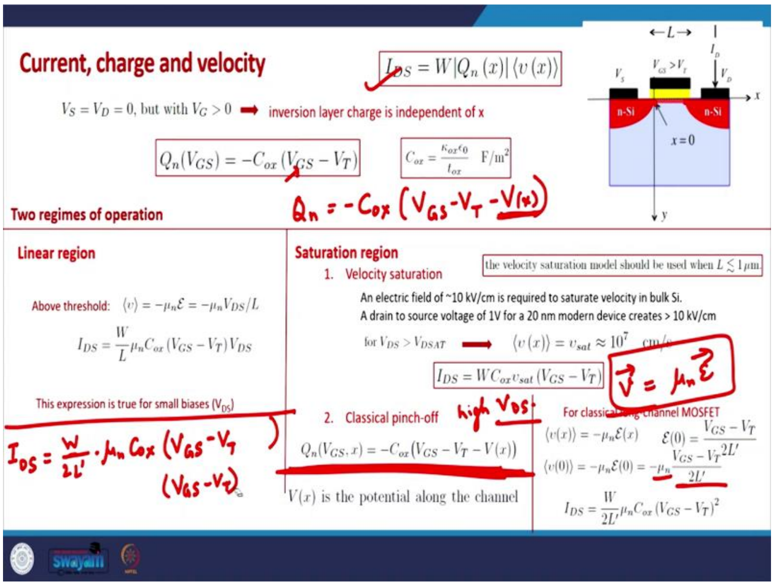
$\langle v(0)\rangle = -\mu_n\mathcal{E}(0) = -\mu_n\frac{V_{GS} - V_T}{2L'}$

So, in this case the charge we need to take. So, in addition to the effect of the gate voltage on the channel we also need to consider the effect of the drain voltage on the channel because now high V_{DS} is there. So, this channel charge will be -C_{ox} (V_{GS}- V_T- V(x)) where this V(x) is the voltage due to the drain voltage ok and this will change across the channel on the source side this will be 0 on the drain side this will be V_{DD} and in between it will be an arbitrary value V(x).

So, this will be the charge and what would be the velocity? The velocity will be from this the velocity will be derived in long channel MOSFET using this the electric field because now the barrier is because the electric field in the barrier is non-linear the electric. So, on one side the voltage is 0, on other side the voltage is V_{DD} and in between the voltage is changing non-linearly. So, this electric field is actually written as $(V_{GS}-V_T)/2L'$ this is after considering all kind of nonlinearities in the channel ok.

We will see where this come comes from as well, but just assume that just in the beginning right at the beginning of the channel this is the electric field. So, in this case the velocity can be written as minus mu n times this electric field the charge is this.

(Refer Slide Time: 20:44)



Current, charge and velocity

$\boxed{I_{DS} = W|Q_n(x)|\langle v(x)\rangle}$ ✓

$V_S = V_D = 0$, but with $V_G > 0$ ➡ inversion layer charge is independent of x

$\boxed{Q_n(V_{GS}) = -C_{ox}(V_{GS} - V_T)}$ $\boxed{C_{ox} = \frac{\kappa_{ox}\epsilon_0}{t_{ox}} \text{ F/m}^2}$

$Q_n = -C_{ox}(V_{GS} - V_T - \underline{V(x)})$

Two regimes of operation

Linear region

Above threshold: $\langle v\rangle = -\mu_n\mathcal{E} = -\mu_n V_{DS}/L$

$$I_{DS} = \frac{W}{L}\mu_n C_{ox}(V_{GS} - V_T)V_{DS}$$

This expression is true for small biases (V_{DS})

$$I_{DS} = \frac{W}{2L'}\cdot\mu_n C_{ox}(V_{GS} - V_T)(V_{GS} - V_T)$$

Saturation region

1. Velocity saturation

$\boxed{\text{the velocity saturation model should be used when } L \lesssim 1\,\mu\text{m}}$

An electric field of ~10 kV/cm is required to saturate velocity in bulk Si.

A drain to source voltage of 1V for a 20 nm modern device creates > 10 kV/cm

for $V_{DS} > V_{DSAT}$ ➡ $\langle v(x)\rangle = v_{sat} \approx 10^7$ cm/s

$\boxed{I_{DS} = WC_{ox}v_{sat}(V_{GS} - V_T)}$ $\boxed{\vec{v} = \mu_n\vec{\mathcal{E}}}$

2. Classical pinch-off high V_{DS}

$$\underline{Q_n(V_{GS}, x) = -C_{ox}(V_{GS} - V_T - V(x))}$$

$V(x)$ is the potential along the channel

For classical long-channel MOSFET

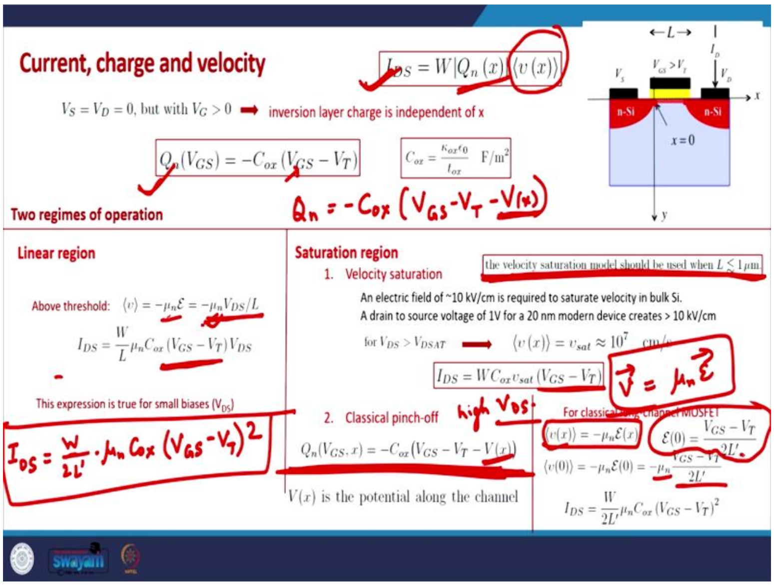
$\langle v(x)\rangle = -\mu_n\mathcal{E}(x)$ $\mathcal{E}(0) = \frac{V_{GS} - V_T}{2L'}$

$\langle v(0)\rangle = -\mu_n\mathcal{E}(0) = \underline{-\mu_n}\frac{V_{GS} - V_T}{\underline{2L'}}$

$$I_{DS} = \frac{W}{2L'}\mu_n C_{ox}(V_{GS} - V_T)^2$$

So, the current from this expression from the general expression of the current from this expression the current in the saturation regime can be written as I_{DS} is equal to $(W/2L')$ prime now times μ_n C_{ox} $(V_{GS}- V_T-V(x))$ $(V_{GS}- V_T)$ ok. And since we are considering the velocity just at the beginning of the channel so, this V x will be 0 at that point. So, this can be removed from here. And finally, $V_{GS}- V_T$ times $V_{GS}- V_T$ becomes $(V_{GS}- V_T)^2$.

(Refer Slide Time: 21:37)



Current, charge and velocity

$I_{DS} = W|Q_n(x)|\langle v(x)\rangle$

$V_S = V_D = 0$, but with $V_G > 0$ ➡ inversion layer charge is independent of x

$Q_n(V_{GS}) = -C_{ox}(V_{GS} - V_T)$

$C_{ox} = \frac{\kappa_{ox}\epsilon_0}{t_{ox}}$ F/m^2

$Q_n = -C_{ox}(V_{GS} - V_T - V(x))$

Two regimes of operation

Linear region

Above threshold: $\langle v\rangle = -\mu_n\mathcal{E} = -\mu_n V_{DS}/L$

$I_{DS} = \frac{W}{L}\mu_n C_{ox}(V_{GS} - V_T)V_{DS}$

This expression is true for small biases (V_{DS})

$I_{DS} = \frac{W}{2L'} \cdot \mu_n C_{ox}(V_{GS} - V_T)^2$

Saturation region

1. Velocity saturation

the velocity saturation model should be used when $L \lesssim 1\,\mu m$

An electric field of ~10 kV/cm is required to saturate velocity in bulk Si.
A drain to source voltage of 1V for a 20 nm modern device creates > 10 kV/cm

for $V_{DS} > V_{DSAT}$ ➡ $\langle v(x)\rangle = v_{sat} \approx 10^7$ cm/s

$I_{DS} = W C_{ox} v_{sat}(V_{GS} - V_T)$

$\vec{v} = \mu_n \vec{\mathcal{E}}$

2. Classical pinch-off

high V_{DS}

$Q_n(V_{GS}, x) = -C_{ox}(V_{GS} - V_T - V(x))$

$V(x)$ is the potential along the channel

For classical long channel MOSFET

$\langle v(x)\rangle = -\mu_n\mathcal{E}(x)$

$\mathcal{E}(0) = \frac{V_{GS} - V_T}{2L'}$

$\langle v(0)\rangle = -\mu_n\mathcal{E}(0) = -\mu_n\frac{V_{GS} - V_T}{2L'}$

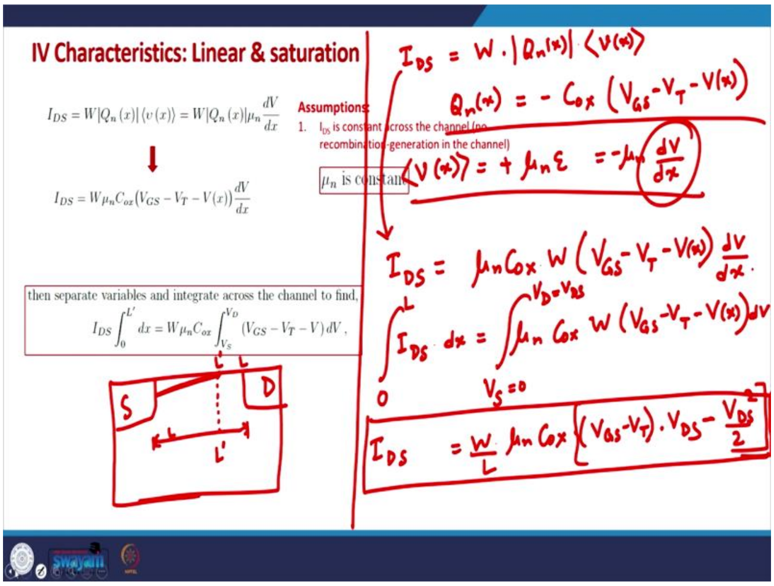
$I_{DS} = \frac{W}{2L'}\mu_n C_{ox}(V_{GS} - V_T)^2$

So, what we will eventually have is $(V_{GS} - V_T)^2$. So, this will be the current in the saturation regime because of the classical pinch off. We have not shown where this comes from, but this is just to show you various extremes of IV characteristics in linear regime electric field is constant because the voltage drop is assumed to be linear, the charge is just because of the gate voltage.

So, the current can be easily deduced using these two expressions; these three expressions from this general expression, this current expression and the velocity expression.

In the saturation regime when the velocity is getting saturated because of the when this the current is getting saturated because of the velocity saturation which is the case for small channels in that case this <v(x)> is taken to be the saturated velocity and Q_n is taken to be charge because of the gate voltage. In the classical pinch off in addition to the effect of gate voltage on the charge we also need to take the effect of the drain voltage and the velocity will be calculated from the electric field, electric field is given by this value and from here we derive the current.

(Refer Slide Time: 23:10)



IV Characteristics: Linear & saturation

$$I_{DS} = W|Q_n(x)|\langle v(x)\rangle = W|Q_n(x)|\mu_n \frac{dV}{dx}$$

$$\downarrow$$

$$I_{DS} = W\mu_n C_{ox}(V_{GS} - V_T - V(x))\frac{dV}{dx}$$

Assumptions:

1. I_{DS} is constant across the channel (no recombination-generation in the channel)

μ_n is constant

then separate variables and integrate across the channel to find,

$$I_{DS}\int_0^{L'} dx = W\mu_n C_{ox}\int_{V_S}^{V_D}(V_{GS} - V_T - V)\,dV\,,$$

$$I_{DS} = W \cdot |Q_n(x)| \cdot \langle v(x) \rangle$$

$$Q_n(x) = -C_{ox}(V_{GS} - V_T - V(x))$$

$$\langle v(x) \rangle = +\mu_n \mathcal{E} = -\mu_n \frac{dV}{dx}$$

$$I_{DS} = \mu_n C_{ox} W (V_{GS} - V_T - V(x)) \frac{dV}{dx}.$$

$$\int_0^L I_{DS} \cdot dx = \int_{V_S=0}^{V_D=V_{DS}} \mu_n C_{ox} W (V_{GS} - V_T - V(x))dV$$

$$\boxed{I_{DS} = \frac{W}{L} \mu_n C_{ox} \left\{(V_{GS} - V_T) \cdot V_{DS} - \frac{V_{DS}^2}{2}\right\}}$$

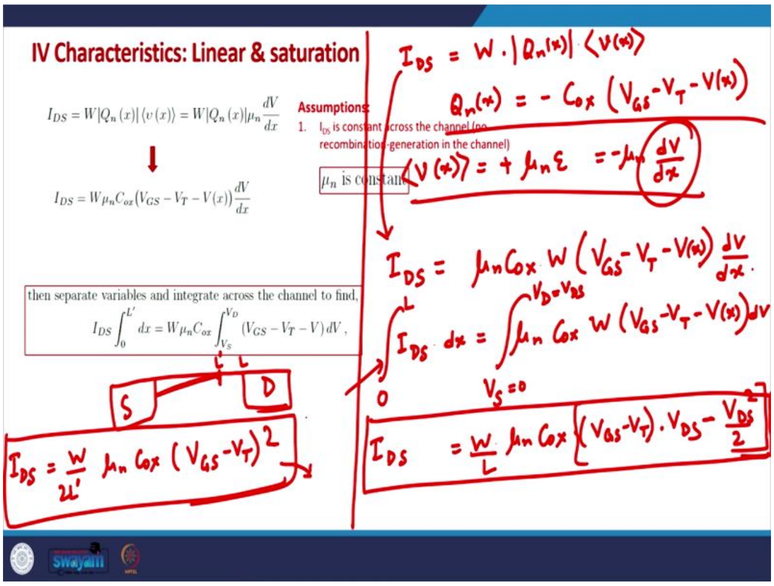
In a more general way this current is given as W times $Q_n(x)$. So, this is the general treatment of the IV characteristics times the velocity ok. So, this in a very general sense this Q_n can be written as minus C_{ox} into (V_{GS} - V_T – V(x)) where V(x) is the voltage because of the drain voltage now. And this V(x) can be written as μ_n times electric field and electric field is –δV/ δx this is the gradient of the voltage in the channel.

So, this general expression of the current can be written as by multiplying these two equations $\mu_n C_{ox}$ W (V_{GS} - V_T – V(x)) δV/ δx ok. So, this dx can be brought to the left side I_{DS} times dx is equal to $\mu_n C_{ox}$ W (V_{GS} - V_T – V(x)) dV ok. So, if we integrate this equation across the channel. So, from 0 to L on the right side the voltage will vary from V_S to V_D. So, this will give us I_{DS} is times L is equal to W times $\mu_n C_{ox}$ (W/L) (V_{GS} - V_T).V_{DS} – V^2_{DS}/2.

So, this voltage will also be from V_S to V_D, V_S is equal to 0 V_D is equal to V DS. So, this is the general expression of the current this L can be taken to the right hand side and in that case it will become W by L. So, this is classically traditionally the way IV characteristics of the MOSFET is derived. There SFET we have a source we have a drain and because of the drain voltage the channel might be pinched off.

So, the channel will terminate at a length let us say L' the total length is L let us say this is L, but the channel will terminate at L' because of the this is L, this is L'. The channel will terminate at L’ because of the drain voltage ok. So, in this case the voltage here is (V_{GS} - V_T) the drain voltage here is (V_{GS} - V_T).

(Refer Slide Time: 27:23)



IV Characteristics: Linear & saturation

$$I_{DS} = W|Q_n(x)|\langle v(x)\rangle = W|Q_n(x)|\mu_n \frac{dV}{dx}$$

$$I_{DS} = W\mu_n C_{ox}(V_{GS} - V_T - V(x))\frac{dV}{dx}$$

Assumptions:

1. I_{DS} is constant across the channel (no recombination-generation in the channel)

μ_n is constant

then separate variables and integrate across the channel to find,

$$I_{DS}\int_0^{L'} dx = W\mu_n C_{ox}\int_{V_S}^{V_D}(V_{GS} - V_T - V)\,dV ,$$

$$I_{DS} = W \cdot |Q_n(x)| \langle v(x)\rangle$$

$$Q_n(x) = -C_{ox}(V_{GS} - V_T - V(x))$$

$$\langle v(x)\rangle = +\mu_n \mathcal{E} = -\mu_n \frac{dV}{dx}$$

$$I_{DS} = \mu_n C_{ox} W (V_{GS} - V_T - V(x))\frac{dV}{dx}.$$

$$\int_0^{L} I_{DS}\,dx = \int_{V_S=0}^{V_D=V_{DS}} \mu_n C_{ox} W (V_{GS} - V_T - V(x))\,dV$$

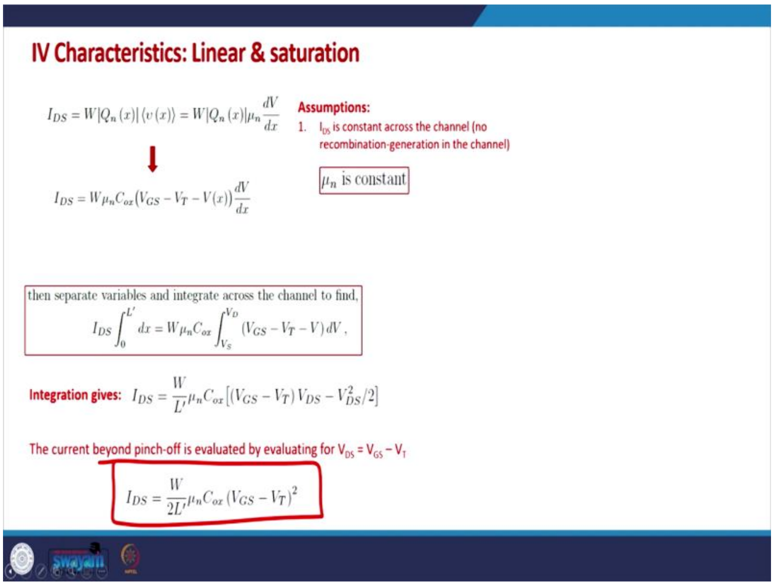
$$\boxed{I_{DS} = \frac{W}{L}\mu_n C_{ox}\left[(V_{GS} - V_T)\cdot V_{DS} - \frac{V_{DS}^2}{2}\right]}$$

$$\boxed{I_{DS} = \frac{W}{2L'}\mu_n C_{ox}(V_{GS} - V_T)^2}$$

And if we integrate this equation from 0 to L’ only this will be I_{DS} times I_{DS} will be equal to W/L’. On the right hand side the limits will be from 0 to V_{DS} sorry (V_{GS} - V_T). So, it will be $\mu_n C_{ox}$ V_{DS} needs to be put as (V_{GS} - V_T). So, it will be $(V_{GS} - V_T)^2/2$ and 2 can come here.

So, this shows the saturation of the current in the channel that the current has almost the current has become independent of the drain voltage in that channel ok. So, this is classically the way the current is typically derived as is also seen here.

(Refer Slide Time: 28:16)



IV Characteristics: Linear & saturation

$$I_{DS} = W|Q_n(x)|\langle v(x)\rangle = W|Q_n(x)|\mu_n \frac{dV}{dx}$$

$$\downarrow$$

$$I_{DS} = W\mu_n C_{ox}(V_{GS} - V_T - V(x))\frac{dV}{dx}$$

Assumptions:

1. I_{DS} is constant across the channel (no recombination-generation in the channel)

$\boxed{\mu_n \text{ is constant}}$

then separate variables and integrate across the channel to find,

$$I_{DS}\int_0^{L'} dx = W\mu_n C_{ox}\int_{V_S}^{V_D}(V_{GS} - V_T - V)\,dV\,,$$

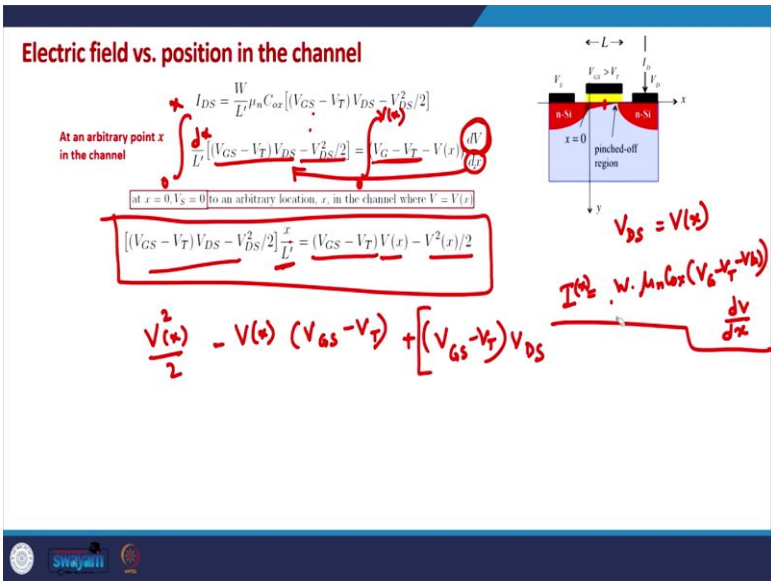
Integration gives: $I_{DS} = \frac{W}{L'}\mu_n C_{ox}\left[(V_{GS} - V_T)V_{DS} - V_{DS}^2/2\right]$

The current beyond pinch-off is evaluated by evaluating for $V_{DS} = V_{GS} - V_T$

$$\boxed{I_{DS} = \frac{W}{2L'}\mu_n C_{ox}(V_{GS} - V_T)^2}$$

In the previous slide we considered the extreme case we derived the current when we take the linear regime just on the saturation regime.

(Refer Slide Time: 28:44)



Electric field vs. position in the channel

$$I_{DS} = \frac{W}{L'}\mu_n C_{ox}\left[(V_{GS} - V_T)V_{DS} - V_{DS}^2/2\right]$$

At an arbitrary point x in the channel

$$\int_0^{x}\frac{dx}{L'}\left[(V_{GS} - V_T)V_{DS} - V_{DS}^2/2\right] = \int_0^{V(x)}\left(V_G - V_T - V(x)\right)dV$$

at $x = 0, V_S = 0$ to an arbitrary location, x, in the channel where $V = V(x)$

$$\boxed{\left[(V_{GS} - V_T)V_{DS} - V_{DS}^2/2\right]\frac{x}{L'} = (V_{GS} - V_T)V(x) - V^2(x)/2}$$

$$V_{DS} = V(x)$$

$$I(x) = W\cdot\mu_n C_{ox}(V_G - V_T - V(x))\frac{dV}{dx}$$

$$\frac{V^2(x)}{2} - V(x)(V_{GS} - V_T) + \left[(V_{GS} - V_T)V_{DS}\right.$$

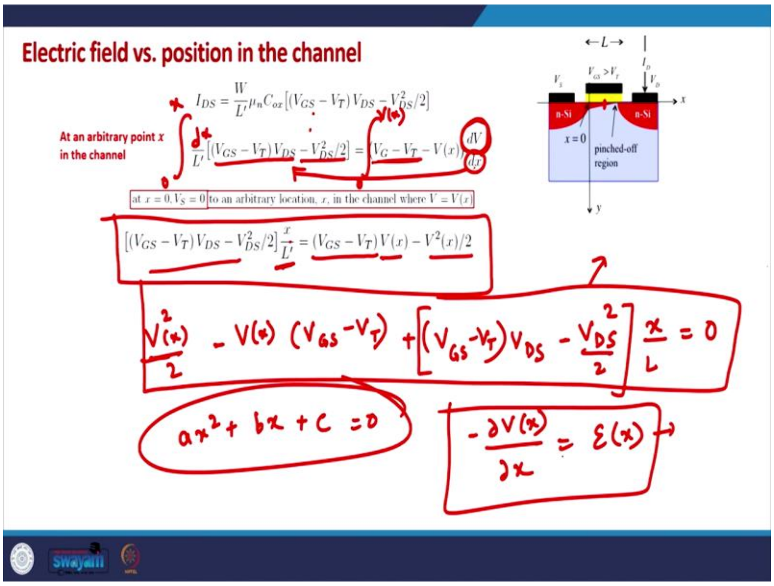
Now, there is another thing important analysis that we need to do that generally is done that is the electric field versus position in the channel, electric field as a function of the position in the channel. So, generally we would like to know the electric field profile in the saturation regime because it is highly non-linear profile in linear regime as we have discussed typically it is a linear one, but quantitatively or precisely how does it look like that is what we will see now. And we will take help from the current expression. So, this is the general form of the current expression.

I_{DS} is equal to $\mu_n C_{ox}$ (W/L) (V_{GS} - V_T).V_{DS} – $V^2_{DS}/2$. At an arbitrary point in the channel, so this current is always constant uniform through the channel in steady state. So, this is the current that we derived previously, but at any arbitrary point in the channel let us say at x, V_{DS} will be V(x) and the current at any arbitrary point will be W times $\mu_n C_{ox}$(V_{GS} - V_T – V(x)) dV/dx.

So, this will be the current at any arbitrary point in the channel. So, if we equate this current with this arbitrary current ideally they should be the same because in steady state the current is uniform, the current is the same all across the channel. So, by equating this expression to this expression we obtain this and in by having a change of variables on both sides if we bring dx on the left side. So, on the left side it becomes dx by L’ this will.

So, if this is brought to the left side it will be dx by L’ and if we integrate this thing now from 0 to x at any point x. So, on the left side what will be there is (V_{GS} - V_T).V_{DS} – $V^2_{DS}/2$ times x/L’. On the right side we will have (V_{GS} - V_T) the integration is with respect to voltage (V_{GS} - V_T) it will be from 0 to (V_{GS} - V_T) times V(x) - $V(x)^2/2$. So, now, this becomes a quadratic equation in the voltage across the channel.

(Refer Slide Time: 32:11)



Electric field vs. position in the channel

$$I_{DS} = \frac{W}{L'}\mu_n C_{ox}\left[(V_{GS}-V_T)V_{DS} - V_{DS}^2/2\right]$$

At an arbitrary point x in the channel

$$\int_0^x \frac{dx}{L'}\left[(V_{GS}-V_T)V_{DS} - V_{DS}^2/2\right] = \int_0^{V(x)} (V_G - V_T - V(x))\frac{dV}{dx}$$

at $x = 0, V_S = 0$ to an arbitrary location, x, in the channel where $V = V(x)$

$$\left[(V_{GS}-V_T)V_{DS} - V_{DS}^2/2\right]\frac{x}{L'} = (V_{GS}-V_T)V(x) - V^2(x)/2$$

$$\frac{V^2(x)}{2} - V(x)(V_{GS}-V_T) + \left[(V_{GS}-V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]\frac{x}{L} = 0$$

$$ax^2 + bx + c = 0$$

$$-\frac{\partial V(x)}{\partial x} = \mathcal{E}(x)$$

So, it is a quadratic equation $V(x)^2/2 – V(x) (V_{GS} - V_T) + [(V_{GS} - V_T).V_{DS} – V^2_{DS}/2](x/L) = 0$. So, this becomes an equation like this $ax^2 + bx + c = 0$. And as all of us know from the basic maths we can solve this quadratic equation and that says that is what your homework assignment is now. So, you need to solve this quadratic equation as a function of V(x) and then $–\delta V/ \delta x$ will give us the electric field at any arbitrary point in the channel.

So, that is how this electric field is calculated at any arbitrary point in the channel in the traditional MOSFET theory. So, please go through this and try solving this quadratic equation in voltage and try to see how this E(x) looks like calculation might become slightly tedious, but it is not a it is a doable calculation. So, just give it an attempt ok. So, we will discuss this point onwards in the next class. I thank you for your attention. See you in the next class.