Physics of Nanoscale Devices Prof. Vishvendra Singh Poonia Department of Electronics and Communication Engineering Indian Institute of Technology, Roorkee

Lecture - 32 The Idea of Mobility

Hello everyone, today we will discuss the Idea of Mobility in ballistic case and in diffusive transport case, because this is the only topic I guess that is left that we did not touch upon in our discussion of the electron transport in nanoscale devices.

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As you might have recalled by now that, we are discussing the or we are sort of contrasting the conventional idea of conductivity conductance and resistivity with the modern ballistic conductivity or diffusive conductivity of the material.

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So, this is what we have seen in last few classes. We started with this discussion that this is the conventional understanding of the resistance and conductivity and in this expression as you can also see that there is a term mu that appears. So, for example, we are since we are discussing a 2D case, in 2D case the resistance is inversely proportional to the conductance which is inversely proportional to the conductance.

It is given as resistance is given as $\frac{1}{\sqrt{2}}$ $n_s q \mu_n$ L $\frac{L}{W}$, where n_s is the sheet carrier density the number of electrons per unit area, q is the electron charged, μ_n is the mobility. So, this idea we have not sort of encountered as yet, although we do not need it in our ballistic in our treatment of ballistic transport or a more general kind of transport.

We do not need the idea of but mobility, but if we want to compare, the conventional understanding of transport with the formalism that we are developing and that essentially can be attributed to Landauer and later on to Supriyo Datta and Mark Lundstrom. (Refer Time: 02:36)

So, in that formalism, if we try to see what is mobility of the electrons or charge carriers will mean; then we need to compare the expression. This classical or this conventional expression of conductivity with the expression that we have derived in last few classes. So, in last few classes what we have seen is that we essentially need to begin with this expression of conductance.

So, this is the I would say this is the fundamental expression that you can easily remember and this comes from the expressions of the steady state charge carriers and steady state current in the system. So, this is the expression to begin with in any case actually, whatever be the. If we encounter a new material, we always need to begin with this expression.

And for this we need to find out the fundamental characteristics of the new material like the number of modes and the transmission coefficient and this transmission coefficient comes from the scattering of electrons in that material and the Fermi window which essentially comes from the contacts, it comes by virtue of the contacts ok.

So, just to sort of quickly review, this is how it looks like $T(E)M(E)(-\frac{\partial f}{\partial E})dE$ ok. And using this expression what we have seen is that the conductance of a ballistic conductor at T equal to 0 Kelvin is given by this expression $\frac{2q^2}{l}$ $\frac{q}{h}$ times the number of modes.

And from here we see this idea of quantum of conductance coming into picture, because this is the if there is only one mode in a device, one mode in the conductor; this is the minimum conductance that the conductor will have. If there are many modes this will be more, but if the conductor is diffusive conductor in that case this conductance might go down.

Because, the transmission coefficient will also come into picture and the transmission coefficient is always less than 1. So, this is the conductance at T equal to 0 Kelvin for higher temperatures at room temperatures case, this is the conductance of ballistic conductor and here we need to make use of the special kind of integrals the Fermi Dirac integrals essentially.

Then we also have seen specially in the last class, the conductance of a diffusive conductor is given by this formula at T equal to 0 Kelvin and this can be related to the conductance of a ballistic conductor, here this new term $\frac{\lambda(E_F)}{L}$ appears in the expression. And at normal temperatures at room temperature this will be the conductance of the diffusive conductor, conductance of a 2D diffusive conductor.

And in addition to this ballistic conductance this has this new term and this new term is a special kind of average of mean free path and this is essentially given by $\lambda(E)M_{2D}(E)(-\frac{\partial f}{\partial E})dE$ divided by $M_{2D}(E)(-\frac{\partial f}{\partial E})$ integrated over dE ok.

So, this is in a way the average mean free path of electrons in the modes in the Fermi window and this is the case at room temperature. This is true at 0 Kelvin ok. So, this is what we have seen and so, we now know what is the conductance. In our previous discussions we have seen how do we define the conductivity as well, the resistivity from this general model of transport.

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But, this idea of mobility we have not encountered yet. And that is what we will briefly discuss in this code in this lecture. So, as always the equation to begin with is this equation, this is the equation of the conductance. So, we need to always begin with this $2q^2$ $\frac{q^2}{h} \int T(E)M(E)(-\frac{\partial f}{\partial E})dE$. And just to sort of remind you that this term $(-\frac{\partial f}{\partial E})$ this ensures that only electrons near the Fermi level contribute to the current flow in our treatment, in the case of our treatment.

And generally for non-degenerate conductors, the electrons in the Fermi window are the all in non-degenerate semi conductor; they are almost all the electrons all the conducting electrons available in the material. So, it essentially is the almost all available electrons for conduction in the non-degenerate conductors, but in the case of degenerate conductors when we have extremely high doping.

In those cases there might be a lot of conducting electrons even outside of the Fermi window. So, if we have this bottom this top of the valence band is V; bottom of the conduction band generally non-degenerate conductors, we have electrons only up to these levels very close to the bottom of the conduction band.

But, in the case of degenerate conductors the electrons are occupied even at or electrons are available even at higher energy values. So, in that case the electrons in the Fermi window which is essentially which will be this window, if this is the Fermi level, this will not account for the all conducting electrons in the case of degenerate semiconductor.

But generally in the case of non-degenerate semiconductors, which is true which is actually true in most of the cases our treatment or this if this Fermi window actually accounts for almost all the conducting electrons available in the semiconductor. So just to sort of if we want to see how the mobility looks like in terms of our treatment.

We need to compare this expression; this expression of the conductance to the conventional expression and this is what it is. Just recall that conventionally this is how it looks like G is $1/R$. So, this will be $1/\rho$; sorry, in the case of 2D materials instead of area we have the width here. So, instead of A we have W. Yeah, So, the conventional understanding of resistance.

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And conductance says that this conductance is inverse of the resistance which is W/L in the case of 2D material it will be $1/\rho$ W/L. So, this is the conventional expression and ρ is

1 $\frac{1}{n_s q \mu_n}$. So, conventionally G is equal to $n_s q \mu_n$ times W/L. So, this we need to equate with the expression here.

So, that way we can sort of calculate the mobility of electron. So, we are considering that only electrons are responsible for conduction in this case. So, that is why we only have written μ_n , which is just the mobility which indicates the mobility of the electrons. So, if we equate this expression with this as it is written here ok. So, in this case we can find out the expression for the mobility.

So, there is one minute point actually that we need to take care of here or not take care of that we need to keep in mind. In our treatment in our general model of transport this is the expression and this is largely this comes from the expression for the current and it has this minus $\left(-\frac{\partial f}{\partial E}\right)dE$ term. So, as I just pointed out to you that in the case of a non-degenerate semiconductors, it mostly accounts for all the conducting electrons available in the conductor.

In the case of degenerate semiconductors, this does not account for all the conducting electrons; it accounts for only some part of those electrons. In the conventional understanding, we take this n_s which is essentially the sheet carrier charge density or sheet electron density, number of electrons per unit area of the 2D conductor.

So, here this nuance is not there. So, it does not have this nuanced picture of the transport. So, here it in conventional understanding it just takes the sheet carrier charge density; however, in our case only the electrons that are participating in the conduction, they are getting taken care of ok.

So, if we equate both of them and if we write it down in terms of M(E); we can write down as W times M_{2D}(E). So, that way the left hand side is $\frac{2q^2}{h}$ $\frac{q^2}{h} \int T(E) M_{2D}(E) (-\frac{\partial f}{\partial E}) dE$ into W is equal to $n_s q \mu_n \frac{W}{I}$ $\frac{W}{L}$. So, this W and W are cancelled and if we bring everything apart from mu term to the left hand side or to the other side, this μ_n can be written as $\frac{1}{n_s}$, q cancels one of the q s it will be $\frac{2q}{h}$.

On this side and we have T(E) times L times $M_{2D}(E)$ times $\left(-\frac{\partial f}{\partial E}\right)$ into dE. So, this is essentially how we can write down the mobility of electrons in a 2 D conductor ok. So, we need to know the sheet carrier charge density, we need to know the length, it is dependent on the length. If T(E) is independent of the length.

But if $T(E)$ is also dependent on the length, they may cancel out; as we will see shortly. So, in our conventional understanding mobility is independent of the length. Mobility is a material parameter, it depends on the temperature definitely.

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So, this is essentially the mobility that we have just written down from this comparison ok. And this is sometimes called as apparent mobility, which means this is the mobility for a general case which is or this will be the or apparently this will be the mobility in our formalism in terms of the fundamental parameters of the material.

And if we recall the mobility from Drude's formula formulation the mobility is given by $q\tau$ $\frac{q_1}{m^*}$ and please remember this τ is not the energy is not the transit time. This is the average time between 2 collisions or this is also known as the mean free time of the electrons and $m[*]$ is the effective mass of the electron. In this case however, things are different as expected.

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So, in ballistic case; so, this becomes the starting point for calculating the mobility of charge carriers in a material. Mobility or the apparent mobility is given as $2q$ $\frac{2q}{n_s h} \int T(E) L M_{2D}(E) (-\frac{\partial f}{\partial E}) dE$. So, in ballistic case we need to replace T(E) by 1 and in that case, the ballistic mobility will be $\frac{2q}{n_sh} L < M_{2D}(E) >$.

L will also come out $M_{2D}(E)(-\frac{\partial f}{\partial E})dE$ or this can be simply written as $\frac{2q}{n_s h}$. This can be written as the average of modes in the Fermi window times L. So, the ballistic mobility apparently from our formulation is directly proportional to the length of the conductor, which is actually not the case if we look into the Drude's formalism in which case it is given as it is directly proportional to the tau and inversely proportional to m star ok.

So, this nuance is not taken care of by the Drude's or conventional formalism of the transport.

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If we look at the diffusive case, in the diffusive case or when the bulk conductor is there this transmission coefficient is given as T(E) is equal to $\frac{\lambda(E)}{L}$ ok. This is the case when purely diffusive transport is there, in that case the transmission coefficient is written as $\lambda(E)$ $\frac{a}{L}$ and that mobility from this expression, if we start with this expression.

The diffusive mobility or mobility in the case of diffusive transport will be $2q$ $\frac{2q}{n_{\rm s}h}\int\frac{\lambda(E)}{L}$ $\frac{E(E)}{L} LM_{2D}(E) \left(-\frac{\partial f}{\partial E}\right) dE$ replacing this T(E) by $\frac{\lambda(E)}{L}$. So, L and L cancels. So, what is left is just this, $\frac{2q}{n-1}$ $\frac{2q}{n_s h} \int \lambda(E) M_{2D}(E) (-\frac{\partial f}{\partial E}) dE$ and this expression at T equal to 0 Kelvin is further simplified at normal temperatures also, this can be written as this, as we have seen in the case of diffusive transport.

And in more general case in a more general case, this T(E) needs to be taken as $\frac{\lambda(E)}{\lambda(E)+L}$ and in that case if we replace $T(E)$ by this expression in that case, this general or the mobility will be more general case it will be a more and this is known as the apparent mobility as we have defined earlier.

So, it will be essentially $\frac{2q}{n_s h} \int \frac{\lambda(E)}{\lambda(E)+}$ $\frac{\lambda(E)}{\lambda(E)+L} LM_{2D}(E)$ (- $\frac{\partial f}{\partial E}$) dE ok. So, this apparent mobility can be written as 1 by μ apparent is equal to 1 by μ diffusion plus 1 by μ ballistic. So, in a way this the general expression of mobility can be written as the inverse or 1 by general

mobility is written as 1 by μ ballistic case plus 1 by μ diffusive case, purely diffusive case ok.

So, this is how we accommodate the idea of mobility in the formalism that we have developed. Although, we do not as I pointed out earlier we do not need it in order to calculate the device characteristics. We can do that from the fundamental expressions of the current and the resistance can be calculated from this expression.

So, this is what essentially we have about the resistance, conductance, conductivity, resistivity and mobility. So, all these parameters that we generally talk about in the electron transport, we have taken care of in this formalism. So, this essentially sort in a way completes the general model of transport and now, we will see some more points.

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For example, we began our discussion with this question with began or in the beginning of this course, we sort of put forward a question where is the power dissipated specially in ballistic transport. So, because in ballistic transport; electron does not collide with anybody in the channel. So, if we have a 2D conductor like this source drain electron starts from here, it directly goes to the drain terminal without any collision in the middle..

So, in such situation where is the power dissipated, because the conventional understanding of the dissipated power says that electrons collide with atoms or electrons undergo a scattering and during the collision electrons transfer their energy to the lattice, and that is how the electron energy is dissipated in the system, and that is how the heat is generated that is how the power is dissipated in the device. That is the conventional understanding of power dissipation.

And the power dissipation according to the conventional expression is given as V square by R; where V is the applied voltage, R is the resistance of the material or I square times R. But, now in ballistic case there is no scattering in the channel and as we will see as we will realize that even now, the power dissipated is V square by R and this is not dissipated in the channel, it is dissipated in the contacts.

And in order to understand this if we have a 2D device or a small channel between the source and drain contacts; and if we have also applied a battery across the device, let us say the voltage of the battery is this, this is how the Fermi level configuration of the device will look like ok.

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So, to sort of have a better understanding of this phenomena, the phenomena of power dissipation. If this is the channel, this is source, this is drain, we have applied a voltage on the drain side. So, the Fermi level on the source side is up, Fermi level on the drain side is a bit down and this is the drain terminal this is the source terminal ok. And in ballistic transport case, the electron here they go directly.

There is a small point that I would like to remind you that these contacts the source and drain contacts, these are large contacts and because of heavy scattering in the contacts the equilibrium is always maintained. So, because of the scattering in the contacts the equilibrium is maintained in the source and drain regions.

This is the source Fermi level E_{Fs} , let us say and this is the drain Fermi level and generally the way things happen is that the source terminal tries to fill all the states in the channel up to the source Fermi level. It tries to bring the channel in equilibrium with the source Fermi function. Similarly, the drain tries to fill all the electronic states up to the drain Fermi level.

So, the states below the drain Fermi level, these are all filled because, both the contacts are trying to fill these states. And the states between the source and the drain Fermi levels, the states between the source and drain Fermi level for these states the source is trying to fill them, the drain is trying to in a way drain electrons out of them try to empty them and this is what is causing current as we have discussed many times by now.

So, if there is an electron that starts from the source side, let us say and the it directly goes to the drain side at the same level and this is the let us say this V_A is the level of difference between the electron energy and the Fermi level. So, because of this drain terminal has Fermi level here, the scattering will bring this electron to the Fermi level. So, this electron will lose energy this q times V_A energy in the drain contact ok. And since, a battery is connected across this device.

Now, the drain contact and there is a battery between the source and the drain. So, battery will supply this electron to the source side and the battery will supply the electron at the energy of the source Fermi level at this energy and because of this electron now, going to the drain side there is a vacancy at this point, at this point there is a vacant state in the source terminal and the battery is continuously supplying electrons at this level.

So, one of the electrons at the Fermi level will make a jump to this level to fill this vacancy and it will dissipate energy equal to q times V minus V_A , because this is V_A and this entire thing is V this is V_A . So, this will be q times V minus V_A ok. So, the total energy that is dissipated in this device is q times V_A , the energy dissipated at the drain and plus q times V minus V_A , which is essentially equal to q times V ok.

That is the energy dissipated when in the ballistic transport case. No scattering in the channel, only the scattering is happening in the contacts. So, the power dissipation will be again the rate of energy dissipation, this is V times if there is a constant voltage, the rate of charge which will be the V times I.

So, it will be V square by R or I square R, the same. Same amount of power will be dissipated in this case as well, as is dissipated in the diffusive transport case ok. And this analysis is true for all the electrons. So, that is why we can say that the only distinction in this case is that now the power is dissipating or the energy is getting dissipated at the contacts.

In the device there is no energy that is getting dissipated ok. On the similar lines I would let you think where the voltage drop happens in the device actually ok. So, using the same line of arguments, please think about where the voltage drop happens and that is what we will begin our discussion with in the next class.

Thank you for your attention, see you in the next class.