**Physics of Nanoscale Devices**
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**Lecture - 27**
**Conductance, Bulk Transport-II**

Hello everyone. So, today we will finish our discussion on Conductance both in ballistic and diffusive transport case and we will see how the ideas that we have discussed can be generalised to Bulk Transport case.

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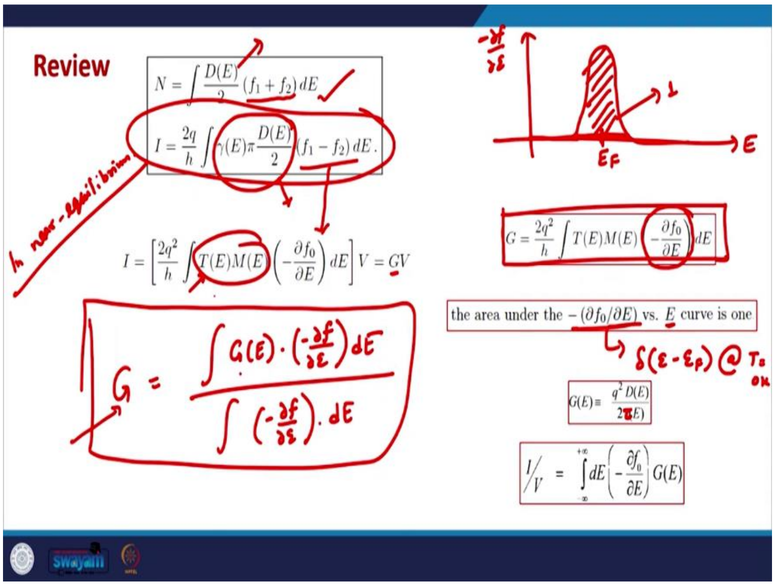


So, what we have seen up to now is that the steady state electronic population in general can be given by this quantity, which essentially depends on the Fermi functions of the context and the density of states of the channel.

The current is given by this general equation and this equation has different interpretations both in ballistic case both different in interpretation in ballistic and in diffusive case because in ballistic case this term is the number of modes and in diffusive case this is number of modes times the transmission coefficient.

In near equilibrium conditions we saw that in near equilibrium this $f_1$ - $f_2$ term can be written as $(-\frac{\partial f}{\partial E})\Delta E_F$ term and from there we could deduce the conductance of the device.

The conductance generally looks like this it is $\frac{2q^2}{h}T(E)M(E)\left(-\frac{\partial f}{\partial E}\right)dE$ and we saw that this thing in the integration $\left(-\frac{\partial f}{\partial E}\right)$ this is an interesting function and this is if we plot this on energy axis $\left(-\frac{\partial f}{\partial E}\right)$ this is a window function around the Fermi level of the material.

So, this looks like a window around the Fermi level moreover the area under minus $\left(-\frac{\partial f}{\partial E}\right)$ versus E is 1. So, the area of this window is 1. Secondly, this function becomes a delta function at extremely low temperatures when temperature approaches is around 0 kelvin ok. And finally, what we saw was that the total conductance of the device can be written as the average of the conductance function over the Fermi window.

So, this is how we can write down the total conductance of the device minus $\left(-\frac{\partial f}{\partial E}\right)$dE, where then this conductance function is given by this general expression the expression $\frac{q^2D(E)}{2\tau(E)}$, but $\tau$ is the characteristic times and both it is different in ballistic and in diffusive cases in both cases it is different ok.

So, this is what we have seen. So, far and one important thing that we need to keep in mind is specially from this expression that the conductance of a device of any material is essentially the average of the conductance function in the Fermi window, which means that only the conduction pathways that are close to the Fermi level only those contribute in the conductance of the device.

The conduction pathways that are far away from the Fermi window from the Fermi level they do not even though there might be conduction pathways there might be more in the device, but they do not contribute in the current and they do not contribute in the conductance.

And then it has a simple intuitive explanation as well because only electrons close to the Fermi level only those electrons are sort of mobile at the bottom of the conduction band and at the top of the valence band ok. And this we can also see once we do things from the right from the bottom up right from the fundamental science of the device ok.

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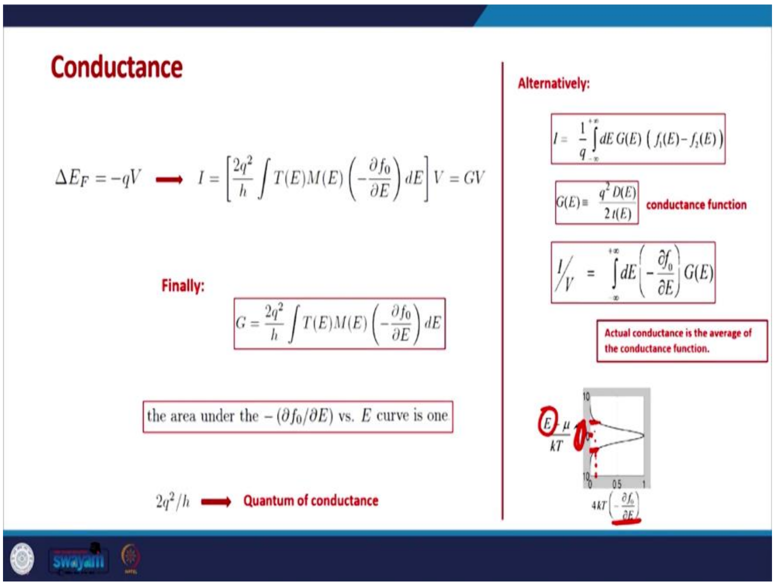


So, this is what we have and more or less this is all about the conductance. Simply this is a plot of minus $(-\frac{\partial f}{\partial E})$ as a function of E, but now instead of directly plotting $(-\frac{\partial f}{\partial E})$ this has been plotted as $(-\frac{\partial f}{\partial E})$ versus $\frac{E-\mu}{kT}$ just for the sake of simplicity and this is how it looks like.

The axis have been flipped now the energy axis is the vertical axis and the horizontal axis is the $(-\frac{\partial f}{\partial E})$ axis this is clearly as we can see a window function this is a computer generated plot essentially ok.

So, at around these values the window becomes the value of this function becomes very small and these values are sort of very close to the this 0 point, 0 point is the Fermi level point generally the width of this window is few kT only few kT wide this is 5 to 6 or maybe 7 kT depends on the temperature as well.

So, only in this small energy range only conduction takes place and conductance is contributed by the channels or the conduction pathways in this range this is true for both ballistic case and diffusive case. So, up to now we have discussed as we saw we have discussed the current in ballistic case in ballistic case we came across the idea of modes. Now, let us see how this can be generalised to the bulk transport case.

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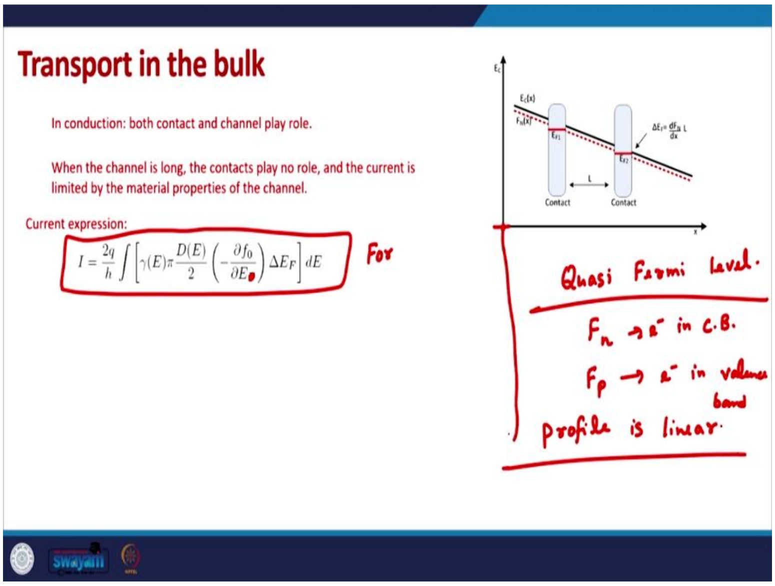


So, generally the bulk transport theory is given by the Drude's model in which the conductivity is derived by assuming electrons to be free particles colliding with atoms in between and taking a average over or considering the classical kinematics of the electrons in the devices.

Now, let us see how these expressions can be generalised to the bulk case. In bulk transport case before going into this let me also quickly tell this that as we are moving from as we are moving away from ballistic transport and going to the diffusive transport case we are seeing that the channel is becoming more important than the contacts.

In ballistic transport case we only had M(E) from the channel the only the conduction pathways in the channel were responsible for the ballistic transport and there was no other contribution of the channel in the transport.

Everything else was getting determined by the contacts, but now in the diffusive transport case due to scattering of electrons in the channel T(E) gets introduced in the I V expression and as the channel starts becoming bigger and bigger this quantity starts becoming more important because this becomes smaller and smaller. And which basically in a way reduces the effects of the contacts or specifically the contacts. Now channel is the determining factor in the current in the device.

So, in bulk transport case this material bulk means now we have a big material we can see it from our eyes it is maybe centimetre or a meter length. In this case what we have when the current is flowing through the device flowing through a material then this device is not in equilibrium. So, we cannot sort of define the Fermi level as such and this idea of quasi Fermi level is used.

So, in bulk transport case we use the idea of quasi Fermi level where we define 2 Fermi levels for electrons one is for the electrons in the conduction band and second is the electrons in the valence band, this is electrons in conduction band. And in steady state there is a gradient in these Fermi or quasi Fermi these are not actual sort of Fermi level this is not an equilibrium concept because as we discussed in beginning that Fermi level is an equilibrium idea.

But on the similar lines these quasi Fermi levels are defined when the system is not in equilibrium system is sort of displaced from the equilibrium and that is why we need to define 2 Fermi levels one is for the electrons in the conduction band second is the electrons in the valence band because these two kind of electrons may behave now differently. And it is this the profile when there is a constant current the profile of quasi Fermi levels is linear this we can see from basic electro statics as well ok

So, now starting with our expression of current this is what we have essentially in near equilibrium in any material we have I is equal to $\frac{2q}{h}\int \frac{\gamma(E)\pi D(E)}{2}\left(-\frac{\partial f}{\partial E}\right)\Delta E_F dE$ ok. So, now, if we for bulk transport case.

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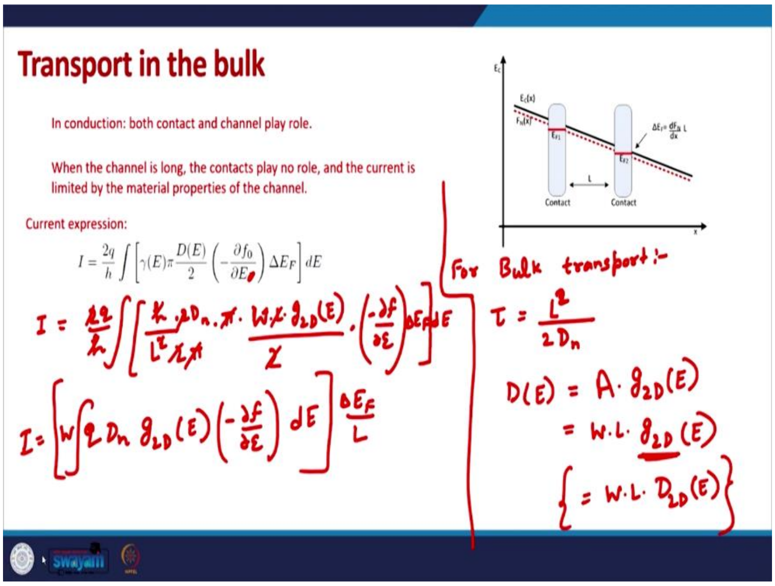

Let me erase everything for bulk transport $\tau$ is essentially $\frac{L^2}{2D_n}$ when Dn is the diffusivity or diffusion coefficient because a bulk is essentially a diffusive conductor.

So, $\tau$ will be given by $\frac{L^2}{2D_n}$ and if we take a 2D conductor in that case this D(E) can be written as area times the density of states or WL$g_{2D}(E)$. In some texts it is also written as W times L times $D_{2D}(E)$. ok.

So, if we put these values here in this expression of current what we see is that I is equal to $\frac{2q}{h}$ , $\gamma$ can be written as $\frac{\hbar}{\tau(E)}$ and $\tau(E)$ is given by $\frac{L^2}{2D_n}$, $\pi$ stays as it is D(E) is written as WL$\frac{g_{2D}(E)}{2}$ times $\left(-\frac{\partial f}{\partial E}\right)\Delta E_F$ and sorry this integral is over E this integral is over E.

We have a $\Delta E_F$ here ok. So, now, $\hbar$ can be written as $\frac{h}{2\pi}$. So, we can instead of writing h bar we can write $\frac{h}{2\pi}$. So, now, looking at everything here $\pi$ *and* $\pi$ cancel L eliminates one of the L's 2 2 here 2 2 h and h. So, finally, what we are left with is we are left with lets take q inside Dn, W outside, I = $\left[W\int qD_n\, g_{2D}(E)\left(-\frac{\partial f}{\partial E}\right)dE\right]\frac{\Delta E_F}{L}$.

So, this is what we can write from the expression of the current ok. So, now we have $\frac{\Delta E_F}{L}$ here this was used in the case of diffusive transport and $\Delta E_F$ was the difference in the Fermi function of the left contact and the right contact.

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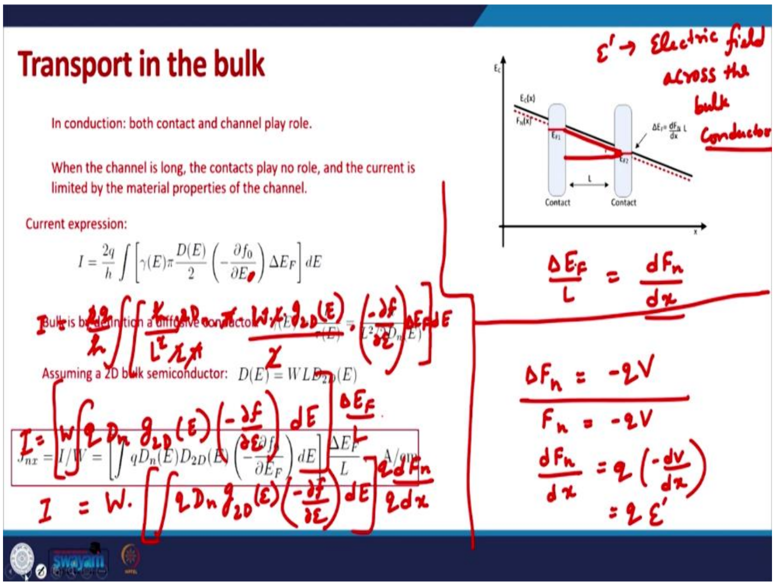

So, in the bulk transport case this $\frac{\Delta E_F}{L}$ can be written as essentially the gradient of the quasi Fermi levels because now in bulk conductors we can define the Fermi level across the entire material across the entire channel. And in steady state this Fermi level is defined by the quasi Fermi levels as I told you just in the beginning of the this in the beginning of this discussion and so this difference or this $\frac{\Delta E_F}{L}$can now be written as this $\frac{\Delta E_F}{L}$ can be written as the gradient of the quasi Fermi levels.

This is one and at the moment we are only considering the transport due to electrons in the conduction band. So, this is one second is generally the way Fermi levels are defined the Fermi levels are dependent on the applied voltage this is how it happens. If we apply a voltage V in the Fermi levels change according to this expression.

And if the reference Fermi level is assumed to be at 0 energy this is how we can define. And the gradient of the quasi Fermi level or even a Fermi level sorry the in case of Fermi level gradient cannot be there because if there is a gradient it will not longer be a equilibrium condition and in that case we only can use the idea of quasi Fermi levels.

So, this can be written as q times $-\frac{dV}{dx}$. And what is this? This is essentially the electric field applied across the device. So, this is the electric field which is also represented by E. Please do not confuse this electric field E by energy E. So, let me put a prime over this, where $E'$ is the electric field. So, $\frac{dF_n}{dx}$ is essentially q times $E'$, $E'$is the electric field across the bulk conductor.

Now, putting everything in here we can replace $\frac{\Delta E_F}{L}$ by $\frac{dF_n}{dx}$. So, I can be written as and please remember we are talking about a 2D channel we are initially considering a 2D conductor $qD_n g_{2D}(E)\left(-\frac{\partial f}{\partial E}\right)dE$ this can be written as $\frac{dF_n}{dx}$. And if we divide and multiply by q, this $\frac{dF_n}{qdx}$ will be $E'$ essentially ok.

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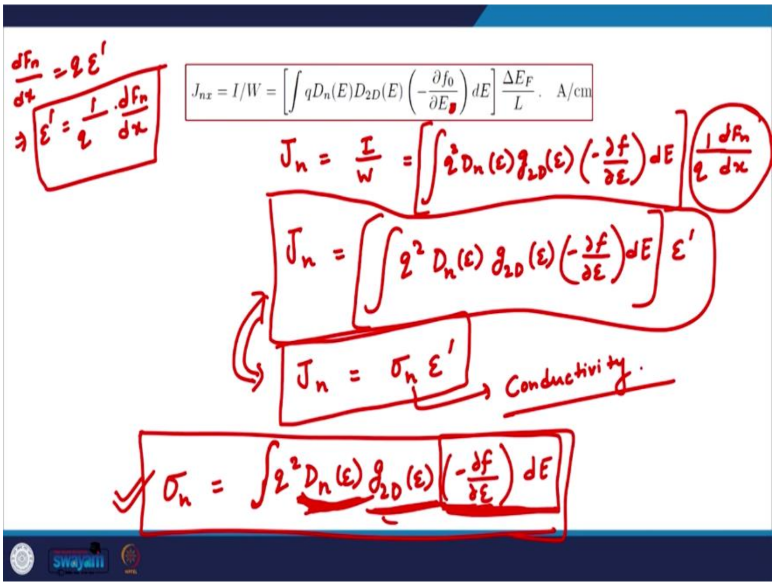


And in a 2D conductor the current density is defined as the I by W is defined by this which will be essentially from the previous expression it will be $\int qD_nD_{2D}(E)\left(-\frac{\partial f}{\partial E}\right)dE$.

Let us bring this q inside 1 by q$\frac{dF_n}{dx}$ and what we saw is that $\frac{dF_n}{dx}$ is q times E prime where $E'$ is the electric field which means $E'$is 1 by $\frac{1}{q}\frac{dF_n}{dx}$. So, this is essentially the electric field in the bulk conductor. So, $J_n$ can be written as integral of $\left[\int q^2D_ng_{2D}(E)\left(-\frac{\partial f}{\partial E}\right)dE\right]E'$.

And if you recall from the classical description of a bulk conductor the current density in the bulk conductor is given as the $J_n$ is sigma times electric field where this sigma is an important quantity in bulk conductors this is known as the conductivity of the material. And if we equate this classical expression with the expression that we have just derived for the current density with, if we equate these two expressions.

The electrical conductivity of electrons in the conduction band only is essentially $\int q^2D_ng_{2D}(E)\left(-\frac{\partial f}{\partial E}\right)dE$ ok. So, this is the conductivity that comes from the fundamental description of the transport ok.

So, now the conductivity instead of deriving it from the classical description of electrons using Drude's model we have derived the conductivity from using a bottom up approach in which we are just restarted with fundamental physics of the device and as expected the conductivity is primarily depends on apart from the Fermi window this function it primary depends on the two quantities one is the density of states in the channel and second is $D_n$ which is essentially which accounts for the diffusion which accounts for the scattering.

So, it depends on two things one is how many electronic states are there in the channel second is the scattering of electrons in those electronic states which is also prima facie we could also sort of imagine which is also in which makes sense. So, this $\left(-\frac{\partial f}{\partial E}\right)$ is again from our previous description of this function this is a Fermi window function and the conductivity of electrons is important only in a small range of energies around the Fermi level that is because of this Fermi window function ok.

So, essentially conductivity from the basic description of the device the conductivity is given by the density of states and the scattering around the Fermi level in a channel that is what these three terms collectively says collectively that is what they convey ok.

So, this is an important idea, now because starting with the basics of quantum mechanics the density of states idea the general model of transport we have now deduce the a bulk property of the material that is the conductivity of the material in terms of the basic parameters of the device.

What is not done yet is the actual description of this $D_n$ which comes from the diffusion and since this comes from the scattering theory I did not want to go directly into this because this will take the discussion in other direction ok. So, this essentially completes the treatment.

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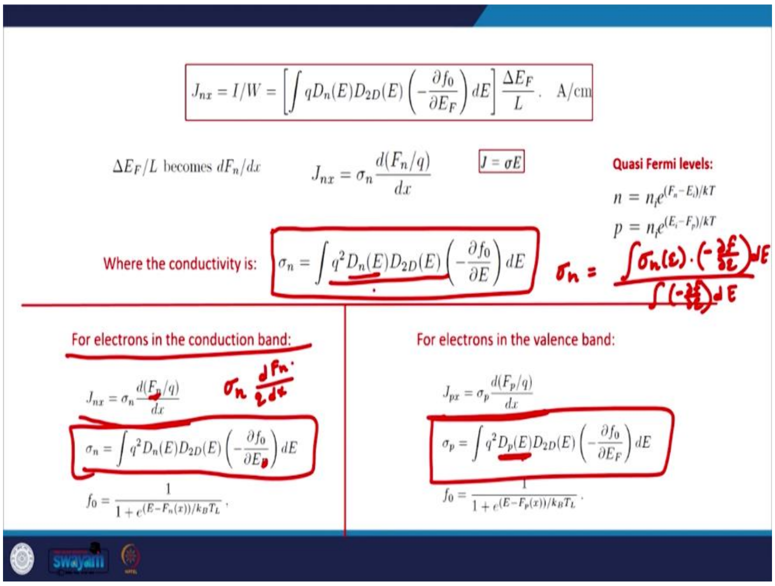


And we can say from this entire discussion that the conductivity is given by this function and the conductivity can also be written as in a way like conductance. It can be written as the average of the conductivity function and the average is to taken over the Fermi window where this conductivity function is $q^2 D_n D_{2D}$ ok.

It does not involve any physical dimensions the physical dimensions may be important in some cases in this ok. So, for the electrons in the conduction band. So, and please remember that we invoked the idea of quasi Fermi levels and the quasi Fermi levels may be different in conduction band and in valence band.

So, for electrons in conduction band the current density for a 2D conductor is essentially the current divided by the width is given as $\sigma_n$ times gradient of the quasi Fermi level where $\sigma_n$ is given by this expression the exactly this expression $\int q^2 D_n D_{2D}(E)\left(-\frac{\partial f}{\partial E}\right) dE$ where f is the Fermi function.

Similarly, the conductivity of electrons in the valence band which essentially indicates the conductive towards the conductivity of force will be given by this expression in which we have instead of $f_n$ the quasi Fermi level for electrons instead we have $f_p$ the quasi Fermi level for electrons in the valence band and here the scattering might be different. So, that is why we have used a different constant here instead of using $D_n$ we have used $D_p$ and apart from that everything else is the same in both cases ok.

So, that is how we define the conductivity and we deduce the conductivity using a bottom up approach. So, this essentially completes our discussion of the conductance function the conductivity function and how do we generalise the ideas of the general model of transport to the bulk transport case.

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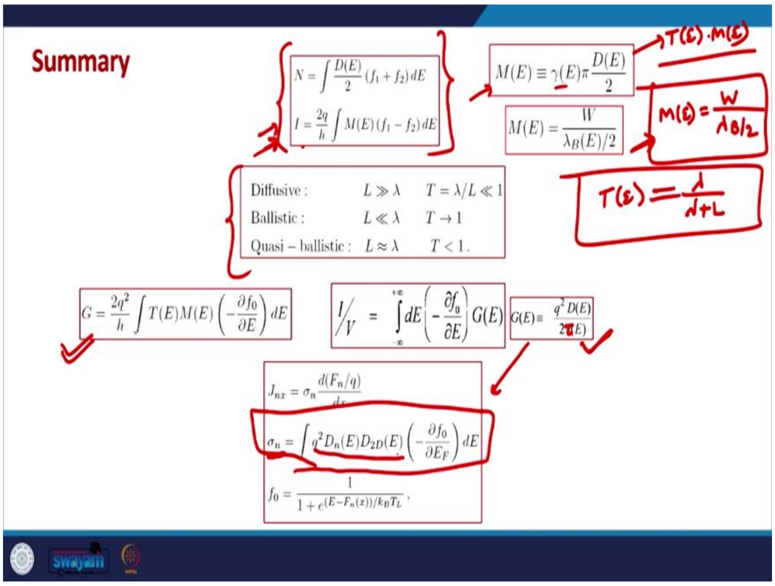

Now, let me give you a brief summary of the entire general model of transport and from by using the basic write equations we deduce these two expressions the expressions for the steady state of the electronic population and the steady state current. The and I said in the beginning that these expressions are extremely important from the electrostatics point of view as well as from the transport point of view.

While dealing with the current while trying to understand current in more details we came across this idea of modes, which is essentially the product of $\frac{\gamma(E)\pi D(E)}{2}$ and this in turn turns out to be if we see in ballistic transport case this turns out to be W divided by $\frac{\lambda_B}{2}$ where W is the width of the 2D conductor and $\lambda_B$ is the de Broglie wavelength of the electrons.

So, M(E) which is the nodes or the conduction pathways is essentially the number of half wavelengths that can fit into the width of the conductor. So, that is what this idea of modes tell us and this is also this makes sense intuitively as well from there we generalised this we try to investigate the current expression for diffusive case.

And in that case this quantity $\frac{\gamma(E)\pi D(E)}{2}$ it becomes T(E) times M(E) where T(E) is the transmission coefficient this transmission coefficient is essentially $\frac{\lambda}{\lambda+L}$ and these are the values of the transmission coefficient for various regimes of transport ballistic, diffusive and quasi ballistic.

From there after that we deduced the conductance of the device which turns out to be this and the conductance can be said to be the average of the conductance function which is given by this expression in the Fermi window. Fermi window is the window around the Fermi level for low temperatures it is like a delta function it is actually a delta function at T equal to 0 kelvin and high higher temperatures it is a window of area 1.

Then we generalised this idea to the bulk transport case and in bulk transport case we again came across a new idea, which is the idea of conductivity of the bulk material and that turns out to be the average of the conductivity function which is essentially this and it is again averaged in the Fermi window.

So, that is how we have developed the theory of transport using bottom up approach next class onwards we will see some practical calculations we will see how the resistance of an actual 2D conductor looks like or a 3D conductor looks like at 0 at low temperatures or at high temperatures.

Thank you for your attention see you in the next class.