Physics of Nanoscale Devices
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Lecture - 28
Resistance: Ballistic and Diffusive Cases-I

Hello everyone. Today, we will do calculations of resistance or conductance in ballistic and diffusive cases, based on the formalism that we have discussed. So, as you know that, we have now discussed the transport in ballistic case, the transport in diffusive case, how the conductance in both cases can be given by the fundamental parameter of the devices.

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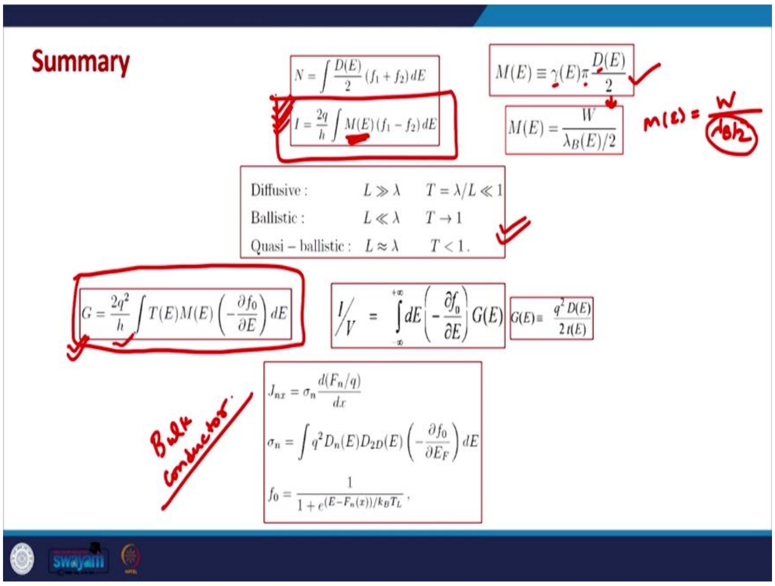

And in this class, we will do some real calculations of resistance and conductance based on the formalism that we developed in last few classes. Let us quickly review what we did in last few classes. From the general model of transport, we deduced the steady state electronic population and steady state current. And the expression for them is given by these two formula.

Here this is an important term, which is known as the number of modes. And it is given by $\frac{\gamma(E)\pi D(E)}{2}$ depends on the density of states, energy broadening and π constant. If we do a small calculation for a 2D channel, we quickly realize that number of modes in a 2D channel is essentially, $\frac{W}{\lambda_B/2}$, where λ_B is the de Broglie wavelength of the electrons. So, it means that, number of modes is the number of half wavelengths that can fit into the conductor ,2D conductor.

So, it is essentially, the conducting pathways in the device. Then, we saw how the current expression changes in case of diffusive transport. And for that we used Fick's law. We could use that, because we assumed E k parabolic relationship ok. And we could assume that the electron wave packet is like a classical particle. Then, from this current expression, we deduce that the conductance at near equilibrium transport, when the applied voltage is a small voltage. In that case, this can be given.

This is given by this expression. Then, we generalized these ideas to a bulk conductor. And for bulk conductor, we derived these expressions. Essentially, in bulk conductor instead of having Fermi level just at the contacts, we can have or we have the notion of quasi Fermi level inside the conductor as well, when the steady state current is flowing.

As you might have observed that up to this point, we did not make explicit calculations of the conductance or current in the device. It is just given in the form of integrals and the constants related to the device ok.

So, that is what we will do. In this and coming classes, we will try to see try to calculate the conductance in real devices ok. So, before going into that, let us quickly review the idea of resistance from our classical understanding of resistance.

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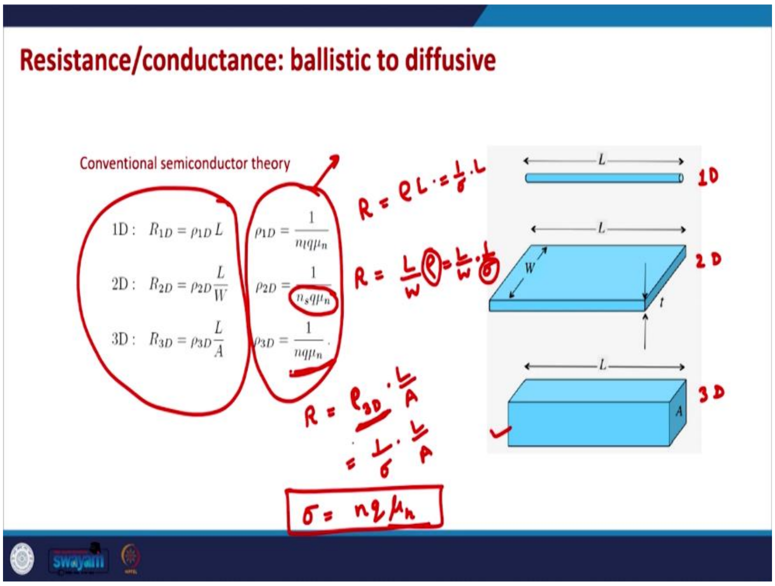

And as we know that, the conductors or the transistor channels can either be a 1D channel, it can be a 2D channel and a 3D channel most popularly. And the classical understanding of resistance or conductance says that, that the resistance is given for a 3D channel let us say, for this a material constant known as the resistivity, which is often represented by $\frac{\rho L}{A}$.

So, the resistance is directly proportional to the length, inversely proportional to the cross section area. And the constant of proportionality is the material constant, which is the resistivity and denoted by ρ ok. So, this is the conventional understanding.

And this is often written as, this ρ can be written as 1 by σ; σ is the conductivity L/A. Where σ is given by $nq\mu_n$. For the case of electrons, the conductivity is given as the product of the concentration, the charge and the mobility of electrons ok.

Similarly, in the case of 2D conductors, the classical understanding says that, that the resistance is proportional to the length, inversely proportional to the width. And the constant of proportionality is the resistivity or inverse of conductivity, where this conductivity or resistivity they are the material constant. And they depend in the case of 2D conductor this is given by $nq\mu$. the same expression, but instead of.

So, in this case, 2D case this n is now the number of charge carriers per unit area instead of number of charge carriers per unit volume, which is the case in 3D materials ok. For 1D conductors the conventional understanding is that, resistance is ρ times L or $\frac{1}{\sigma}L$, where sigma is given by $nq\mu$, n is now the charge carriers per unit length of the conductor.

This is the classical understanding in this as you can clearly see that, the resistivity or the conductivity rho and sigma parameters those are the material constants, if the number of charge carriers are not changing in the material are the same.

But based on our formalism, when we do actual calculations, we realize that these expressions are not correct. Actually, they are not correct in all limits, in some limits, in some situations these expressions do not hold true.

And that is why this theory this classical theory as one of the main motivations for this course, this does not hold for the nanoscale devices and that is why we need to start from the scratch and study the transport from the basics. So, that is what we will see, that in some cases even the conductivity or resistivity they depend on the length or the dimension could be width as well ok.

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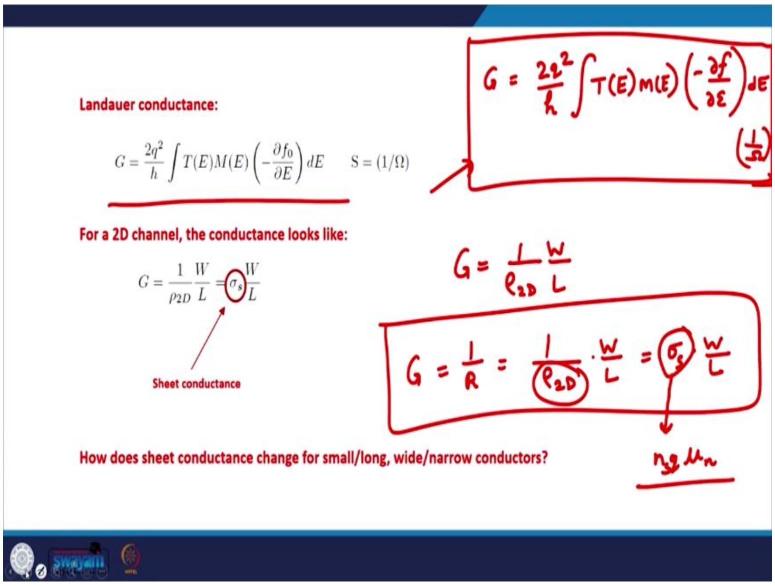

So, our calculations will start with this basic formula of conductance. So, we will start with this conductance formula which is essentially, $\frac{2q^2}{h}\int T(E)M(E)\left(-\frac{\partial f}{\partial E}\right)dE$. The unit of this is 1 by Ω this is the conductance of the device ok. So, this is where we will start our calculations with. And in this case, what we also see is that by equating this with this expression.

So, as all of us are aware that, conductance is inverse of resistance. And from the resistance formula, if we put down the classical formula, it is given by σ times W by L, where σ is now the sheet conductivity or the conductivity of a 2D conductor ok.

So, by equating this expression we will see that, we can deduce the sheet conductivity and even the resistivity from our treatment from this expression. And we will see that, it is not exactly it is not always $nq\mu$, it may be different as well. That is what we will see. So, we begin our analysis with a 2D resistor or 2D conductor and ballistic case.

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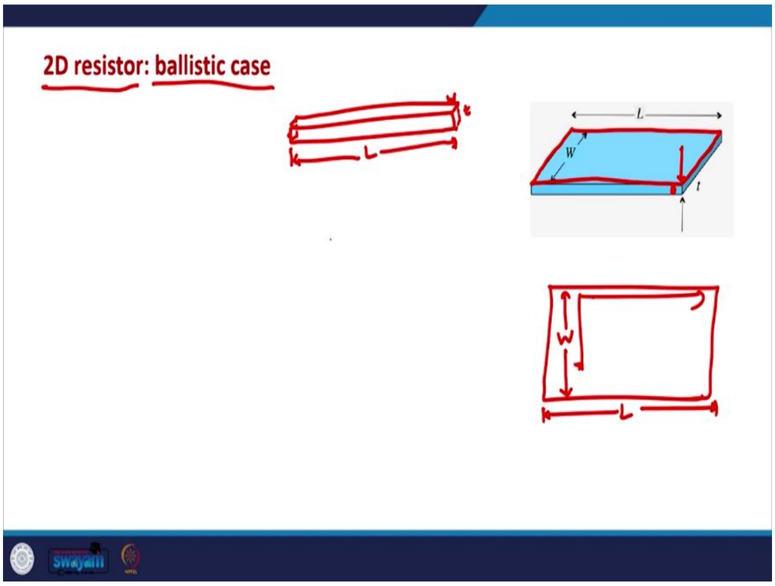

So, we begin our calculations with as usual a 2D conductor, because 2D conductor first is easy to visualize, second is most of our treatments most of our discussions have been centered around the 2D conductor. And so, that is why, we will it is easy to generalize to the it is easy to sort of analyze a 2D conductor.

So, this is how a practical 2D device looks like. It has a certain length L it has some width W. Ideally a 2D conductor is assumed to be having just the length and the width. But practically it also has a finite thickness. So, in a 2D conductor in practice the carriers can move freely in 2 dimensions 2 directions.

But they are confined in the 3rd direction ok. And that is actually the case even with similar is the case with a 1D conductor. If we have a 1D conductor like this, a practical 1D conductor has a finite it has certain length L, but it also has finite width and finite thickness.

So, in a 1D conductor in practice electrons are free to move in one direction, but they are confined in the other two directions. But in a 3D conductor in on the similar lines we can say, that electrons are free to move in all three directions ok. So, we are at the moment interested in the conductivity of a ballistic 2D conductor.

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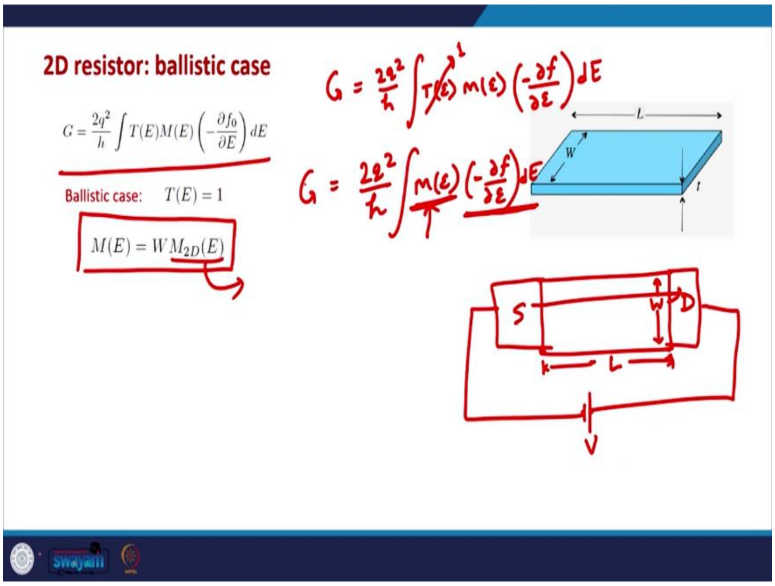

So, we have a 2D device let us say, the source, the drain, the length and width are there. And ballistic means that, the electrons are not colliding they are directly going from source to the drain side, which also implies that the length of the conductor is not large it is a small length.

So, this is the conductance the general formula of the conductance at in near equilibrium transport case, when the applied voltage is not large; small applied voltage is there across the device in ballistic case. So, if we begin with this formula $\frac{2q^2}{h}\int T(E)M(E)\left(-\frac{\partial f}{\partial E}\right)dE$. In ballistic conductor case, this T(E) tends to 1 actually, T is actually 1. So, G is $\frac{2q^2}{h}\int M(E)\left(-\frac{\partial f}{\partial E}\right)dE$ ok.

And M(E) is in a 2D conductor is width times $M_{2D}(E)$, where $M_{2D}(E)$ is a parameter also known as the modes per unit width so to say ok. So, there are two terms basically, two terms here one is M(E) second is this, M(E) is the number of modes. And we know the expression for the number of modes in a 2D conductor.

So, its we can just directly put the values from the expression that we derived while we discuss the number of modes. And second term is $\left(-\frac{\partial f}{\partial E}\right)dE$. And as you might recall, we have also had a discussion on this term.

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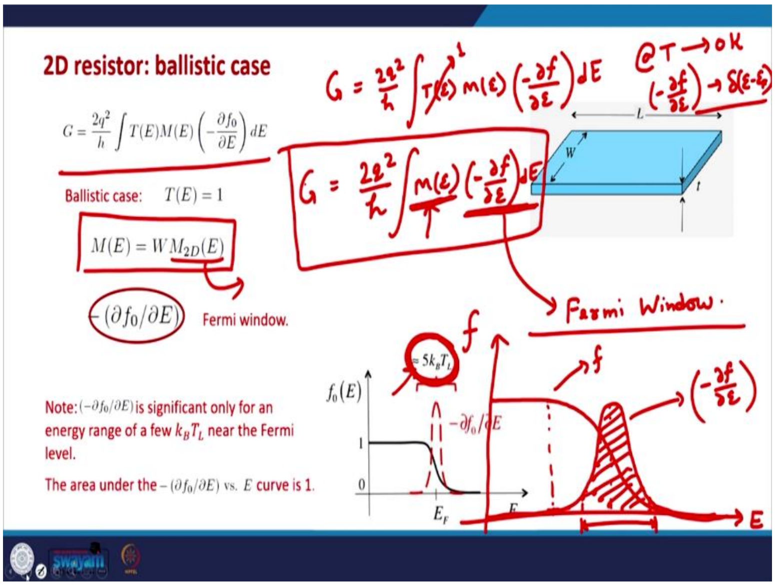

This second term, which is this also known as the Fermi window ok. So, if the Fermi function is plotted as a function of energy. And if the Fermi function looks like this, the Fermi window function will be something like. So, it will be 0 up to this point then, it will increase and again decay to 0. So, this will be broadly how the Fermi window function will look like and this is the f ok.

So, one important property, that we also studied in the case of Fermi window function was that, the area under this function is 1 on the energy axis. So, the area covered by the function $\left(-\frac{\partial f}{\partial E}\right)dE$ versus E is 1. And the value of the Fermi window function is significant at; for example, at room temperature it is significant only in few k T values of energy.

So, it has significant value only typically in this range, only in the energy range of 5 k T this function has non-zero and significant value ok. So, that is why we also deduced that, the conductance is contributed by the modes only in a small energy range around the Fermi level.

Now, you might ask, where is the Fermi level of the device? There is a Fermi level of the source, there is a Fermi level of the drain in near equilibrium transport. These two Fermi levels are not actually very far away from each other, because we have applied only a small amount of voltage here.

So, the Fermi level of the device will be actually, the average of these two Fermi functions. That can be assumed to be the Fermi level of the device in steady state. And around that value, in few kT energy range only this Fermi window function will be significant. Also, it is a function of temperature as well, at higher temperatures this Fermi window will be broadened, at lower temperatures this Fermi window will be shrunk.

So, at very low temperatures at around 0 Kelvin, this Fermi window becomes a delta function actually. So, at T approaching 0 Kelvin $\left(-\frac{\partial f}{\partial E}\right)$ approaches or delta function $\delta(E - E_F)$ on the Fermi level of the device ok.

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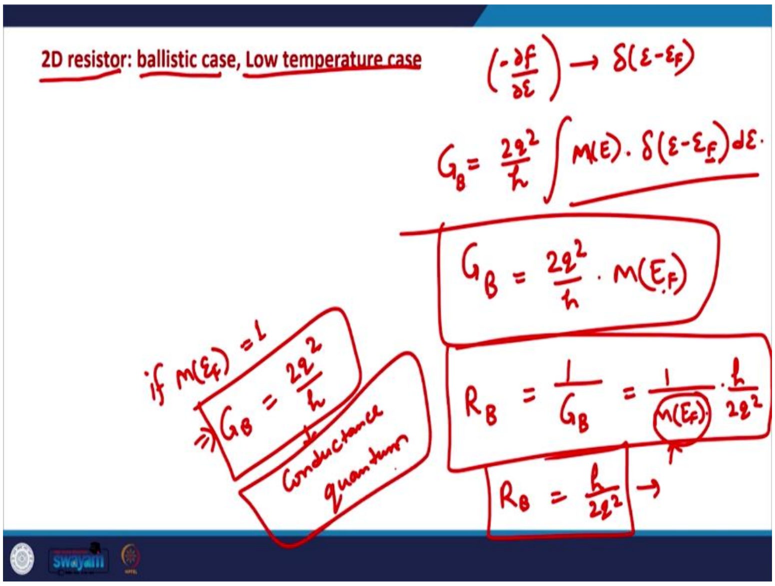

So, this is what we have finally. And so, we begin the calculation of the resistance of a 2D conductor, ballistic case and we first discuss the low temperature case. Because low temperatures things are easy at low temperatures, we can say that this $\left(-\frac{\partial f}{\partial E}\right)$ tends to $\delta(E - E_F)$ ok. So, in this case, the ballistic conductance is $\frac{2q^2}{h}\int M(E)\delta(E - E_F)\, dE$.

So, just to quickly review, we started our discussion of the calculation of resistance of the device ok. We started with 2D device, ballistic case and we are looking at the low temperature case right now. Then, we will see the 2D device ballistic case at normal temperatures higher temperatures. Then, we will see the 2D device diffusive case both at low temperature and high temperature.

So, that way we will have a idea of all kind of all possibilities, all cases of the resistance in both ballistic and diffusive cases. Actually now, M(E) in this integration has a delta function along with it at E_F the Fermi level. So, this integration now simplifies to $\frac{2q^2}{h}M(E_F)$ essentially, this one. So, the value of the modes at Fermi energy is now the important factor in the conductance.

And the resistance of the ballistic 2D conductor at low temperature limits will be 1 by the conductance, which will be essentially, $\frac{1}{M(E_F)}\frac{h}{2q^2}$. And if this value is 1 in that case, the ballistic resistance is $\frac{h}{2q^2}$. This is also known as the quantum of resistance or this $\frac{2q^2}{h}$, if $M(E_F)$ is 1; for example, in a device if $M(E_F)$ turns out to be 1 in that case, the ballistic conductance will be $\frac{2q^2}{h}$. And this is known as the quantum of conductance or conductance quantum.

This in a way, is the conductance of the single channel single conducting channel single mode in the device. And that is why it is known as the quantum of the conductance or conductance quantum, ok.

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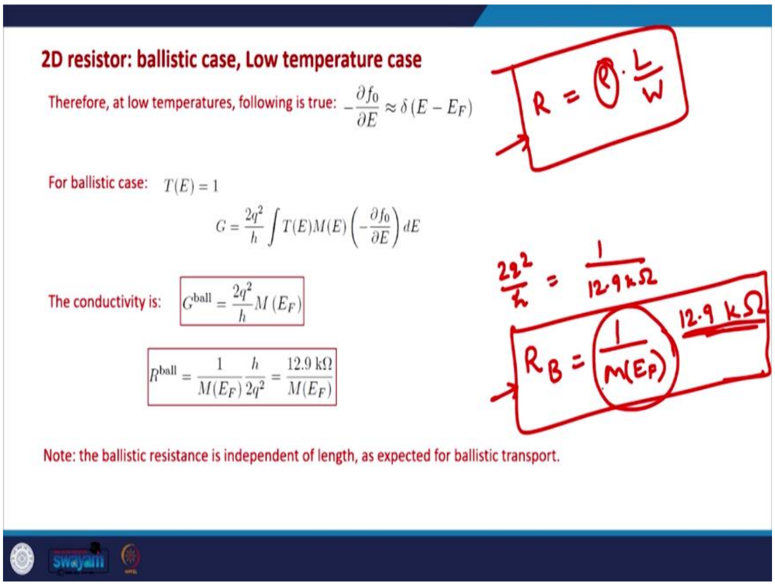

So and the value of this turns out to be this $\frac{2q^2}{h}$ is essentially, $\frac{1}{12.9k\Omega}$. So, the value of the ballistic resistance is, $\frac{1}{M(E_F)}$ into $12.9k\Omega$, ok. So, as you can see that, the ballistic resistance is now independent of the length of the conductor. So, if the as long as the conductor is ballistic whatever be its length, the resistance remains the same. It does it only depends on the modes in the device, which is also independent of the length it depends on the width ok.

So, this is. So, just to recall that, this is in direct contrast to the classical understanding of resistance, where the resistance is directly proportional to the length, inversely proportional to the width. And we also have a constant rho, ok. Here in ballistic case, the resistance is independent of the length and that is a key difference and that is what actually, is the main highlight I would say of this formalism. It is also experimentally observed.

Actually, in nanoscale devices experimentally it has been observed that, as we increase the width it will increase the number of modes and the resistance is decreased and the resistance decreases in quantas. So, it something like this is observed.

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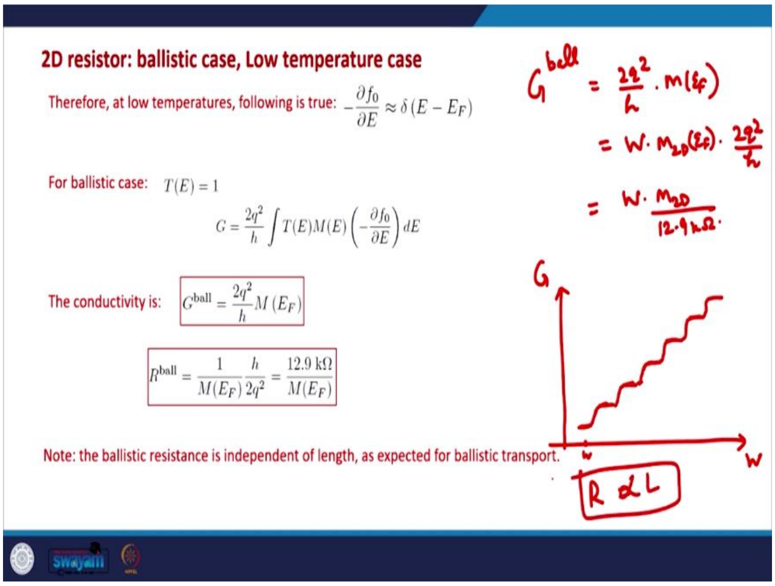

So, we have the conductance expression to be the ballistic conductances, $\frac{2q^2}{h}$ $M(E_F)$ and M is directly proportional to the width. Let us say, W times $M_{2D}(E_F)$ times $\frac{2q^2}{h}$. This $\frac{2q^2}{h}$. is 1 by 12.9 kilo ohms.

And experimentally, it has been observed that, in ballistic conductors in the conductors of few times of nanometers, if we plot the conductance as a function of width, the conductance follows this kind of relationship initially, it is constant for certain for this width. And as the width is increased, it increases in steps.

So, it essentially conveys that, as the number of modes are increasing in a device, the conductance is increasing in steps like this. So, this is the typical observation that we have about the ballistic conductors in experiments as well.

And this is in direct contrast to the classical understanding, which says that, resistance is directly proportional to the length always, which is not the case here ok. So, this is our first sort of important observation about the ballistic conductors and the resistance and conductance of the ballistic conductors.

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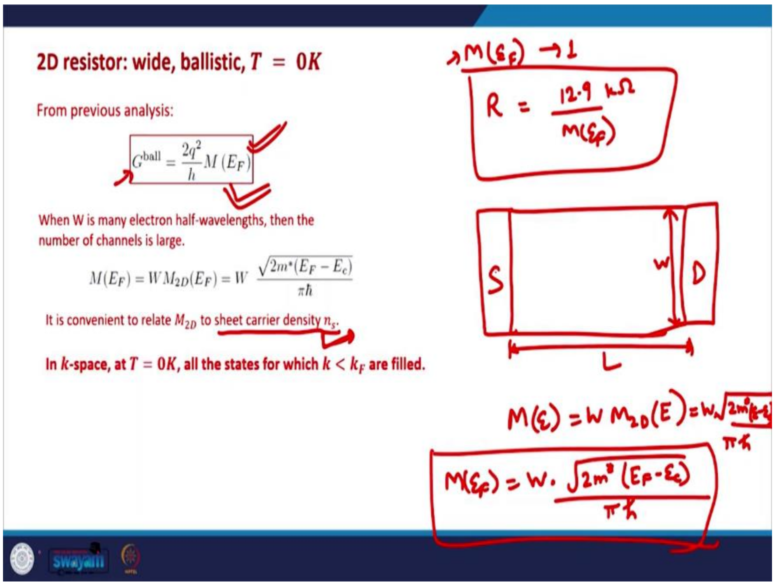

In our previous slide, we have assumed that the modes may be, let us say modes may be, 1 or there is small number of modes in the device. So, the resistance will be 12.9 kilo ohms by modes. But let us take a more general case. Let us take a wide ballistic conductor, which means that now, there could be many number of modes many modes in the device.

In our 2D device, source, drain, length is such that it is a ballistic conductor, the width may now be more such that, the number of modes could be large in number. And now let us see, how the resistance of the device looks like. In this case, one way is to use the expression for modes, which we have already derived in our previous discussions.

This is the expression for the conductance that we just saw. And the number of modes for a 2D conductor is, M(E) is W times $M_{2D}(E)$, which is W times $\sqrt{\frac{2m^*(E-Ec)}{\pi\hbar}}$. So, this M($E_F$) will be, W times square root of $\sqrt{\frac{2m^*(E_F-Ec)}{\pi\hbar}}$.

So, that is one way of calculating the number of modes. And if we put this expression in the expression for conductance, we will obtain the conductance of the material. But we generally try to correlate it with experimental parameters, we generally try to correlate our mathematical derivations, our equations to the experimental parameters that we might observe while conducting experiments on the device. And one such parameter is the sheet carrier density.

Generally, in many experiments, we can observe or we can calculate the sheet carrier density or in general the carrier density inside a semi conductor or a conductor. For example, using Hall effect experiment, we can calculate the; if we are aware of the Hall coefficient of the material, we can calculate the sheet carrier density or the carrier density in the material.

So, we will try to correlate this number of modes with the sheet carrier density. And let us see and try to see how the conductance is related to the sheet carrier density. For this, we need to go back to our understanding of the k space the inverse space. Because we are assuming that, the temperature is very low approaching 0 Kelvin.

And we are assuming that, the conductor is a ballistic conductor. But it might be a wide conductor, many modes might be there in this. So, at 0 Kelvin all of us know that, all the states up to the Fermi level are filled in a material ok.

And all the states above the Fermi level are empty. So, in order to have a mathematical form of this experimental parameter a mathematical relation of this experimental parameter n_s, we need to go back to the k space.

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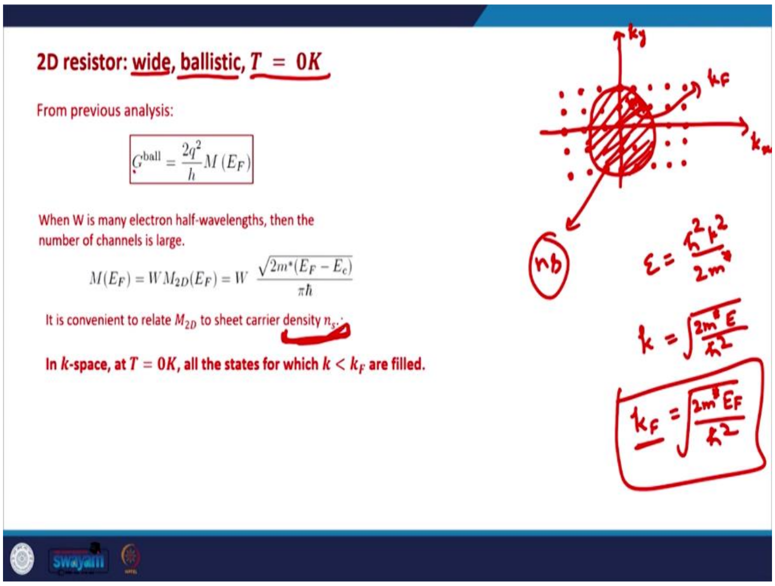

And in the k space, if you recall from our discussion on the density of states, this is the k space k_x. These are the allowed k points in a 2D conductor. Allowed points mean that, the electrons can have valid wave functions at these k values.

And if we plot a circle in the k space and the circle radius is k_F let us say, the radius is k_F then, at where E is $\frac{\hbar^2 k^2}{2m^*}$. And k is $\sqrt{\frac{2m^* E}{\hbar^2}}$. So, k_F will be, the wave vector at Fermi level $\sqrt{\frac{2m^* E_F}{\hbar^2}}$.

So, at t equal to 0 Kelvin all the k states up to this k_F circle will be filled. And this will give us the sheet carrier density. If we count the number of states in this circle, this will tell us about the sheet carrier density. And I will leave this as a small assignment to you before the next class.

So, please make this calculation, what will be the sheet carrier density in terms of the this k vector. And we will continue our discussion this point onwards. And in next class, we will see how the conductance can be given in terms of the sheet carrier density n_s ok.

So, thank you for your attention, see you in the next class.