Physics of Nanoscale Devices
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Lecture - 31
Resistance: Diffusive Case

Hello everyone, today we will continue our discussion on calculation of resistance and conductance. And today we will do calculation of resistance in the case of diffusive transport and if we have a quick review of what we have done so far.

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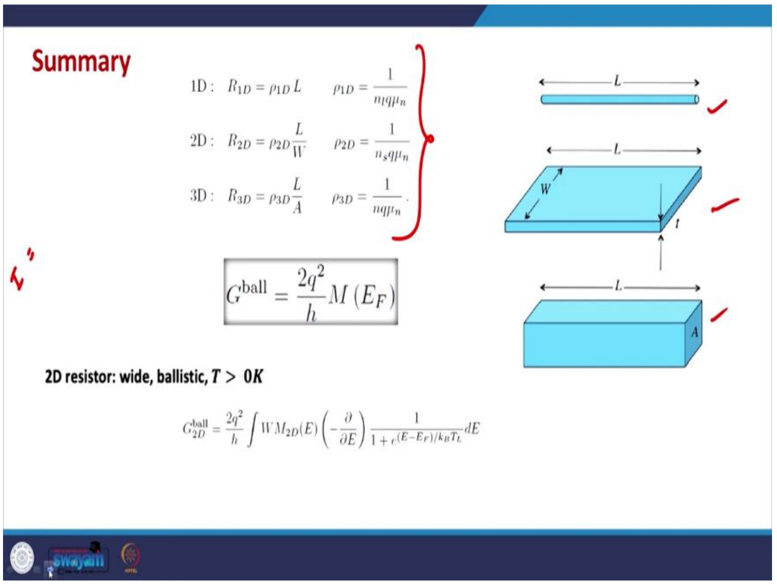

This is what we started our discussion with, we started with these expressions these are conventional expressions for conductivity and resistance for 1D , 2D and 3D conductors. These essentially comes from the conventional theory of transport which has roots in roots model of transport. And in last class or in last few classes the emphasis has been on the point that the starting point should be this or the starting point for these kind of discussions should be this expression, the expression for the conductance from the ballistic transport case ok.

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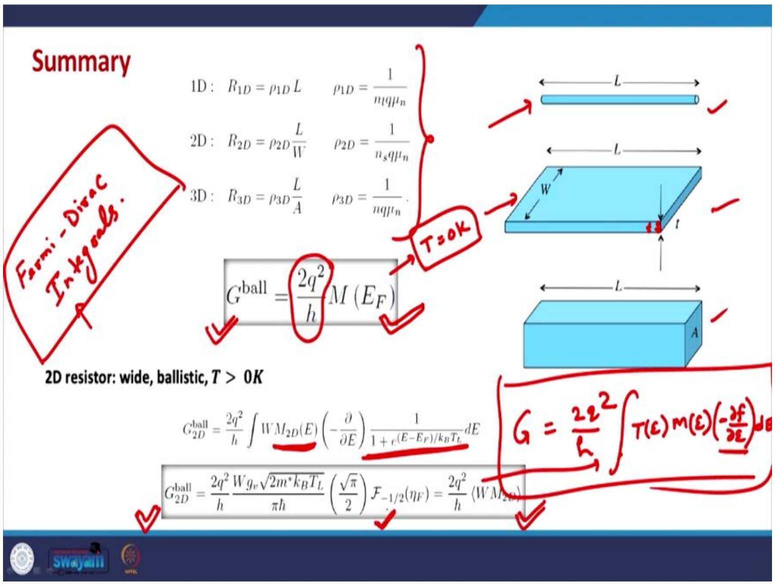

So, we should start the discussion with this expression $\frac{2q^2}{h}\int T(E)M(E)(-\frac{\partial f}{\partial E})dE$. And from here in our last couple of classes last class particularly, we calculated the conductance in the case of ballistic transport to be this. And as you can see this constant term which is known as the quantum of conductance multiplied by the number of modes in the conductor at Fermi level.

So, this was the case when T was 0 kelvin in a ballistic conductor. And when we operate at higher temperature at room temperature may be, in that case we need to do exact calculation of this integral as well as this differential in this as well as this derivative in this equation. And so this becomes first we need to do integration and then we need to take differentiation and it becomes quite a complex expression, because we need to integrate this expression.

And number of modes is also; in some cases it is not a straight forward function. So, this is tackled by what is known as Fermi Dirac integrals, so that is what we also saw in our previous classes. So, we have seen calculation for ballistic transport case with the help of Fermi Dirac integrals.

Today we will see how things happen in the case or what are the conductance and resistance in the case of diffusive transport and that will essentially conclude our discussion of the transport theory. We will briefly have a look at the practical 1D, 2D conductors because, we are assuming till now we are assuming that these 1D and 2D conductors are ideal 1D ,2D conductors. But, actually they are not because, a 2D conductor has a finite thickness this thickness t which leads to quantum confinement in this direction.

And a 1D conductor has confinement in two directions the directions apart from the long dimension in two other dimensions there will be electron confinement. And that will introduce some that we also need to take into account while doing the final calculation for practical conductors ok.

So, that is what we will see, and this was the expression for conductance for a 2D conductor in ballistic case when we have a 2D ballistic conductor. This is the final expression and we studied some properties of the Fermi Dirac integrals with those with the help of those properties we can actually calculate this value ok.

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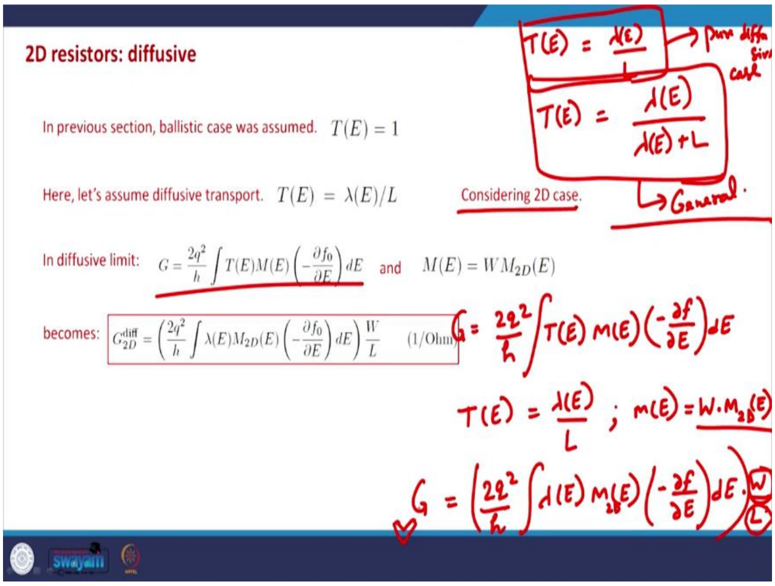

So, today let us see how this looks like in the case of diffusive transport. So, generally in diffusive transport case; we assume that a conductor is a bulk conductor purely diffusive transport in that case this transmission coefficient is equal to $\frac{\lambda(E)}{L}$. T(E) is equal to $\frac{\lambda(E)}{L}$.

But, in intermediate cases when the conductor has electron scattering, but the conductor is not extremely long it is not a bulk conductor it is in the intermediate length scale. In that case the transmission coefficient needs to be taken to be T(E) is equal to $\frac{\lambda(E)}{L+\lambda(E)}$.

So, this is the pure diffusive limit, T(E) is equal to $\frac{\lambda(E)}{L}$ is the pure diffusive case, and this is actually a more general case. So, this will hold true even in the case of ballistic transport and also in the case of diffusive transport.

So, again we take the channel to be a 2D channel, because of the sake of simplicity to visualize transport and also there is some simplicity in calculations as well, and the starting point like always for us is this expression. So, the conductance is $\frac{2q^2}{h}\int T(E)M(E)(-\frac{\partial f}{\partial E})dE.$

So, in the case of pure diffusive transport in that case this T(E) is equal to $\frac{\lambda(E)}{L}$ and generally this number of modes can be written as W times number of modes per unit width M_{2D}(E). So, that is how we can generally write the number of modes ok. So, if we put that these two expressions in this the conductance turns out to be $\frac{2q^2}{h}\int \lambda(E)M_{2D}(E)(-\frac{\partial f}{\partial E})dE\,\frac{W}{L}$ and this M(E) can be replaced by W times M_{2D}(E).

So, we will write it in terms of M_{2D}(E)dE and W from here can be taken outside and similarly L can also be taken outside L by λ(E) from the T(E) can be taken outside. And now, as you can see that now the conductance is inversely proportional to the length of the conductor directly proportional to the width of the conductor which is actually true in the case of conventional expressions of conductance and resistance as is written here.

So, for a 2D conductor the conductance should be according to the conventional theory of transport it should be inversely proportional to the length. So, that is true in the case of purely diffusive transport in the conductors. Now, the conductance has become inversely proportional to the length, but as we have seen earlier in ballistic case the conductance or the resistance is independent of the length.

So, we cannot use our conventional theory of electron transport in ballistic case or even in diffusive case or in intermediate cases when we have some scattering. But, the conductance, but the conductor is also not extremely long it is not a pure bulk conductor it is in the intermediate length scale in that case also we need to actually start the treatment of electron transport from the basics like here ok.

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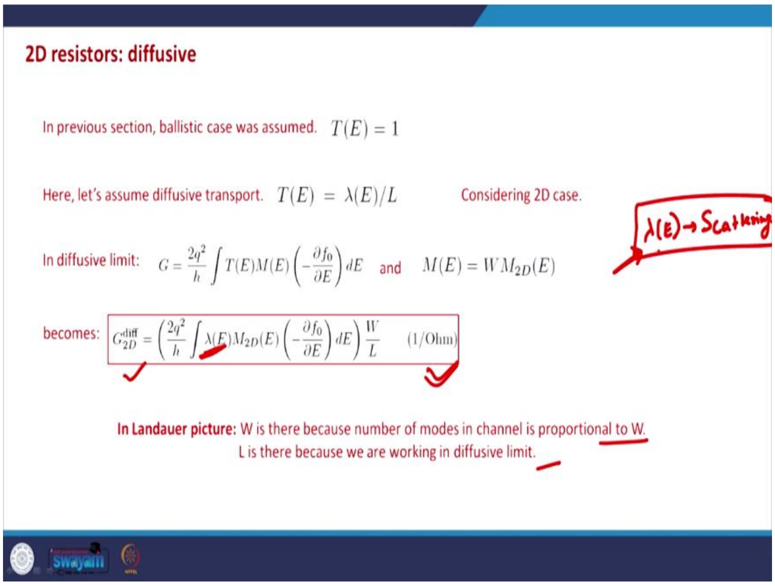

So, if we further simplify this expression this will be actually we finally, we need to find out the this number of this scattering of electron this lambda essentially comes from the scattering in the channel. And this is the topic that we have not dealt in detail. The scattering and how the scattering length the mean free path of the electrons can be calculated as a function of energy.

So, we will see if there is time left we will also revisit the scattering and try to calculate λ in terms of as a function of E. But at the moment let us assume that we are aware of the mean free path or the scattering mechanisms in the channel. And from there if we put the expression of λ(E) in this conductance expression we can calculate the conductance in the diffusive case ok.

So, we again like a ballistic conductor we will see how it looks like when T is equal to 0 kelvin at extremely low temperatures. Because the calculation is much simpler in those cases and how it looks like at practical operating temperatures the room temperature or even higher temperatures ok.

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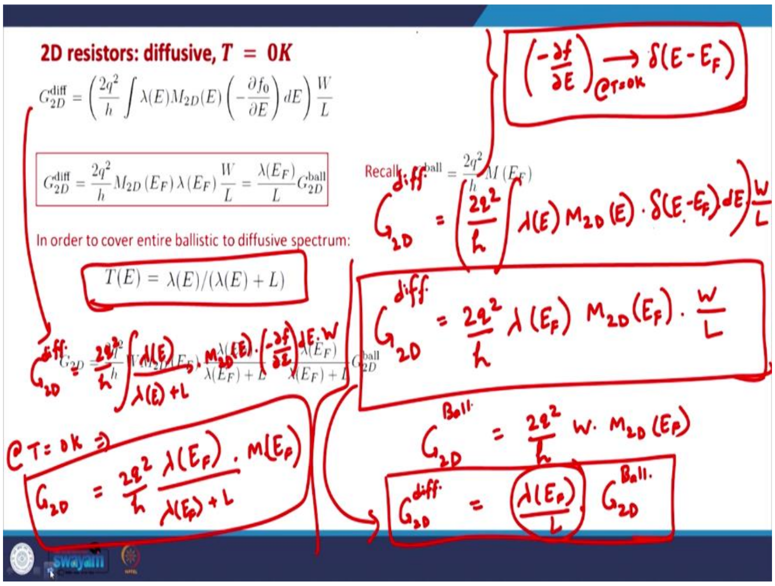

So, let us first see what happens at T equal to 0 kelvin and like we know this Fermi window function $(-\frac{\partial f}{\partial E})$ function this boils down to a delta function at Fermi level at T equal to 0 kelvin. So, at T equal to 0 Kelvin this becomes a delta function and this greatly simplifies the expression or this integrations. So, now, a 2D diffusive conductor the conductance can be written as $\frac{2q^2}{h}\int \lambda(E) M_{2D}(E)(-\frac{\partial f}{\partial E})dE$ can be written as $\delta(E - E_F)$dE and all this times W/L ok.

So, if we have a delta function in the integral this essentially simplifies the integral like this; now this $\lambda(E_F) M_{2D}(E_F)$ will be there. So, this is essentially a conductance in the case of diffusive transport when the transport is purely diffusive ok.

So, and if we try to write it down in terms of ballistic transport the ballistic conductance is given as $\frac{2q^2}{h}$ at 0 kelvin $M_{2D}(E_F)$ this is the ballistic conductance. So, this diffusive conductance for a 2D conductor can be written in terms of ballistic conductance as $\frac{\lambda(E_F)}{L}$ times G_{2D} ballistic.

So, the only difference is of this term. Now, we need to in case of diffusive conductor we need to know the scattering at Fermi level scattering length of electrons at the Fermi level, when we are trying to calculate the conductance or resistance at 0 kelvin. And this T(E) needs to be replaced by $\frac{\lambda}{L}$. And in a more general case when we are treating or when we are considering both ballistic and diffusive conductors, we need to replace T(E) by $\frac{\lambda(E)}{L+\lambda(E)}$.

And in that case this expression will simplify to instead of this expression what we will have is G_{2D} diffusion will be $\frac{2q^2}{h}\int \frac{\lambda(E)}{\lambda(E)+L} M_{2D}(E)(-\frac{\partial f}{\partial E})dE$ times W. So, that is what we will be having in this case W ok. And at T equal to 0 kelvin this will essentially be the conductance of a diffusive conductor or any conductor in a more general case.

Let me write instead of writing a diffusive here, in a more general case the conductance at 0 kelvin is $\frac{2q^2}{h}\frac{\lambda(E_F)}{\lambda(E_F)+L}$ times $M(E_F)$. Where $M(E_F)$ is W times M_{2D} (E_F); so, this will cover both ballistic and diffusive cases ok.

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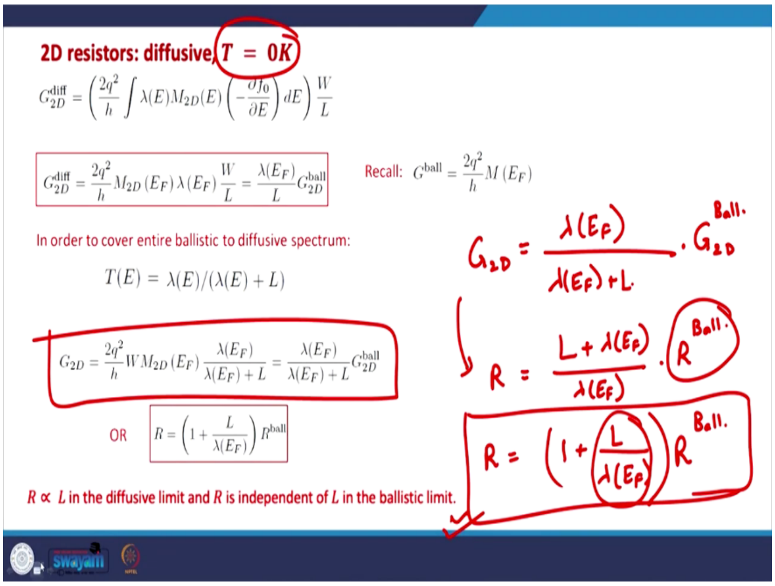

So, as always the calculations at T equal to 0 Kelvin are not difficult and this is how it would look like. So, we can also write it down as G_{2D} is $\frac{\lambda(E_F)}{\lambda(E_F)+L}$ times G_{2D} ballistic conductance. Or the resistance which is essentially the inverse of the conductance is $\frac{\lambda(E_F)+L}{\lambda(E_F)}\ \frac{1}{G_{2D} ballistic}$ will be the ballistic resistance. So, this will be essentially 1+ $\frac{L}{\lambda(E_F)}$ into ballistic resistance that is the case when we have T equal to 0 Kelvin.

And as you can see that this ballistic resistance is independent of length, but the diffusive resistance is not independent of the length. And as we go deep into diffusive limit as we make the conductor large enough in that case this term dominates and resistance becomes directly proportional to the length of the conductor.

As is the case in conventional transport theory that we sort of with which we began our discussion ok. So, this was a low temperature case extremely low temperature case actually. And as we saw in the case of ballistic transport, things are bit difficult when we have high temperature or normal temperature because the max is difficult in that case.

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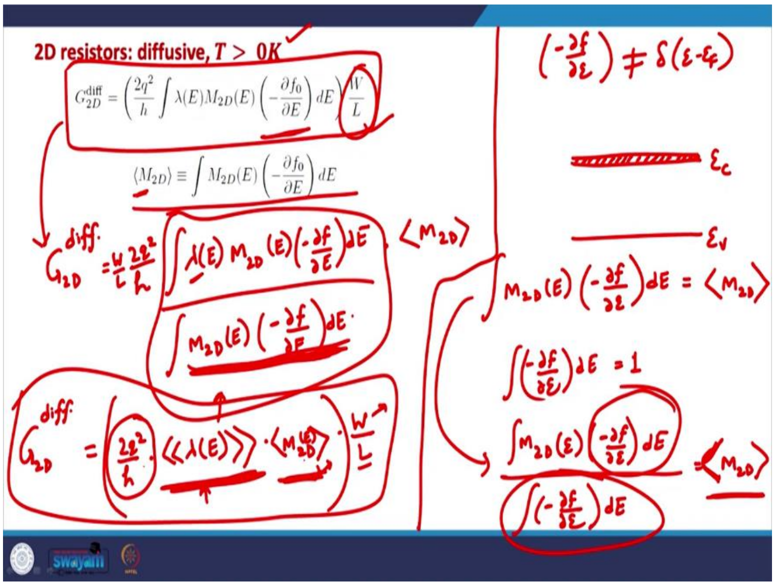

Because in that case this in this expression of conductance, this $(-\frac{\partial f}{\partial E})$ function this cannot be replaced by the delta function. And we need to explicitly do this calculation in order to calculate the conductance.

And but as we have seen that with some mathematical tricks we can manage these calculations as well particularly in the case of non degenerate semiconductors. Because in the case of non degenerate semiconductors most of the electrons are close to the bottom of the conduction band. So, if we have the valence band ,the conduction band most of the conduction electrons are quite close to the bottom of the conduction band.

And in this case even the Fermi Dirac function can be approximated by the exponential function. Things are more difficult when we have to deal with degenerate semiconductors or extremely highly doped semiconductors ok. So, at T greater than 0 kelvin temperatures at room temperature also we begin with this expression. And now we sort of make this replacement; let us say that we write it down we write this term $M_{2D}(E)$ $(-\frac{\partial f}{\partial E})$. This we can write down as the average of M_{2D} over all energy values. And the reason we can do that is because this integral is $\left(-\frac{\partial f}{\partial E}\right)$by this the area under Fermi window is 1.

So, this can be written as $M_{2D}(E)$ $\left(-\frac{\partial f}{\partial E}\right)$dE divided by $(-\frac{\partial f}{\partial E})$ dE; because this denominator is anyway 1. So, we can always put this term in denominator and this is essentially the average of the function $M_{2D}(E)$. This average is taken over all energies, O in the Fermi window. And what it essentially means is as we have also discussed at some point earlier that this means that this is the number of modes averaged in the Fermi window ok.

So, now, what we do is we divide and multiply this term by this average. So, G_{2D} diffusion conductance is $\frac{2q^2}{h}\int \lambda(E)M_{2D}(E)(-\frac{\partial f}{\partial E})dE$ we multiply and divide by this average function. And in the denominator we write the expanded version of this average which is $M_{2D}(E)(-\frac{\partial f}{\partial E})dE$ and of course, we have W/L as well from here ok.

So, now this is the conductance at normal temperatures. If we have a closer look at this expression, this expression is also like an average of a function average of essentially this function λ(E). And now, this average is taken over this function $M_{2D}(E)(-\frac{\partial f}{\partial E})dE$. So, what it intuitively mean is that we can write it down as G_{2D} diffusion conductance at normal temperatures to be $\frac{2q^2}{h}$ average.

We have used a different sign for this average just to make a distinction between the average of $M_{2D}(E)$ and λ(E). Because these 2 averages are of different kind, this average is taken over the Fermi window and this average is taken over the modes in the Fermi window times W/ L.

So, now, what we can essentially, intuitively say is that, the conductance depends. Conductance is; obviously, is given by the quantum of conductance multiplied by the average or average number of modes in the Fermi window times the average mean free path of electrons in the modes in the Fermi window.

So, this function intuitively means this is the average mean free path of electrons in the modes in the Fermi window. So, from this expression we can intuitively say that this is the average mean free path of electrons in the modes where conduction happens. And the modes where conduction happens is the modes in the Fermi window times W/L.

So, as is evident here conductance is inversely proportional to the length, directly proportional to the width as we expect in the case of a diffusive conductor ok. So, this is the core sort of result here and these calculations if we know the exact expression for lambda E we can do these calculations and this can be calculated with the help of the Fermi Dirac integrals ok.

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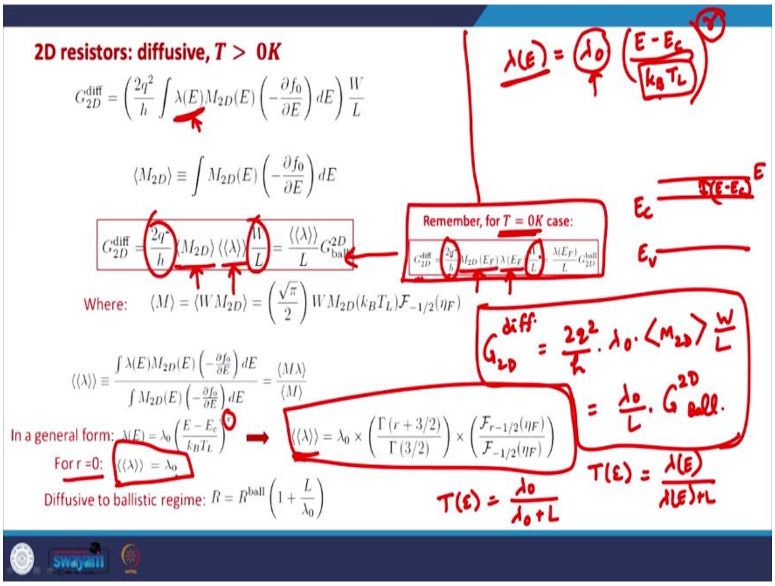

So, this is essentially how it looks like we will not do the exact calculations, because this function λ(E) is yet not very clear to us. We have not derived this because we have not dealt with the scattering in our analysis yet. And so, this is if you recall the T equal to 0 Kelvin case, this was the conductance of a diffusive conductor at 0 Kelvin and it was given by $\frac{2q^2}{h}\lambda(E_F)M_{2D}(E_F)\frac{W}{L}$.

But now, in this case the only difference that we sort of see is this $\frac{2q^2}{h}\frac{W}{L}$ is there these 2 terms are there. And instead of $M_{2D}(E_F)$ we have average of $M_{2D}(E)$ in the Fermi window. And instead of mean free path at Fermi level energy, we have the average mean free path of electrons in the modes in the Fermi window. So, this is sort of a generalization of the T equal to 0 Kelvin case ok. And again this is average of $M_{2D}(E)$ for a non degenerate conductors is given by this value.

This we have already seen and this average of or this double average sort of mean free path can be calculated from this expression. And here we need to put we might need to use the Fermi Dirac integrals and the expression for the number of modes. And so, roughly we will sort of we observe that as I pointed this to you that we have not gone into the scattering theory.

So, this expression the expression for λ(E), we have not explicitly derived yet. But generally, this expression λ(E) looks like this,this is dependent on the electron energy above the bottom of the conduction band and there is an exponent R here in this equation. There is a constant λ_O which is actually not constant per se it is independent of energy, but it depends on the temperature. And this is this k_B is Boltzmann constant, T_L is the lattice temperature.

So, this scattering essentially depends on the difference between the electron energy and the bottom of the conduction band. So, if this is roughly the band structure and the electronic energy is this, this difference between these the electron energy in the bottom of the conduction band this will govern the mean free path of the electrons in the crystal or in the channel. And this exponent actually depends on the kind of scattering mechanism that is present in the crystal.

So, for example, so, the scattering can happen because for example, because of the phonons because of the lattice vibrations scattering can happen. And the scattering can also happen because of the interaction with external let us say electromagnetic field or external or impurity atoms, ionic impurities which are present inside the crystal or may be surface atoms.

So, there are multiple kind of scattering mechanisms and this R is dependent on them. Generally using this formula for λ(E), this average of lambda is given by this expression. This is a detailed calculation we are not going into the details of this calculation, but this is how it would look like. And you can try to see where this comes from and generally for the acoustic phonons when we have phonons travelling like sound waves this exponent R is 0.

And in that case this average of λ is equal to λ_O for R equal to 0 case. And in that case this entire calculation actually simplifies and in that case we can write the conductance or the diffusive conductance to be $\frac{2q^2}{h}\lambda_O < M_{2D}(E_F) > \frac{W}{L}$ or $\frac{\lambda_O}{L}$ G_{2D} in ballistic case.

And in ballistic case we have already done the calculation; so, this calculation is actually becomes quite straight forward. And if we take this general expression for transmission coefficient which is T(E) is equal to $\frac{\lambda(E)}{L+\lambda(E)}$ in that case. For R equal to 0 case for just for the acoustic phonon scattering case T(E) becomes equal to $\frac{\lambda_O}{L+\lambda_O}$.

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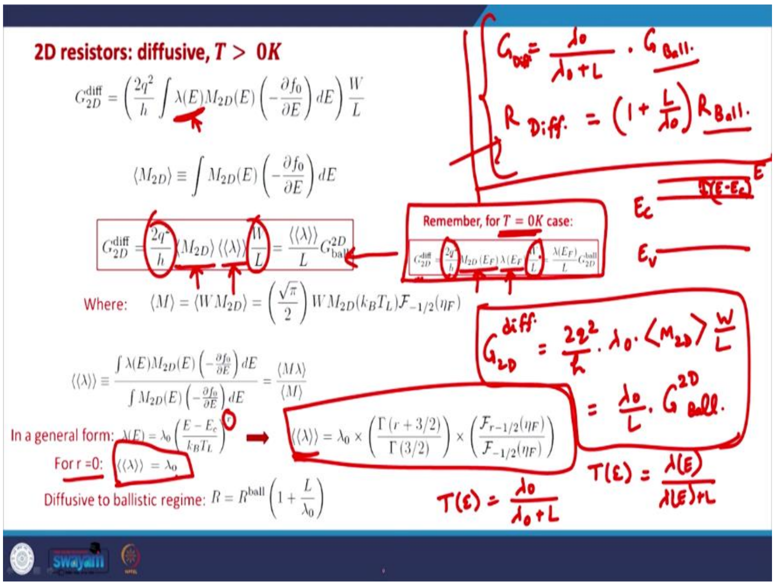

And this the conductance will be essentially $\frac{\lambda_O}{L+\lambda_O}$ into G ballistic conductance or the resistance in diffusive case will be inverse of that which is essentially $1 + \frac{L}{\lambda_O}$ into the ballistic resistance. And these calculations we have already done the calculation of ballistic conductance and resistances at room temperature. So, these calculations can be done similarly.

So, this essentially completes our discussion on the calculation of conductance and resistance for ballistic and diffusive conductors both at low temperature limits where calculations are easier and high temperature limit where calculations are bit difficult. But, with the help of Fermi Dirac integrals the calculations can be simplified, and especially for non degenerate semiconductors these calculations can be easily done.

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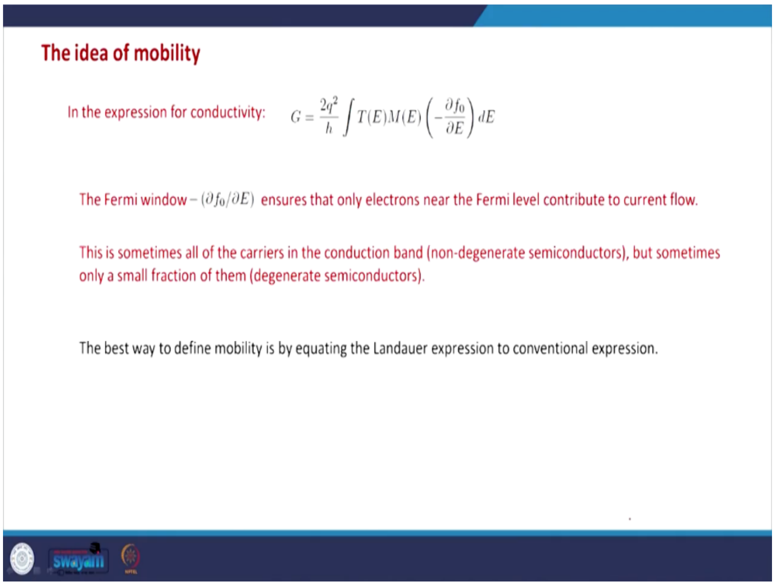

So, with this, the final idea will be the idea of mobility that we will start in our next class. So, thank you for your attention, see you in the next class where we will discuss this idea of mobility in ballistic and diffusive cases.

Thank you.