Physics of Nanoscale Devices

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Lecture - 24
Diffusive Transport

Hello everyone. As you know we had already started our discussion on Diffusive Transport in the previous class and before that we discussed the ballistic transport and specifically the idea of modes in ballistic transport. So, today we will discuss the idea of diffusive transport in somewhat more details. Before going into that let me quickly review what we have seen so far.

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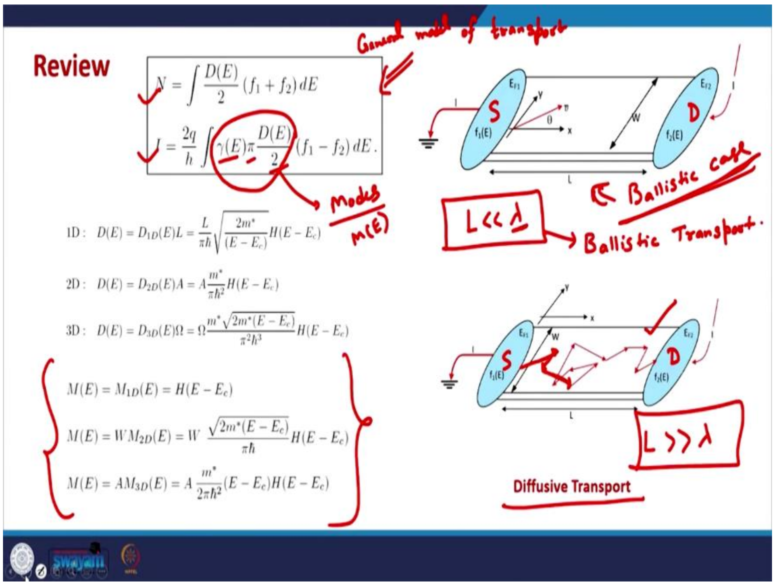

So, what we saw from our discussion of general model of transport. From the general model of transport we saw that this is the carrier concentration and this is the expression for the current in a small conductor when the channel length is small. And here as you might have already noticed that the carrier concentration and the current is in terms of the basic fundamental parameters of the material and the electrons ok.

Building on top of this, we saw that when there is a steady state current in the conductor. Specially in the ballistic case in that case we can define this quantity to be the modes and modes are like conduction pathways in the channel. They are actually the pathways the broadened energy level in one way in through which the electrons basically transport.

So, this is defined as the modes and this is represented as M(E) and these are the expression for the modes in 1D, 2D and 3D conductors. Then we started our discussion of the diffusive transport. So, this is the ballistic case, this figure is the ballistic case in which the electron starts from the source contact from the left contact and it directly goes to the drain contact to the right contact without any collision in the middle and in this kind of transport electron only traverses through one energy level.

So, to say or one conduction pathway, but this is true when the conductor is small in length when the channel is small in length. More specifically when the channel length is less than the mean free path of the electrons, then in that case ballistic transport condition holds true.

What is the mean free path? Mean free path is the distance or average distance traveled by the electron between 2 collisions. So, when we have a large conductor when we have a large channel for example, in that case electrons will collide with atoms and other particles or may be other vacancies.

And the transport will be something like this. So, electron will start from the source contact, it will collide with something, it might deflect backward as well or maybe in some cases forward as well. And some of the electrons that start from the source side some of them eventually would reach to the drain side when there is a voltage applied across the device.

Many electrons may not reach to the drain side actually. So, in this mechanism electron is continuously colliding with other particles in the system and the average distance between 2 collisions is known as the mean free path. And when the channel length is greater than the mean free path in that case the transport will look something like this.

Some of the electrons will reach to the drain side, but some of the electrons may not reach to the drain side and this transport is known as the diffusive transport in devices ok. So, we have already seen that we can define modes in the case of ballistic transport, because in ballistic transport the electron does not change the conduction pathway.

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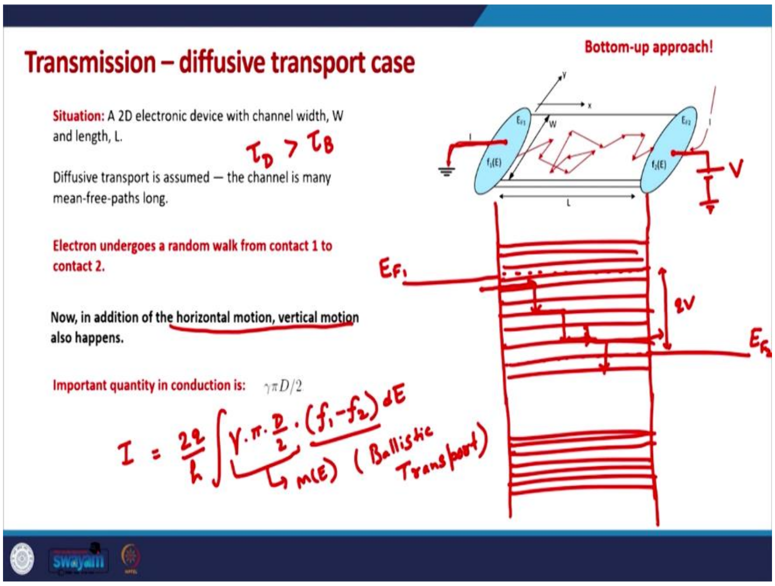

But in the case of diffusive transport electrons may undergo in the change of pathway. So, if this is the long conductor in which electron is undergoing diffusive transport. And in this case let us say we have a positive voltage applied. So, the situation will look something like this we have E_{F1}, E_{F2} these are the E_{F1} and E_{F2} levels, the applied voltage across the device is q times V.

Let us say we have applied a voltage sorry we have grounded the source side and we have applied a battery on the drain side battery of voltage V, the source side is grounded. So, in this case and there are various conduction pathways in the channel that the electrons can take. So, let us say these are various electronic states, there might be a band gap as well there might be a disallowed energy range.

For example this is the disallowed energy range, that the electrons cannot take this is again allowed energy range. So, this might be a typical situation that we would have in a practical device. So, in the case of diffusive transport the electron may start. For example, the electron may be sitting here in the source side it may start it might collide with something else and it might lose some of its energy and it might go down as well to this level.

So, it might again travel forward may lose energy, may go down. Sometimes it may also gain energy, because of the sometimes it may change the momentum in such a way that it may make transitions some of the transitions. So, some of these electrons may reach to the ultimately to the drain side, but some of the electrons may ultimately go below E_{F2} and that is why they will not reach to the drain side ok.

So, this is how the diffusive transport case looks like when in the energy levels, this is the energy level picture of the diffusive transport ok. So, in addition to the horizontal motion in energy levels a vertical motion also takes place. Please keep in mind that since the electron is changing conduction pathways.

So, we cannot define or we cannot say that electron is travelling through one mode throughout. Electron may be travelling through different modes while it is going from the source terminal to the drain terminal. So, that is why the notion of modes does not hold exactly as it holds in the case of ballistic transport ok. And as we have already seen that the current is given as $\frac{2q}{h}\int\frac{\gamma\pi D}{2}(f_1-f_2)dE$.

So, this is known to us this we know from our understanding of the contacts. This quantity is what we essentially need to know in order to find out the current in any kind of transport and this quantity turns out to be the number of modes in the case of ballistic transport. Now in this case we can guess that when the electrons are colliding with many intermediate atoms or other entities the transit time of electrons will increase in this case.

So, we can definitely say that this transit time of electrons in the case of diffusive transport is more than the transit time of electrons in the case of ballistic transport. So, at least this we can say; because in the case of ballistic transport the electron was directly going to the drain side without changing the mode.

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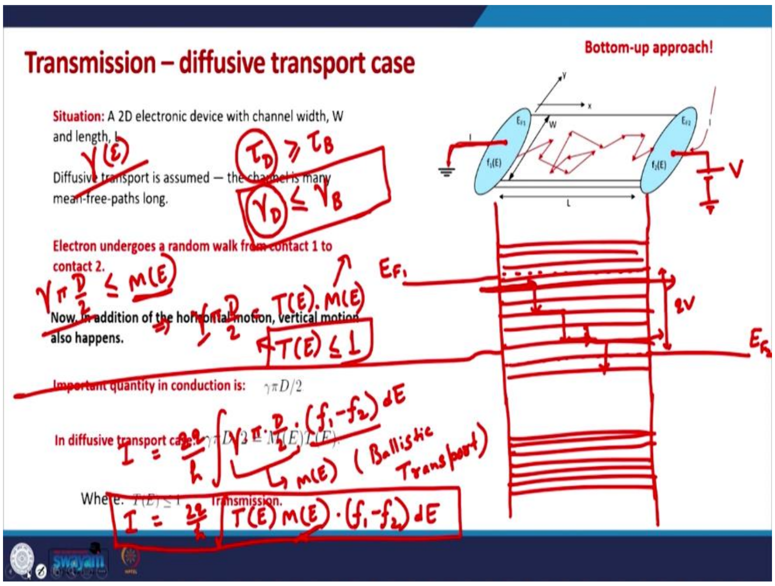

Now, electron is colliding so it is always greater than or equal to τ_B, which means that γ in the diffusive transport case will always be less than γ in the case of ballistic transport case less than or equal to. So, this quantity in the diffusive transport case this quantity $\frac{\gamma \pi D}{2}$. So, which means that now $\frac{\gamma \pi D}{2}$ will be less than equal to the number of modes. Because this is $\frac{\gamma \pi D}{2}$ in the case of ballistic transport.

Now, since γ in the case of diffusive transport is less than the γ in the case of ballistic transport case. Please keep in mind that this γ is sort of finally, γ is a function of energy. So, at different energy levels different energy values this γ is different. In general when we talk about γ we talk about an average γ. So, that is why we could write γ in the case of diffusive transport is less than the γ in the case of ballistic transport case.

So, in the diffusive transport case this quantity $\frac{\gamma \pi D}{2}$ will be less than equal to M(E). Or we can write it down as $\frac{\gamma \pi D}{2}$ is equal to T(E) times M(E) where this T(E) is a coefficient which is always less than equal to 1 and this T(E) is known as the transmission coefficient in diffusive transport case ok.

So, if T(E) is 1 then we can say that the transport is purely ballistic, electrons are not colliding with anything in between and they are directly going to the drain side. But if T(E) is less than 1 or if the channel is long and the electrons are colliding in between T(E) will always be less than 1.

So, this is in a way this parameter T(E) captures the or it accounts for the collisions or it accounts for the diffusive transport, it accounts for the intermediate collisions of electrons. So, in order to understand the diffusive transport case we ultimately in a way need to know what is T(E). Because we already know what is M(E) and so, we just need to know what is T(E) and all other things are known to us.

So, in that case also in that case we would be able to find out the current. Because the current can now be written as in diffusive transport case the current can be written as 2 q by h integration of T(E) times M(E). Because this quantity can be written as T(E) times M(E) times $(f_1 - f_2)$dE. So, this will be the current in the case of diffusive transport ok alright.

So, all other things we already know, this M(E) is same as the modes that we derived in the case of ballistic transport. What we need to know is just the transmission coefficient and that is what we will try to figure out. And in order to find out the transmission coefficient we need to in a way know what is γ in the case of diffusive transport case ok.

And in order to find out γ we need to find out what is τ in the case of diffusive transport case or what is the transit time what is the τ_D in the case of diffusive transport case. So, that will tell us about the transmission coefficient and ultimately we would have an expression for current in the case of diffusive transport. So, we will use exactly the same methodology that we used for the ballistic transport case.

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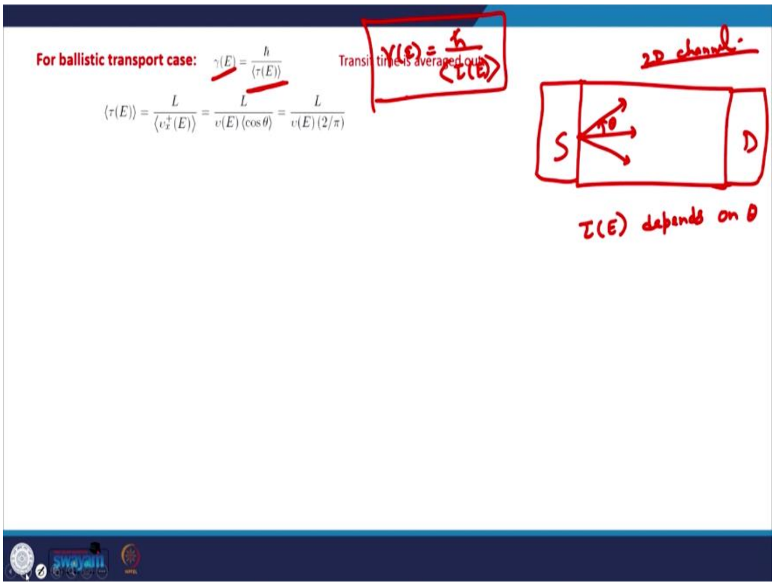

In the ballistic transport case in order to find out γ we found out the average value of $\tau(E)$. So, $\gamma(E)$ is essentially $\frac{\hbar}{\langle\tau(E)\rangle}$. So, there might be various electrons and we took a 2D channel. So, there might be various electrons that might start from the source side at different angle at the same energy.

So, this $\tau(E)$ will is dependent on theta the angle with which they start from the source side. So, that if that is the channel this is the source, this is the drain, $\tau(E)$ depends on the θ.

So, this $\gamma(E)$ in the case of ballistic transport is defined as $\hbar$ r divided by average of all the transit times at energy E ok. So, that is why we averaged over all the angles at which the electron starts from the source side.

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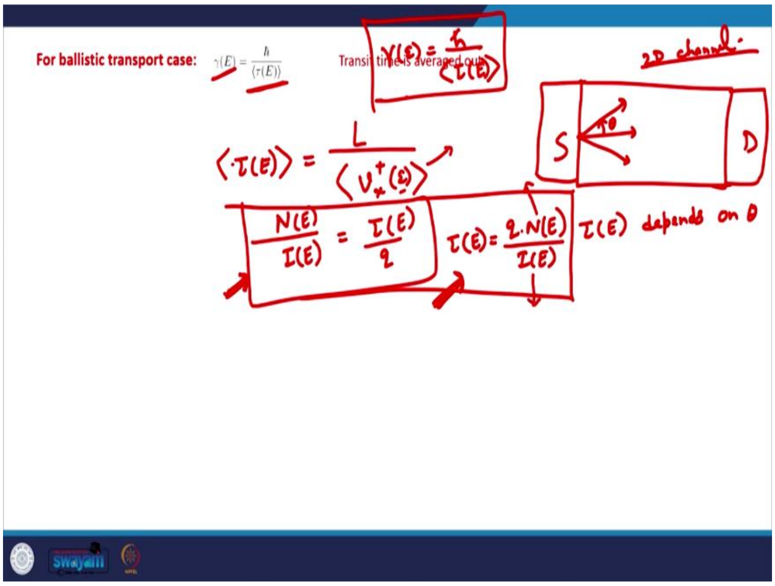

In order to calculate $\gamma(E)$. We found out average of $\tau(E)$ and the average of $\tau(E)$turns out to be the, this average of $\tau(E)$ actually turns out to be L divided by average of velocity of electrons in x direction at energy E ok.

And this was found out from the ratio of the steady state number of electrons to the steady state current which was essentially the steady state number of electrons τ. So, please remember this was the expression that we used or this was sort of a thought experiment that we did in order to find out the $\tau(E)$time.

So, this $\tau(E)$ was essentially q times N(E) divided by I(E). So, that was the case when we had the ballistic transport, but now in the case of diffusive transport this thing does not hold actually. Because now the electrons are electrons all the electrons are not reaching to the drain side, electrons are continuously scattering in the middle and the angle is changing.

So, we cannot find out $\tau(E)$ this way, but we will use make use of this expression in order to find out the tau time for diffusive transport case. Now the situation that is different in diffusive transport is from the ballistic transport is scattering.

Electrons are undergoing scattering in the case of diffusive transport and that is why it is actually called as diffusive transport. Because electrons are diffusing from the high concentration which is source side to the area of low concentration which is the drain side and that is the way electrons are traveling from the source to the drain ok.

So, in order to use this expression we actually need to find out the steady state electronic concentration and the steady state current in diffusive transport case ok. And that is what we will do using a well known result which is known as the Fick's law, Fick's diffusion law essentially.

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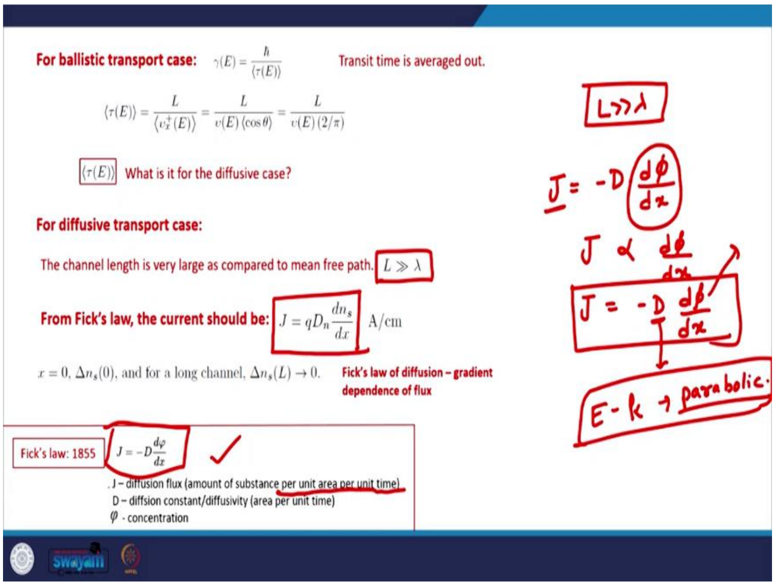

So, in the case of diffusive transport it is understood that the channel length is extremely large as compared to the mean free path. And the Fick's law of diffusion says that when we have something that has a population gradient that has a high concentration area and a low concentration area a concentration gradient. In that case the flux of that thing is given by this expression or more formally Fick's law is given as J is equal to -D times $\frac{d\phi}{dx}$.

So, it was given by Fick in 1855 that was the time when Coulombs law around that time when the Coulombs law and very basic building blocks of electrical engineering were being formulated. This is also the same time when the Fick's law was given.

And according to the Fick's law the flux or the amount of substance per unit area per unit time. So, this J is the amount of substance per unit area per unit time. So, that is sort of flux per unit flux of the substance, that is flowing from high concentration to the low concentration area that is directly proportional to the gradient of the concentration ok.

So, phi is the gradient, so the flux is directly proportional to the gradient of the concentration or it can be written as -D $\frac{d\phi}{dx}$, where D is the diffusion constant or diffusivity of the substance material. And generally in 3 and this is actually this was given for a 3D for a 3D system when we have concentration gradient in one direction, one dimension.

And this the unit of J is the amount of substance per unit area per unit time. The unit of D is area per unit time and ϕ is essentially the concentration of the substance. So, as you know that by invoking the idea of effective mass, we can treat electron as a classical particle with now with a modified mass that is the effective mass.

So, electron can be treated as a classical particle with mass equal to effective mass when the E k relationship is parabolic. So, when E k is parabolic which means effective mass does not change with energy or momentum in some region it is constant. In that case electron can be treated or electron wave packet can be treated as a classical particle.

So, which means that the conduction electrons that are the electrons very close to the bottom of the conduction band, which are essentially the conduction electrons those electrons. In most of the cases those electrons have parabolic E k, those electrons can be treated as classical particles and the diffusion for classical particles will be given by the Fick's law.

There are some scenarios in which the Fick's law of diffusion does not hold true and for example, when the fluid is going through the porous medium. But generally when there is no restriction in between in the flow the diffusion of classical particles follow the Fick's law. So, we can use the Fick's law for electrons as well, when the electrons can be treated as classical particles of mass equal to effective mass ok.

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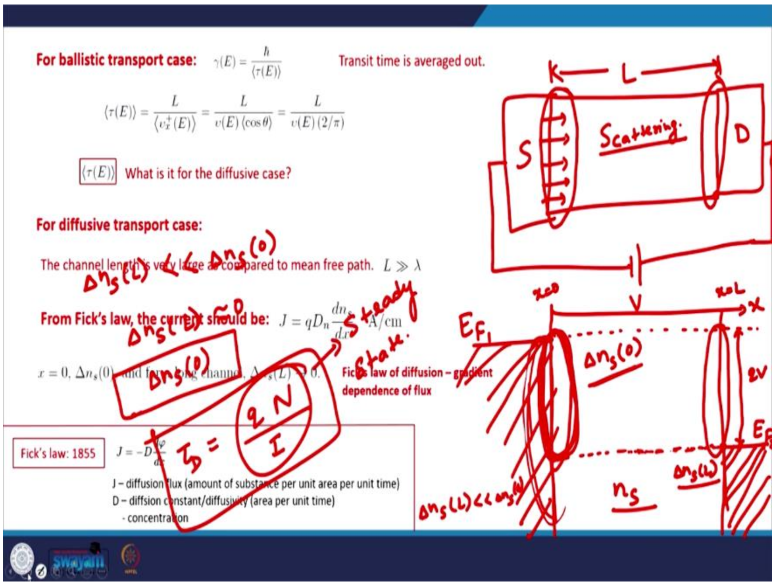

Now, the situation that we have at our hands is that we have a 2 dimensional channel, we have a source contact, we have a drain contact. The source is injecting a lot of electrons because let us say we have applied a voltage and across the device the drain is at the high voltage source is at the low voltage. And so that is why the source might be injecting a lot of electrons, all the all of these electrons will not make it to the drain side.

They will undergo scattering in the channel. So, this area of the channel becomes the area of high electronic concentration, this area of channel becomes the area of the low electronic concentration and that is why the diffusion essentially takes place ok. In terms of energy levels we have E_{F2}; E_{F1} the Fermi levels of source and drain are E_{F1} and E_{F2} respectively.

And when we have an applied voltage q times V in that case there will be a difference between E_{F1} and E_{F2} and that will essentially make the source to inject electrons in the channel. So, this area will become, this area of the channel will become the area of high electronic concentration and this area of the channel will become the area of low electronic concentration because in this side all the states up to this energy level are filled.

So, the source contact is trying to fill all the electronic states up to this energy level, the drain contact is trying to fill all the electronic states up to this energy level. So, the electrons are available up to E_{F1} here. So, there is an axes of electrons as compared to the drain side ok.

In steady state let us say there is a steady state concentration of electrons. Let us say may be steady state concentration of electrons is n_s, that is the steady state sheet charge density in the channel.

On this side we have an axes of electrons. So, let us say the axes of electrons on the source side is $\Delta n_s(0)$; where this is the x axis, this is x equal to 0, this is x equal to L let us say. Because if the channel length is L in that case this is what it would be the case some of these electrons, some of these axes electrons recombine. And ultimately the channel has some electronic population here on the right terminal as well.

So, we will have maybe some axes electrons at $\Delta n_s(L)$ at the drain end as well. Generally in steady state, it can be assumed that this delta n s in near equilibrium situations. When we do not apply high voltage to the material or even in other cases this population is extremely small Δn_s is extremely small as compared to the $\Delta n_s(0)$.

So, yeah let me write it here. So, $\Delta n_s(L)$, the axes electrons at the drain side is extremely small as compared to the axes electrons on the source side $\Delta n_s(0)$ or what we can say is that $\Delta n_s(L)$ is almost equal to 0, when we talk about the, when we compare this to the axes electronic population on the source side, n_s is the sheet carrier density sheet charge density in the material ok.

So, this is what we have, we have an electronic population here which is a $\Delta n_s(0)$ on the left side, on the left contact and 0 on the right contact ok. Then now we need to see what is please just recall that we need to find out τ which is essentially $\frac{qN}{I}$ in steady state in steady state. This is what we need to find out; this is the transit time in the case of diffusive transport.

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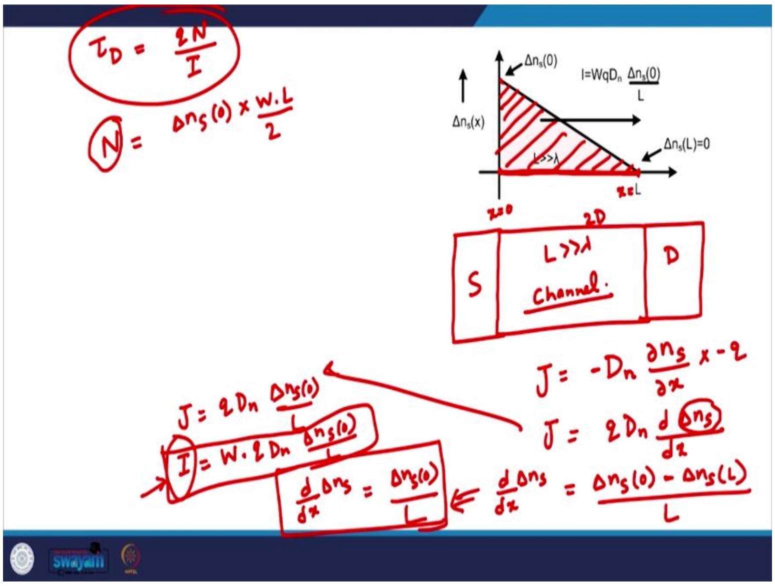

So, now this all these things can be summarized. This is the x equal to 0, this is x equal to L, this is the source contact point x equal to 0 and x equal to L is the drain contact point. In between from x equal to 0 to x equal to L we have the channel ok which is a long channel may be and this L is very very greater than λ; where λ is the mean free path.

So, now in this case when we apply the Fick's law we will see that the current density or current per unit, since this is a 2D channel we are taking a 2D channel. So, the current density is essentially the current per unit width this is equal to the flux of charge into -D. Let us say $\frac{\partial n_s}{\partial x}$ times the electronic charge which is -q. So, this current density will be qD_n, this will be ultimately dx because Δn_s tells us about the axes electronic population on the left side and on the right side.

Now, this from the recombination or from carrier statistics when there is no recombination generation of carriers. Then just in the case of diffusion this profile of electron is linear actually.

If there is in addition to this diffusive transport in addition to just collisions in the channel. If we also have recombination generation of electrons then this profile of electrons, will this profile will be sub linear, this will no longer be a linear profile ok.

So, this $\frac{d}{dx}\ \Delta n_s$ in the case of linear profile it can be written as $\frac{\Delta n_s(0)-\ \Delta n_s(L)}{L}$. So, this becomes $\frac{d}{dx}\ \Delta n_s$ becomes $\frac{\Delta n_s(0)}{L}$ ok. So, that will give us the current density equal to from this expression the current density will be equal to J is equal to $qD_n\frac{\Delta n_s(0)}{L}$.

So, this is the current per unit width current density in the case of 2D devices current per unit width. So, the current in steady state becomes width times the current density, which is essentially $\frac{\Delta n_s}{L}$. So, this is what we obtain as the steady state current and please also recall that in order to find out τ_D, we need steady state electronic population divided by the current.

Now, the current we have obtained from by application of the Fick's law. This steady state electronic population is essentially the number of electrons in the channel and this will be the area under this curve will be the number of electrons in the channel in steady state. So, this will be essentially $\Delta n_s(0)$ into $\frac{WL}{2}$; the area is W into L by 2 because this profile is linear.

So, we now have n the steady state population. The steady state current we can now find out the, this τ_D just by putting these 2 in this expression ok that will give us the expression for the current.

Because now from τ_D we can find out the γ, the broadening and from γ we can find out the current expression. That will also tell us about the transmission coefficient in the diffusive transport case. So, that is what that is what we will see in the next class. I would recommend you to please go through the Fick's law and the application of Fick's law in this case see you in the next class.

Thank you.