Physics of Nanoscale Devices
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Lecture - 25
Diffusive Transport, Conductance

Hello, everyone. We will today finish our discussion of Diffusive Transport, we will try to find out the expression for current in the case of diffusive transport and then we will discuss the next important topic that is the conductance in the case of ballistic transport and in the diffusive transport.

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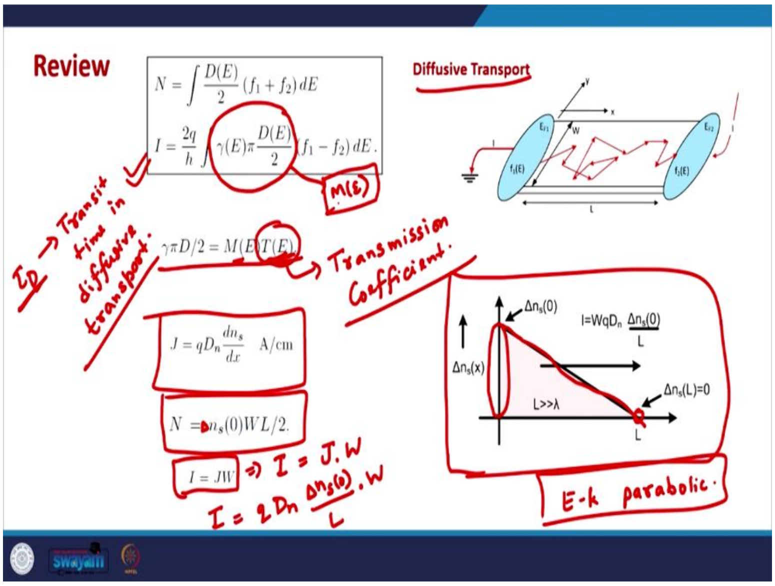

Before going into that let me quickly review what we were discussing, essentially we had this with us we had this expression for the steady state electronic population and steady state current from the general model of transport.

We could see that in the case of ballistic transport this term is the number of modes in the device. But when we have a diffusive transport scenario in which electrons are colliding with intermediate atoms and they may be following a zigzag motion the electrons energy may also change in the channel during the transport.

In that case, this notion of modes does not hold true because modes are the conduction pathways and now since the electrons are not going through unique conduction pathways.

So, in this case we cannot say that this quantity is the modes and that is why this quantity is defined as the modes times a coefficient which is known as the transmission coefficient.

And, if we need to find out the current expression in the case of diffusive transport we would need to find out the expression for T(E) essentially. And in order to find out T(E) we need to find out $\gamma(E)$ in order to find out $\gamma(E)$ we need to find out the τ_D essentially which is the transit time in the case of diffusive transport case. You might have realized this that we are building the transport theory using a bottom up approach basically.

Because we have started with the idea of density of states, Fermi function, Fermi level of contacts and from that we are trying to build the theory of transport, we are trying to find out the expression for the currents IV characteristics and finally, the conductance of the material.

So, in the diffusive transport case we need to use we have the electronic profile in the device which looks like this we have access of electrons on the left side which is the source contact side, we have a low concentration on the drain side.

And because of this concentration gradient in the device the electrons will travel from the source side to the drain side. The profile is linear in absence of recombination generation and we can use if the electrons follow this parabolic E k relationship in that case the diffusion can be assumed or can be approximated by the Fick's law essentially which looks like this.

So, this will help us in finding out the τ_D time which is what we want to find out because using Fick's law we can find out the steady state current in the device which looks like this.

So, Fick's law gives us the current density and in a 2D channel the current density is the current per unit width and so the total current will be the current density times the width. And from the Fick's law this turns out to be J is essentially q times $D_n \frac{d}{dx}\ n_s$ turns out to be delta n s 0 by L times W. And the steady state electronic population is $\frac{\Delta n_s(0).W.L}{2}$essentially, ok.

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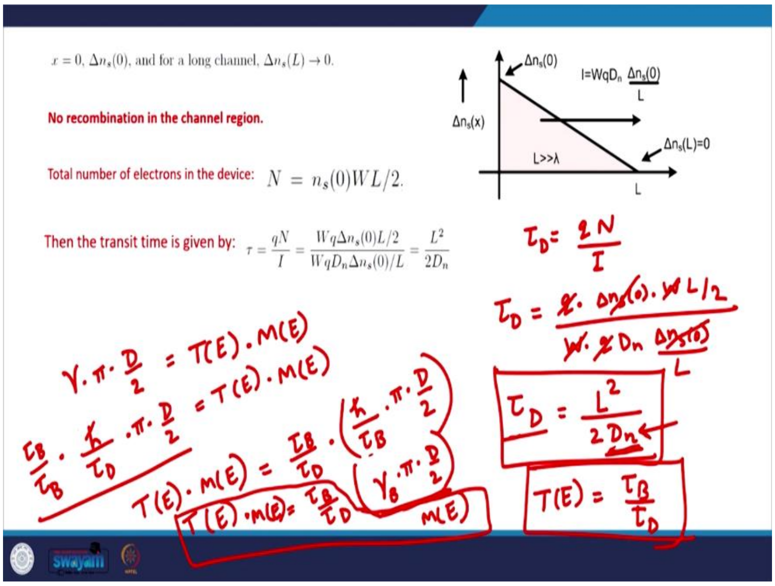

So, now, we are in a position to directly use this expression for τ we can essentially use τ is equal to q times steady state electronic population divided by the steady state current. And since we are talking about the diffusive transport case we will put a D with a τ. So, this τ_D will be equal to $\frac{q\Delta n_s(0).W.L}{2}$ divided by I will be equal to W times J and J is $\frac{\Delta n_s(0).qD_n}{L}$.

So, this is what we have in the diffusive transport case and as you can see that $\Delta n_s(0)$ $\Delta n_s(0)$ will cancel out q with q W with W what is left here is this τ_D will be equal to $\frac{L^2}{2D_n}$.

So, this is the transit time this is the characteristic time in the case of diffusive transport case ok. As you can see that, we now need to find out the or we now need to know the this D_n parameter in order to find out the τ_D parameter, ok.

This D_n parameter actually comes from the scattering theory from the, if we study the scattering of electrons in the channel by the other atoms. And in the scattering there are interaction between the electron and the other atoms there is energy and momentum exchange and that is sort of a separate topic to discuss about. But just with this expression at the moment if we have a look at this term $\frac{\gamma\pi D}{2}$ in the case of diffusive transport this is T(E) times M (E).

On the left hand side, we would have γ will be equal to $\frac{\hbar}{\tau_D}$times π times $\frac{D}{2}$, on the right hand side, we have this M(E) if we multiply and divide by τ_B on the left side. Then this can be rewritten as let us bring the right hand side to the left side T(E) times M(E) is equal to $\frac{\tau_B}{\tau_D}$ times $\frac{\hbar}{\tau_B}$ times π times $\frac{D}{2}$ ok.

So, this is then this is the γ in the case of ballistic transport π times $\frac{D}{2}$. So, all this is essentially number of modes. So, this T (E) into M (E) is equal to this. So, rewriting this T(E) is equal to $\frac{\tau_B}{\tau_D}$. So, this transmission coefficient is essentially the ratio of the transit time in the ballistic case to the transit time in diffusive case ok.

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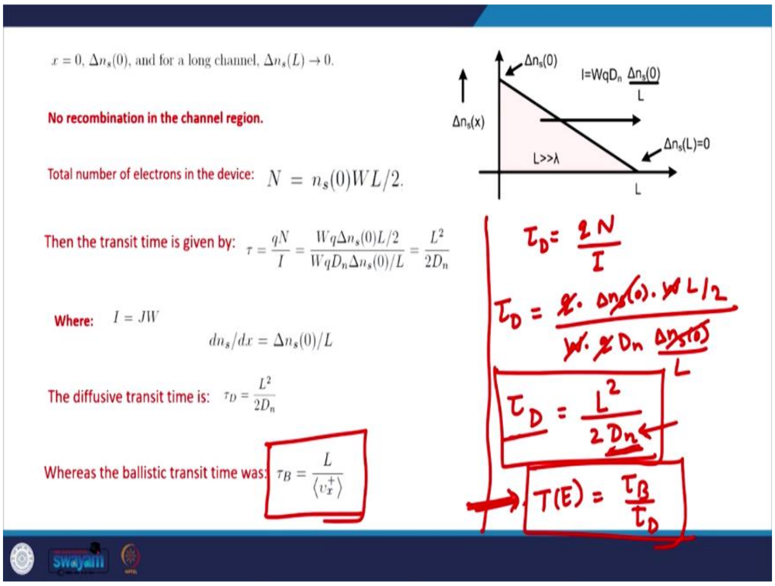

And, as we clearly know that the transit time in the ballistic case is essentially $\frac{L}{\langle v_x^+ \rangle}$. So, this is the expression for the transit time in ballistic case τ_B is equal to essentially $\frac{L}{\langle v_x^+ \rangle}$. And the transmission coefficient is the ratio of $\tau_B$to $\tau_{D.}$ τ_D is $\frac{L^2}{2D_n}$ where L is the length D_n is the diffusivity or diffusion constant of the material ok.

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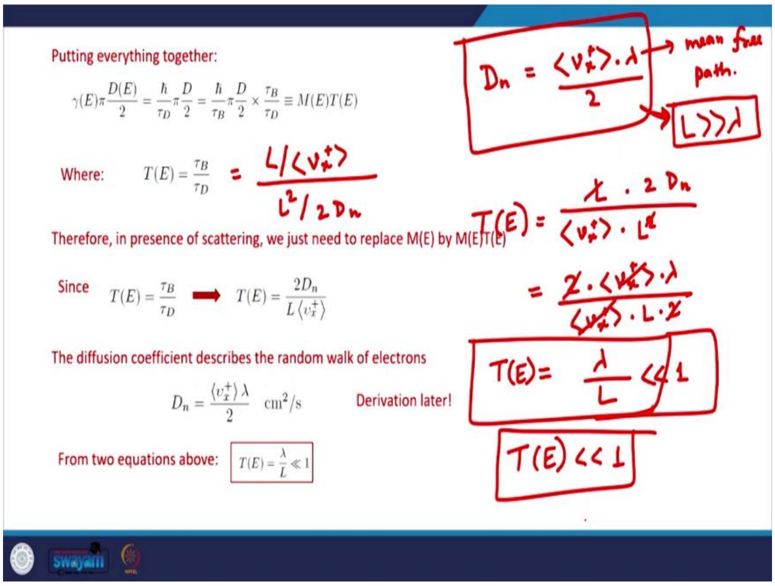

Now, if we put everything there in the expression for T(E) what we see is that T(E) which is equal to $\frac{\tau_B}{\tau_D}$. τ_B is $\frac{L}{\langle v_x^+\rangle}$ divided by τ_D is $\frac{L^2}{2D_n}$. Now, from the theory of scattering which is in a way a separate topic this coefficient diffusion coefficient turns out to be $\langle v_x^+\rangle$ times $\frac{\lambda}{2}$.

Average velocity in x direction positive x direction times λ, where this λ is the mean free path of the electrons. So, just at the moment please assume that this is true. So, if we consider this to be the value of the diffusion constant diffusivity then if and if you put this in the this expression the transmission coefficient turns out to be $\frac{L}{\langle v_x^+\rangle}$ into $\frac{2D_n}{L^2}$ ok.

So, what is left is 2 times $\langle v_x^+\rangle$ into λ divided by $\langle v_x^+\rangle$ L into 2, which comes from here. So, what is left is this, this is the case when the channel is extremely large as compared to the mean free path. So, this thing comes from the scattering theory when L is extremely large as compared to the mean free path of the electron which means that the channel length is or the transport is purely diffusive transport ok.

So, in that case this will be very very less than equal to 1 when the L is extremely larger than λ in that case this transmission coefficient will be very very less than 1 ok. Now, this expression was for a limiting case when the L is extremely large as compared to the λ as compared to the mean free path.

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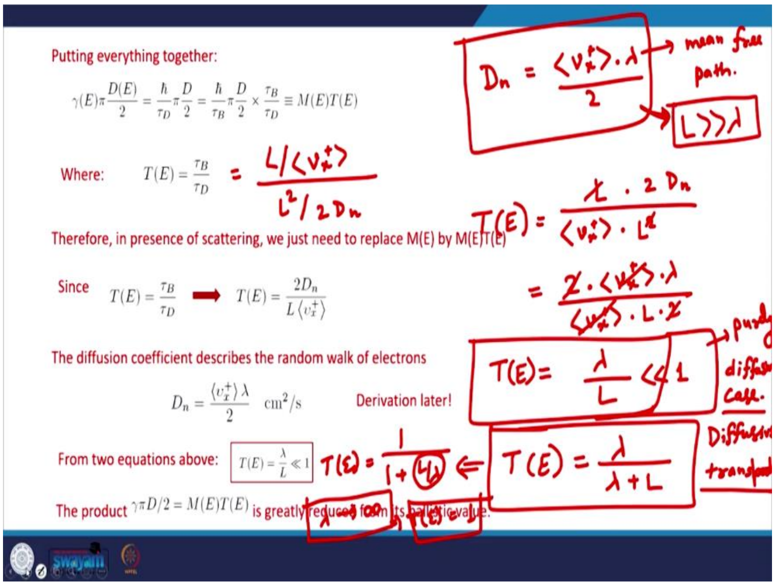

But in more general way this expression this transmission coefficient can be written as T(E) equals $\frac{\lambda}{\lambda+L}$. So, this is the more general way of more general expression for the transmission coefficient. And this holds true both for the diffusive case and the ballistic case ok because in ballistic case this λ tends to infinity and so in let me sort of rewrite this expression for T(E).

So, this T (E) can be written as $\frac{1}{1+L/\lambda}$, in ballistic case lambda tends to infinity which makes this second term tend to 0. So, this means this T(E) is equal to 1; this is what we expect in the case of ballistic transport, but in the case of diffusive transport we have a finite λ.

In the case of ballistic transport since there is no collision in the channel we can say that the electrons are not colliding with anybody in the channel and that is why we say that λ tends to infinity.

So, in the case of diffusive transport this expression holds true and this boils down to or this is equal to 1 in the case of ballistic transport case. So, this is a general expression this is true for all cases this expression is true only for purely diffusive case when the channel is extremely large as compared to the mean free path ok.

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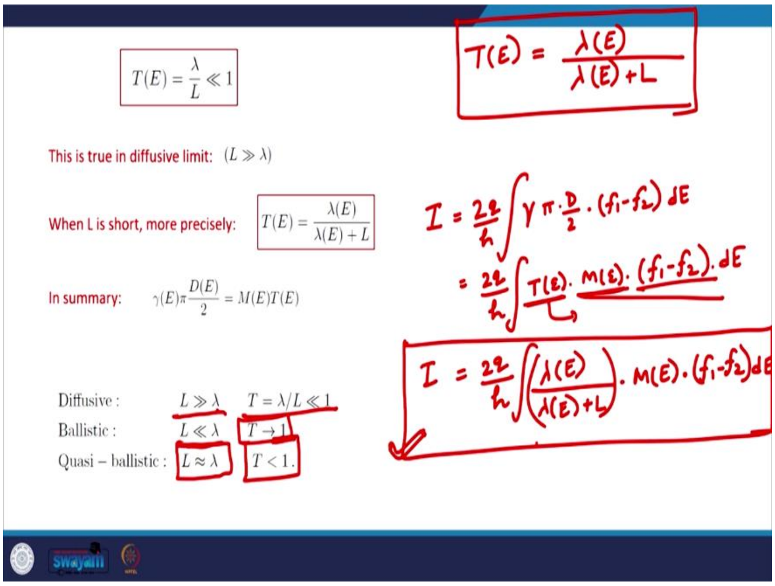

So, I hope this point is clear. More generally, as a function of energy this transmission coefficient can be written as the mean free path which is also a function of energy divided by λ(E) + L ok. So, now, as you can see that we have now obtained the expression for the transmission coefficient. So, we can now easily deduce the or we are now in a situation to find out the exact expression for the current in the case of diffusive transport.

So, the expression for the current in the case of diffusive transport as you might recall is $\frac{\gamma\pi D}{2}$ times $(f_1 - f_2)$integrated over all energy values this was written as T(E) times number of modes times $(f_1 - f_2)$ times dE. And now we have an expression for T(E) as well; writing the expression for T(E) , so the current in the case of in a general case I would say current can be written as M(E) times $(f_1 - f_2)$dE. So, this is the expression for the current in the case of diffusive transport ok.

And this is an important expression because this expression will be or can be generalized to bulk conductors as well or we can find the bulk conductivity the bulk IV characteristics using this expression. And as you can see that, we have built from the bottom we like roots model we did not assume that lambda is there lambda is given. We assumed that there is a density of states of electrons in the channel in diffusive case electrons might be scattering.

Then we had an expression for the diffusion constant from there we obtain the expression for the transmission coefficient we have the modes we have the Fermi functions. Essentially, the parameters from the basic physics of the device ok. So, this is a general expression for current in the case of devices ok. And as we have already seen that when L is extremely large as compared to λ in that case this transmission coefficient is very very less than one.

When we have the ballistic case when lambda is extremely larger than L then the transmission coefficient is 1 tends to 1 and in the quasi ballistic case when L is almost equal to λ in that case T is less than 1, but it is not very small it is around 1, but not exactly equal to 1.

Because sometimes in quasi ballistic case sometimes the electron might scatter might get scattered in the channel and sometimes they might not get scattered in the channel ok.

So, now, we in a way we have a general expression for the current which covers both ballistic and diffusive transport cases. And now we are in a situation to sort of look at an another important parameter or I would say one of the most important electrical parameter of the material which is essentially the conductance or more generally conductivity.

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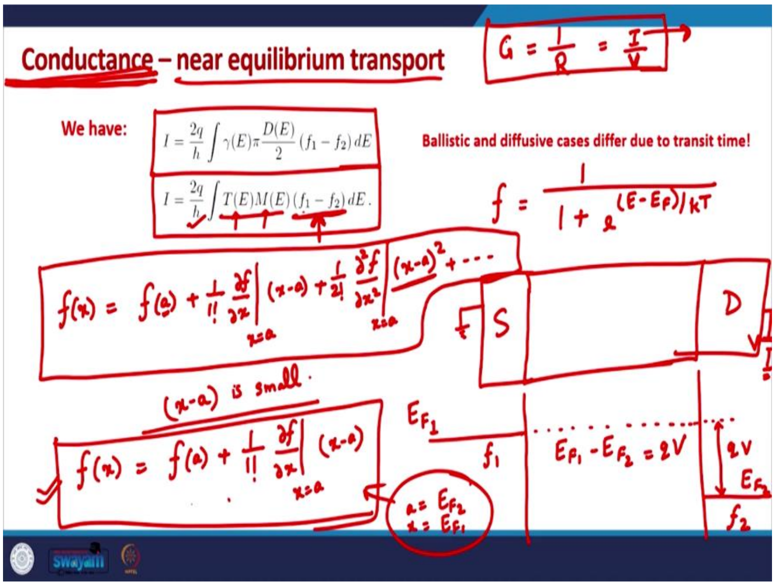

So, that is what we will see now. And as you might be aware that conductance is denoted by G, it is essentially the inverse of the resistance. So, if the resistance of a material is R or of a channel is R the conductance will be 1/R and from the IV characteristics R is V/I. So, G will be I/V. So, this conductance can be found out from the expression of the current from the IV characteristics of the device.

And now we are in a position to find out the IV characteristics as well because we know the expression for the current. So, this is the general form of current in the devices $\frac{2q}{h}\int\frac{\gamma\pi D}{2}(f_1-f_2)dE$ in more general in other way it can be written as T(E) times M(E) times $(f_1-f_2)dE$ into $\frac{2q}{h}$. If we assume a near equilibrium situation in that case we can find out (f_1-f_2) in a clean way actually.

Because we know what is M(E) we know what is T(E) now, this is just a constant we just need to find out the (f_1-f_2) this difference term. And if you recall, f is the Fermi function $\frac{1}{1+e^{\frac{E-E_F}{kT}}}$. So, the Fermi function is a function of energy it can be written as a function of energy, it is also a function of Fermi levels.

The Fermi function changes with energy the Fermi function also changes if the Fermi level change. And in order to find out this difference we need to know actually what would be the Fermi function at two contacts at the source contact f_1 and what is the Fermi function on the drain contact and then we need to find out the difference of the two.

So, just for the sake of clarity let me again draw, what we are up to here. We have a source, we have a drain, on the source side the Fermi level is E_{F1} on the drain side when we have applied a voltage the Fermi level is E_{F2} the Fermi function on source side is f_1, the Fermi function on the drain side is f_2. The voltage that we have applied is q times V, if the source side is grounded and on the drain side we have a battery of voltage V, ok. So, the $(E_{F1}-E_{F2})$ difference is q times V.

Now, this Fermi function is also a function of Fermi level and while going from the source contact to the drain contact what is changing is the Fermi level. So, in order to find out this difference we will make use of we will take help of Taylor series expression.

So, what the Taylor series expression says that any arbitrary function can be written as in this form f(x) = f(a) + $\frac{1}{1!}\frac{\partial f}{\partial x}|_{x=a}(x-a)$ + $\frac{1}{2!}\frac{\partial^2 f}{\partial x^2}|_{x=a}(x-a)^2$ +…. and similarly there are higher order terms as well. So, this function can be approximated by this Taylor series.

If we know the value of the function and the value of its derivative at a certain point we can find out the general form of the function in this way, ok. And if this difference the difference or if we find if we want to find out the function very close to point a if the values of x are very close to point a in that case this difference x - a is very small ok.

In that case, the second order and higher order terms can be ignored because these will be very small coefficients and they will make everything to almost tend to 0. So, when x - a is very small in that case this function can be written as or in a close vicinity of a point the function can be a continuous function can be approximated by this just using the Taylor series expansion up to first order terms, ok.

So, in the case of near equilibrium transport this E_{F1} and E_{F2} are close to each other because the applied voltage is not too much is less. So, this qV is not a big number it is a small number and if we expand the Fermi function or if we put a is equal to E_{F2} and x is equal to E_{F1} which means in this which means that we are trying to find out the Fermi function very close to E_{F2}, ok.

And in near equilibrium transport E_{F1} and E_{F2} are close to each other, in that case we can use this approximation ok. For that we need to know the derivative of f with respect to x or derivative of f with respect to E_F because in this case we are assuming the Fermi function to be the function of Fermi level, ok. So, let us try to do that.

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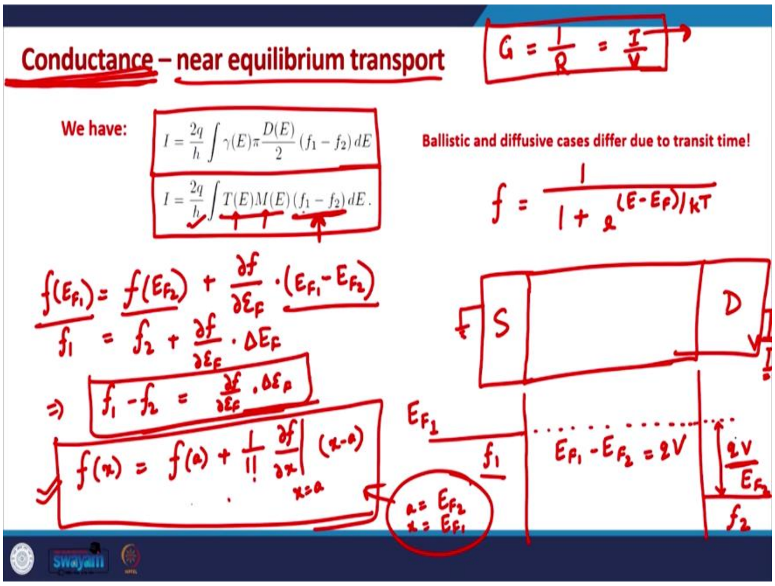

This is the so what we can see from here is that f at E_{F1} if we put x equal to E_{F1} and a equal to E_{F2} is equal to f at E_{F2} plus $\frac{\partial f}{\partial E_F}$, let us say this E_{F1} and E_{F2} might also change so let us not put the E_{F2} value here E_{F1}-E_{F2} ok.

So, this is E_F of E_{F1} is essentially, f_1 Fermi function of the source contact this is equal to f of E_{F2} is Fermi function of the drain contact this is $\frac{\partial f}{\partial E_F}$ times ΔE_F where ΔE_F is the difference between the source Fermi function and the drain Fermi function. So, this is what we have.

And from here, $(f_1 - f_2)$ is equal to $\frac{\partial f}{\partial E_F}$times ΔE_F ok and ΔE_F is essentially q times V. Had the electron be in a positive charge $(E_{F1} - E_{F2})$ had be in a negative number because in that case, E_{F2} would have gone up, but in the case of electrons because of an applied positive voltage on the drain side E_{F2} goes down when we apply a positive voltage on the drain side. So, this ΔE_F is equal to q times V.

So, finally, we have obtained $(f_1 - f_2)$ in the case of near equilibrium transport and that can give us the exact form of current or conductance in near equilibrium transport case. So, we have $(f_1 - f_2)$ is equal to $\frac{\partial f}{\partial E_F}$ times ΔE_F .

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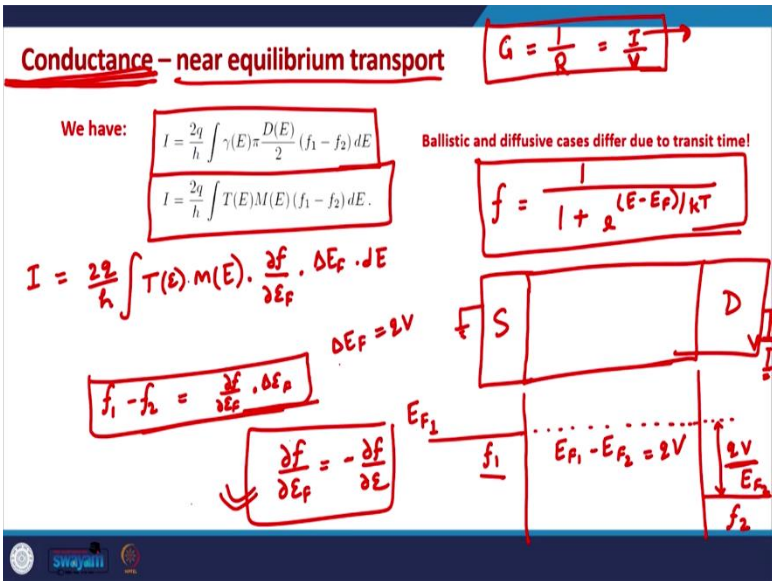

And if we put, this in this expression the current expression what we obtain is I equal to $\frac{2q}{h}$ integral T(E) times M(E) into $\frac{\partial f}{\partial E_F}$ times ΔE_F into dE. And if you look at this form of the Fermi function we can easily see from here that $\frac{\partial f}{\partial E_F}$ is equal to $(-\frac{\partial f}{\partial E})$.

So, if we take the derivative of Fermi function with respect to E and with respect to E_F, there will be only a difference of negative sign ok. So, we will keep in mind this fact and ΔE_F is equal to q times V. So, that is what we will put in this expression. So, let me erase this extra part.

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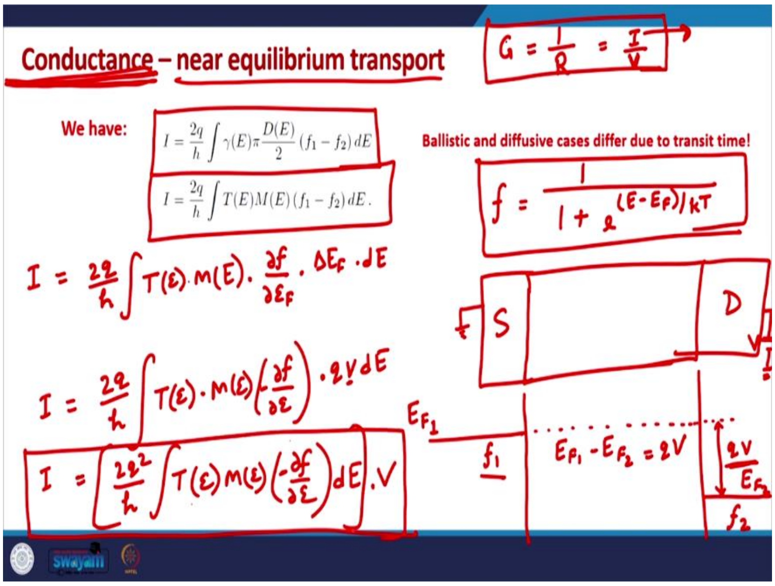

So, finally, what we have is we have I is equal to $\frac{2q^2}{h}$ T(E) into M(E) into $\frac{\partial f}{\partial E_F}$ can be written as $(-\frac{\partial f}{\partial E})$ and ΔE_F is q times V dE. So, we can take this V out so I becomes this q also comes out, $\left[\frac{2q^2}{h}\int \mathrm{T(E)M(E)}(-\frac{\partial f}{\partial E})\ \mathrm{dE}\ \right]V$. So, this is the direct relationship between the current and the voltage applied across the device.

And please remember that, this is true when we assume the near equilibrium transport because only in that case we can approximate $(f_1 - f_2)$ by the first just by the first order terms in the Taylor series expansion ok. So, that will that brings us to the expression for the conductance.

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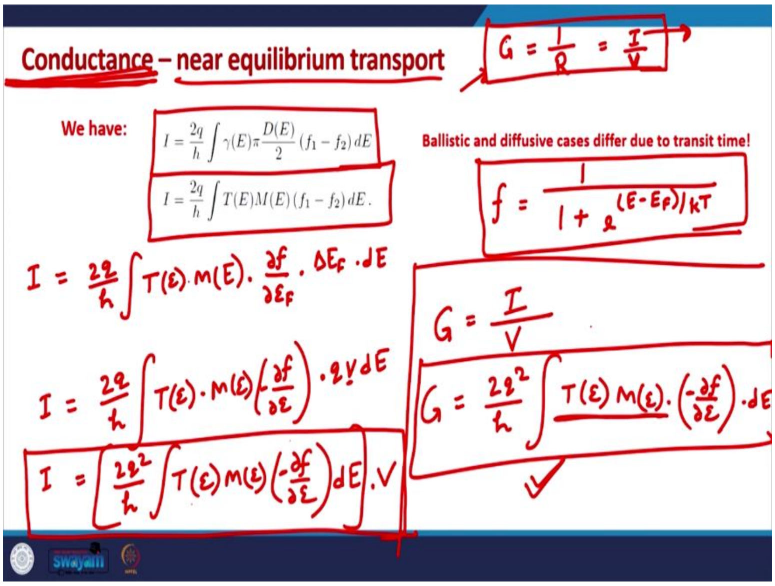

So, the conductance of the channel is the ratio between the current and the voltage and so this will be $\frac{2q}{h}$ integration of T(E)M(E) into $-\frac{\partial f}{\partial E}$ into dE. So, this is finally, the expression for the conductance of the material and we can also find out the expressions for the conductivity from this expression.

So, this is what we also measure in experiments ok and this is a general expression because for ballistic transport case this T(E) will be 1 and we will be left just with M(E) for the diffusive transport case T(E) will be given by $\frac{\lambda}{\lambda+L}$ and this expression will hold true. So, this expression is true for right from very small devices up to large devices.

We will discuss more about this conductance in our coming class and I would recommend all of you to go through this derivation again this is an important concept the conductance and conductivity of the material of the channel see you in the next class.

Thank you for your attention. See you in the next class.