Physics of Nanoscale Devices
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Lecture - 26
Conductance, Bulk Transport-I

Hello everyone today we will discuss the idea of Conductance using the bottom up approach that we have been discussing since last few classes. And we will generalize this entire idea this entire general model of transport and see, how can we use this in the bulk. Basically bulk, how can we deduce bulk transport using the general model of transport that we have been discussing so far?

So, just to sort of remind you in the beginning itself that in Bulk Transport the idea of conductivity is quite important. So, that is why we have started looking at the idea of conductance right from the ballistic and diffusive transport cases.

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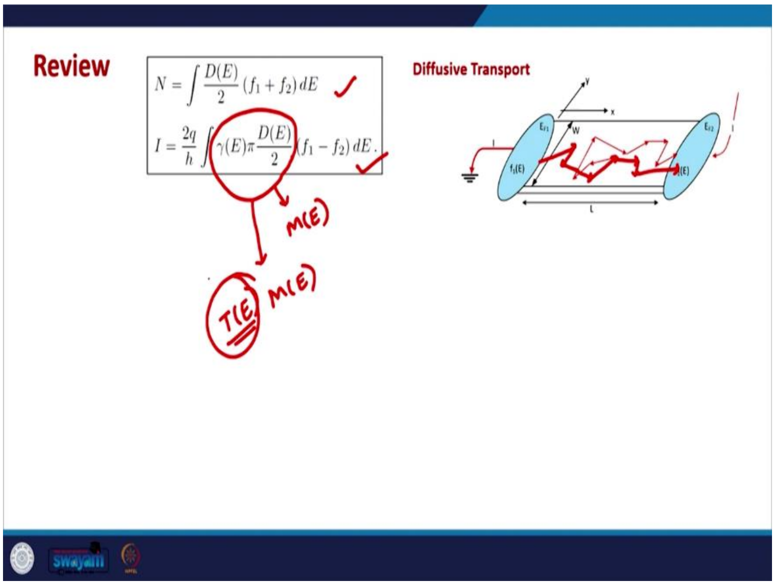

So, let me quickly review what we have been seeing in our discussion. We saw that that in steady state the electronic population looks like this the current looks like this. The only difference in ballistic and diffusive transport is, this quantity essentially. In ballistic transport case, this quantity is the number of modes in the device.

And in the diffusive transport case this becomes the number of modes times the transmission coefficient, where this transmission coefficient accounts for the scattering in the channel; accounts for the scattering because now the channel is long in diffusive transport.

And electron travels like this on a zigzag path from the source to the drain and some of the electrons may reach to the drain side and some of them may not reach to the drain side. So, this might be one of the paths one of the possible paths that the electrons can take. So, this is accounted for by this transmission coefficient T(E).

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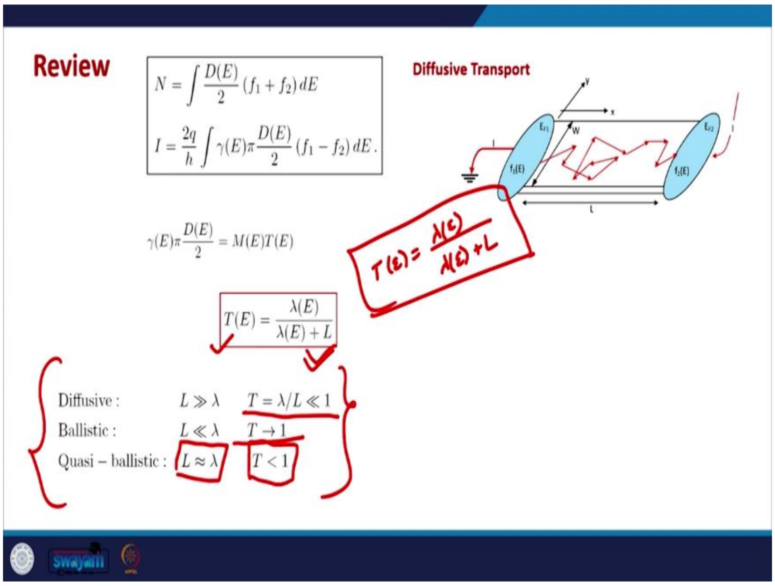

And we saw that using Ficks law by applying Ficks law in this case in the transport case, we saw that the transmission coefficient turns out to be this. So, T(E) is essentially $\frac{\lambda(E)}{\lambda(E)+L}$, where $\lambda(E)$ is the mean free path of an electron with energy E. And in various limits we saw that in various limits for example, in ballistic diffusive and quasi ballistic case we saw, what are the values of the transmission coefficient?

In ballistic case it automatically from this expression turns out to be T almost equal to 1. In diffusive case it is very very less than 1. And in quasi ballistic case when the length of the channel is of the order of the mean free path of electrons it is quite close to 1, but less than 1 ok.

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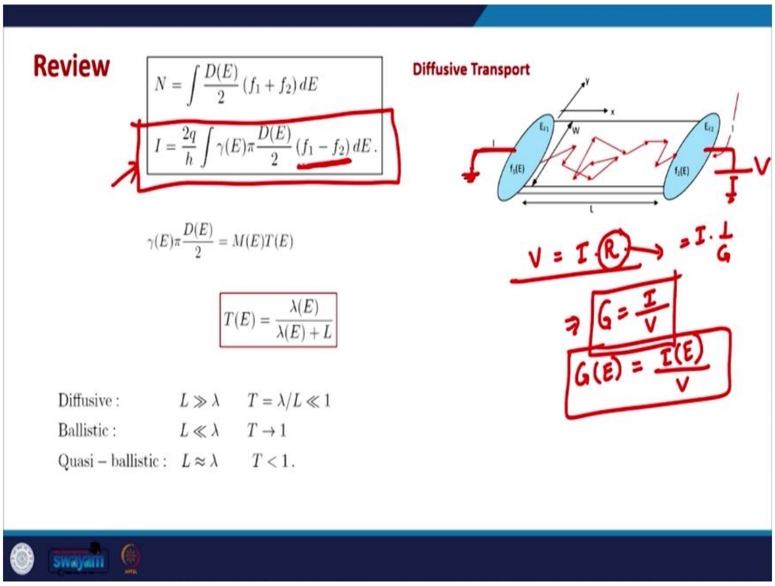

Then we started the idea of conductance and as all of us know, that very basic relationship between the voltage and the current is this. So, the voltage and currents are related to each other by the resistance of the device and inverse of the resistance is known as the conductance. So, which means that the conductance is essentially current divided by voltage.

So, conductance of an electron at energy E will be the current that is contributed by electrons of that energy divided by the applied voltage. So, in order to sort of derive this expression of conductance we need to explicitly obtain a relationship between the voltage and the current from this expression, this equation basically.

And in this equation voltage applied across the device if we apply a voltage for example, if the source side is grounded in the green side we have applied a voltage V that is accounted for by the difference of the Fermi functions f_1 - f_2.

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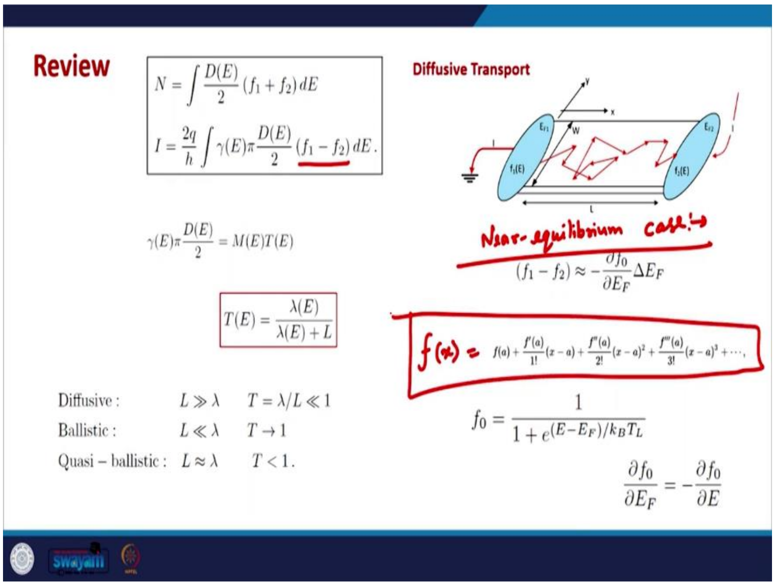

And what we saw in the near equilibrium case in near equilibrium case, and when is the near equilibrium case? Near equilibrium case is when we apply a short voltage across the device, the applied voltage is very small. And in that case the difference between f_1 and f_2 is there, but it is not very large.

And in that case we can by using Taylor series expansion of a function for example, according to the Taylor series a function can be expanded in this form. We could deduce that the difference between the Fermi functions is essentially f_1 - f_2 is equal to.

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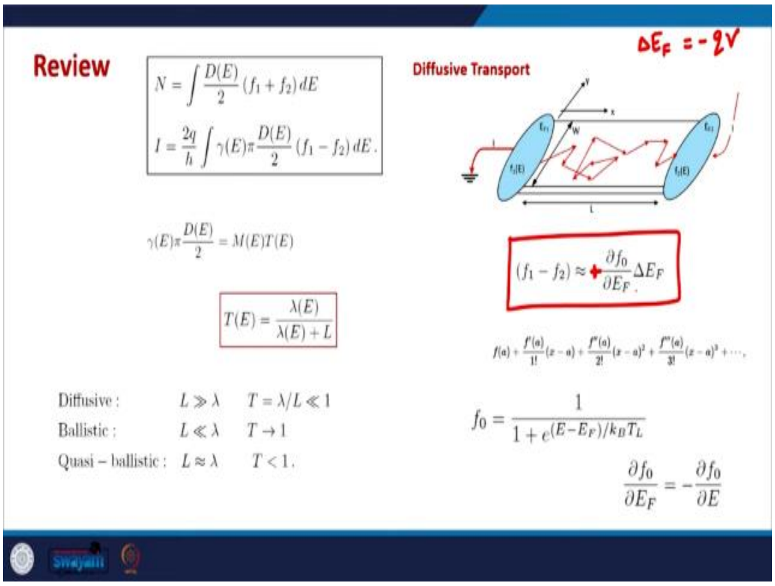

Actually it is a matter of convention in some cases it is assumed that ΔE_F is at some reference is it is assume that it is minus q times V. And in that case this difference will be $(-\frac{\partial f_0}{\partial E_F})\Delta E_F$.

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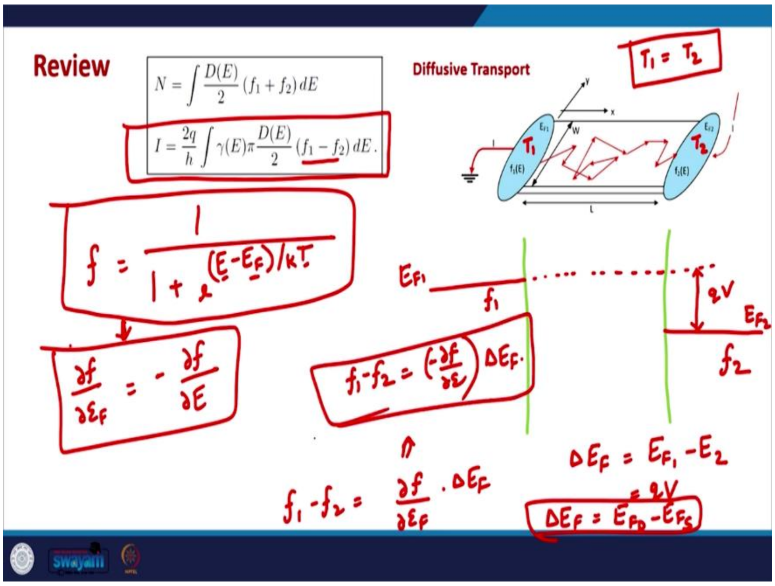

But using our convention which is which looks like this in which case we assume that, we have this kind of a device, these are the contacts. On the source side we have E_{F1}, Fermi function f_1, on the drain side we have a Fermi level E_{F2} the Fermi function is f_2. And the applied voltage basically brings down the drain side Fermi function and the drain side Fermi level and this difference between E_{F1} and E_{F2} is given as q times V.

So, in our case ΔE_F which we define to be E_{F1} - E_{F2} is q times V. In some sources in some books and papers ΔE_F has been defined as E_F drain side minus E_F source side. In that case this ΔE_F becomes minus q times V and $f_1 - f_2$ becomes, in that case this becomes $\left(-\frac{\partial f}{\partial E_F}\right)\Delta E_F$. But in our case this negative sign is not there ok.

This expression can be written as $f_1 - f_2$ can be written as $-\frac{\partial f}{\partial E}$. So, $\frac{\partial f}{\partial E_F}$ can be written as $(-\frac{\partial f}{\partial E})\Delta E_F$. And the reason for that is that the Fermi function has a general form like this it is $\frac{1}{1+e^{\frac{E-E_F}{kT}}}$.

So, it means that if we take a derivative of f with respect to E_F it will be just an negative of derivative of f with respect to E, this can be a small exercise just right out it is a simple derivative technique. So, from here what we see is that the Fermi function is a function of energy it is also a function of Fermi level also the Fermi function is a function of temperature as well.

So, in our treatment in so far we have assumed that the temperature of source contact and the temperature of the drain contacts are the same. So, T1 is equal to T2 is assumed, because if the temperature is different in that case also there might be a difference in the Fermi function and that might result in a current conduction. Those are known as the thermal effects or thermoelectric devices, that we will also discuss once we complete this discussion of general model of transport and then a brief of MOSFET devices ok.

So, this is all what we have, now we are in a position to basically calculate the conductance. Because in the current equation in this equation we just need to replace $f_1 - f_2$ by this.

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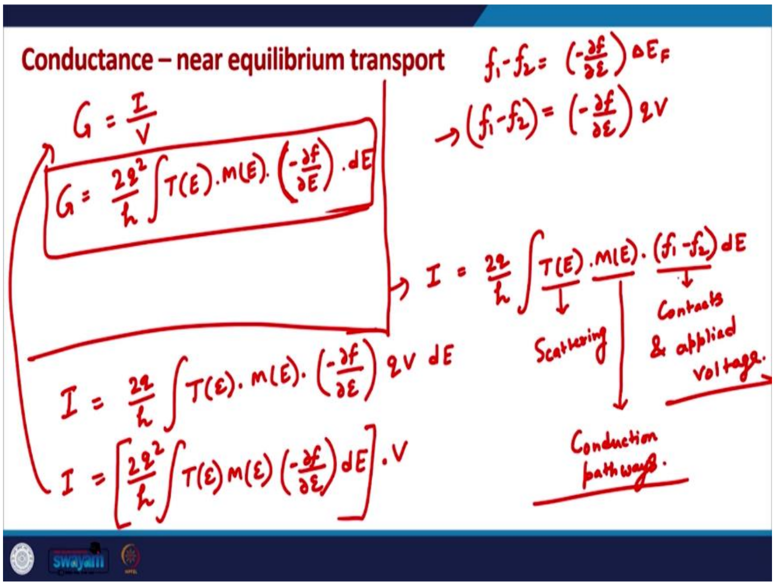

So, if we do that, I is equal to $\frac{2q}{h}\int T(E)M(E)\,(f_1 - f_2)\,dE$ ok. In this equation as well you can see there are broadly three terms, one is T(E) second is M E) and third is $(f_1 - f_2)$. These three terms actually account for different processes or different physical phenomena that go into the current conduction. T(E) accounts for the scattering of electrons in the channel and that is why T(E) is equal to 1 in case of ballistic transport, because in ballistic transport there is no scattering.

Any accounts for the conduction pathways in the channel. How many conduction pathways are available for electrons to travel in the channel? This is a fundamental property or that comes from the fundamental physics of the channel. $(f_1 - f_2)$ describes the contacts and the applied voltages and applied voltage.

And ultimately current is and a cumulative I would say a cumulative effect of all these three things that go on in a device. So, using this expression we can now see that if we put $(f_1 - f_2)$ to be equal to from the Taylor series expansion if ye put this to be $(-\frac{\partial f}{\partial E})\Delta E_F$ and ΔE_F is q times V. So, $(-\frac{\partial f}{\partial E})qV$.

So, if we do this if we use these two expressions this one and this one what we obtain is I is equal to $\frac{2q}{h}\int T(E)M(E)\,(-\frac{\partial f}{\partial E})qVdE$. So, what we can see from here is I is $\frac{2q^2}{h}\int T(E)M(E)\,(-\frac{\partial f}{\partial E})VdE$.

Because V can be taken out of the integral because V is not dependent on the electron energy, it is an external parameter that we can control from the battery. So, the ratio or the from this expression from here what we can see is the conductance which is essentially the ratio of current and voltage is $\frac{2q^2}{h}\int T(E)M(E)\,(-\frac{\partial f}{\partial E})dE$ this integral essential. So, this turns out to be the conductance of the channel of the device.

And this is true for the case for the ballistic case and as well as for the diffusive transport case ok. And please also remember that this approximation holds true, when we have the near equilibrium conditions because in that case only this $(f_1 - f_2)$ can be approximated by the $(-\frac{\partial f}{\partial E})\Delta E_F$ ok.

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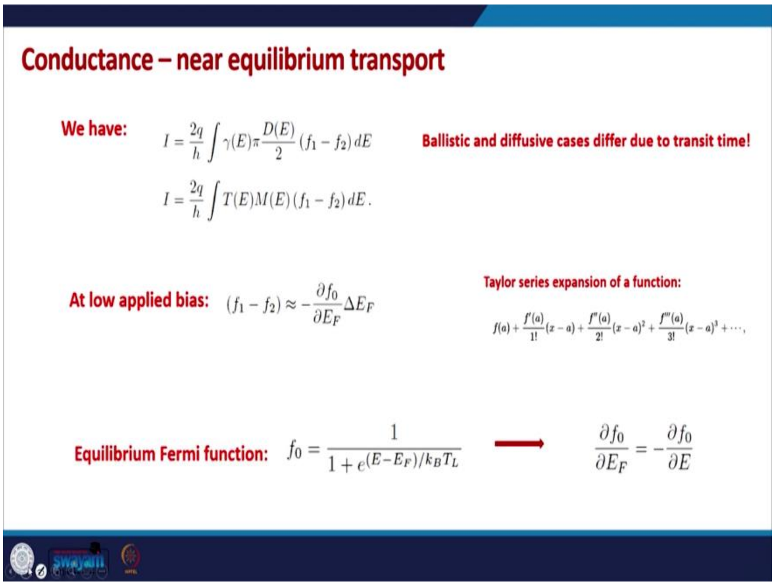

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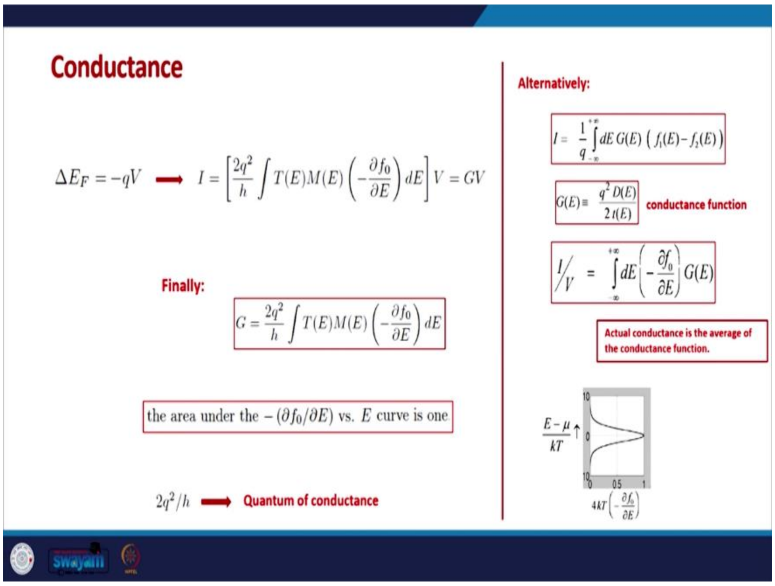

So, this is what we obtain, the conductance finally this is the expression for the conductance ok. Now we need to see few important things here actually.

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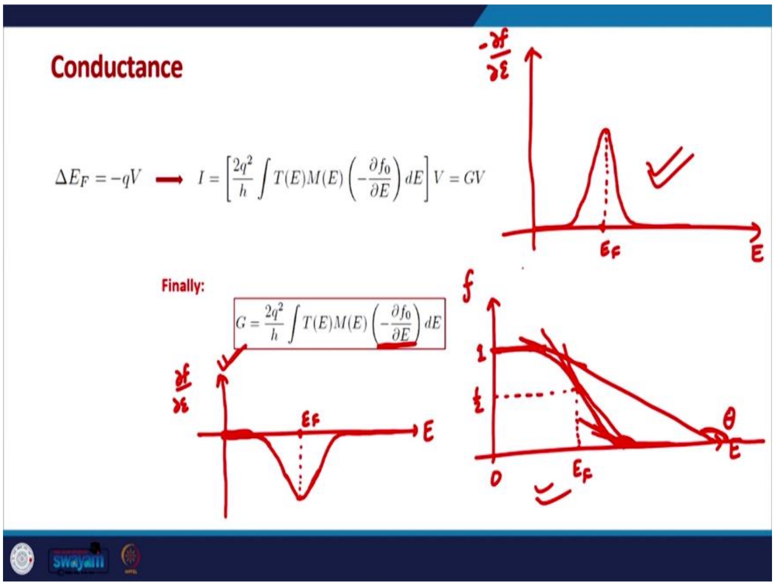

This is finally, the expression for the conductance. Now, this quantity $(-\frac{\partial f}{\partial E})$ is an interesting quantity. Here it is written as $(\frac{\partial f_0}{\partial E})$ which is fine. So, what f is a Fermi function, so the way it looks like is $\frac{1}{1+ e^{\frac{E-E_F}{kT}}}$.

So, this is an important mathematical part here which we always need to keep in mind. This is an important quantity and let us see how it looks like. Because this is going to appear in conductance this will be there in the current expression, so we need to properly see what it is.

So, let us start with how the Fermi function essentially looks like, so this is the form of the Fermi function. So, if we plot the Fermi function as a function of energy this will look something like this, so the value of the Fermi function changes right from 1 to 0.

So, up to a certain energy it is 1 and the energy where it is half that energy is known as the Fermi level, at low energies it is 0 at high energies the Fermi function is 1 ok. So, now, if we try to plot what $\frac{\partial f}{\partial E}$ look like, let us see how it may look like. So, this is E this is $\frac{\partial f}{\partial E}$ and if we have E F here for example, at this point. So, $\frac{\partial f}{\partial E}$ versus E will just be the gradient function of this function the function that is plotted here on the right hand side.

So, the gradient of this function at low energies at these points is 0 almost. So, this function will be 0 at low energy values, so this will be the. So, at low energies the energies which are way below then the Fermi level that $\frac{\partial f}{\partial E}$ function is 0 as the energy starts approaching E_F level we start seeing some gradient there.

So, the gradient at this point roughly is this and this angle theta is greater than 90 degrees, so the gradient is negative. So, initially the gradient was 0 then gradient starts becoming non-zero negative. So, this is how gradient at this point the gradient is negative, but a with high magnitude, at this point also gradient is negative.

So, generally this function will be, but as soon as we start going. So, up to E_F energy level this magnitude of the gradient starts increasing it increases essentially. After E_F level as you can see the gradient at this point is again the magnitude is again decreasing. So, it is again decreasing after E_F and at high energy values the energy values that are far away from the Fermi level the gradient again becomes 0.

So, this function will look something like this and $-\frac{\partial f}{\partial E}$ function will just be the inverse of this function, we will just need to say change the sign of this function we just need to flip this function with respect to X-axis that will give us minus $-\frac{\partial f}{\partial E}$. So, finally, $-\frac{\partial f}{\partial E}$ function turns out to be like this. So, if this is the Fermi level initially this function is 0, so it is kind of a window function.

And this is a window around the Fermi level and this function is 0 when the Fermi level is either 1 or 0 when the Fermi level is constant 1 or constant 0, up to those energy values or up to those energy points this function is 0. So, this is kind of a window function ok.

In addition to this if the temperatures are very low, let us say at extremely low temperatures the Fermi level will be step function as we all of us know. So, the Fermi level at very low temperature says is a step function. So, this is the way it looks like. So, let me plot both of them close to each other on the same reference.

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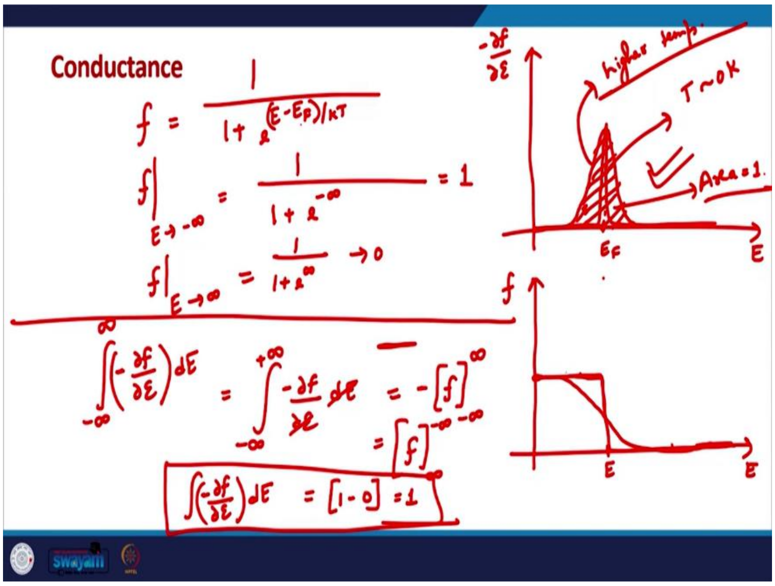

So, when we have a very low temperature in that case if we plot f versus E, this is kind of a step function. And in that case this function becomes like a delta function. So, this holds true when T is close to 0 Kelvin or extremely low temperatures this is at higher temperatures.

So, that is one thing that is there that this $-\frac{\partial f}{\partial E}$ function is a window function. It is 0 for extremely small values of E and it is 0 for extremely large values of G, but around the Fermi level it is kind of a window function. Second important property of this function is that, if we take sort of the derivative of this function if we take this derivative $-\frac{\partial f}{\partial E}$ over all possible values of energy let us try to calculate this derivative.

So, if you do this is a simple maths the derivative $-\frac{\partial f}{\partial E}$ times dE, dE and ∂E goes away, because at the moment we are only considering that the Fermi level is constant temperature is constant. So, the Fermi function is just the function of energy and. So, $\frac{\partial f}{\partial E}$ becomes $\frac{df}{dE}$ and this goes away.

So, what we are left with is $[-f]_{-\infty}^{\infty}$ or f if we change the limits. Now the Fermi function mathematically is this $\frac{1}{1+e^{\frac{E-E_F}{kT}}}$. And if we assume that E_F and T(E) are constant for a given system if the Fermi level and the temperature are constant. Then f at E tending to $-\infty$ will be 1 divided by $-\infty$, E to the power $-\infty$ is 0, so it becomes 1.

And the Fermi function at extremely high energy values E tending to ∞, if we evaluate the Fermi function at extremely high energy values that is 1 plus e^{∞}, so it tends to 0. So, this upper limit –f at $-\infty$ at extremely small values of energy f is 1 and 0 at extremely high energy values which is also clear from the plot,

If we have a general plot of Fermi function; it is 1 at very low energy values and it is 0 at extremely high energy values ok, we do not even need to do this maths. So, this is essentially 1, so the integral of $-\frac{\partial f}{\partial E}$ over E is 1 ok. So, these are two things that we need to keep in mind one is that $-\frac{\partial f}{\partial E}$ is a window like function and second is that the area that this function sort of covers with the energy access is 1.

So, which means that the area in this window is 1, this area is area of this window on the energy access with respect to energy access is 1. With these two things with us, now let us come back to the expression of the conductance that we derive.

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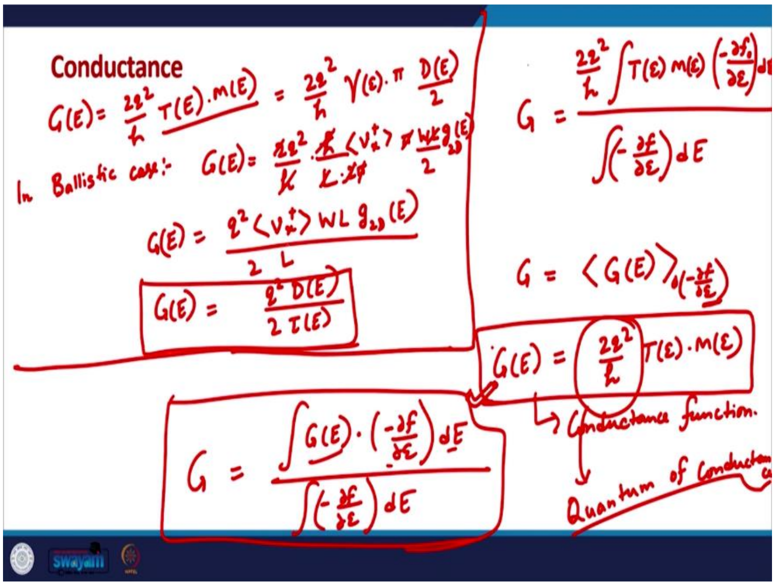

So, the conductance is $\frac{2q^2}{h} T(E)M(E)\left(-\frac{\partial f}{\partial E}\right) dE$. This expression can equivalently be written as like this. So, we can write this expression to be like this as well, because the integral that we are writing in the denominator is 1 as we have just calculated. The value of this integral is exactly equal to 1, so we can always divide by this integral not a problem at all. Now if you see this expression this can be written as sort of the average of over the this average is taken over $-\frac{\partial f}{\partial E}$.

So, it means where G(E) is essentially $\frac{2q^2}{h}T(E)M(E)\left(-\frac{\partial f}{\partial E}\right)dE$, this is known as the conductance function. And if we take the average of conductance function over this window, which is also known as Fermi window $-\frac{\partial f}{\partial E}$ is also known as Fermi window. If we take the average of this conductance function over Fermi window, that will give us the total conductance of the device.

And that is an interesting result this turns out to be the conductance of the electrons at energy E and it is known as the conductance function . So, the total conductance can be written as G = $\frac{\frac{2q^2}{h}\int T(E)M(E)\left(-\frac{\partial f}{\partial E}\right)dE}{\int\left(-\frac{\partial f}{\partial E}\right)dE}$ we are taking average of conductance function over Fermi window ok.

It also has an interesting interpretation which we will do in our coming classes as well. That this is also the conductance of one conduction pathway and that is this quantity this constant number, this is that is why known as the quantum of conductance. This is the conductance of one conduction pathway, it is just this quantity $\frac{2q^2}{h}$.

And the total conductance of the device will be in near equilibrium conditions please remember that. It will be the average of the conductance function over the Fermi window ok. Now in the case of diffusive transport, this conductance function is G(E) is $\frac{2q^2}{h}$ T(E) times M(E), this term is essentially $\frac{2q^2}{h}\frac{\gamma(E)\pi D(E)}{2}$.

Let us try to see how the conductance function looks like in the case of ballistic transport and in the case of diffusive transport. So, in the case of ballistic transport, in ballistic case the conductance function is $\frac{2q^2}{h}$, γ is $\frac{\hbar}{\tau}$ and τ is $\frac{L}{<v_x^+>}$. πD is for a 2-D channel we are since we are considering we are doing all the discussion for the 2-D channel. This D is WL$\frac{g_{2D}(E)}{2}$.

This $\hbar$ can be written as $\frac{h}{2\pi}$. So, h h go away 2 and 2π with π, L by L, so G(E) is essentially $\frac{q^2\langle v_x^+\rangle WLg_{2D}(E)}{2L}$. So, this L is if we keep the L we need to divide by L or this can alternatively be written as $\frac{q^2D(E)}{2\tau(E)}$ because $\tau(E)$ is $\frac{L}{<v_x^+>}$.

So, this is the conductance function for the case of ballistic transport. This is also true in the case of diffusive transport just in case of diffusive transport the τ(E) will be instead of $\frac{L}{<v_x^+>}$ it will be $\frac{L^2}{2D_n}$ ok. So, let us do a small calculation we could not start the bulk transport case today, but we will do that in the coming class.

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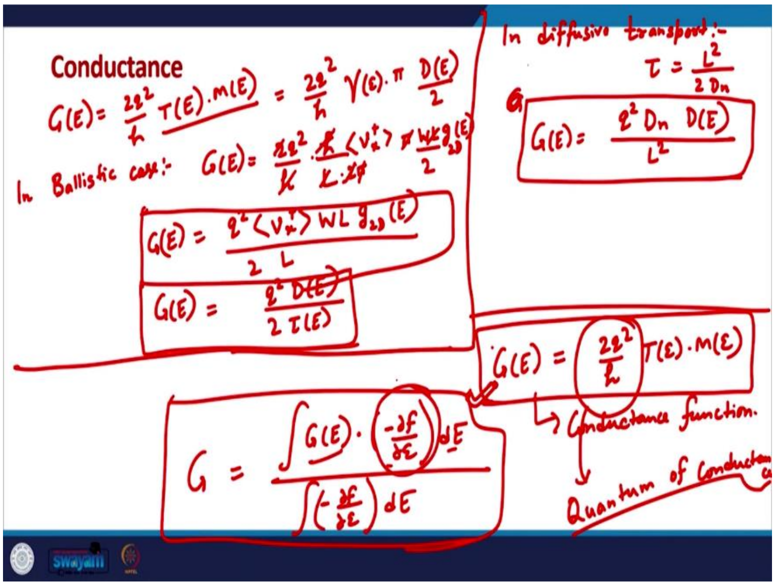

In diffusive transport case this will be the same tau can now be written as τ is essentially $\frac{L^2}{2D_n}$. So, this conductance function is $\frac{q^2D_nD(E)}{L^2}$ and these 2 will cancel with each other ok. And in the case of ballistic transport this will be the conductance function ok

So, just to sort of briefly recall what we discussed. We discussed that in near equilibrium transport, f_1 - f_2 can be written as minus $(-\frac{\partial f}{\partial E})\Delta E_F$. From there we can deduce that the conductance of the system both in ballistic and diffusive transport case is the average of the conductance function over a Fermi window function over the Fermi window, where Fermi window is $(-\frac{\partial f}{\partial E})$ ok.

So, please keep these ideas in mind these are important ideas; it means that from our plot of Fermi window that the conduction pathways that are far away from the Fermi level they do not contribute in the conductance. So, we will discuss more about this in coming classes.

Thank you for your attention, see you in the next class.