Physics of Nanoscale Devices
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Lecture - 33
Voltage Drop in Ballistic Conductor

Hello everyone. Today, we will continue the discussion that we were having in the last class that was on the Power Dissipation in the Ballistic Device, and today we will see how the Voltage Drops in a Ballistic Conductor.

And then, we will discuss some practical 2D and 1D devices, and how the transport is different in a real 2D and 1D conductor.

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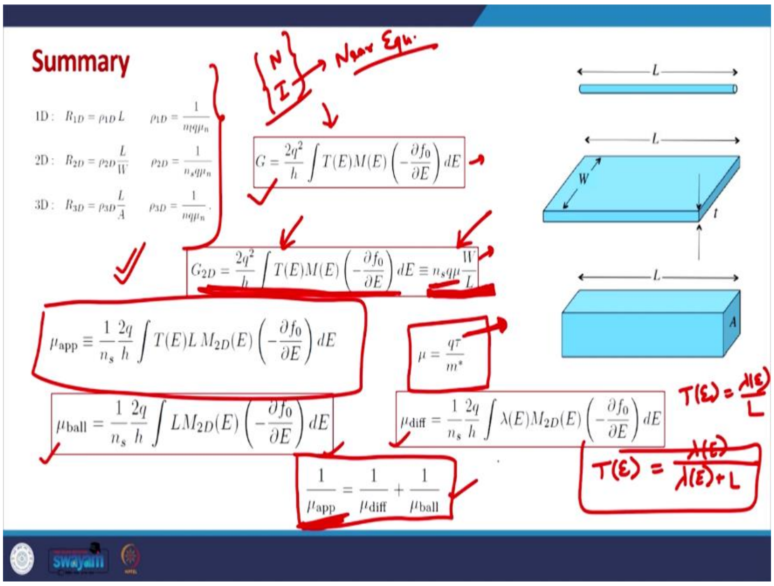

So, before going into those details, let me quickly review what we have been seeing. We saw that the classical understanding of resistance is this. This comes from the Drudes model of transport, electron transport or electron conduction in conductors. And this is also generally used for semiconductors and in our model of transport, we begin with this equation and this we obtained from the current equation.

So, recall, if you quickly recall from the general model of transport, we derived the equation for both N and I. And from the equation of I in near equilibrium transport, in near equilibrium case we could obtain this equation of conductance. So, then after this we had seen that this is how this can be calculated, in ballistic case, and it can be calculated in diffusive transport case.

Apart from this, we also saw the notion of mobility how to sort of make sense of the notion of mobility in our formalism. So, for that we compare this with the classical understanding of the conductance.

So, this comes from the general model of transport, this comes from the classical understanding, and if we compare these two terms, we obtain the expression for the mobility of the electrons in semiconductors. And we saw that now the mobility is in terms of the fundamental physics, in terms of the fundamental device parameters, and not in terms of the phenomenological parameters that we obtain from the experiments or from external observations.

For example, in the classical understanding of mobility where the mobility is given by $\frac{\tau q}{m*}*$, we need to know the τ time which is essentially the mean free time, the average time between two consecutive collisions. From here we could see that the ballistic mobility is given by this expression, and the diffusive, the mobility in the case of pure diffusive transport in which the transmission coefficient is taken to be $\frac{\lambda((E))}{L}$is given by this expression.

And finally, in a more general case when we take the diffusion transmission coefficient to be $\frac{\lambda(E)}{\lambda(E+L}$, in that case the mobility which is now written as apparent mobility because this mobility comes from our this analysis by comparing this fundamental equation of conductance with the classical equation of conductance.

So, that is why it is the apparent; it does not come from the from the basic physics. It is defined using a class by comparing a classical expression to the expression that we obtained from our analysis. So, that way it is called as apparent mobility. And this apparent mobility in the general case can be written as $\frac{1}{\mu-apparent}-=\frac{1}{\mu-ballistic}+\frac{1}{\mu-diffusive}$.

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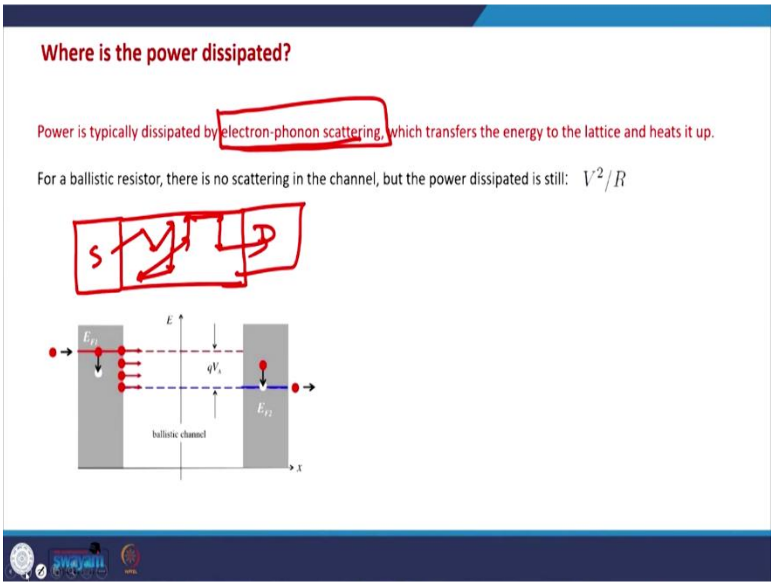

So, this is what we have seen and we were discussing or we were wondering where the power is dissipated in a ballistic conductor.

Generally, in conventional devices the power is dissipated by electron phonon scattering in the channel itself. So, for example, if we have a two-terminal device, we have the source, we have the drain in the classical case. Generally, the electron transport involves collision of electrons with a lot of atoms in between and that way, because of these collision of electrons with the atoms, this is known as the electron phonon scattering, and it is in this electron phonon scattering that the energy of the electron is transferred to the lattice.

And that is how, and that is why the lattice heats up when the electron transports through the lattice, but in the ballistic conductor, in the ballistic case electron does not collide with anything in the channel and it directly goes to the drain side from the source side. So, this is the source terminal, this is the drain terminal.

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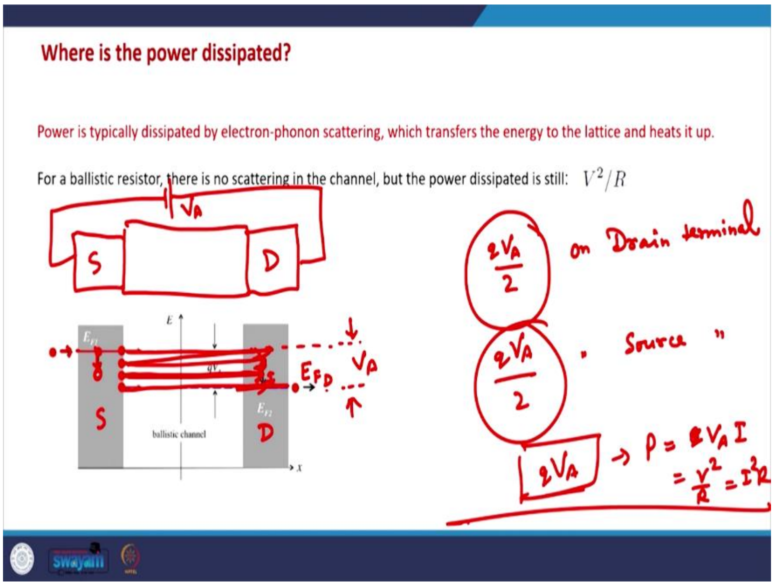

So, the mechanism of or the way power is dissipated is different in this case as is the case in classical theory or in macroscopic devices. Here what happens is that here the contacts are large this we know from the beginning that the contacts are large. So, an electron that starts from the source contact it goes to the drain contact. So, as soon as is it reaches to the drain contact, the drain contact tries to again come back into equilibrium because in equilibrium all the energy states up to Fermi level are filled.

So, by various mechanisms of scattering it tries, again tries to come back into a near equilibrium state. So, as soon as one electron enters into the drain terminal, it dissipates its access energy the energy above the Fermi level of the drain contact in collisions. So, this much energy is dissipated in the drain side and in our device we have; so, this is source, this is drain, this is the channel region in between, this is the channel in between. So, typically we have a battery across the device and the voltage V is applied and thus that is also equal to the difference in the Fermi levels of the two sides.

So, the battery again supplies an electron to the source side and it supplies the electron to the source side at the Fermi level of the source. Now, this electron sees a vacancy from where the electron went to the drain side, and it dissipates, it quickly dissipates its energy to fill this vacancy and this much energy is dissipated in the source terminal, ok.

So, on an average, we can see that almost for electrons numerous electrons going from the source side to the drain side almost on an average half of the energy is dissipated on the source terminal and half of the energy is dissipated on the drain terminal.

So, this much energy is dissipated on the drain side and the same amount of energy is dissipated on the source terminal. In total, the total energy dissipated is q times V_A and so the power dissipation is q time sorry V_A times I which is V square by R or I square R, ok.

So, even in ballistic case because of the scattering mechanisms in the contacts the power dissipates exactly the same amount as it dissipates in the diffusive case.

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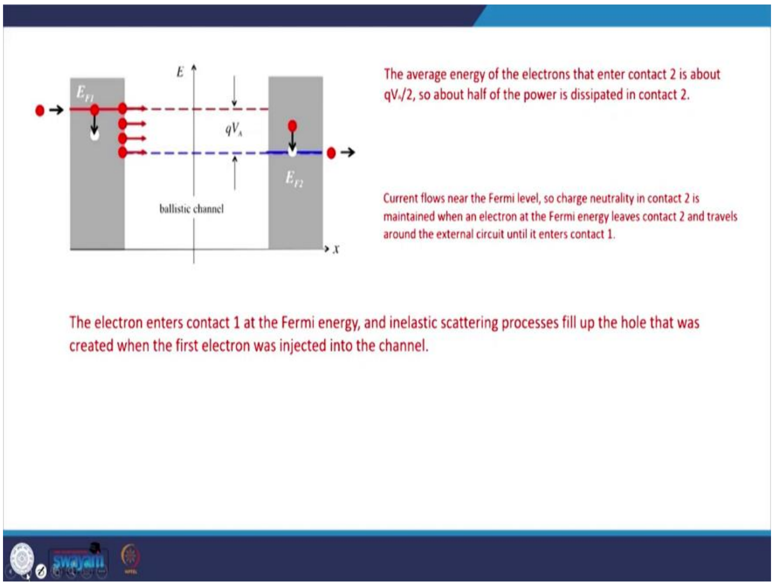

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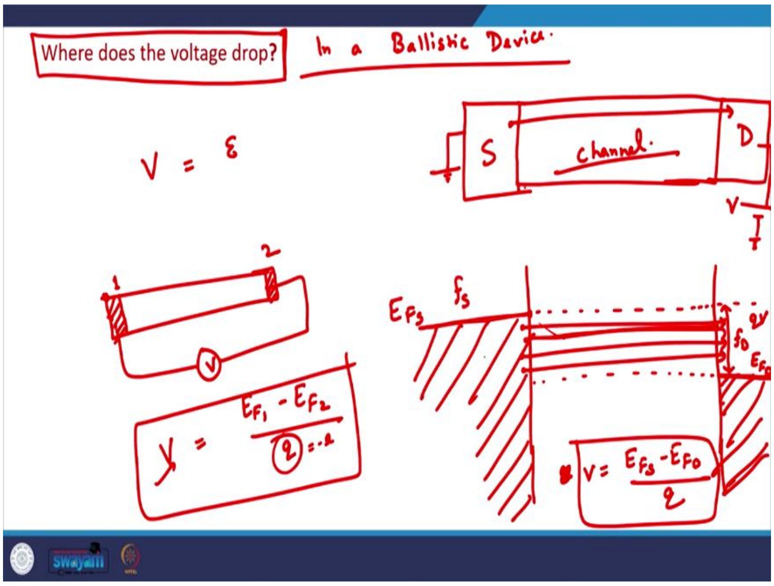

So, now the second question is and the related question is where does the voltage drop in a particularly in a ballistic device. In a ballistic device, where does the voltage drop? So, we have a device like this, again we have a a two terminal device which is like this and in the ballistic case electron directly goes to the drain side. We might have applied a voltage on the drain terminal, ok. The source is grounded, the drain is at high voltage, so that is why there is an electron flow from source to drain or a current flow from drain to source.

Now, if we plot this system this is E_{FS} in the source Fermi level, this is the drain Fermi level E_{FD}. From the basic principles of carrier statistics, this contact tries to fill all the states in the channel up to E_{FS} and tries to bring the channel in equilibrium with the source Fermi function.

Similarly, the drain contact tries to fill all the states in the channel up to the drain Fermi level and tries to bring the channel in equilibrium with the drain Fermi function. As such the channel is small, the channel is generally a microscopic channel or a mesoscopic channel. So, we cannot per se, we cannot define a Fermi level inside the channel. And typically, the voltage that we measure across any device especially in these semiconductor devices, the voltage that is measured by the voltmeter that is exactly equal to the difference in the Fermi level divided by the electronic charge.

So, the voltage measured is always, so if this is the conductor and if this is the conductor and we are measuring voltage between this point and this point, point number 1 and we are connecting it with a voltmeter. So, this measured voltage will be $(E_{F1} - E_{F2})/q$, the electronic charge which is essentially minus e. So, the voltage that is measured by an external device that is essentially the difference in the Fermi levels.

So, now the question is, so the question this question particularly where does the voltage drop this translates in the question where does the Fermi level sort of change or how does the Fermi level change across the device. And according to the change in the Fermi level the voltage will be directly dependent on that. That will also be the voltage profile of the device, ok.

So, we see that the Fermi level on the left side is here, the Fermi level on the right side is here. So, the voltage; obviously, the voltage that will be measured will be this q times V which is or this V is E_{FS} minus E_{FD} divided by charge of the electron. So, that is the total voltage difference in this device and that is also equal to the voltage that is beings voltage of the battery that is connected to the drain terminal.

Now, the question is how does it sort of drop across the device, ok. And in the ballistic case electrons are making direct transitions. Actually, they are not losing any energy in the channel. So, we cannot say that a particular electron is undergoing change in energy which means that throughout this channel, throughout the channel the energy of the electrons is constant.

So, the voltage will not drop through the channel because the energy of the electrons is not dropping, energy of any electron that is standing here that is starting from the source side going through the drain side is not changing.

So, and in the microscopic channel in a small channel the notion of Fermi level can also not be properly defined because a Fermi level is a statistical concept and generally it is true when we talk about bulk materials. In these cases in which we have a small channel between two bulk contacts what we can at best do is we can approximate the Fermi level of the channel. And once we know the Fermi level of the channel then we can easily find out the voltage drop where the voltage drop is happening across the device.

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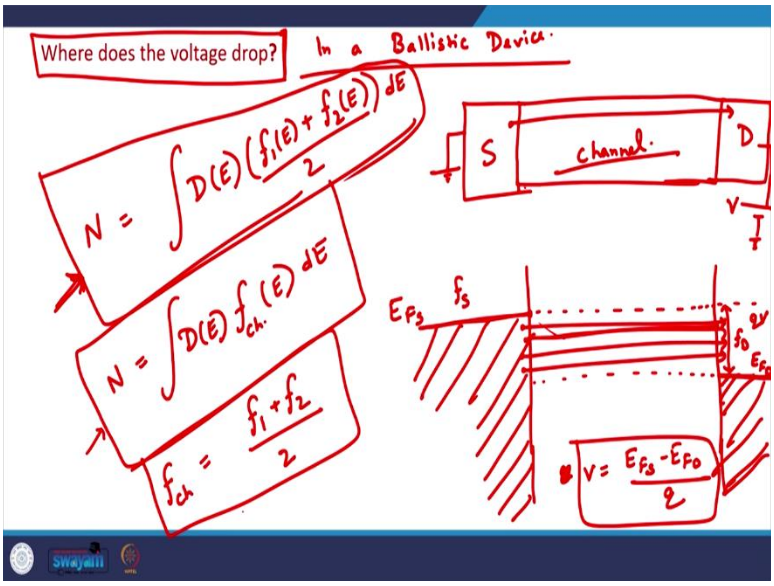

So, if you recall our discussion in the beginning of the general model of transport and when we derived the steady state number of electrons in the channel, in that case, if you remember the expression, the expression looks something like this. Number of electrons is f_{1E} D(E), where D(E) is the density of states of the channel f_{1E} is the source Fermi function, f_{2E} is the drain Fermi function, and it was this.

So, the steady state number of electrons in the channel, number of charge carriers in the channel is given by this expression and if we equate this expression with let us say a more conventional expression of calculating the carriers in a electronic device. This is how we would do it.

Let us say that the Fermi function or the Fermi function of the channel is f_{ch} and density of states is D(E), then the number of electrons in the channel will be given by this number. This expression from our general calculation we obtain, then steady state number of electrons as is shown in this expression. So, by equating these two in a way this Fermi function can be said to be the average of the two Fermi functions.

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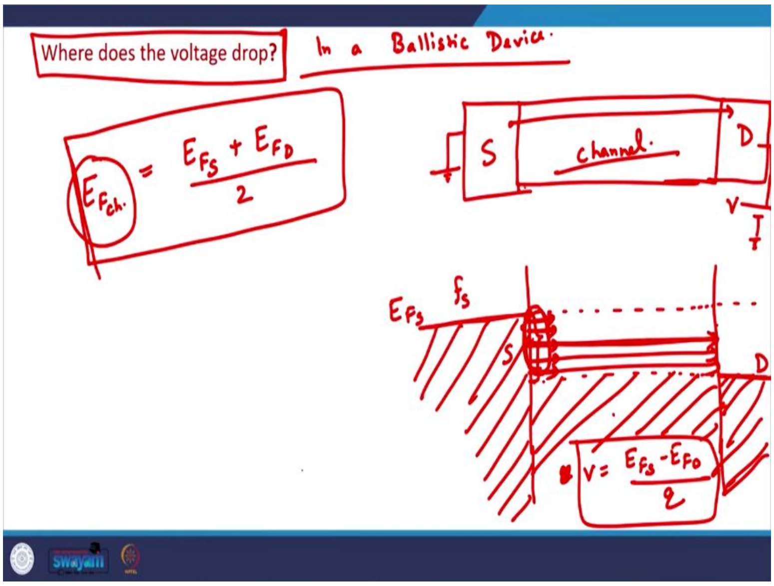

Or in other words,what we can   visualize from our these from here. So, anyway these states are all filled because both the source and drain contacts are trying to fill these states. Channel is trying to fill even these states as well. So, all these states are being filled by the source terminal. The, so sorry the source terminal is trying to fill all these states in the channel and all these states in the channel are being emptied by the drain terminal, ok.

So, in steady state what we can assume is that on an average, on one side we are trying one side there is a push to fill all the available states on other side there is a push to fill, push to sort of empty all the states. On an average what we can see is that almost half of the states are filled in the steady state. So, in steady state what we can assume is that half of the available states in the channel are filled. So, in a way the channel Fermi function the Fermi function, sorry the channel Fermi level can be assumed to be in the middle of the source and drain Fermi levels, ok.

And although this does not come from the carrier statistics, this does not come the channel Fermi level does not come from the statistics of electrons in the channel in equilibrium because we can do that only in equilibrium, but this is just an approximation, ok.

So that way and the channel Fermi level does not change through the channel, this, length of the channel.

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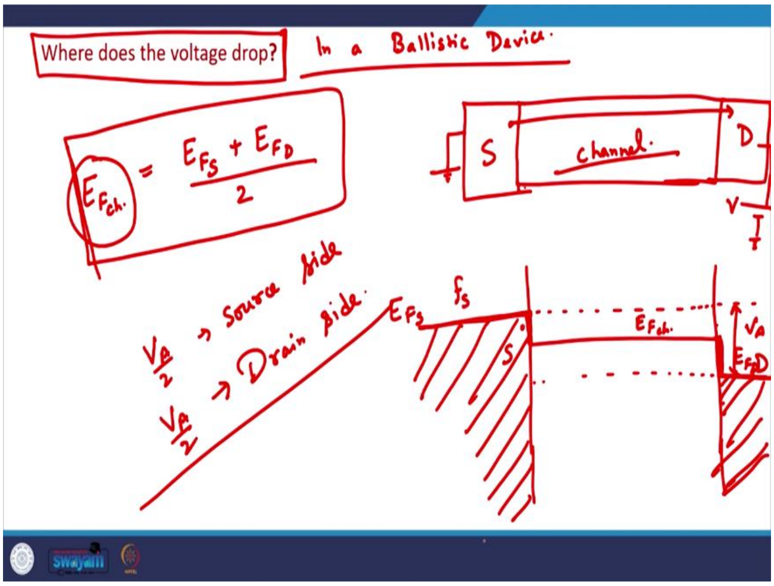

So, what we can finally say is that the source Fermi level is on one side, the drain Fermi level is on the other side and there is a sort of channel Fermi level which lies in the middle of these two Fermi levels E_{Fch}.

And the voltage drop which is by definition the difference in the Fermi levels happens on the contacts. So, almost half of the voltage drops on the source contact and second half of the voltage drops on the drain contact, ok.

So, the total voltage, if the total voltage drop is V_A, V_A /2 drops on the source side and V_A/ 2 drops on the drain side, ok.

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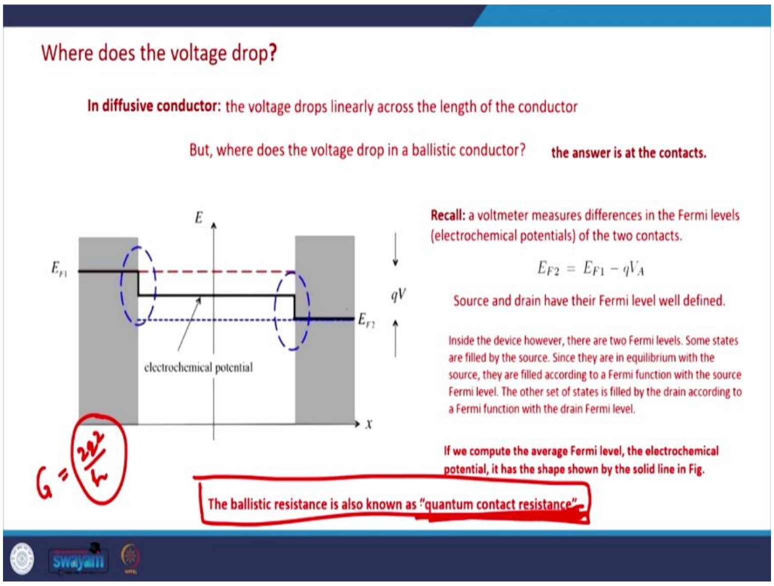

So, this is the overall picture of the voltage drop in ballistic transport and again like power dissipation, the voltage drop also takes place at the contacts. And this is a new concept which we can only see in mesoscopic or nanoscopic devices. When the electron is not colliding with anything in the channel and it is directly going from the source terminal to the drain terminal, ok.

So, this essentially concludes our discussion on the electron transport and there is this minor point that we generally need to know is that the ballistic resistance the ballistic quantum conductance is this and inverse of this is the ballistic resistance. It is also known as the quantum contact resistance. And the reason for this is that this resistance arises because of the contacts, it is not because of the channel. That is why it is also known as the contact resistance in ballistic case, ok. So, this essentially completes our discussion on the ballistic and diffusive transport.

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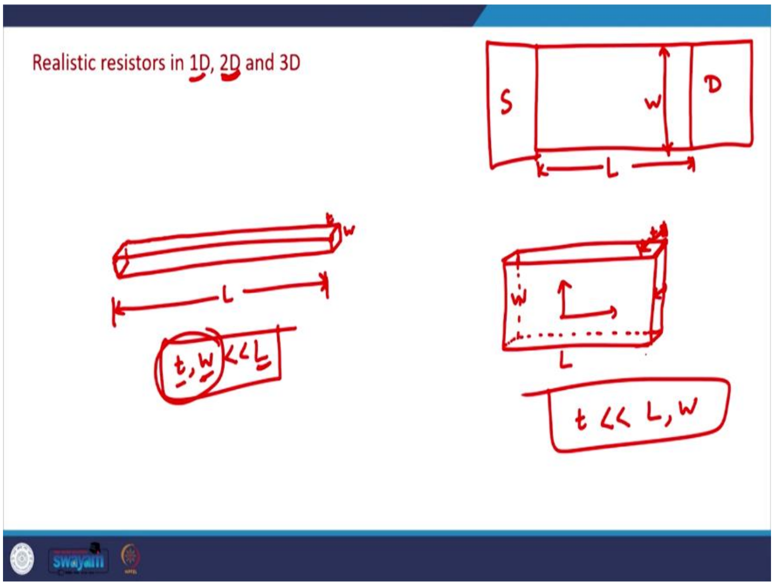

Now, we also need to see or there is this important point that we need to keep in mind that while talking about 1D and 2D conductors as we have done in our discussions, right from the discussions on density of states, the discussion on number of modes and then further on the current equation, and the carrier number of electrons in steady state in the channel. So, generally we assume that these are ideal 2D channels.

So, generally the picture that we have in mind is most of our discussion was focused on the 2D channel in which we had the length of the channel to be L, the width of the channel to be W, ok. And we assumed that in this 2D channel the third dimension of the channel is not there or it is negligible.

But in a real 2D channel, in a real 2D device it may have significant extent in two directions, it may have significant length and width, but practically all 2D materials will also have a finite thickness as well. So, all 2D material also have a finite thickness. And this thickness is known as the ,this generally represented as t, and typically this thickness of a two practical 2D material is extremely smaller than its length and width, ok.

So, practically a 2D material is not exactly a 2D material, it is a 3D material in which one-dimension is extremely small as compared to the other two-dimensions. It is almost of the atomic scale, ok. This thickness is of the atomic scale.

Similarly, a 1D conductor or a 1D channel is not precisely a 1D channel actually, it is we generally assume that a 1D channel has length L, but invariably it will also have a finite width and finite thickness.

So, generally we assume this to be a linear kind of conductor a 1D conductor is assumed to be like this, but invariably it will have, it will look something like this. It will have certain length, it will have a width as well and thickness as well. And in a practical 1D channel this thickness and width will be extremely smaller than the length of the conductor. So, and 3D is essentially 3D, 3D is actually the in real 3D is also the way that we treat it in maths in our calculations.

In a real 1D conductor, the thickness and the width are extremely smaller than the length they are of the atomic dimensions. So, practically everything is in 3D, in our Euclidean world, in our sorry in our real world everything is 3D at least in space, if we do not consider the time dimension everything is 3D.

So, for 2D channels generally the thickness is less and for 1D channel the thickness and width are of atomic scale, ok. So, this we have not taken into account. These small dimensions for example, in the case of a 2D conductor this small extent in the third dimension is not taken into account in our analysis. And that is what we actually need to take into account similarly for 1D channel as well.

So, now, what happens is that in a 2D conductor generally, in a 2D channel the electrons or charge carriers are generally free to move in two directions, ok. But they are confined in the third direction, and as we have seen from the basics of quantum mechanics in our discussion on quantum mechanics in the beginning of this course, the confinement leads to discretization of energies, ok.

So, let us take the case of a 2D conductor a realistic 2D conductor. So, in a realistic 2D, the realistic 2D conductor will look something like this. It will have length, finite length, finite width and a very small thickness.

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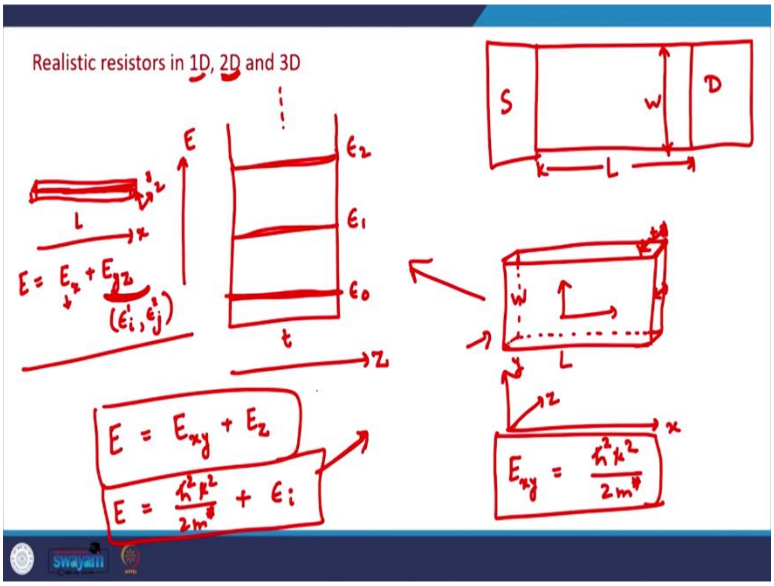

Now, let us say if this is the length is in x direction, width is in y direction, and thickness is in z direction, and this is x, this is y, and this is z, ok. So, now, electrons are confined in the z direction, and electrons are free to move in x and y direction.

And electronic energy in x and y direction is typically given by this expression $h^2k^2/2m^*$ as we have seen previously from our analysis. But in z direction the electrons are confined and electrons will have certain energy in z direction as well. So, this energy we need to calculate from, the so if this is now the z direction let us say this is the thickness the electronic energy will be discrete like for a particle in a 1D box. Now, it may have energy levels like this and so on.

So, y axis is the energy axis, x axis is the z axis. And in this practical 2D conductor electrons are confined in the z direction, so the electrons will have discrete energies in the z direction, ok. The total energy of the electrons in this system will be Exy + Ez. So, that will be the total energy of electrons. And this total energy will be given by in x-y direction electrons are free to move.

So, this we can calculate from our conventional E k relationship and in z direction this is discrete and so it would be one of the εi's. So, if in z direction the energies are ε0, ε1, ε2, it would be εi for if the electron is in let us say ith excited state. So, ε0 is the ground state, ε1 is the first excited state, ε2 is the second excited state, so to say, ok. So, this we need to take into account.

So, while doing all the calculations all the calculation of modes, even the calculation of charge, steady state charge the calculation of current the calculation of conductance and finally, the calculation of conductivity or mobility as well, we need to take this fact into account and this is an important thing that we need to keep in mind.

Similarly, for 1D conductors, for 1D realistic resistors, electrons are confined in two directions. So, electrons will be like in a 2D potential well, ok. And in that case, let us say that the 1D conductor is, our 1D conductor is like this, it has finite length in x direction, ok. This is the x direction and electrons are confined in these two directions y and z directions.

So, the electronic energy, total electron energy here will be energy because of the motion in x direction and energy because of the motion in y and z direction. And this energy may be continuous, this may be given by $h^2k^2/2m^*$, but this energy the energy where the electrons are confined is discrete. And this will be given by epsilon maybe by a combination of ε1i, ε2i or j.

So, these things in realistic resistors in 1D or 2D we always need to keep in mind. This is an important point. And generally in nanoscale devices nowadays when we talk about using carbon nanotubes or graphene nano ribbon, these points we need to keep in mind, the confinement in other directions apart from the long direction, ok.

So, with this I let you think more about this. And in next class, we will conclude this discussion and begin with the discussion of MOSFETS, ok.

Thank you for your attention. See you in the next class.