Physics of Nanoscale Devices
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Lecture - 30
Resistance: Ballistic and Diffusive Cases-III

Hello everyone, we have been discussing the idea of resistance in Ballistic and Diffusive cases. And, since last class we started doing actual calculation of resistance and comparing that to the conventional idea of resistance that we generally have from our classical theory of transport.

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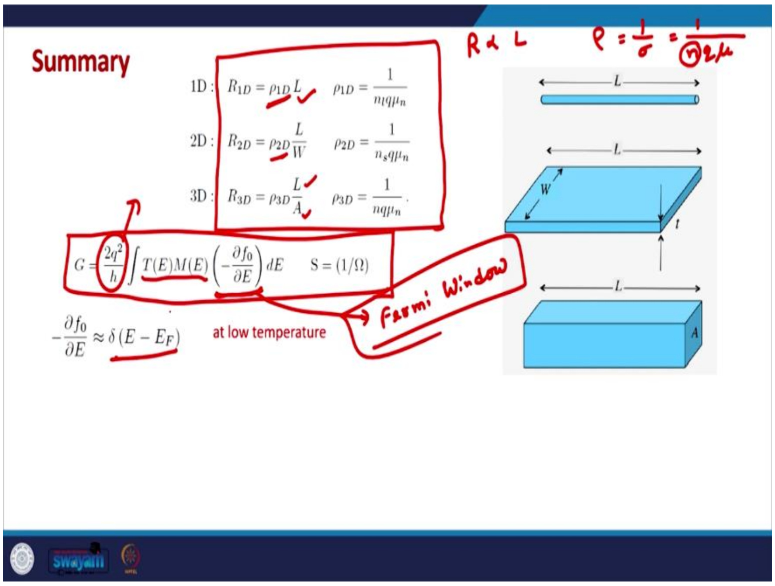

And this is what we understand from the classical theory of transport. That the resistance is given is generally proportional to length; resistance is considered to be proportional to length in 1D case. In 2D case, resistance is directly proportional to length inversely proportional to width and in 3D case resistance is directly proportional to length and inversely proportional to the area.

And the constant of proportionality is known as the resistivity denoted by ρ which is essentially the inverse of conductivity that is our classical understanding of resistance and conductivity is given as n times q times μ where μ is the mobility of the carriers and this n will be the number of carriers per unit length in 1D conductor it will be the number of carriers per unit area in 2D conductor and it will be number of carriers per unit volume in 3D conductor.

In contrast to this our discussion of conductivity starts with this expression which we obtain from the general theory of transport or the general model of transport and in this expression we have this constant which is known as the quantum of conductance and in integral we have T(E) M (E) integrated over this function which is also known as the Fermi window function.

So, the Fermi window function is $-\frac{\partial f}{\partial E}$. And, with this expression we started doing calculation of resistance for ballistic case in our previous discussion and there we saw that at 0 kelvin at extremely low temperatures this function the Fermi window function boils down to a delta function which essentially simplifies this integral greatly and the conductance that we obtain is essentially this.

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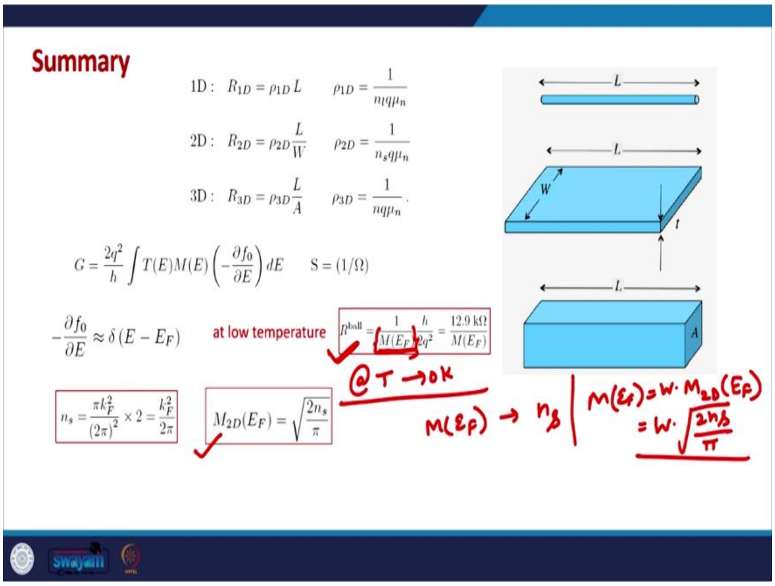

So, at low temperatures when T approaches 0 kelvin in that case this is the ballistic resistance of the device. So, in this case what just needs to be done is, we just need to calculate this $M(E_F)$ parameter which is the number of modes at energy E_F at 0 kelvin.

And we also saw that this term $M(E_F)$ term this can be related to the sheet charge density number of electrons per unit area in a 2D conductor and the relationship looks like this.

So, the M(E$_F$) value can be written as w times M$_{2D}$(E$_F$) where M$_{2D}$(E$_F$) actually turns out to be $\sqrt{\frac{2n_s}{\pi}}$. This we saw from our previous discussions.

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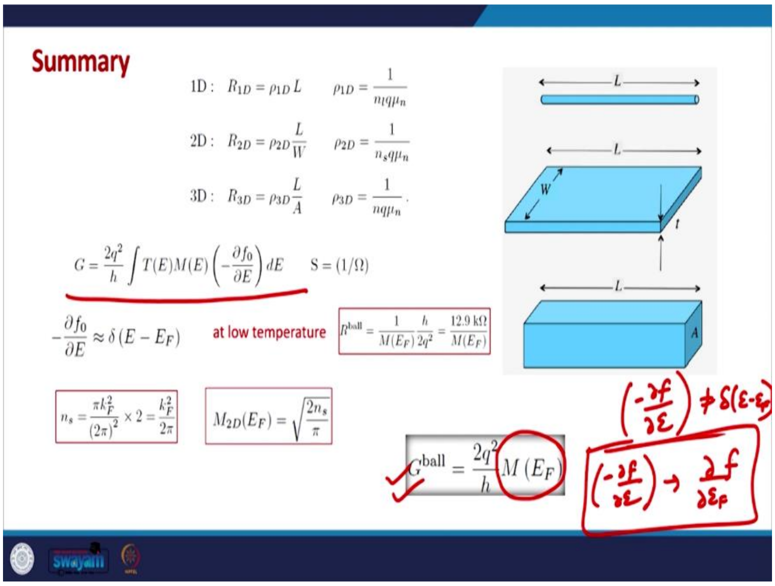

And finally, this is the conductance that we obtain and by putting an appropriate value of M(E$_F$) we obtain the value of the conductance. So, all this was true for low temperatures when T approaches 0 which means Fermi function is approximately a unit function and Fermi window is a delta function; in those cases the calculations are easy.

Then we started doing calculation for above 0 temperature cases, at room temperature cases. So, at those temperature values we realize that this Fermi window function can no longer be approximated by a delta function. And, in this case this integral needs to be evaluated properly and now this integral also has a derivative with respect to E which can be converted to derivative with respect to E$_F$ ok.

And after doing calculations at room temperature for the conductance we realize that we come across a special type of integral functions those one we defined as Fermi-Dirac integral function those are the Fermi-Dirac integral functions. So, generally in these calculations we need to know about the Fermi-Dirac integral functions.

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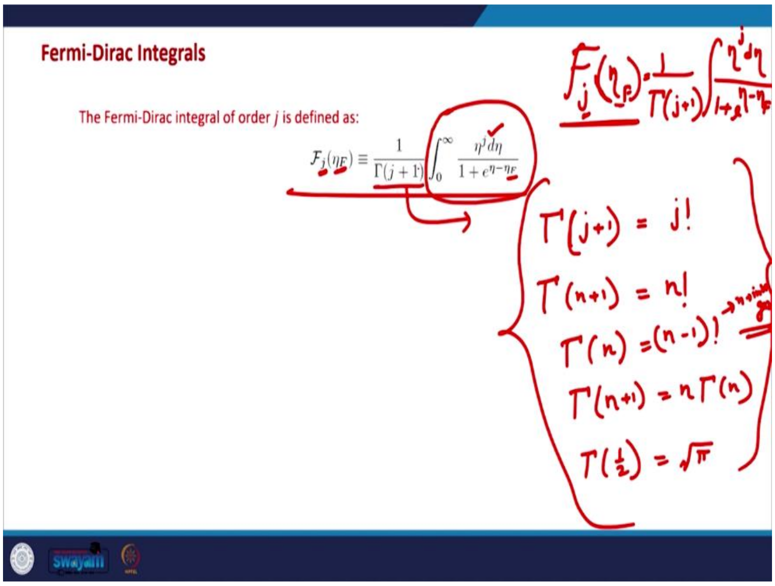

And that is what we will quickly go through a Fermi-Dirac integral function looks like this. It has a Fermi Dirac integral function of order j-- $F_j(\eta_F) = \frac{1}{\Gamma(j+1)} \int \frac{\eta^j d\eta}{1+e^{\eta-\eta_F}}$. This is how a Fermi-Dirac integral function of order j is defined.

The parameter or this Fermi-Dirac integral function has the variable η_F in its argument. Please remember this, on the right hand side we have both η and η_F, but in the argument of the Fermi-Dirac integral function η_F is there ok.

The order j depends on the exponent of η in the numerator on the right hand side of this function. So, if we closely look this j the order of the Fermi Dirac integral function it depends on the exponent of the parameter η and the argument of the function is η_F.

This Fermi-Dirac integral has this gamma function as well in its expression. And, most of us would be aware that a gamma function. If j is an integer this is equal to factorial j or if n is an integer gamma function of n plus 1 is essentially factorial n or more popularly it is written as gamma function of n is (n-1)! ok. There is another property of gamma functions which is gamma function of n plus 1 is n times gamma function of n and gamma function of half is essentially root pi ok.

So, this is essentially the summary of the gamma function; and with this gamma function we can define the Fermi-Dirac integral function in which there is an integral which looks like this which is essentially integral over parameter eta of the term $\frac{\eta^j}{1+e^{\eta - \eta_F}}$. And along with this we have a normalization factor of $\Gamma(j+1)$.

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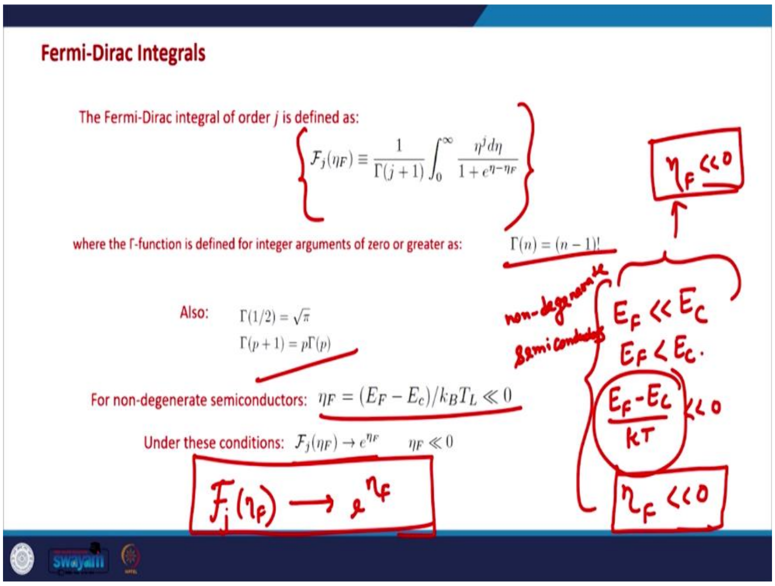

So, this is what we have. This is what we just discussed for the gamma functions and there is for non degenerate semiconductors. So, there are generally depending on the doping of semi conductors, semi conductors are divided in two types one is the degenerate semiconductors and second is non degenerate semiconductors.

Generally in moderate doping situation, when the doping is not too much which means that the dopant atoms are far away from each other they are not interacting with each they are not talking to each other, the potential due to one dopant atom is not felt by the another dopant atom. In that case the doping is known as the or the semiconductor in that doping is known as the non degenerate semi conductor.

And generally the Fermi function is way below than the bottom of the conduction band. So, in that case in non degenerate cases E_F is less than E_C generally or at least E_F should be sufficiently below E_C which in other words is written as E_F minus E_C can be written as kT is less than 0 very it is much smaller than 0 ok.

And, this thing we if you remember this is what this parameter η_F is. So, in non degenerate semi conductors, generally η_F is quite smaller it is quite it is a negative number and this is less than 0 and in those cases this Fermi-Dirac integral of order j in this case in non degenerate case this please remember that this is the case of non degenerate semi conductors.

And in these cases η_F parameter is less than 0 . So, in these cases this Fermi-Dirac integral can be approximated by exponential of η_F. So, this is an important approximation which is true in most of the cases because in most of the cases our semi conductor is non degenerate.

So, this can be approximated by. So, these are few mathematical I would say bits that if we keep in mind these calculations those the calculations that look so difficult, those calculations will become easier ok. So, just keep these all these things in mind otherwise this evaluation of the integral in conductance becomes extremely difficult ok.

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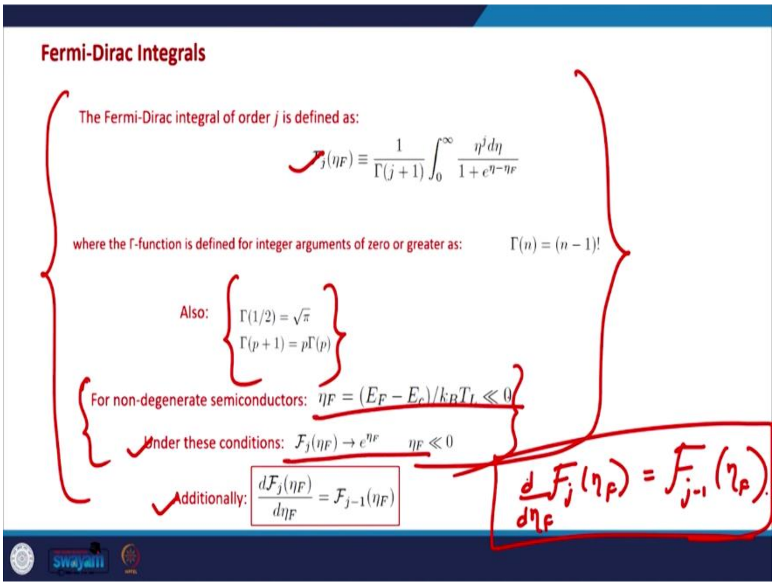

In addition to all these properties I also mentioned last time that if we take a derivative of Fermi-Dirac integral with respect to parameter η_F, the order of the Fermi-Dirac integral decreases by 1 just that. And this is useful because in addition to integral we also have a derivative in our conductance expression.

So, this thing and this along with these, they will greatly simplify all our calculation. So, just keep these things in mind; you do not need to remember these things because not to complicate things in too much of details, but these mathematical bits are discussed just so that the calculations do not look too difficult to you.

If we remember these few points the calculations involving the Fermi-Dirac integrals are not extremely are not that difficult, we can manage them ok; specially in the non degenerate semiconductor cases ok.

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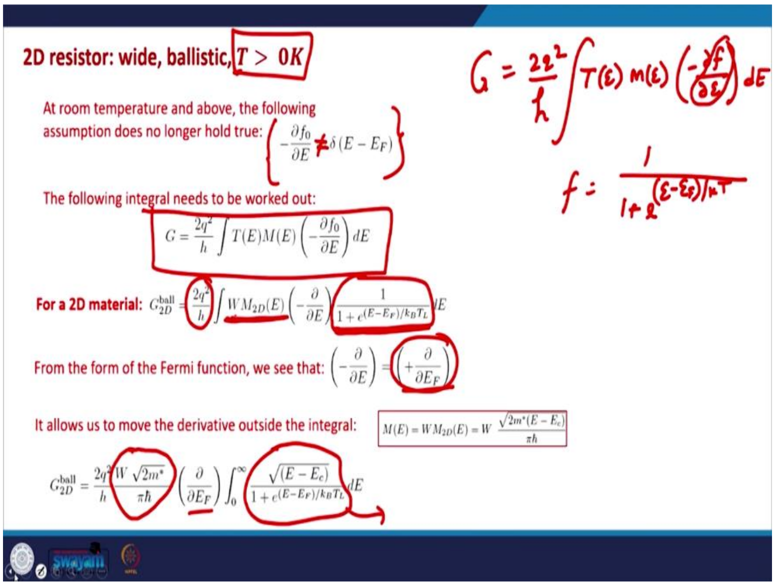

So, with this we continue our calculation that we were doing last time in last class we were discussing the resistance of a ballistic conductor, a wide ballistic conductor at room temperatures I would say or higher than 0 temperatures. So, in this case we were doing the calculations and we saw and I also mentioned this that in this case this Fermi window can no longer be approximated by this delta function.

And so, that is why we again need to start from this expression of the conductance. Please remember that the expression of conductance to begin with in any cases this $\frac{2q^2}{h}\int T(E)M(E)(-\frac{\partial f}{\partial E})dE$.

And, if we explicitly write down the Fermi function in this derivative in this Fermi window this is how it looks like Fermi function is $\frac{1}{1+e^{\frac{E-E_F}{k_BT_L}}}$ in these expressions where k_B is the Boltzmann constant and T_L is the lattice temperature, we are in short writing kT ok.

So, with this we make this replacement $(-\frac{\partial}{\partial E})$ is replaced by $(\frac{\partial}{\partial E_F})$ which allows us to bring this $(\frac{\partial}{\partial E_F})$ outside the integral. So, inside the integral what is left is this constant is outside and inside the integral we are left with terms involving this $M_{2D}(E)$ function this number of modes in a 2D conductor and this Fermi function.

And, this is essentially all these terms apart from these terms these were constants and these constants come out of the integral. So, finally, this can be written down in terms of Fermi-Dirac integral and as you can see here η to the power half will be there.

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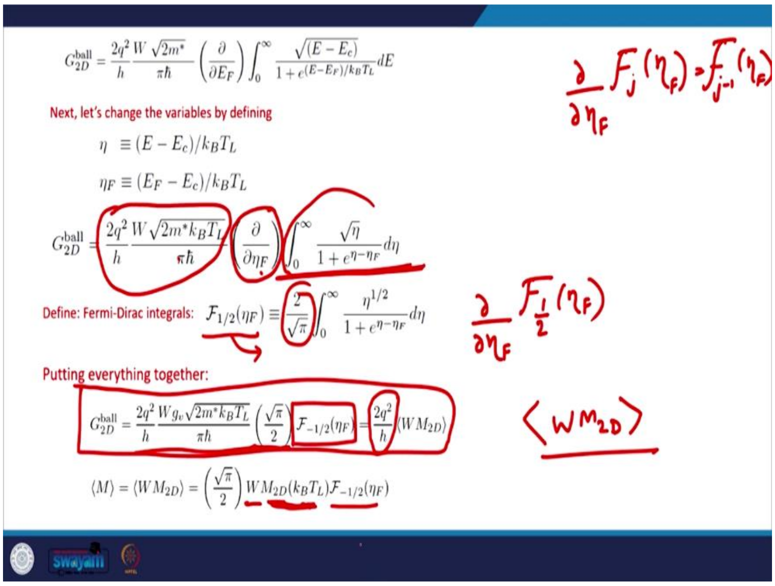

So, if we make this replacement η is equal to $\frac{E-E_C}{kT}$ and η_F is $\frac{E_F-E_C}{kT}$, and if we do this replacement apart from this constant term what is there is we are left with a derivative with respect to η_F and in integral with respect to $\frac{\sqrt{\eta}}{1+e^{\eta\,-\,\eta_F}} d\eta$ this we saw in our last class as well.

And, today with a bit of background of Fermi-Dirac integrals this thing is now more clear, I hope this is more clear now. So, as we can clearly see this is the Fermi Dirac integral of order half and with some constant unit and if we make the replacement if we replace this integral by the Fermi-Dirac integral of order half then what is left is apart from this constant term.

Now, in addition to this constant we also have $\frac{\sqrt{\pi}}{2}$, we are left with $\frac{\partial}{\partial \eta_F}$ Fermi-Dirac integral of order half $F_{\frac{1}{2}}(\eta_F)$ ok. And as we saw that if we take a derivative of the Fermi Dirac integral of order j with respect to η_F then it becomes the Fermi-Dirac integral of order j minus 1. So, the order is reduced by 1.

So, ultimately this becomes Fermi-Dirac integral of order minus half ok. So, finally, the conductance of a ballistic conductor a void ballistic conductor at normal temperatures above 0 kelvin temperatures is given by this expression.

And, in this expression please keep in mind that apart from this constant term $\frac{2q^2}{h}$, everything else is written as some sort of average of W times M_{2D} where this average of W times M_{2D} is $\frac{\sqrt{\pi}}{2}$ W M_{2D}(kT) $F_{-\frac{1}{2}}(\eta_F)$ ok.

And as we saw from our as I just mentioned few minutes back that in the case of non degenerate semiconductors generally the Fermi-Dirac integral function can be evaluated quite easily it becomes just the exponential of that order.

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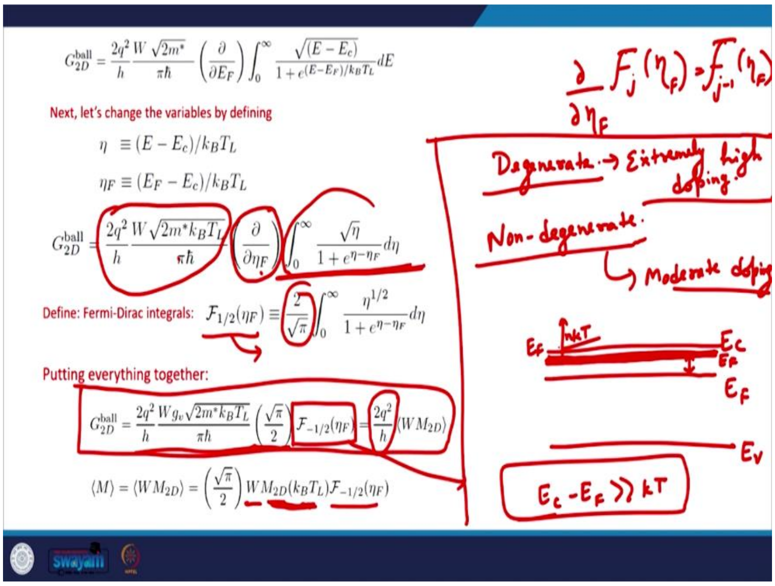

There are also few things that I would like to sort of highlight here I mentioned that the semi conductors can be of two type degenerate or non degenerate semiconductors.

So, when in moderate doping, in realistic doping situations the semiconductors are non degenerate semiconductors and when we have extremely high doping generally we have degenerate semiconductors. And, in degenerate semiconductors the Fermi level lies quite close to the conduction band it may even lie inside the conduction band.

So, if the Fermi level is if this is the top of the valence band this is the bottom of the conduction band and this is the Fermi level, Fermi level is somewhere in between or slightly closer to the or if this is the Fermi level. This kind of conductor is semi conductor is non degenerate semi conductor because this distance this E_C minus E_F is many kT ’s ok.

But if the doping is extremely high, the dopants are placed very close to each other in the semi conductor this E_F comes very close to the conduction band edge or it may even go inside the conduction band. So, the Fermi level may lie here. So, in the regenerate semi conductor case, as you can imagine that the electrons in the conduction band will be spread in many or the electrons will be the conduction band will be more filled in a way and electrons will occupy many kT energy state.

So, it will be n kT many kT energy states, but in the case of non degenerate semi conductors the electrons in the conduction band are not too many . So, if we talk about the non degenerate normal cases.

In non degenerate semi conductor cases there are only electrons are only there in a small range of energy values or around 1 or 2 kT energy range. So, just above the bottom of the conduction band only up to 1 or 2 kT energy values, the charge carriers are there in the non degenerate semi conductor cases.

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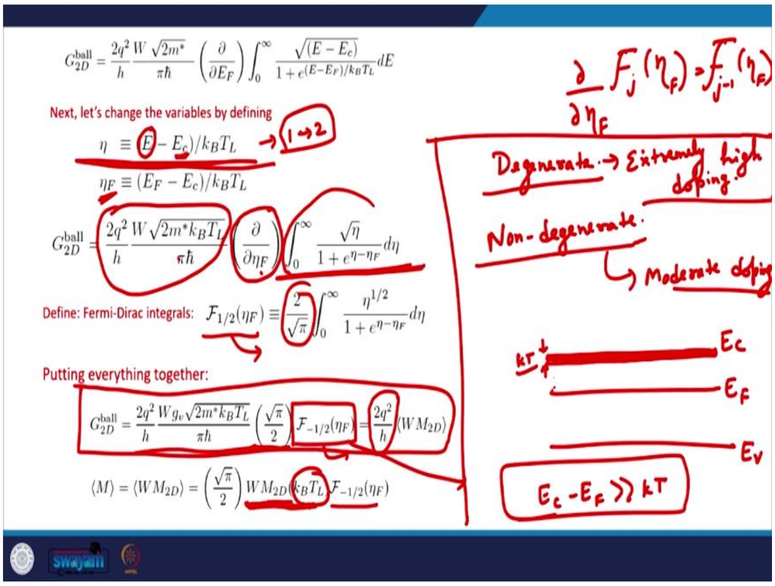

So, that is why these Fermi-Dirac integral in the case of non degenerate semi conductor cases that is why we can replace this η_F to be quite less than 0 or we can evaluate or this can be this number of modes can only be evaluated for certain kT values of energy ok. So, this is sort of a restatement of the fact that we discussed in the last slide that is this condition essentially.

So, in this case in non degenerate semi conductor cases even eta is quite close to 1, because now the energy values of electrons is quite close to E_C it is only 1 or 2 kT within 1 or 2 kT values of E_C. So, this value is also within 1 or this is around 1 to 2, mostly around 1.

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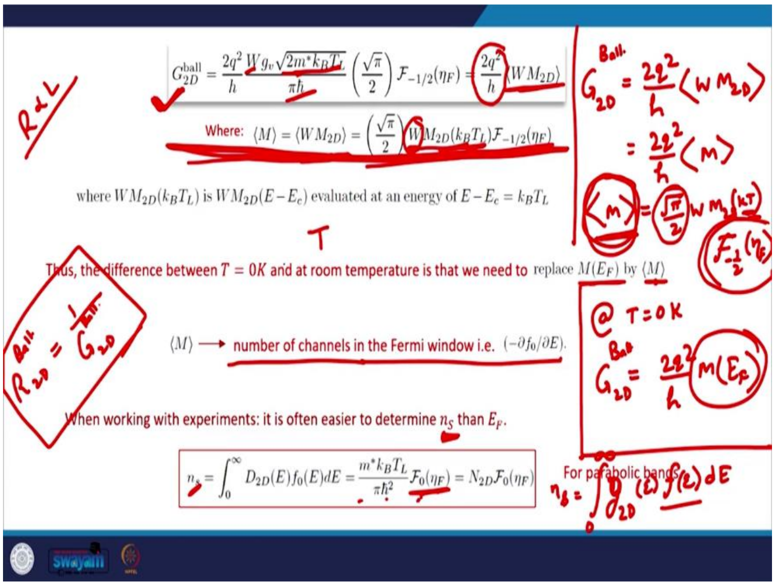

So, with this; now this thing this expression of conductance of a ballistic conductor at above 0 kelvin temperatures this is what we obtain. And, we have now written it down as $\frac{2q^2}{h}$ times average of W into M_{2D} and the reason and this average of W into M 2D is essentially $\frac{\sqrt{\pi}}{2}$ W M_{2D}(kT) and this W M_{2D}(kT) is actually W times this one ok.

So, what I mean to say is that this constant here in our expression; this constant is essentially W times M_{2D} evaluated at kT and this is justified in the case of non degenerate semiconductors because electrons only occupy around 1 or 2 kT or in most of the cases around 1 kT of energy range in the conduction band. So, this accounts for most of the electrons in the non degenerate semi conductors. So, that is why these constant terms can be written as average of W times M_{2D} ok.

So, what we finally realize is that the conductance in a ballistic conductor at room temperature let us say can be written as $\frac{2q^2}{h}$ W times M_{2D} or it is written as $\frac{2q^2}{h}$ average of M where this average of M is essentially M_{2D} (kT) $F_{-\frac{1}{2}}$.

So, what this means is that this average of M is the average of modes in the Fermi window in the energy range of kT that is true for the non degenerate cases. And if you recall that at T equal to 0 kelvin the conductance was essentially $\frac{2q^2}{h}$ $M(E_F)$. So, what changes at room temperature is this. So, instead of $M(E_F)$, now we need to use M average of M.

So, this average of M is the average of modes inside the Fermi window essentially. And the Fermi window also at room temperature generally is only around 1 kT above the conduction band edge ok. So, that is the only difference that we see in our treatment of or in our calculation of conductance at room temperature. We just need to replace $M(E_F)$ by average of M where average of M is given by this expression and these are the number of modes or number of conducting pathways in the Fermi window.

And as we also saw in our previous case, in the case when we just had this experimentally available parameter ns sheet carrier density of electrons we could calculate the conductance and resistance in terms of ns.

Similarly, in this case we can calculate or we can have a relation between the conductance and the sheet carrier density in this way the sheet carrier density is defined as is pretty evident sheet charge density is this comes from this is the density of states in a 2D conductor times the Fermi function D(E)f(E) and now this needs to be evaluated for all energy values at room temperature.

And this turns out to be $\frac{m^* kT}{\pi h^2} F_0(\eta_F)$. And, by sort of dividing these two expressions we can have a relation between the two. So, the resistance will be just the inverse of the resistance of a ballistic conductor at room temperature will be just the inverse of the conductance.

And, as we saw from our classical theory of transport that the resistance is always directly proportional to length, but in this case both at 0 kelvin and at room temperature resistance is independent of the length in the case of 2D conductor.

And similarly we can also see for 1D and 3D conductors that resistance is independent of the length of the conductor ok. It depends, it is inversely proportional to the width because the conductance is directly proportional to the width. So, the resistance will be inversely proportional to the width of the conductor ok.

And that is also understandable because more the width more will be the number of modes and less will be the resistance or more will be the conductance. So, this is how we can do calculation for conductance and resistance in realistic situations for a ballistic conductor.

Now, the next topic that we will begin with we will quickly calculate the conductance and resistance of a diffusive conductor the calculation is pretty similar to what we did for the ballistic conductor case. In that case , in addition to everything else we would also have this transmission coefficient.

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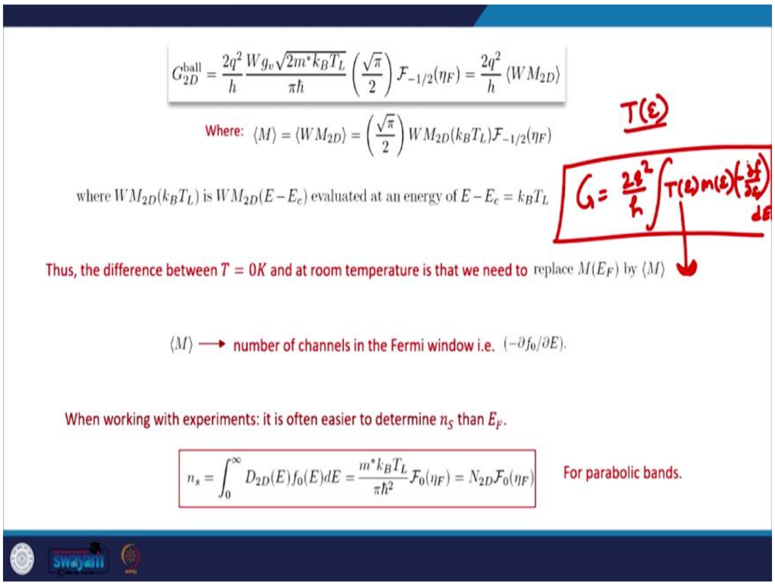

So, in this expression of conductance $\frac{2q^2}{h}\int T(E)M(E)(-\frac{\partial f}{\partial E})dE$, we would also have to include the transmission coefficient in the case of diffusive conductor and the those calculations we can now quickly do because we are now familiar with the Fermi-Dirac integrals we are now familiar with how to sort of deal with these kind of calculations ok. So, that will that is what we will see in the next class.

Thank you for your attention, see you in the next class.