Physics of Nanoscale Devices
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Lecture - 34
1D and 2D Realistic Conductors

Hello everyone. In the last class, we started seeing how practical 2D and 1D channels look like, how do we sort of need to analyze those things in those channels in more with more nuances and. So, today we will complete this part and we will begin with the introduction of MOSFETs ok.

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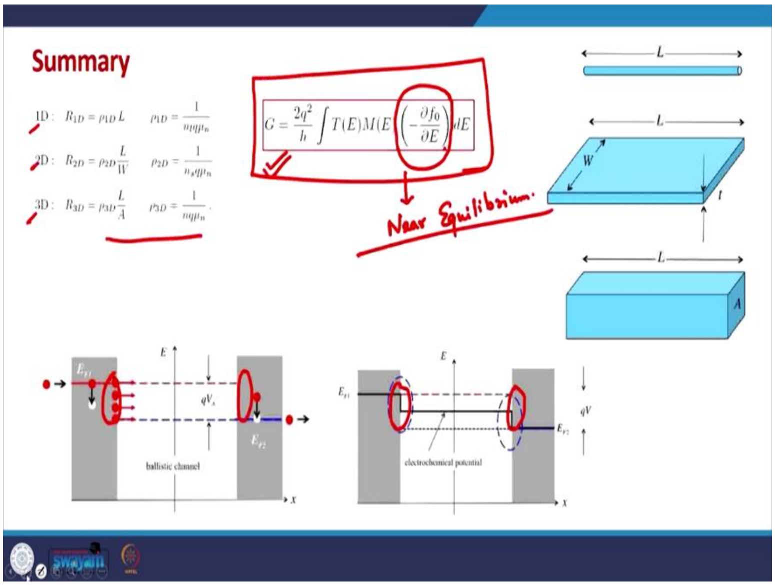

So, let me quickly review what we have been doing. In this in last few classes, we have been discussing how the notion of resistance and conductance is different in classical understanding in the conventional understanding, and how it is different in the general model of transport both for 1D, 2D and sorry for all 1D, 2D and 3D conductors.

Apart from this we saw that in a ballistic conductor the power dissipation takes place at the contacts half of the power is dissipated at the left contact and half is dissipated on the right contact. Similarly the voltage drop also takes place on the contacts. And this is actually quite opposite to what happens in the conventional macroscopic devices ok.

So, these are some new things. So, ultimately the equation to begin with or the central equation to calculate the conductance is this. And please keep in mind that this is true for near equilibrium transport; near equilibrium means that we are not applying high voltages because in that case $(f_1 - f_2)$ cannot be approximated just by the first order of the Taylor series ok.

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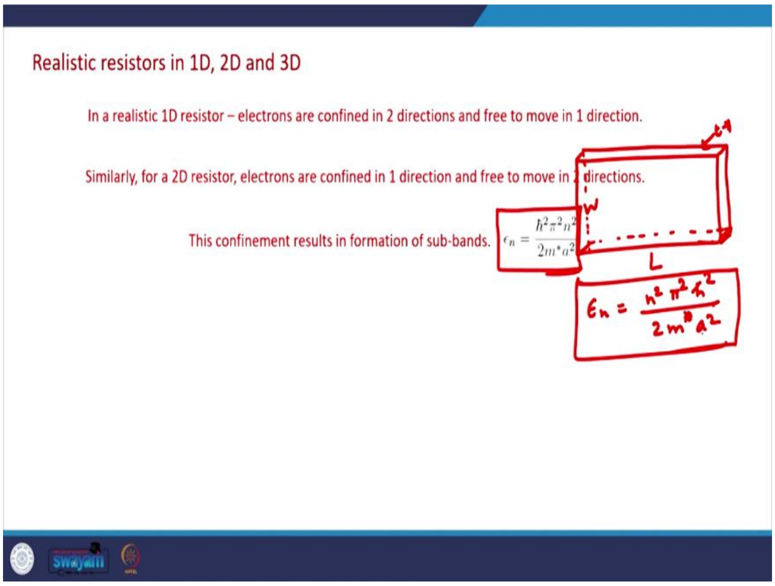

So, with this background we started discussing how 1D and 2D resistors, practical 1D and 2D resistors are different from the ideal cases. In a realistic 2D conductor apart from its length and width, we also have a finite thickness and that we need to actually take into account while doing the calculations for the current in a channel that is 2D channel ok.

So, there is a finite thickness. So, the electrons will be confined in this direction because this is of the atomic scale and because of the quantum mechanical confinement electrons energy will be discrete. So, in the confined direction generally the energy of the electrons is given by this number. So, the ε_n energy is $\frac{n^2\pi^2\hbar^2}{2m^* a^2}$ where a is the extent to which electrons are confined the carriers are confined. So, in this case instead of a, we might have t because t is the thickness in which electrons are confined.

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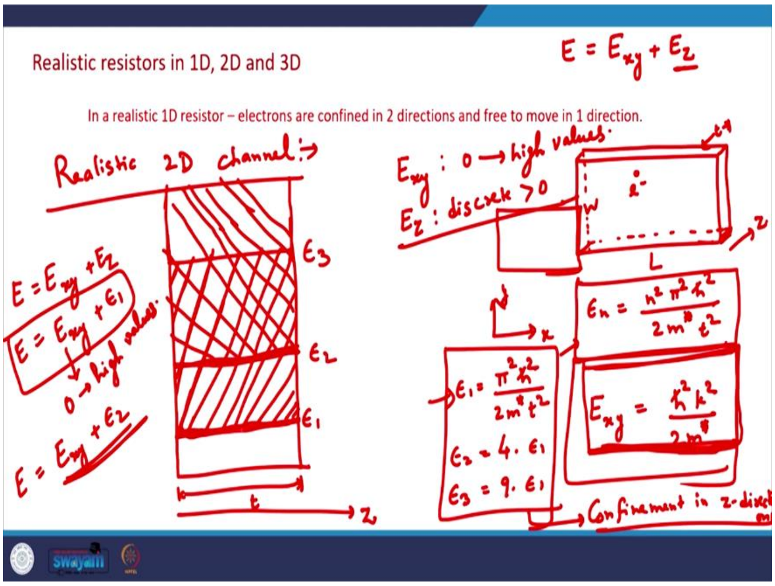

So, and in 2 directions, if we just consider 2 directions if we just if we consider ideal 2D conductor, the energy of the electrons can be in 2-dimension the energy can be $\frac{k^2\hbar^2}{2m^*}$ ok. So, let us consider the case of a practical or a realistic 2D channel. In a realistic 2D channel the electrons are confined again the electrons are confined in z direction, if this is the z direction electrons are confined in this direction and electrons are free to move in x and y direction.

Now, if we just see the confinement in the z direction, this is the thickness t and because of this confinement in this dimension t, the energies will be discrete the energy of the electrons will be discrete and this energy will be given by these numbers. And if we start this calculation, the energy of the first level let us say ε_1 of the second level call it ε_2, for third level let us call it ε_3. So, ε_1 is essentially $\frac{\pi^2\hbar^2}{2m^*t^2}$. Similarly ε_2 is 4 times ε_1 and ε_3 is 9 times ε_1 ok.

And please keep in mind that these are the energies because of the confinement in z-direction, ok. In x y direction there is no confinement electrons are free to move or we can say that relatively the length and width of the semiconductor are large. So, electrons energy is not properly discrete it may have bands and band gaps because of x y direction.

And there will be from the E-k diagram we or from the E-k relationship we might have this parabolic relationship specially near the bandages. So, now, any electron that exists inside this device, any electron this energy of this electron will be the summation of the two components the energy because of the motion in x y direction and energy because of the motion in z direction where the motion is confined ok.

So, now there is an interesting fact here. In x y direction the energy can be as low as 0 actually. So, energy can go to very low levels ok. So, this E xy typically it can be from 0 to any high values, but Ez the energy in the confined direction this is the discrete values and this is always greater than 0.

So, it is never 0 even the lowest possible state has a non-zero energy which is given by this value ok. So, if let us say if the electron has energy epsilon 1 in z direction or electron is in the lowest possible state in the z direction then because of the energy in x y direction electrons energy, the total energy of the electrons which is essentially the sum of Exy + Ez.

And here if we put Ez to be ε_1; so, it would be Exy + ε_1. So, for these electrons the energy can be anywhere from epsilon because this Exy can go from 0 to high values very high values as well or a certain value let us say. So, the electrons may have energies from. So, the electrons that have energy ε_1 in z direction may have total energy anywhere from ε_1 to a high value alright.

So, these lines essentially show the that the electrons can have any energy from epsilon 1 to high values and, but the electrons let us say the electron if the electron is sitting in ε_2 state or in z direction the electronic energy is ε_2. In that case the total energy of the electron will be Exy + ε_2 and this value can go from ε_2 to a very high value.

So, the electrons sitting in this state in z direction may have any energy starting from ε_2 to a very high value. So, this may be the energy of electrons in the total energy of the electrons, those electrons who are sitting in the ε_2 state. So, now, an interesting situation arises the electrons that are in ε_1 state may have energies here in this regime.

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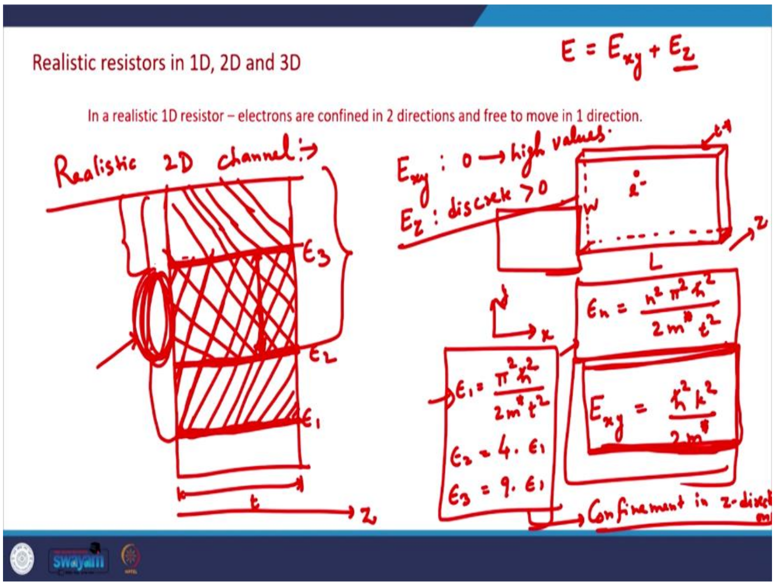

So, in this regime also the electrons with energy or electrons sitting in state ε_1 can have energies in this regime. Similarly, the electrons sitting in state ε_2 can also have energies in this regime. Electrons sitting in state ε_3 can have any energy above ε_3. So, they can have any energy here or electron sitting in ε_1 can have any energy total energy to be this, ok.

So, now this overlap here in this range, in this range electrons can exist in two configurations in this range. They can be sitting in ε_1 state and have energy sufficient to be sufficient so that their total energy is in this range or they can be sitting in ε_2 state and have the energy equal to the difference between ε_2 and energy in this range.

So, these 2 are different configurations and these 2 are allowed configurations for electrons; and this leads to a sort of degeneracy in the systems ok because now there are 2 type of electrons that can take this energy in this range energy in this range.

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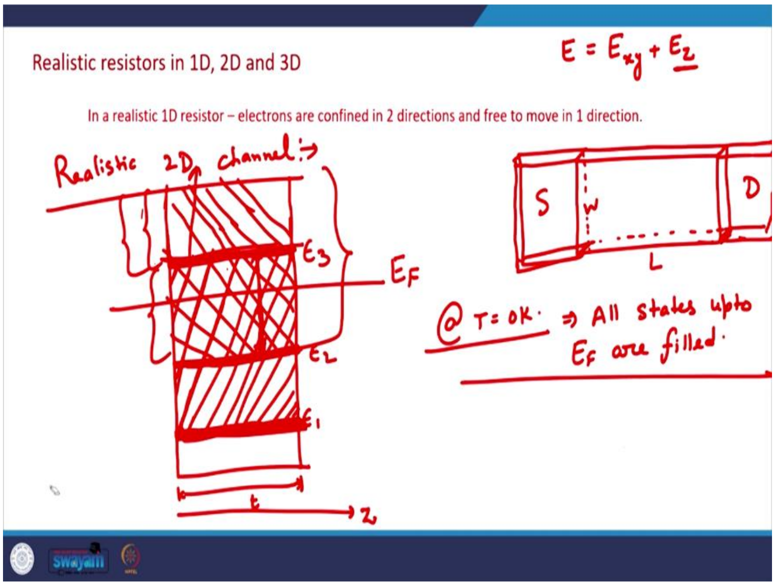

And these are known as the sub bands, overlapping sub bands. Similarly in this range, electrons sitting in ε_1 can also have energy, electrons sitting in ε_2 can also have energy and electrons sitting in ε_3 can also have energy in this range. And here the degeneracy or there might be three configurations of electrons above ε_3.

So, in other words what we can say is that these kind of solids have sub bands starting from the discrete energy levels because of the confinement. So, the discrete energy levels are because of the confinement in one direction that gives ε_1, ε_2, ε_3 in the direction of confinement.

Now, because of the free motion in other two directions electrons energy can be from 0 to any value; and in that case the electrons might have an energy configuration in which their energy can be ε_1 in z direction and Exy in or any arbitrary energy in x y direction. So, electrons might have a sub band at ε_1, another sub band at ε_2, another sub band at ε_3.

And some of these sub bands may be overlapping with each other if these energies are not sort of far apart from each other. Now, something interesting happens in this case. In this case let us say we have this is the channel this is a real 2D device as has been shown here.

This we are making a two terminal device with this 2D conductor this realistic 2D conductor. So, what it means is that we are putting contacts on 2 sides of it and please always keep in mind that these are bulk contacts. So, we are putting contacts on this device. This is the source contact, this is the drain contact.

Now, the Fermi level of the system is let us say here somewhere here E_F is this point ok. Now, if we need to calculate the total number of electrons in this system in this channel at T equal to 0 kelvin, let us say which means that all the states all states up to the Fermi level are filled ok. So, all the states up to this Fermi level are filled which means that, and how many states can be there up to the Fermi level?

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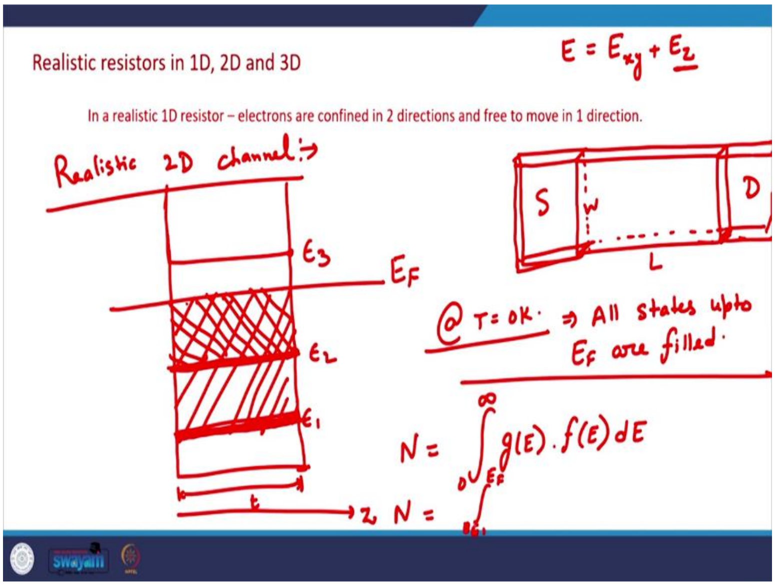

So, let us say this was our original potential box this is epsilon 3, this is epsilon 2 and all the states below the Fermi level are now filled. There cannot be any allowed electronic state below ε_1 that is disallowed by the quantum mechanical nature of the electrons.

So, it means that in this particular case there are 2 states 2 discrete states below E_F. So, there can be 2 sub bands in the system and the number of electrons in the system will be because of this sub band and because of this sub band, which means that there are 2 configurations of electrons possible in this system.

One is those electrons that have energy epsilon 1 in z direction and any energy from 0 to E_F - ε_1 in x y direction. Similarly, the second configuration can be that the electrons might have energy epsilon 2 in z direction and any energy from 0 to E_F - ε_2 in x y direction.

So, that is why we need to calculate generally the formula to calculate the number of electrons is the density of states times the Fermi function dE. Now, in order to account for both the sub bands and this is from 0 to infinity for all possible energy states. Now, we need to do this from 0 to or ε_1 to E_F because of the first sub band.

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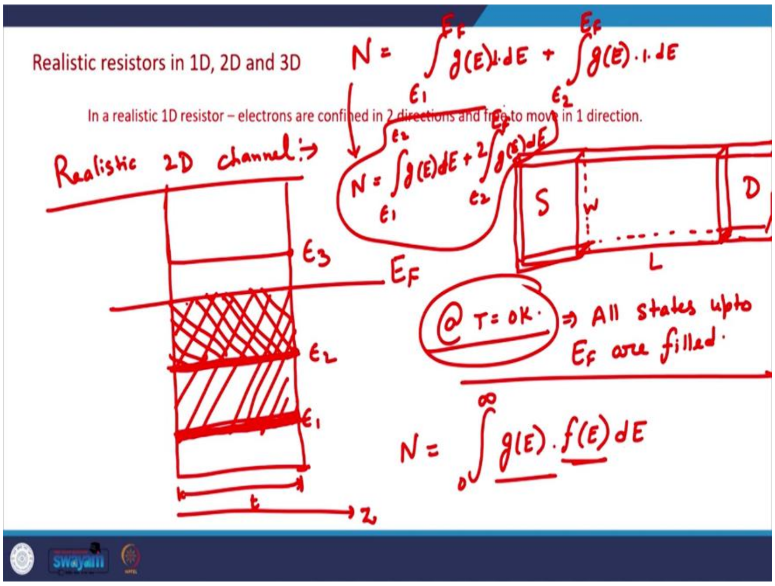

Or let me write it in a more cleaner way. The number of electrons in the system will be ε_1 to E_F because of the first sub band and g(E) is the density of states. So, g(E) is the density of states; f(E) the Fermi function below Fermi level will be one at T equal to 0 kelvin. We are assuming the case to be T equal to 0 kelvin. So, it will be 1 times dE plus yeah.

Because of the second sub band from ε_2 to E_F g(E) times 1 times dE ok. So, in a way this will be $N = \int_{\varepsilon_1}^{\varepsilon_2} g(E)dE + 2\int_{\varepsilon_1}^{E_F} g(E).1.dE$. So, this could be the total number of electrons in the system. So, this we need to keep in mind actually while doing calculations for the realistic 1D and 2D conductors that now the sub bands are formed because of the confinement in the smaller atomic dimension and those sub bands may result in degeneracy in the system and that we need to keep in mind and that we need to take in account in all our calculations ok.

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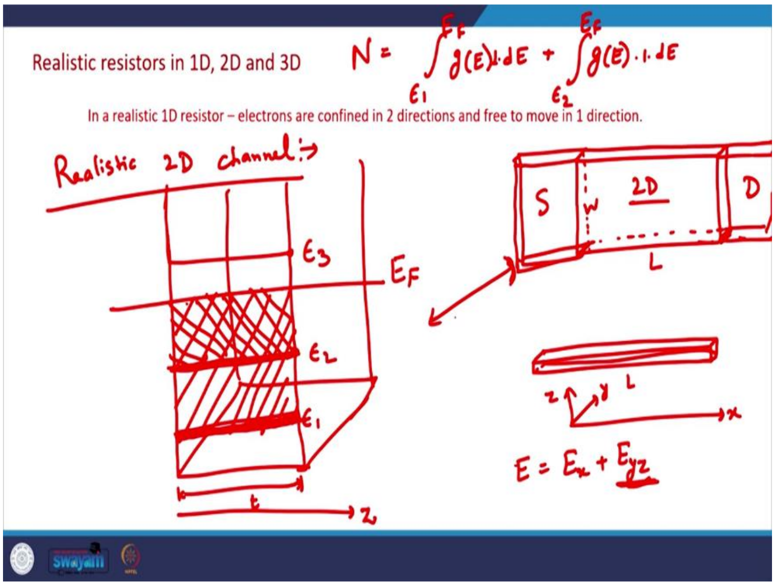

So, similarly this was the case of a practical 2D conductor, a real 2D conductor. Similar analysis will hold for a 1D conductor as well. In 1D conductor we will have confinement in 2 directions. So, the potential well will be in 2 directions and electrons are free to move in 1 direction. So, that the total energy. So, in a real 1D conductor let us say this is x direction, this is y direction, this is z direction.

So, the electrons are confined in y and z direction and the total energy is Ex + Eyz. And, now this Eyz is now a discrete value will be a discrete value and that is what essentially we need to take into account now ok like we did for. So, there will be sub bands and now the sub bands will be even more because this potential well now be in 2 directions and we need to solve for the potential for electron confined in 2 directions.

So, accordingly we will obtain the discrete energy states. And on based on those discrete energy states the sub bands will be there and all those sub bands need to be taken into account while doing calculations ok. This analysis is pretty much similar to the analysis of the 2D channel ok.

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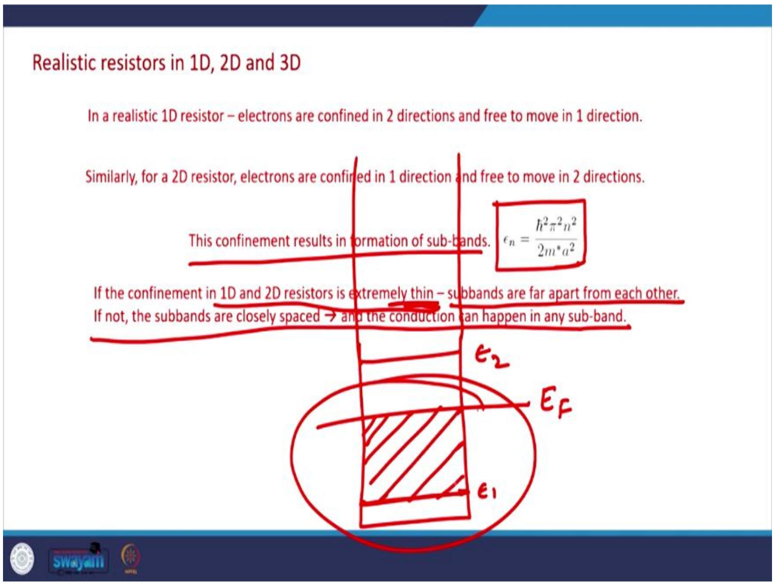

So, just to sum it up the confinement results in the formation of sub bands and these sub bands are based on the discrete energy levels that arises because of the confinement. If the confinement in 1 and 2D resistors or if the confinement direction is extremely thin sub bands are far apart from each other; and if not sub bands are very closely spaced.

So, if a 2D conductor is extremely thin it might happen the other sub bands are very far apart from each other and it does not make any difference in that case. So, for example, if ε_1 is here, ε_2 is here or maybe the higher energies are even higher. And if the Fermi level of the system is here in that case there will be just this sub bands will not come into picture, because yes there will be only 1 sub band that will affect the system and that will be this one.

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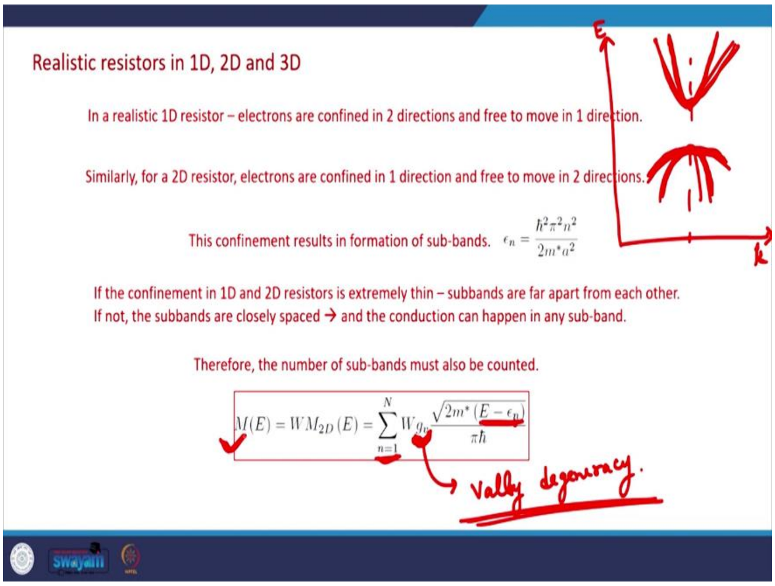

By taking sub bands into account the expressions that we derived previously will modify like this. So, the expression for the modes will modify in this way, we need to take into account all the sub bands and in the even in the expression of even with energy corresponding to a sub band we need to take into account this epsilon n number here.

There is this another number that is known as that is written as g_v this is known as the valley degeneracy. So, typically what happens is that sometimes in the E-k diagram of the semiconductor we might have a situation like this is the E-k diagram or energy axis can be here as well.

So, in this case what happens is that at the bottom of the conduction band or top of the valence band there are multiple solutions or multiple E-k plots overlap with each other generally at the band edges. So, in this case what happens is that we need to. So, the electron can be sitting in, electron can either follow this curve or it may sit in this band or it can also follow this curve.

So, at this energy electron has two choices to take for the E-k relationship for the wave function and this introduces this kind of degeneracy that is known as the valley degeneracy. And, this is given by g_v ok. So, if there are two E-k plots two E k curves or two bands that are overlapping with each other right at the bottom of the conduction band then the valley degeneracy will be g_v ok.

So, there are two kind of degeneracies that can come into picture; one is, because of the overlapping bands and second is because of the sub bands and both of them we need to account while doing calculations for the number of modes in our practical devices in our realistic devices. So, this summation is over the sub bands and corresponding to a sub band there is this ε_n energy.

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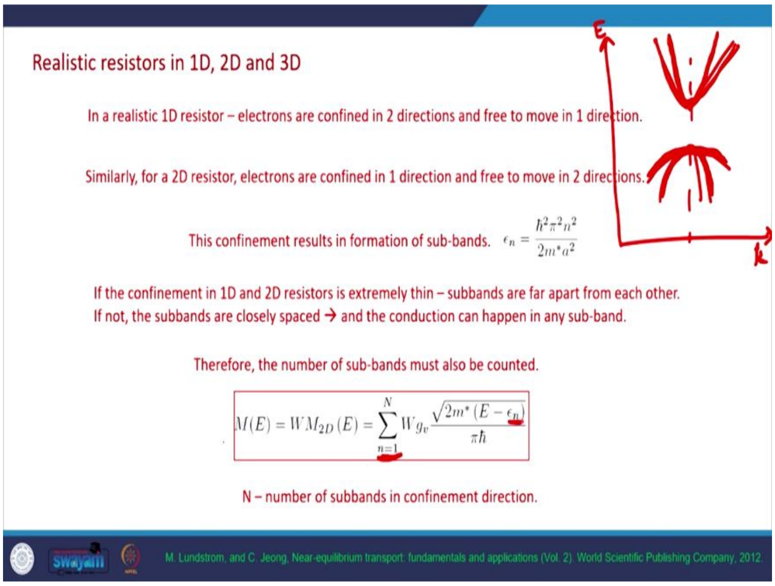

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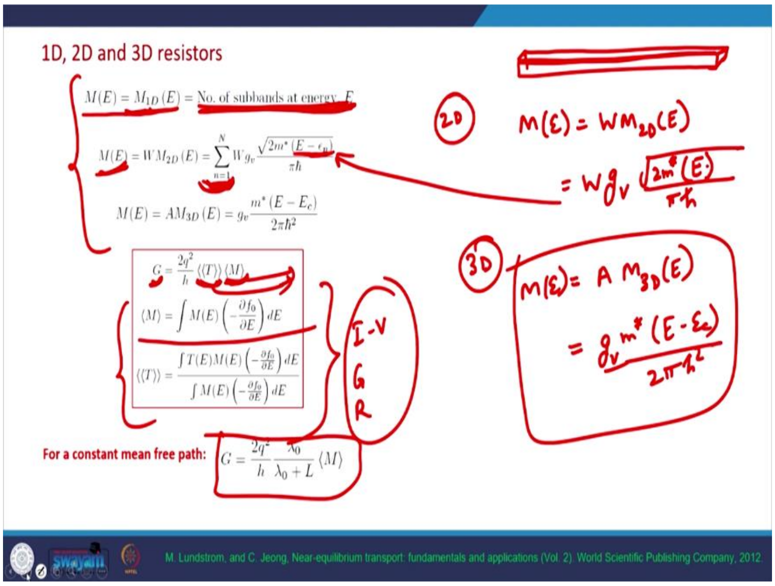

So, finally, just to sum up everything. The number of sub bands or the number of modes in a 1D conductor, this number of modes in a 1D conductor is actually 1 in 1D conductor, but because of the practical constraints because of the confinement in two directions the number of modes will be equal to the number of sub bands ok.

In ideal 1D conductor it is 1, but in a practical realistic 1D conductor it might be it is equal to the number of sub bands at energy E. In a 2D conductor, the number of the modes are given by this expression modes are M(E) is in a 2D conductor M(E) is given as W times M_{2D}(E) or it is written as M g_v if g_v is the valley degeneracy $\sqrt{2m^*E}$ or generally like this.

But, with now with sub bands we need to modify this expression like this we need to sum over all possible sub bands and in addition we need to introduce this energy term here in the square root as well ok. Similarly in the 3D practical resistors the this is for the 2D case. In 3D case the number of modes expression is actually given by the area of times M_{3D}(E) which is $\frac{m^*(E-E_C)}{2\pi\hbar^2}$.

And if g_v is the valley degeneracy this g_v will appear here. And this will not change actually because the practical 3D conductor is actually the way an ideal 3D conductor will look like. So, with these modifications in number of modes, now we need to modify the expressions for the conductance, because here we need to take the average of modes in the Fermi window and we take the average of the transmission coefficient or average of the scattering mean free path in each mode over the Fermi window.

So, this will also get modified accordingly this will be all these things will be modified. And finally, in a practical when we sort of have a new device when we have a new material and we want to make a device out of it we first need to calculate its density of states then number of modes from there we can calculate the and we in addition we also need to calculate its transmission coefficient if it is a bulk device or if scattering is happening we need to calculate the mean free path or transmission coefficient.

Then depending on the carrier confinement in other directions we need to find out how many sub bands arise in this system and by taking all that into account we can finally, calculate the device parameters device I-V characteristics and conductance or resistance of the device. So, this is how things actually work in real life ok. And ok so, this is the expression for the conductance in a more general case. So, all these things will be modified if we take the sub bands into account.

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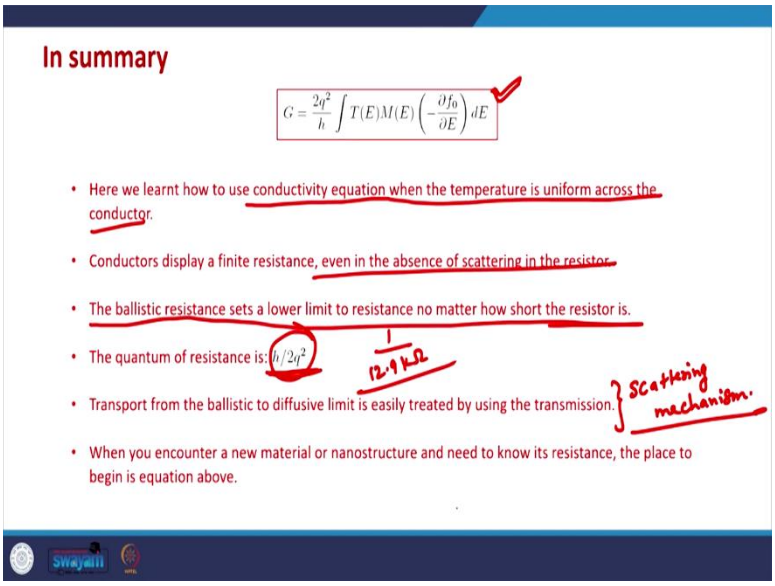

So, in summary what we have is while we need to calculate the conductance of the system, we always start with this expression ok and this is true for the near equilibrium transport. If the transport is not near equilibrium then we will start with the current equation ok.

Then we have seen here that conductors have finite resistance even in the absence of scattering. So, even in the ballistic case the conductance conductors might have finite resistance and that is because of the scattering in the contacts ok. We also saw how to use conductivity equation when temperature is uniform across the conductor.

If the temperature is varying that will result in variation in the Fermi functions and that will also need to be taken into account that we will see while discussing the thermoelectric systems. This ballistic resistance that is the resistance in the absence of scattering in the channel that sort of sets a lower limit to the resistance no matter how short the resistor is ok. And this is also known as the quantum of resistance and this is given by this formula or 1/12.9 $k\Omega$.

Transport from the ballistic to diffusive limit can be easily calculated if we know the scattering mechanisms. So, by knowing scattering mechanism I mean we need to know the mean free path and we need to know the transit time in the channel. So, if we finally, if we encounter a new material or a new nanostructure and we need to know its resistance or its I-V characteristics we need to begin with these set of equations that we discussed in our general model of transport.

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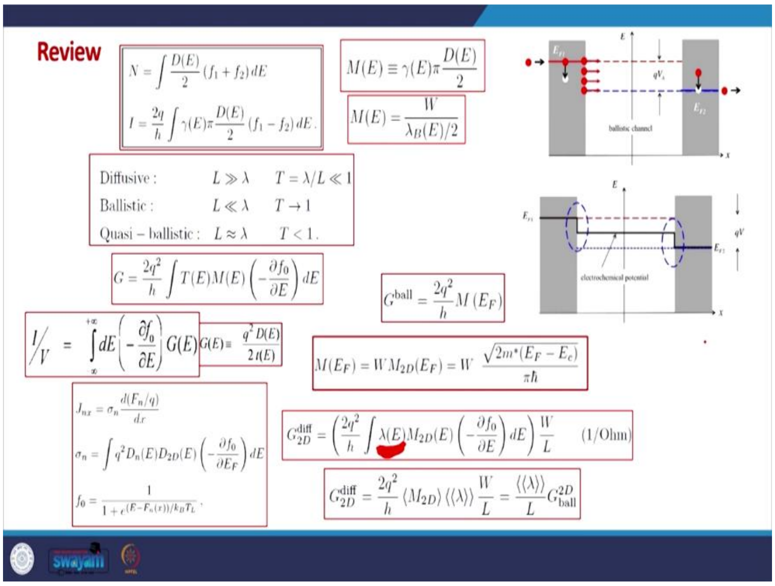

And this slide essentially reviews all those equations that we discussed in detail. So, with this we will now see how MOSFET actually works and specially a nano scale MOSFET. In this there is one missing point that we have not discussed and that is the derivation for this mean free path ok.

So, if time permits we will do that later, but that is not extremely essential in order to understand this entire dynamics ok. So, in the next class we will begin with the discussion on the basics of MOSFETs and then we will move to the nanoscale MOSFETs.

Thank you for your attention, see you in the next class.