Physics of Nanoscale Devices
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Lecture - 29
Resistance: Ballistic and Diffusive Cases-II

Hello everyone, as you know we are having a discussion on Resistance as derived from the formalism that we discussed and as we understand from our classical understanding of semi conductor devices. So, we are contrasting these two type of or the resistance from these two formalisms.

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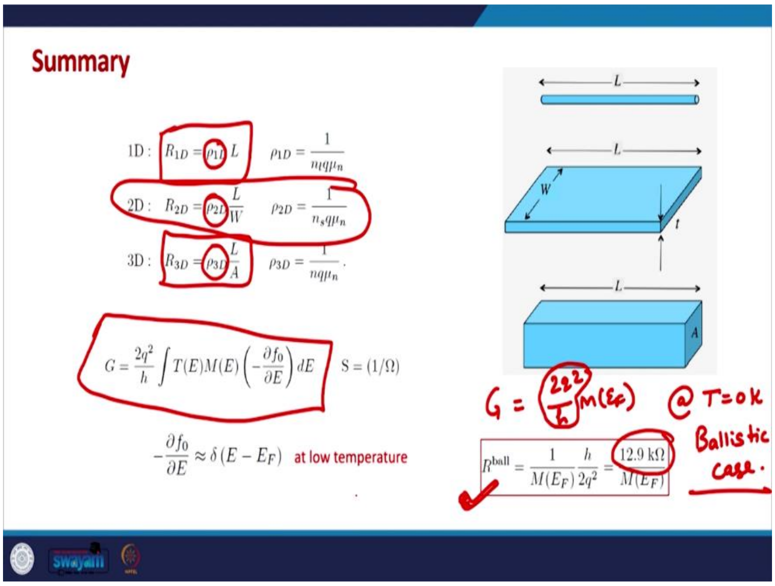

And, the classical understanding of resistance says that in a typically in a 1D, 2D, or 3D in any kind of conductor the resistance is directly proportional to the length of the conductor like this. In case of 1D conductor it is just proportional to the length of the conductor, in case of 2D conductor it is directly proportional to the length inversely proportional to the width.

Similarly, in 3D conductor it is directly proportional to the length inversely proportional to the area. This constant of proportionality is known as the resistivity and the inverse of resistivity in the material is known as conductivity that is the classical picture of transport that we have in our mind.

From our formalism of the general model of transport we deduced this expression of the conductance of the device. It is G is equal to $\frac{2q^2}{h}\int T(E)M(E)\left(-\frac{\partial f}{\partial E}\right)dE$.

And, we started the discussion for low temperature cases, what happens at low temperatures in a ballistic conductor and at low temperatures in a ballistic conductor, the conductance turns out to be $\frac{2q^2}{h}$ $M(E_F)$ this is at T equal to 0 kelvin for ballistic case ok.

This term $\frac{2q^2}{h}$ is known as the quantum of conductance it is the conductance of the single channel of the electron in the devices and its value is $\frac{1}{12.9k\Omega}$. From this sort of derivation or this analysis we could see that for a 2D conductor 2D ballistic conductor the resistance is independent of the length as opposite to the our classical understanding, our conventional understanding of the resistance.

So, that is the key difference. So, that is why in our modern nanoscale devices we can no longer use the conventional understanding of electron transport we need to have a fundamentally different kind of theory of transport and that is what we are doing in this course. After that we are currently studying the transport in studying the resistance in ballistic conductor at 0 kelvin, but now a wide conductor that is where our discussion was stopped in the last class.

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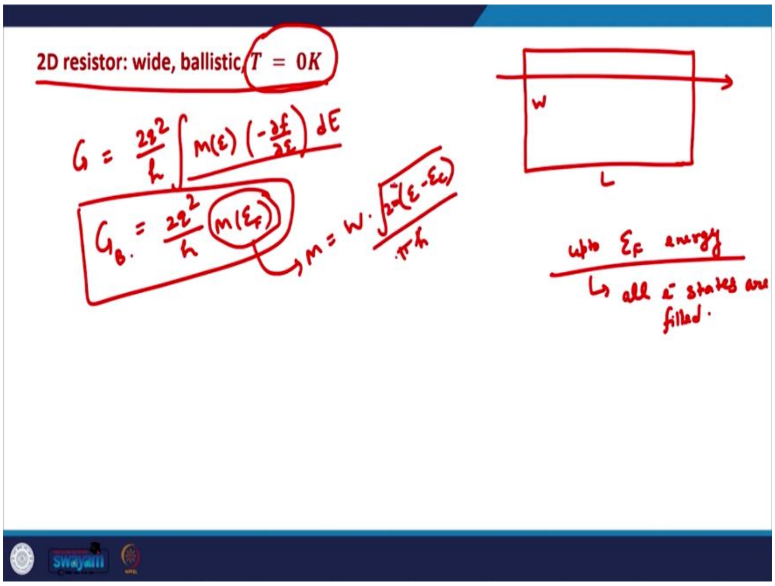

So, we have a conductor let us say of length L, width W and it is a ballistic conduction which means that electrons do not collide with anything in between they directly go from the left terminal to the right terminal and because of the temperature being 0 kelvin. Now we can say that all the electronic states up to energy E_F are filled all electronic states are filled.

So, if we need to calculate the resistance here we again begin with the same expression of the conductance which is $\frac{2q^2}{h}$ in ballistic case T(E) is 1. So, we are just left with M(E) and essentially $\left(-\frac{\partial f}{\partial E}\right) dE$ and at 0 kelvin this expression essentially boils down to M(E_F) this is the ballistic conductor at 0 kelvin. Now, this term this M(E_F) this we can put it the form of this from our derivation which is essentially W times $\sqrt{\frac{2m^*(E-Ec)}{\pi\hbar}}$ as well.

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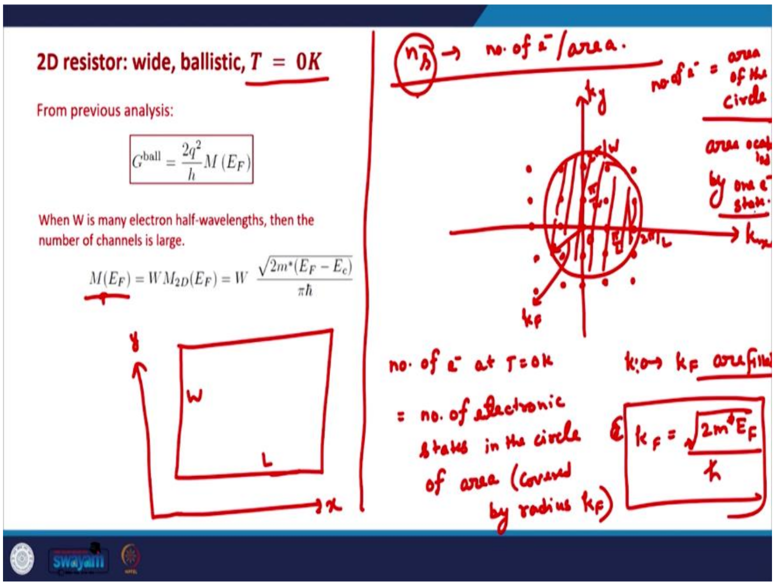

Or we can correlate this term to an experimental parameter which is generally the sheet carrier density in a 2D conductor which is the number of electrons per unit area. So, this is given this or this can be deduced from the experiments for example, I gave you an example of Hall Effect experiment. So, if we know the hall coefficient of the material we can deduce the sheet carrier density by measuring the hall voltage of that particular material.

So, this parameter is generally available from the experiments and we would like to correlate this $M(E_F)$ with this parameter and for that we need to go back to the k space. So, this is k_x, k_y the conductor physically looks like this. In x direction it has certain length, in y direction it has certain width.

So, the allowed k points or the values of the k points where a valid electronic wave function exists in this material is given by the solution of the Schrodinger equation and we obtain these kind of points these are essentially $\frac{\pi}{L}, \frac{2\pi}{L}$, 0 on the y axis we have $\frac{\pi}{W}, \frac{2\pi}{W}$ and so on.

And, since the temperature is 0 kelvin all the energy states up to E_F are filled which means that all the k points up to k equal to k_F are filled k from 0 to k_F are filled where, k_F is given by $\sqrt{\frac{2m^* E_F}{\hbar^2}}$, this comes from the E k relationship ok.

So, the total number of electrons will be inside this circle of radius k_F, if this is the radius of the this circle is k_F, the total number of electrons or the sheet carrier density in the material will be given by the area of this circle in the k space and what is the area here or number of electrons at T equal to 0 kelvin will be number of electrons more precisely electronic states in the circle of area covered by radius k_F ok.

Now, how many electrons are there in this circle? That will be given by the number of electrons will be the area of the circle divided by the area occupied by one electron or one electronic state. So, if we divide the area of the circle by the area of one electronic state that will give us the total number of electronic states in the or total number of electrons in the system.

So, essentially this is what we need to calculate in this case and, the area of the circle is simply πk_F^2, but what is the area that a single electronic state occupies. So, in the k space if we have a close look on the k space these are the ok.

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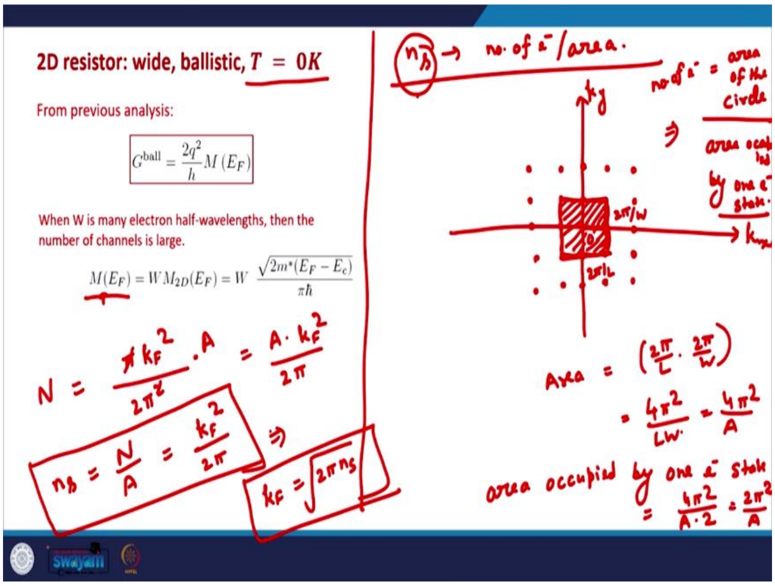

So, generally this is the area occupied by one electronic state; however, we also saw in our discussion of density of states that this set of k values and this set of k values and this set of k values and this set of k values they essentially represent the same wave function. So, what we can say is that the area occupied by an electron is this or if we consider the spin this is the area occupied by two electron.

So, two electrons, one with up spin one with down spin will occupy this amount of area. So, area of one electron will be and what is the area and this is $\frac{2\pi}{L}$ length and $\frac{2\pi}{W}$ width. So, the area will be $\frac{2\pi}{L}$times $\frac{2\pi}{W}$. So, it will be $\frac{4\pi^2}{LW}$ or $\frac{4\pi^2}{A}$, this is the area occupied by two electronic states if we consider the spins as well. So, the area occupied by one electronic state is $\frac{4\pi^2}{2A}$, so it will be $\frac{2\pi^2}{A}$ ok.

Now, we can sort of see the total number of electrons in the system from this expression. So, total number of electrons will be the area of the circle which is πk_F^2 divided by the area of the single electron which is $\frac{2\pi^2}{A}$. So, pi, so, it is $\frac{A{k_F}^2}{2\pi}$ and the sheet carrier density will be the total number of electrons divided by area.

So, it will be N/A which means n_s can be written as $\frac{{k_F}^2}{2\pi}$ ok. So, that way we have a relationship between n_s and k_F vector ok. In other words this k_F is essentially $\sqrt{2\pi n_s}$.

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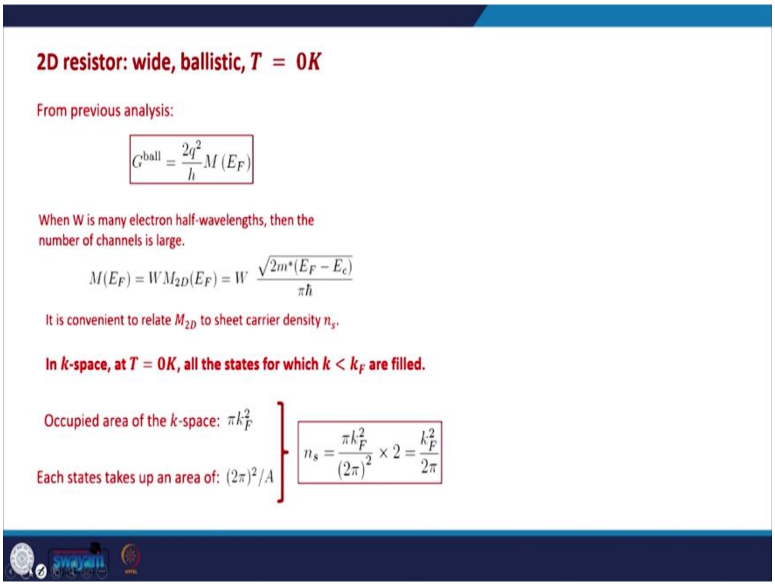

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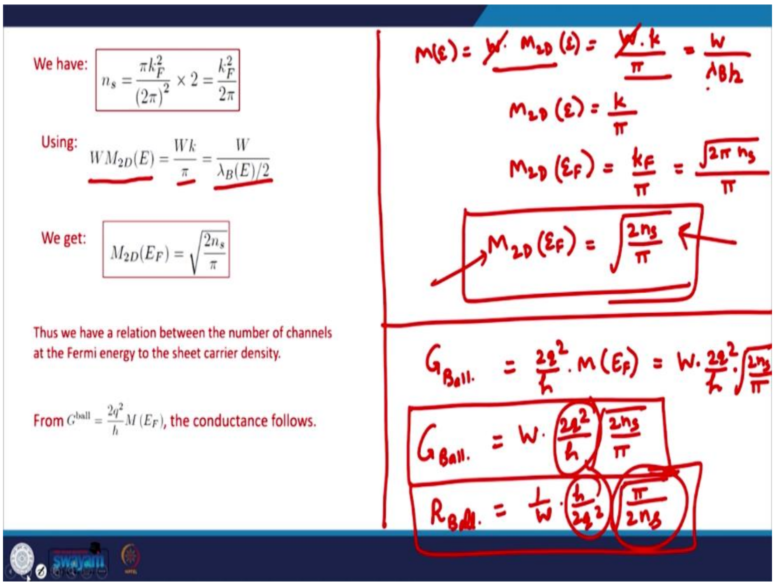

So, let us just keep this equation with us k_F is $\sqrt{2\pi n_s}$ and if we use this expression for a 2D conductor which is essentially the number of modes in a 2D conductor is W times $M_{2D}(E)$ also written as $\frac{Wk}{\pi}$ or $\frac{W}{\lambda_B/2}$ where, λ is the de Broglie wavelength of the electrons ok. So, using this expression now this $M_{2D}(E_F)$ from here we can see that $M_{2D}(E)$ is actually W and W will go away $\frac{k}{\pi}$.

So, $M_{2D}(E_F)$ is $\frac{k_F}{\pi}$ and from the previous analysis k_F is $\frac{\sqrt{2\pi n_s}}{\pi}$. So, $M_{2D}(E_F)$ now is $\frac{\sqrt{2\pi n_s}}{\pi}$. So, we now have this number of modes parameter in terms of the basic or experimental parameter of the device in terms of n_s.

And if we replace this in the conductance expression, the ballistic conductance at 0 kelvin of the conductor is $\frac{2q^2}{h}$ $M(E_F)$ where M is W times $M_{2D}(E_F)$. So, it becomes W times $\frac{2q^2}{h}\sqrt{\frac{2n_s}{\pi}}$.

So, this is the finally ballistic conductance of a 2D conductor $\frac{2n_s}{\pi}$ or the resistance ballistic resistance will be the inverse of this which is essentially $\frac{1}{W}\frac{h}{2q^2}\sqrt{\frac{\pi}{2n_s}}$.

Now as is also clear from this expression that the ballistic resistance is independent of the length as we also expect because the electron is not colliding throughout the length. So, that is why the length is not important in resistance it is however, inversely proportional to the width because more the width more will be the number of modes in the transistor, more will be the conductance less will be the resistance.

Apart from that it is inversely proportional to the sheet carrier density which is also expected because more the number of electrons available in the conductor for transport less will be the resistance or more will be the conductance. And, apart from that we have a fundamental constant or the quantum of conductance constant in these expressions ok.

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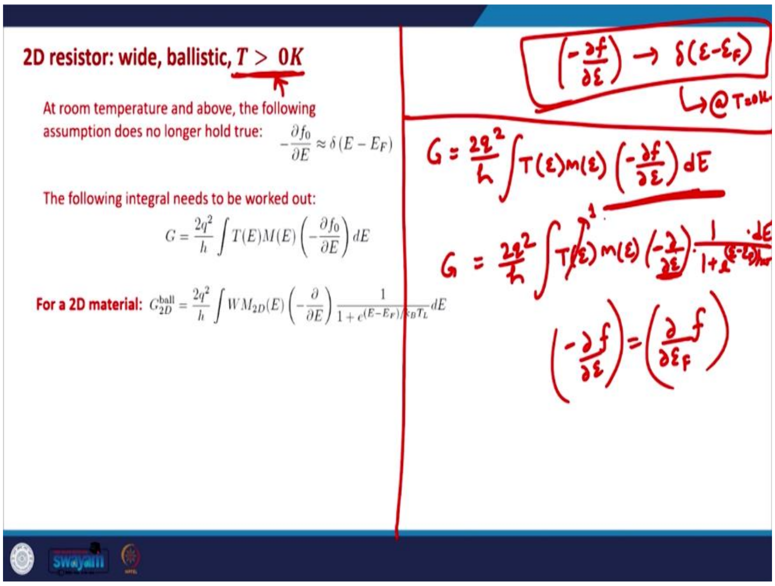

So, this is pretty much what we have in our 2D conductor at 0 kelvin and with a finite width ok. Up to now we have only considered the case of 0 kelvin at extremely low temperatures. But now let us consider a more realistic case. So, the case when the temperature is room temperature or it is beyond or it is above 0 kelvin temperature.

And because of this, because of the room temperature case or higher temperature scenario we can no longer take this assumption we cannot approximate the Fermi window by or delta function because this is only possible at 0 kelvin. So, now, this needs to be explicitly calculated and the this becomes one of the main things to do in these calculations actually.

So, here we have again we to in order to calculate the resistance of a 2D resistor at room temperature let us say we start with the formula of the conductance which is $\frac{2q^2}{h} \int T(E)M(E)\left(-\frac{\partial f}{\partial E}\right) dE$. And now this term needs to be explicitly calculated and this is a tedious calculation I would say ok. So, if we put the Fermi function explicitly in this expression it becomes G is $\frac{2q^2}{h} \int T(E)M(E)\left(-\frac{\partial}{\partial E}\right)\frac{1}{1+e^{\frac{E-E_F}{kT}}} dE$.

This derivative $\left(-\frac{\partial}{\partial E}\right)$. So, this $\left(-\frac{\partial}{\partial E_F}\right)$ is equivalently can be it can be written as $\frac{\partial}{\partial E_F}$ of f. So, $\left(-\frac{\partial f}{\partial E}\right)$ can be written as $\frac{\partial f}{\partial E_F}$. And by making this substitution, we can essentially bring this derivative term outside the integral.

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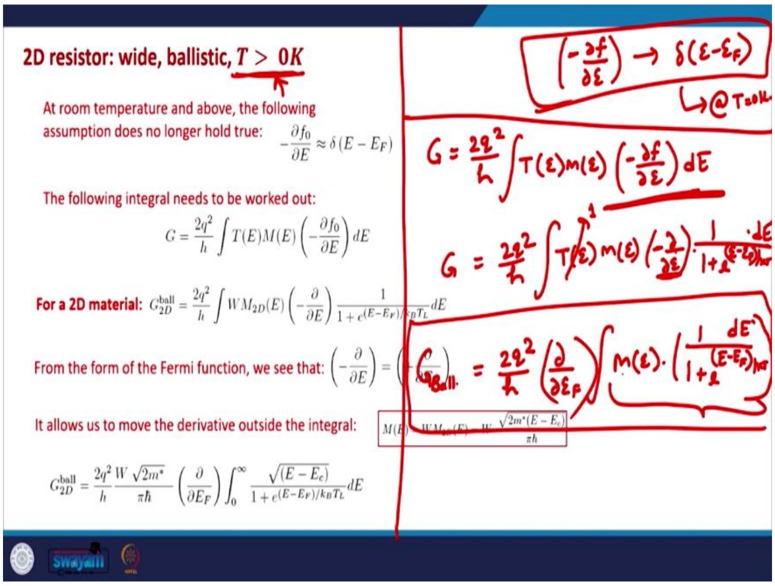

In ballistic case T(E) is 1. So, if we make this substitution the ballistic conductor at this the conductance of the ballistic conductor at higher temperatures is it becomes if we bring $\frac{2q^2}{h}(\frac{\partial}{\partial E_F}) \int M(E) \frac{1}{1+e^{\frac{E-E_F}{kT}}} dE$.

Now, this entire calculation, this entire integral needs to be evaluated along with this differential and this becomes a non trivial calculation actually for most of the practical applications this become a known, it becomes a difficult thing to do. So, that is why this is finally, the if we also write it down explicitly M(E) which is essentially so, if we write down M(E) to be W times $\sqrt{\frac{2m^*(E-Ec)}{\pi\hbar}}$.

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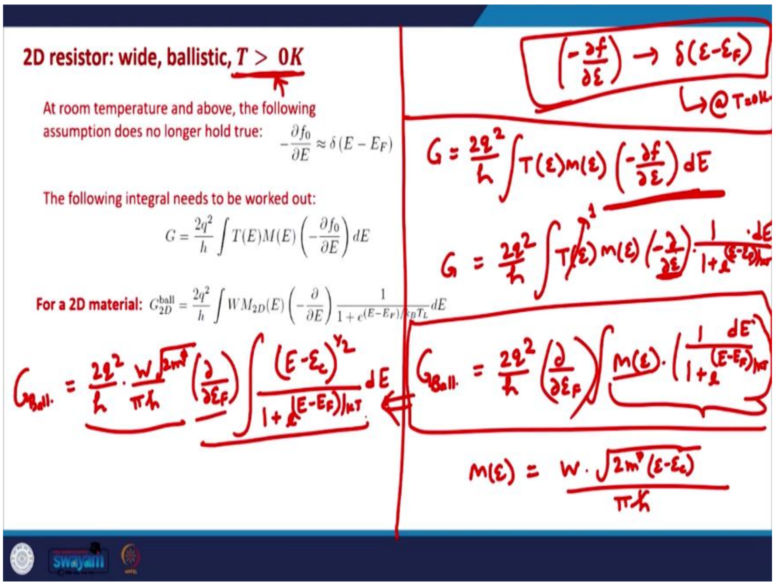

So, then this ballistic conductance formula essentially becomes this formula can be written down as G ballistic is equal to $\frac{2q^2}{h}(\frac{\partial}{\partial E_F})$. Now if we put instead of M (E) we put this then W by. So, $\frac{W}{\pi\hbar}$ will come out and even $\sqrt{2m^*}$ can also be taken out, we are left with this derivative term $(\frac{\partial}{\partial E_F})$ and the $\int (E - E_c)^{1/2} \frac{1}{1+e^{\frac{E-E_F}{kT}}} dE$.

So, this is finally, this formula will be or the conductance of the ballistic conductor will become, it has range of constants here then a derivative and then an integral. So, this is combination of. So, we need to first integrate then take a derivative with respect to E_F.

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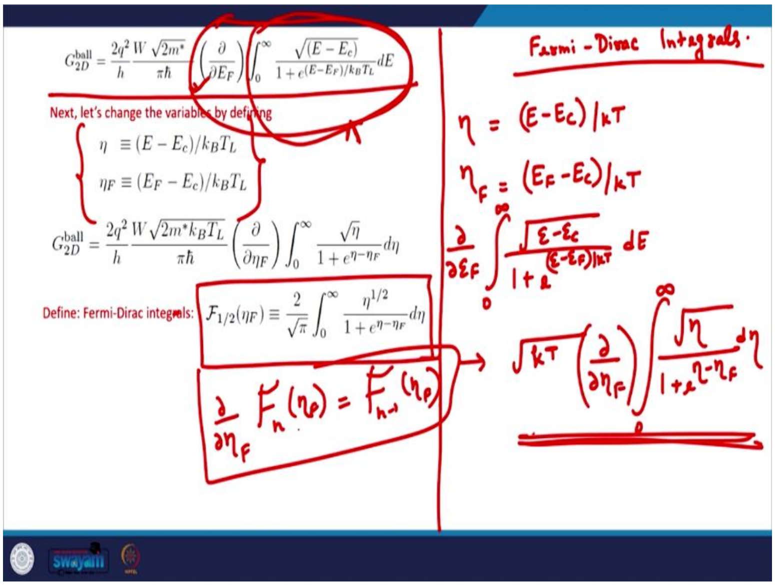

And as I mentioned that it is a difficult thing to do. So, and that is why generally this expression which is essentially the same that was there in the, this is evaluated this kind of integrals are separately evaluated and they are separately categorized as Fermi Dirac integrals Fermi Dirac integrals.

Because, these kind of integrals they appear quite often in solid state physics, specially in the ballistic transport and in the calculations of the conductance or even in the calculations of the I V characteristics at room temperature similar kind of integrations appear there ok.

So, these are known as the Fermi Dirac integrals and the way to evaluate them is to make this change of variables. So, a new variable η is defined which is given as $\frac{E-E_C}{kT}$, generally in these derivations it is written as k_B times $T_L k_B$ is the Boltzmann constant T_L is the lattice temperature which we are in short writing as k times T.

And another parameter η_F is defined as $\frac{E_F-E_C}{kT}$. So, if we make this change of variables in this equation then this integral essentially this part of the integral $(\frac{\partial}{\partial E_F})$ this derivative and $\int (E-E_c)^{1/2} \frac{1}{1+e^{\frac{E-E_F}{kT}}} dE$. So, this changes to $\sqrt{kT}\ (\frac{\partial}{\partial \eta_F})$ integral this if this limit is 0 to ∞ this is also 0 to ∞. *This will be equal to* $\sqrt{kT}\ (\frac{\partial}{\partial \eta_F}) \int (\eta)^{1/2} \frac{1}{1+e^{\eta-\eta_F}} d\eta$.

Just consider this to be a small exercise just put these variables $E-E_c$ replace E by ηkT and E_F-E_c by $\eta_F kT$ and then see how this derivative and integral changes this is the answer actually, this is how it changes. So, just do it yourself that will give you a more hands on feel of this expression. So, ultimately in the calculation of the conductance of the material this derivative along with the integral it changes to this.

So, finally, we can write down the ballistic conductivity of a 2D conductor $\frac{2q^2}{h}$W times $\sqrt{\frac{2m^*kT}{\pi\hbar}}\left(\frac{\partial}{\partial\eta_F}\right)\int_0^{\infty}(\eta)^{1/2}\frac{1}{1+e^{\eta-\eta_F}}d\eta$.

So, these kind of integrals are known as the Fermi Dirac integrals and depending on the power of η in the numerator here, inside the integral the order of the Fermi Dirac integral is defined. So, in this case the power of η is half. So, this is the Fermi Dirac integral of order half ok.

So, and this is how precisely the Fermi Dirac integral of order half is defined. It is $\frac{2}{\sqrt{\pi}}\int(\eta)^{1/2}\frac{1}{1+e^{\eta-\eta_F}}d\eta$. So, it does not involve the derivative here yeah. So, earlier I mentioned that this entirely is defined as the Fermi Dirac integral, but it is only this part that is defined as the Fermi Dirac integral.

And there is an interesting property of the Fermi Dirac integral that, if we take a derivative of the Fermi Dirac integral with respect to η_F. So, if we do this with a Fermi Dirac integral let us say of order n then generally the Fermi Dirac integral of order, it becomes a Fermi Dirac integral of order n -1.

So, in the case of calculation of conductance for a 2D conductor 2D ballistic conductor at room temperatures, we have Fermi Dirac integral of order half and we are taking a derivative with respect to η_F. So, finally, it will be a Fermi Dirac integral of order minus half and these maths we are not going in this in the in the detail of the maths the Fermi maths of Fermi Dirac integral.

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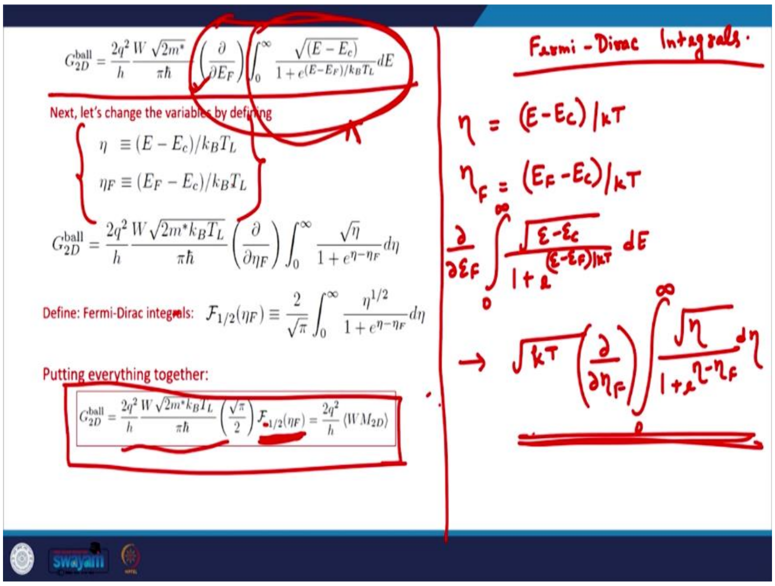

But, this is how finally, everything will look like and this should be order minus half actually. So, finally, the conductance of a 2D ballistic conductor at room temperature will be given by a set of constants and a Fermi Dirac integral of order minus half ok.

So, I will let you think about this replacement of variables by this change of variables through which we obtained this Fermi Dirac integral formulation and, I would recommend you to go back and look at the Fermi Dirac integrals independently I will also share some materials on them and, we will start the next class this point onwards.

We will have slightly more deeper understanding of this final expression and that will essentially conclude the discussion of the conductance of the ballistic conductor at normal temperatures.

Thank you for your attention. See you in the next class.